Entire-Domain Spectral Basis Functions for the Efficient Design of Modulated Metasurface Antennas on Circular Domains

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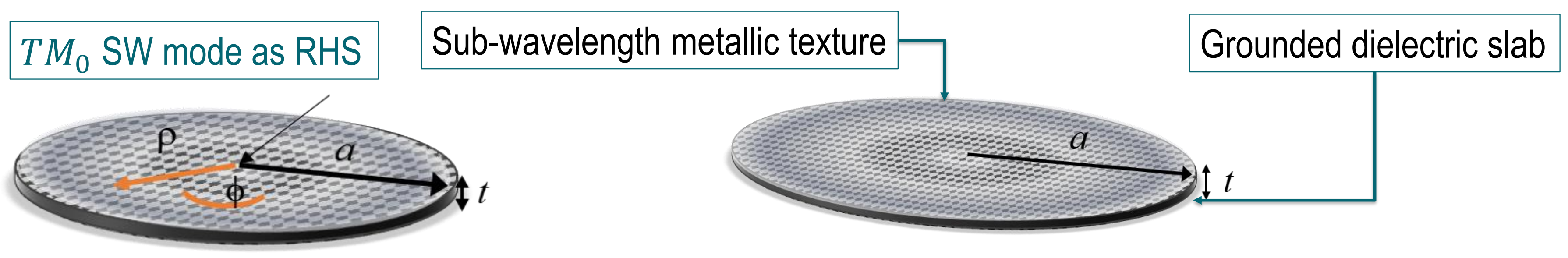
Research context and motivation

- **Metasurfaces** are thin metamaterial layers characterized by unusual **reflection/refraction properties** of plane waves and/or **dispersion properties** of **surface/guided waves.**
- Metasurfaces are composed of a ***dense periodic*** texture of *small elements* (in terms of wavelength, λ) printed on a **grounded dielectric slab**.

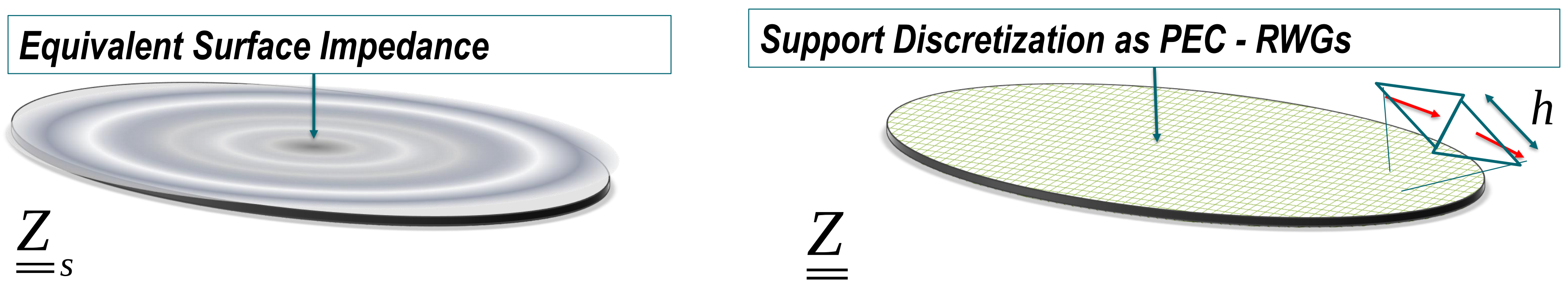


TM_0 SW mode as RHS

Sub-wavelength metallic texture

Grounded dielectric slab

- For Metasurfaces, instead of PEC boundary condition, the exact b.c. can be approximated by the Leontovich b.c., which is also known as **Impedance Boundary Condition** (IBC).
- The IBC can also be applied to model thin dielectric sheets, perfect conductors with thin dielectric coatings, corrugated surfaces, rough surfaces, and other configurations. Because of this, the IBC approximation has been **widely used in industry**.



Equivalent Surface Impedance

$\underline{\underline{Z}}_s$

Support Discretization as PEC - RWGs

$\underline{\underline{Z}}$

Challenges, strategies and advantages

Numerical challenges related to domain discretization:

- **Large-size problems**: geometry discretization sensitive to dielectric properties
- **MoM** based on classical mixed element discretization (i.e. via RWG basis functions) can not be used in practice in optimization processes to determine the proper IBC required for a specific antenna pattern

Strategy:

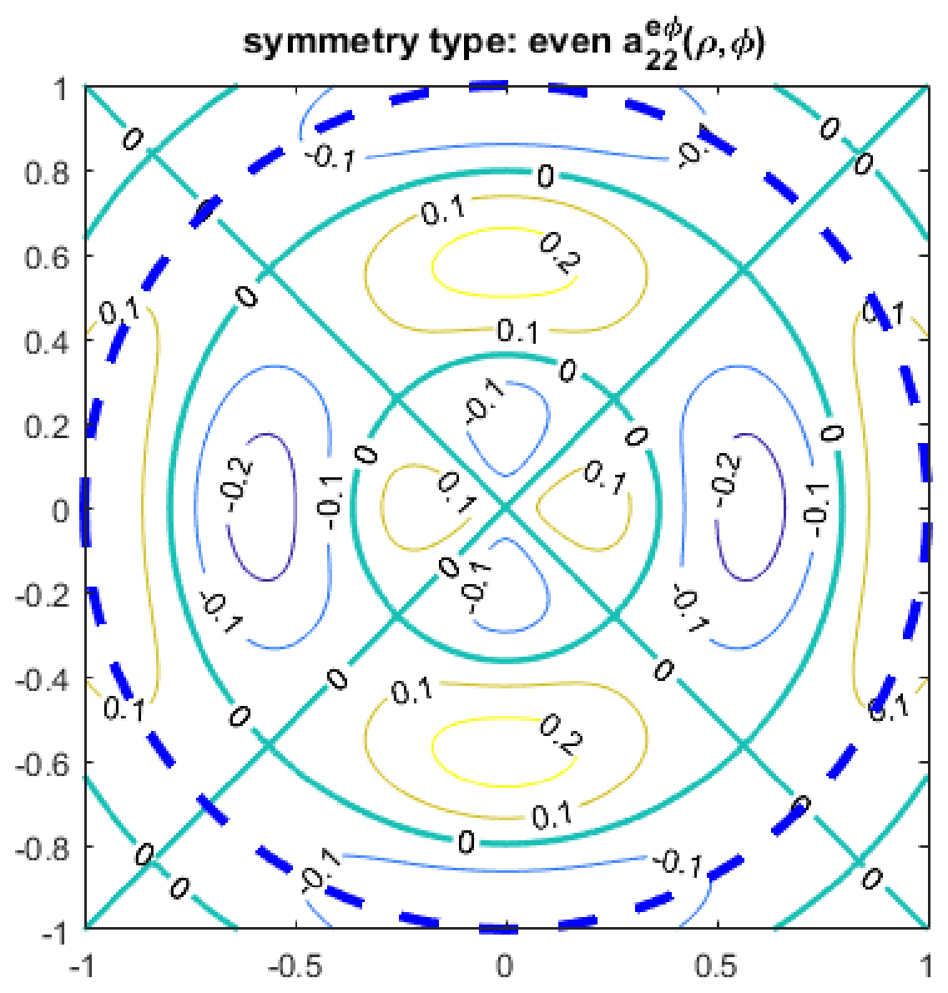
- The use of the entire domain basis functions implies reducing storage requirements for the system matrix and often allows for a direct inversion of the matrix system: this is especially desirable when the linear system needs to be solved numerous times, for example in an **optimization process**.

Advantages:

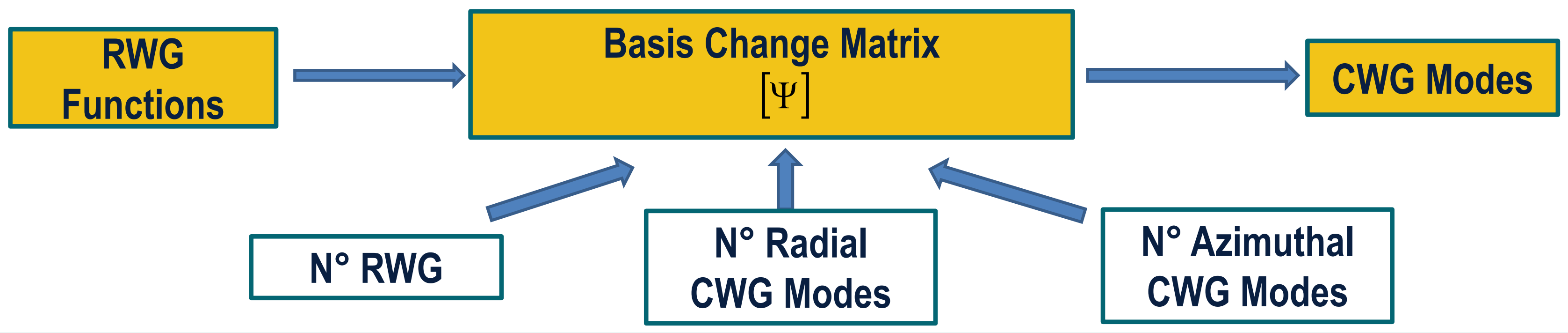
- Domain discretization not necessary
- **Compression** of the problem size and reducing of storage requirements
- **Regolarization** of the problem: improved conditioning properties of matrix system in iterative solution
- Suited in **optimization** problems

Entire domain Functions: Circular WaveGuide (CWG) Modes

$$\mathbf{h}^{e,i}_{mn}(\rho,\phi) = c^{e,i}_{mn}\left[h^{e\rho}_{mn}(\rho)h^{e\rho,i}_{m}(\phi)\hat{\rho} + h^{e\phi}_{mn}(\rho)h^{e\phi,i}_{m}(\phi)\hat{\phi}\right]$$
$$\mathbf{h}^{h,i}_{m,n}(\rho,\phi) = c^{h,i}_{mn}\left[h^{h\rho}_{mn}(\rho)h^{h\rho,i}_{m}(\phi)\hat{\rho} + h^{h\phi}_{mn}(\rho)h^{h\phi,i}_{m}(\phi)\hat{\phi}\right]$$



- The superscripts ‘e’ and ‘h’ stand for e-modes (TM) and h-modes (TE) respectively
- The superscript ‘i’ accounts for the symmetry type of the angular Φ dependence, which can be even or odd
- If correctly used, the CWG modes can be written as div-conforming basis



RWG Functions

Basis Change Matrix $[\Psi]$

CWG Modes

N° RWG

N° Radial CWG Modes

N° Azimuthal CWG Modes

Application in Optimization Problem

Looking for proper IBCs to produce surface currents on antenna which radiate a **required pattern**

$$\underline{\underline{\psi}}^{T}_{N_{cwgm}\times N_{rwg}}\left(\underline{\underline{Z}}^{RWG}_{N_{rwg}\times N_{rwg}} - \underline{\underline{Z}}^{RWG}_{s}\right)\underline{\underline{\psi}}_{N_{rwg}\times N_{cwgm}} I^{\psi}_{N_{cwgm}\times 1} = \underbrace{\underline{\underline{\psi}}^{T}_{N_{cwgm}\times N_{rgw}}\underline{\underline{V}}^{RWG}_{N_{rwg}\times 1}}_{\underline{\underline{V}}^{\psi}_{N_{cwgm}\times 1}}$$

Basis Change Matrix

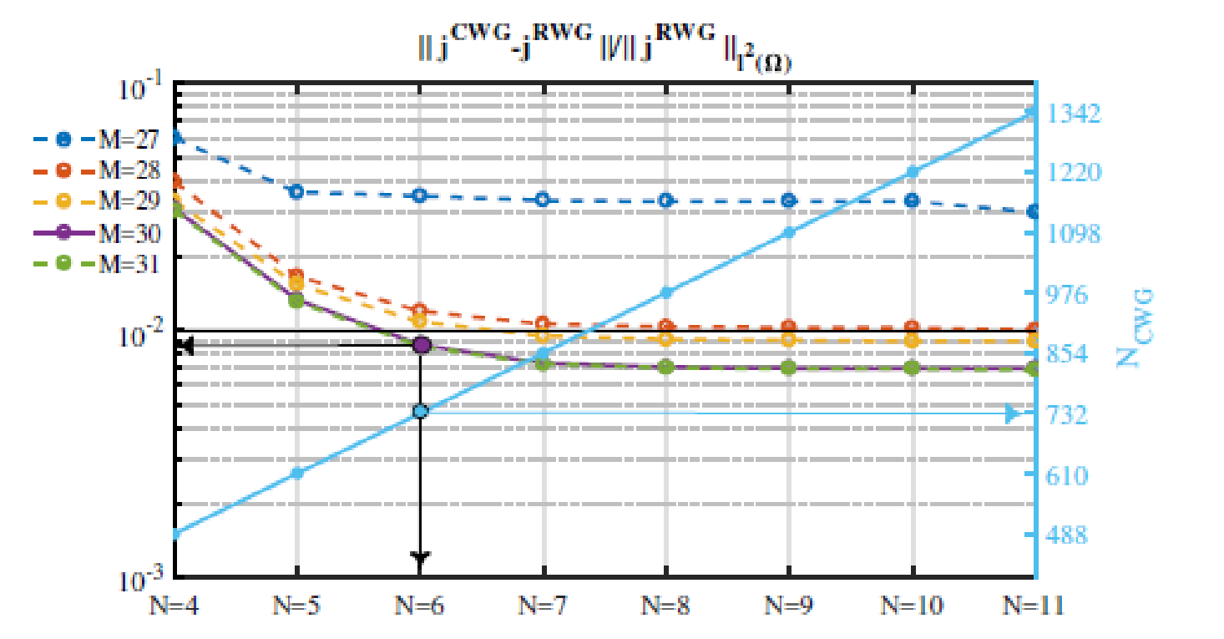
$$\left[\underline{\underline{\psi}}^{T}_{N_{cwgm}\times N_{rwg}}\underline{\underline{Z}}^{RWG}_{N_{rwg}\times N_{rwg}}\underline{\underline{\psi}}_{N_{rwg}\times N_{cwgm}} - \underline{\underline{\psi}}^{T}_{N_{cwgm}\times N_{rwg}}\underline{\underline{Z}}^{RWG}_{s}\underline{\underline{\psi}}_{N_{rwg}\times N_{cwgm}}\right] I^{\psi}_{N_{cwgm}\times 1} = \underline{\underline{V}}^{\psi}_{N_{cwgm}\times 1}$$

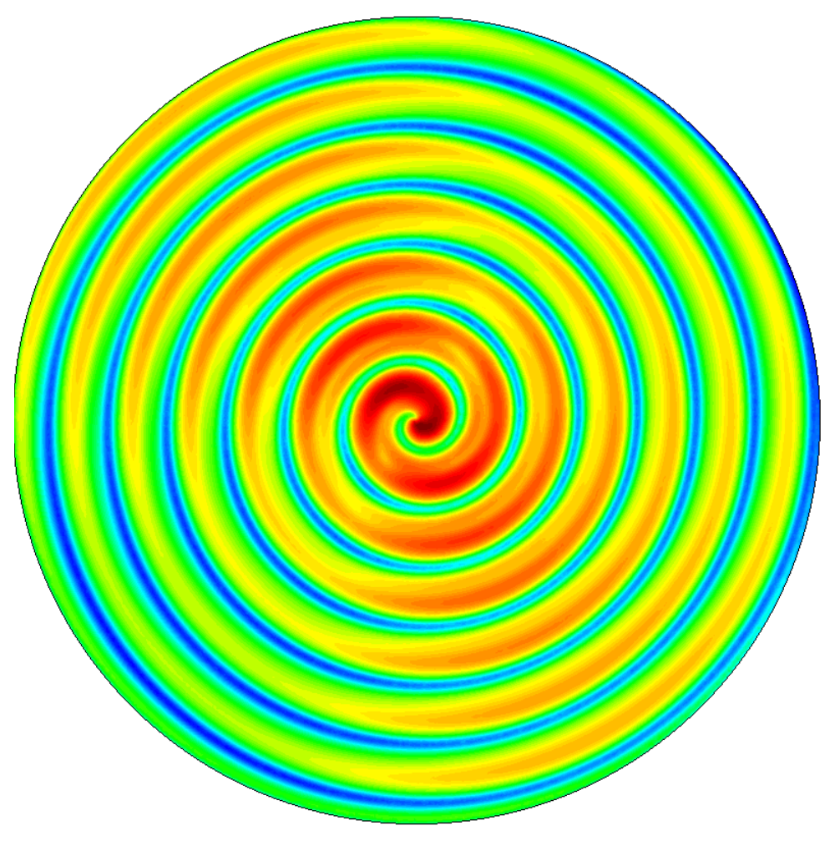
- Computed once
- Reduced size

IBCs changed during optimization process

Currents to look for to radiate the required pattern

Preliminary Results





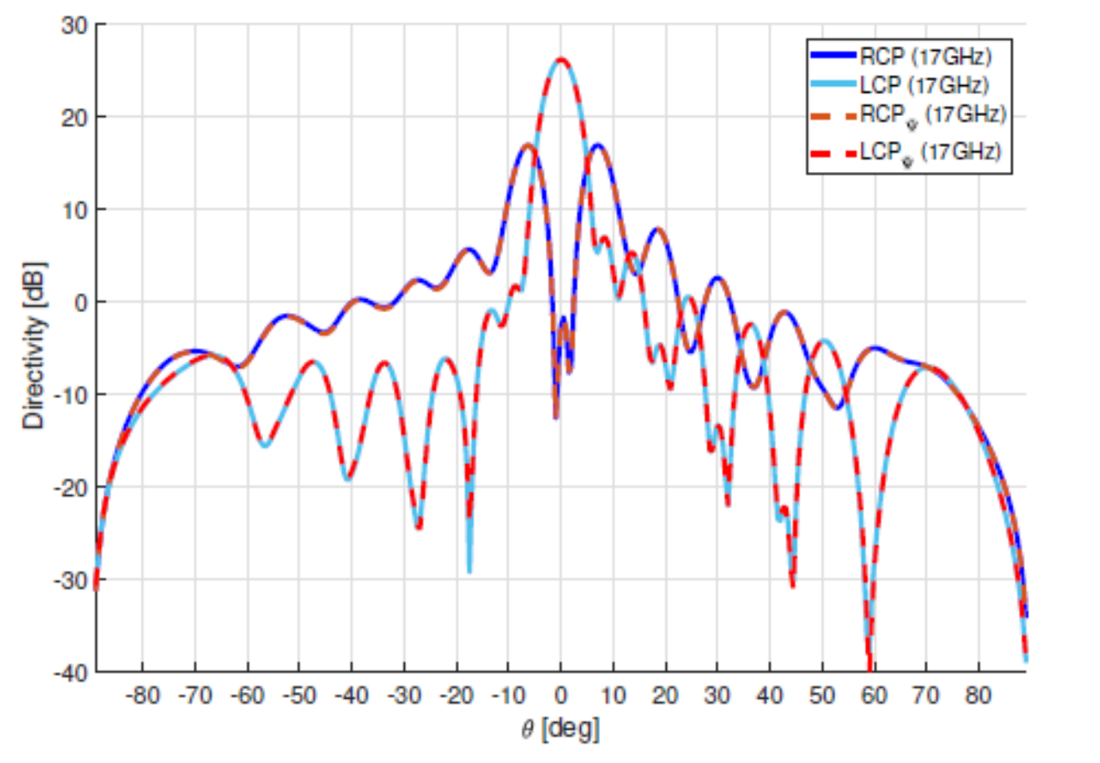


Fig. 2. The right-plot shows the $l_2(\Omega)$-error between CWGs and RWGs with respect to (m, n). In the left-plot (cyan) is shown the number of CWGs with respect to (m, n)

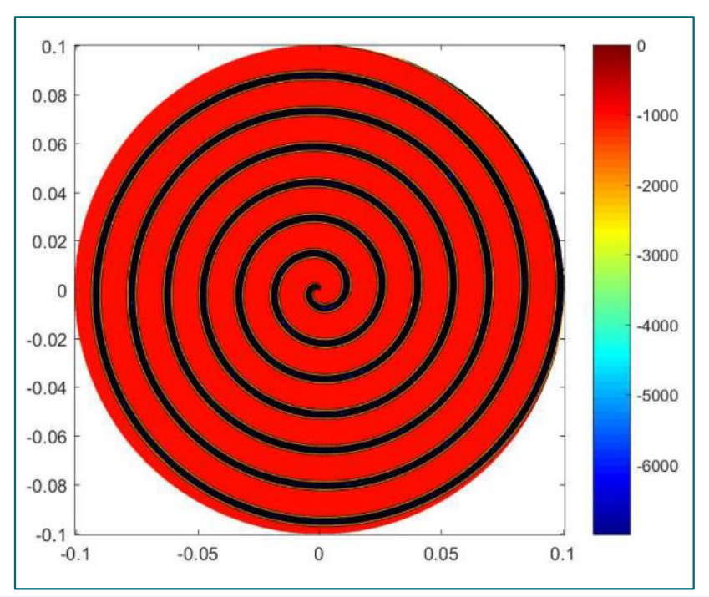
Fig. 3. Co-polar and cross-polar components of the field radiated by the RWGs (solid) and the CWGs (dash), when $(M, N) = (30, 6)$

Operative frequency: 17GHz
Dielectric substrate parameters: 3.66

N° of RWG: 37680
N° of CWG modes : 350

$$X(\rho,\phi) = X_s\left[1 + M\sin(\beta_{sw}\rho - \phi)\right]$$

$$X_s = 0.67\eta_0, M = 0.27$$

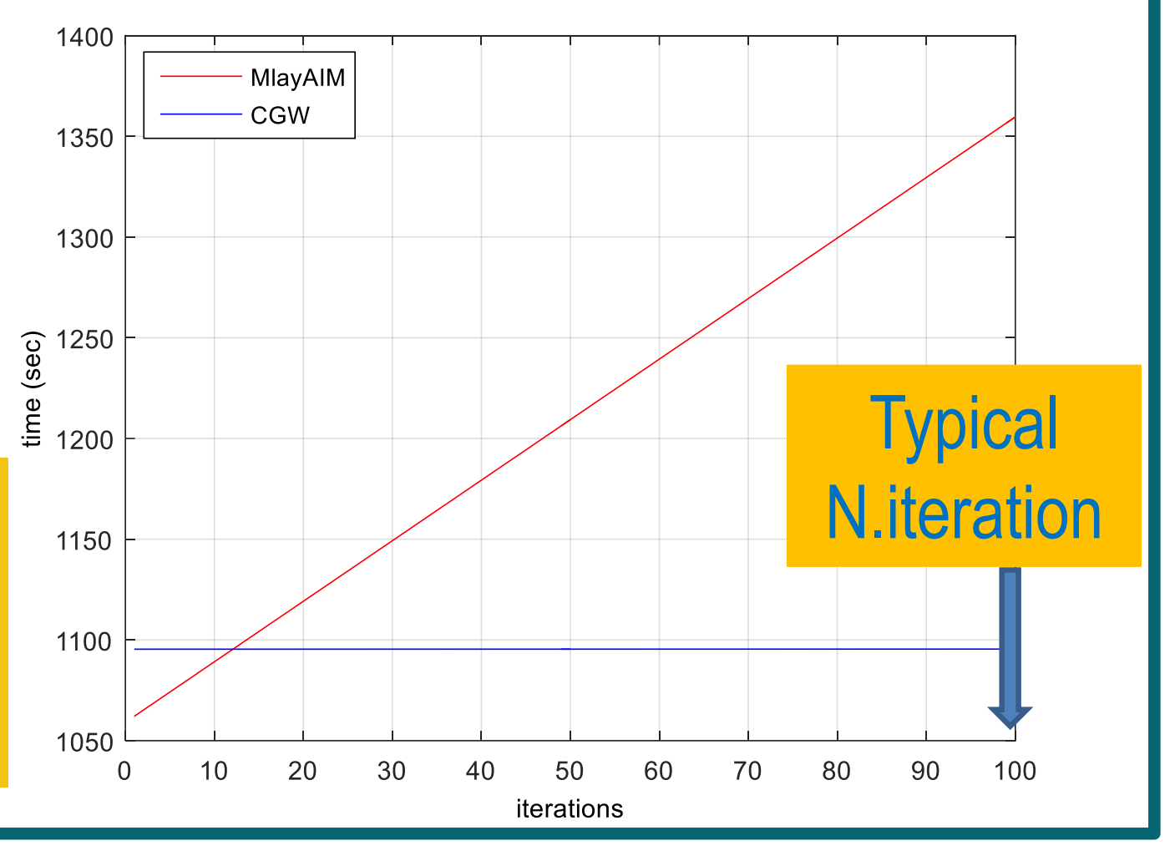


$$h \approx 0.1\lambda_g = 0.1\lambda_0/\sqrt{\varepsilon_r}$$

$$Time = T_s + nT_r$$

T_s : Setup time
T_r : Run time
n : N° of optimization steps

Ts{MlayAIM} = 1059.1 sec : MlayAIM Set-up time
Tr{MlayAIM} = 3.0049 sec : MlayAIM Run Time
Ts{CWGM} = Ts{MlayAIM} + 36.3 sec: CWGM Set-up time
Tr{CWGM} = 0.000522 sec CWGM Run time



Typical N.iteration

Summary and Future work

- Use of entire domain basis functions in MoM for solving the electromagnetic problem associated metasurface antennas modelled as impedance surfaces
- Expressing the unknown surface currents using **div-conforming** entire domain basis functions, obtained from **orthogonal eigenmodes**, allows a large **reduction in the number of unknowns** while maintaining the **accuracy** of the solution.
- **Compression** of the problem size, **regularization**, improving **conditioning** properties of matrix system in **iterative solution** have been highlighted through numerical results
- Results which show benefits in terms of computational costs in **optimization** problems have been presented. The cost function is related to the field distribution, which therefore has to be known for **several impedance profiles**.
- This **reduction** is mandatory to handle the large systems we have in practical applications
- Apparently limited in scope, because regular geometries for which modes can be defined are necessary. Although, the method described can be **generalized** to **geometries of arbitrary shape**.

Future works:

- Modes of coaxial cable can be exploites to properly model the geometry **with feeding region**
- **Generalize** the method exploring different kind of simmetries and entire-domain basis functions
- Work on the optimization method for the **synthesis** of a generic pattern.
- Regularization of IBC-EFIE for **polarization control** (cascaded **tensorial impedance**)

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