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A novel mathematical model to localize a multi-target modular probe for large-volume-metrology applications

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Recent studies show that the use of multiple Large-Volume Metrology (LVM) systems can lead to a systematic reduction in measurement uncertainty and a better exploitation of the available equipment. This is actually possible using a recently developed modular probe, which is equipped with different typologies of targets and integrated inertial sensors.

The goal of this paper is to present a new mathematical/statistical model for the real-time localization of this probe. This model is *efficient*, as it is based on a system of linearized equations, and *effective*, as the equations are weighed with respect to their uncertainty contribution.

Keywords: Large-volume metrology; Multi-target probe; Real-time localization; Generalized least squares.

1. Introduction

Typical industrial applications in the field of *Large-Volume Metrology* (LVM) concern dimensional verification and assembly of large-sized mechanical components [1]. LVM systems are usually equipped with *sensors* that perform local measurements of *distances* and/or *angles* [2]. Even though the existing measuring systems may differ in technology and metrological characteristics, two common features are: (i) the use of some *targets* to be localized, which are generally mounted on a hand-held probe in direct contact with the measured object's surface, and (ii) the fact that target localization is performed using the local measurements by sensors (e.g., through *multilateration* or *multiangulation* approaches).

Recent studies show that the use of multiple LVM systems can lead to a systematic reduction in measurement uncertainty and a better exploitation of the available equipment [3]. Unfortunately, the sensors of a specific LVM system are able to localize only specific targets and not necessarily those related to other systems. To overcome this obstacle, the authors have recently developed a new *modular probe*, equipped with targets related to different systems, and a *tip* in contact with the point of interest (P), which allows to localize P in a single turn [4, 5]. This probe has several innovative features: the number and typology of targets can be varied depending on the specific application, and the probe can integrate additional *inertial sensors* (i.e., two-axis inclinometer and compass), which are able to provide additional data.

The goal of this paper is to present a new mathematical/statistical model to localize the probe in measurements involving combinations of different LVM systems, i.e., systems equipped with sensors of different nature and metrological characteristics. In a nutshell, the model consists of a set of linearized equations that are weighted with respect to their uncertainty contribution.

The remainder of this paper is organized into three sections. Sect. 2 summarizes the technical and functional characteristics of the probe. Sect. 3 illustrates the mathematical/statistical model for the probe localization. Sect. 4 summarizes the original contributions of this research, focusing on its practical implications, limitations, and future development.

2. Multi-target modular probe

The probe has a modular structure. The main or *primary* module consists of a bar with a handle for the operator, two ends with several calibrated holes (in predefined positions), in which different types of *secondary* modules can be plugged in, and a power-supply and data-transmission system [4, 5]. There are different types of secondary modules: sphere mounts where to put spherically mounted retroreflectors (SMRs) for laser trackers; targets of different nature – such as those for rotary-laser automatic theodolites (R-LATs) or photogrammetric sensors; variable-length *extensions*, to be interposed between the primary module and the previous secondary modules; *styli* with a tip in contact with the point of interest.

An important requirement is that these secondary modules are coupled on the primary module, quickly, precisely and with a certain repeatability. This requirement can be achieved by adopting different technical solutions, such as providing the calibrated holes and shafts with threads or adopting quick coupling systems with magnetic lock.

The primary module has appropriate housings to lodge some integrated inertial sensors, such as two-axis inclinometer and compass, and is also equipped with a trigger for the acquisition of the point of interest: when the trigger is pressed, the probe tip is localized on the basis of the data collected by the probe targets/sensors at that time.

Once the primary and secondary modules are assembled, the relative positions between the probe targets and tip can be measured using a standard coordinate measuring machine (CMM). At this stage, a local Cartesian coordinate system ($o_P x_P y_P z_P$) – with origin (o_P) in the probe tip, and axes perpendicular to some reference planes on the surface of the primary module – can also be defined [4].

3. Model for probe localization

In general, each i-th LVM system (S_i) includes a number of *distributed* sensors, which are positioned around the measurement volume; we conventionally indicate the generic j-th sensor of S_i – or, for simplicity, the ij-th sensor – as s_{ij} (e.g., s_{i1}, s_{i2}, …, s_{ij}, …). The probe includes a number of targets of different nature and a tip, in contact with the points of interest on the surface of the measured object. T_k conventionally denotes a generic k-th target that is mounted on the probe. Sensors can be classified in two typologies: *distance* sensors, which are able to measure their distance (d_{ijk}) from the k-th target, and *angular* sensors, which are able to measure the azimuth (θ_{ijk}) and elevation (φ_{ijk}) angle, which are both subtended by the k-th target.

The subscript "ijk" refers to the local measurements (of distances or angles) by the ij-th sensor with respect to the k-th probe target. It is worth remarking that each ij-th sensor is not necessarily able to perform local measurements with respect to each k-th probe target, for two basic reasons:

- The communication range of the ij-th sensor should include the k-th target and there should be no interposed obstacle.
- Even if a k-th target is included in the communication range of the ij-th sensor, local measurements can be performed only if this target and this sensor are compatible with each other.

In the case of compatibility between the ij-th sensor and the k-th target, we can define some (linearized) equations related to the local measurements:

$$\begin{aligned} &\boldsymbol{A}_{ijk}^{\text{dist}} \cdot \boldsymbol{X} - \boldsymbol{B}_{ijk}^{\text{dist}} = \boldsymbol{0} \text{ (i.e., one eq. for each } ij\text{th distance sensor and } k\text{th target)} \\ &\boldsymbol{A}_{ijk}^{\text{ang}} \cdot \boldsymbol{X} - \boldsymbol{B}_{ijk}^{\text{ang}} = \boldsymbol{0} \text{ (i.e., two eqs. for each } ij\text{th angular sensor and } k\text{th target)} \end{aligned}, \quad (1)$$

where $\boldsymbol{X} = [X_P,\ Y_P,\ Z_P,\ \omega_P,\ \phi_P,\ \kappa_P]^T$ is the (unknown) vector containing the spatial coordinates (X_P, Y_P, Z_P) of the centre of the probe tip (P) and the angles (ω_P, ϕ_P, κ_P) of spatial orientation of the probe, referring to a global Cartesian coordinate system $OXYZ$. Matrices related to distance sensors are labelled with superscript "dist", while those related to angular sensors with superscript "ang". The matrices $\boldsymbol{A}_{ijk}^{\text{dist}}$, $\boldsymbol{B}_{ijk}^{\text{dist}}$, $\boldsymbol{A}_{ijk}^{\text{ang}}$ and $\boldsymbol{B}_{ijk}^{\text{ang}}$ contain:

- the position/orientation parameters ($X_{0_{ij}}, Y_{0_{ij}}, Z_{0_{ij}}$, ω_{ij}, ϕ_{ij} and κ_{ij}) related to the ij-th sensor;
- the distance (d_{ijk}) and/or angles (θ_{ijk}, φ_{ijk}) subtended by the k-th target, with respect to a local Cartesian coordinate system $o_{ij}x_{ij}y_{ij}z_{ij}$ of the ij-th sensor.

Since the "true" values of the above parameters are never known exactly, they can be replaced with appropriate estimates, i.e., $\hat{X}_{0_{ij}}, \hat{Y}_{0_{ij}}, \hat{Z}_{0_{ij}}$, $\hat{\omega}_{ij}$, $\hat{\phi}_{ij}$ and $\hat{\kappa}_{ij}$, resulting from initial calibration process(es), $\hat{d}_{ijk}$, resulting from distance measurements, and $\hat{\theta}_{ijk}$ and $\hat{\varphi}_{ijk}$, resulting from angular measurements.

As already said, the probe can also be equipped with some integrated inertial sensors (two-axis inclinometer and compass) which are able to perform angular measurements for estimating the spatial orientation of the probe, through the following linearized equations:

$$\boldsymbol{A}^{\text{int}} \cdot \boldsymbol{X} - \boldsymbol{B}^{\text{int}} = \boldsymbol{0} \text{ (i.e., three eqs. related to three angular measurements).} \quad (2)$$

Matrices $\boldsymbol{A}^{\text{int}}$ and $\boldsymbol{B}^{\text{int}}$ contain local measurements of three angles (ω_I, ϕ_I, κ_I), depicting the orientation of the integrated sensors with respect to a ground-referenced coordinate system ($x_I y_I z_I$).

The probe-localization problem can therefore be formulated through the following linear model, which encapsulates the relationships in Eqs. 1 and 2:

$$\boldsymbol{A} \cdot \boldsymbol{X} - \boldsymbol{B} = \begin{bmatrix} \boldsymbol{A}^{\text{dist}} \\ \boldsymbol{A}^{\text{ang}} \\ \boldsymbol{A}^{\text{int}} \end{bmatrix} \cdot \boldsymbol{X} - \begin{bmatrix} \boldsymbol{B}^{\text{dist}} \\ \boldsymbol{B}^{\text{ang}} \\ \boldsymbol{B}^{\text{int}} \end{bmatrix} = \boldsymbol{0}, \quad (3)$$

where blocks $\boldsymbol{A}^{\text{dist}}$, $\boldsymbol{A}^{\text{ang}}$, $\boldsymbol{B}^{\text{dist}}$ and $\boldsymbol{B}^{\text{ang}}$ are defined as:

$$\boldsymbol{A}^{\text{dist}} = \begin{bmatrix} \vdots \\ \boldsymbol{A}_{ijk}^{\text{dist}} \\ \vdots \end{bmatrix}_{ijk \in I^{\text{dist}}}, \ \boldsymbol{A}^{\text{ang}} = \begin{bmatrix} \vdots \\ \boldsymbol{A}_{ijk}^{\text{ang}} \\ \vdots \end{bmatrix}_{ijk \in I^{\text{ang}}}, \ \boldsymbol{B}^{\text{dist}} = \begin{bmatrix} \vdots \\ \boldsymbol{B}_{ijk}^{\text{dist}} \\ \vdots \end{bmatrix}_{ijk \in I^{\text{dist}}}, \ \boldsymbol{B}^{\text{ang}} = \begin{bmatrix} \vdots \\ \boldsymbol{B}_{ijk}^{\text{ang}} \\ \vdots \end{bmatrix}_{ijk \in I^{\text{ang}}},$$

I^{dist} and I^{ang} being the sets of index-pair values (ijk) relating to the ij-th distance/angular sensors that can see the k-th target.

We remark that all the equations of the system in Eq. 3 are referenced to a unique global Cartesian coordinate system, *OXYZ*. These equations therefore include suitable roto-translation transformations to switch from other reference systems (e.g., the local reference system related to each distributed sensor, that one related to the probe, or the ground-referenced system of the inertial sensors that are integrated into the probe) to *OXYZ*.

The six unknown parameters in $\boldsymbol{X}$ can be determined solving the system in Eq. 3, which is generally *overdefined*, i.e., there are more equations than unknown parameters: one for each combination of ij-th distance sensor and k-th target, two for each combination of ij-th angular sensor and k-th target, and three for the integrated sensors (i.e., two for the two-axis inclinometer and one for the compass).

The equations of the system may differently contribute to the uncertainty in the probe localization. Four important factors affecting this uncertainty are:

- *Uncertainty in the position/orientation of distributed sensors* ($\hat{X}_{0_{ij}}$, $\hat{Y}_{0_{ij}}$, $\hat{Z}_{0_{ij}}$, $\hat{\omega}_{ij}$, $\hat{\phi}_{ij}$ and $\hat{\kappa}_{ij}$), resulting from initial calibration process(es);

- *Uncertainty in the local measurements* ($\hat{d}_{ijk}$, $\hat{\theta}_{ijk}$ and $\hat{\varphi}_{ijk}$) *by the distributed sensors* with respect to probe targets, which depends on their metrological characteristics;
- *Uncertainty in the relative position between the probe targets and the tip* (P), which may depend on the accuracy of the manufacturing and calibration processes of the probe modules.
- *Uncertainty in the angular measurements* ($\hat{\omega}_I$, $\hat{\phi}_I$ and $\hat{\kappa}_I$) *by the inertial sensors integrated into the probe*, which depends on their metrological characteristics.

Consequently, it would be appropriate to solve the system in Eq. 3, giving greater weight to the equations that produce less uncertainty and *vice versa*. To this purpose, a practical method is that of *Generalized Least Squares* (GLS) [6], in which a weight matrix (***W***), which takes into account the uncertainty produced by the equations, is defined as:

$$\boldsymbol{W} = \left[\boldsymbol{J} \cdot \Sigma_{\xi} \cdot \boldsymbol{J}^T\right]^{-1}, \tag{4}$$

where ***J*** is the Jacobian matrix containing the partial derivatives of the elements in the first member of Eq. 3 (i.e., ***A***·***X*** – ***B***) with respect to the parameters contained in the vector ***ξ***, i.e., the position/orientation of distributed sensors, the local measurements by the distributed sensors available, the angular measurements by the integrated sensors, and the relative position of the probe targets with respect to the tip. For details, see [5]. Σ_{ξ} is the covariance matrix of ***ξ***, which represents the variability of the parameters in ***ξ***.

The parameters in Σ_{ξ} can be determined in several ways: (i) from manuals or technical documents relating to the distributed/integrated sensors in use, or (ii) estimated through *ad hoc* experimental tests. We remark that these parameters should reflect the measurement uncertainty of the elements of ***ξ***, in realistic working conditions – e.g., in the presence of vibrations, light/temperature variations and other typical disturbance factors.

By applying the GLS method to the system in Eq. 3, we obtain the final estimate of ***X*** as:

$$\hat{\boldsymbol{X}} = \left(\boldsymbol{A}^T \cdot \boldsymbol{W} \cdot \boldsymbol{A}\right)^{-1} \cdot \boldsymbol{A}^T \cdot \boldsymbol{W} \cdot \boldsymbol{B}\,. \tag{5}$$

For further details on the GLS method, see [6].

We remark that the metrological traceability of the probe localization is ensured by (i) initial calibration processes to determine the spatial position/orientation of the distributed sensors and (ii) another calibration process to determine the relative position of probe targets. In fact, these processes are generally based on the use of physical artefacts (such as calibrated bars with multiple reference positions) or measuring instruments (such as CMMs), which are traceable to the measurement unit of length [1].

4. Conclusions

This paper has described a novel mathematical/statistical model for the real-time localization of a modular and multi-target probe. The model is *efficient*, as it is based on a system of linearized equations, and *effective*, as the equations are weighed with respect to their uncertainty contribution, through the GLS method.

For the model to be viable, some parameters relating to the sensors in use should be known in advance, e.g., uncertainties in the position/orientation or local measurements; this can be done through *ad hoc* experimental tests or using manuals or technical documentation of the measuring systems. The model is automatable and could be a key tool to promote the combined use of LVM systems.

Regarding the future, we plan to extend the use of the probe from the measurement process to the distributed-sensor calibration process.

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