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Level Zero Core Curriculum and Entrance Examination: An Italian Experience

PAOLO BOIERI & ANITA TABACCO

SUMMARY *In this paper the mathematical part of the entrance examinations at the Politecnico di Torino is considered; the topics covered are compared with the SEFI Mathematical Working Group Level Zero Core Curriculum. The structure of the examinations is described, and the global results of the examination, in particular Algebra and Trigonometry, are analysed.*

1. Introduction

The SEFI Mathematical Working Group (MWG) in its analysis of the role of mathematics in the engineering curriculum has elaborated a 'Core Curriculum in Mathematics for the European Engineer'. The aim of this proposal is to offer to educational institutions a guideline for preparation of a syllabus for engineering studies.

One of the main problems that arose during the elaboration of the core curriculum was the definition of a common entry base level. Indeed, there are wide variations in the mathematical background in different European countries, and also inside the same country. Nevertheless, the MWG proposed a 'Level Zero Core Curriculum' containing a reasonable list of assumed prerequisite topics (see the appendix).

In this paper, we consider the Italian experience, examining the results of the mathematical part of entrance examinations given at the Politecnico di Torino in the last 2 years.

In Section 2, we describe the structure of the exam and the relation between its contents and the Level Zero Core Curriculum; the main point to observe is the lack of calculus in the topics of the examinations.

Section 3 is devoted to the analysis of the results; the questions are grouped in eight units. For each of them, we define a confidence index and a knowledge index, which show the attitude of the students towards the prerequisite mathematical knowledge.

Finally, in the last section, we describe only two of these units (trigonometry and algebra) which we think are good representatives of the entire work we have done. We relate the questions of these subjects and the associated results. A detailed analysis is done for the most interesting problems.

We conclude by remarking that the results seem to be significant due to the large number of students involved. We also think that this kind of analysis can be useful for at least two reasons. High school teachers can check the effectiveness of their work and its compliance with the needs of universities. Engineering school teachers can obtain a

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precise idea of the real entry level of the students and plan, in a coherent way, first year mathematics courses.

2. Mathematics in Entrance Examinations and the Level Zero Core Curriculum

2.1 Entrance Examinations at the Politecnico di Torino

The examinations we are considering (see [1]) were proposed to students entering the Politecnico in 1991 and 1992 (about 3200 students each year). These entrance examinations were organized for different reasons.

The first aim was to have a more balanced distribution of the students in Piemonte and Lombardia and to avoid overcrowding the two big engineering schools in Milano and Torino.

In 1991 the test was prepared by the Politecnico di Milano. The positive results led us to repeat this experience the following year. This time, the Politecnico di Torino actively collaborated in the preparation of the exam with the other universities involved.

A second reason was to check the students' entry level and to offer them the possibility of testing their attitude towards technical studies. None the less, a poor result did not preclude the possibility of entering the engineering school in Torino, while in Milano only a fixed number of students was accepted.

2.1 The Structure of the Examination

Each exam is composed of 100 multiple choice questions. Each of these has five proposed answers, among which only one is correct.

The questions are divided into four different series. Each series is proposed to the students separately in a pre-assigned time (see the following table) and it is separately graded.

Logic	30 questions	35 minutes
Text comprehension	20 questions	45 minutes
Mathematics	20 questions	35 minutes
Physics and chemistry	30 questions	35 minutes

The grading policy is the following: one point for a right answer, $-1/4$ of a point for a wrong answer and zero for an unanswered one.

We are interested only in the analysis of the mathematics series results, even if it would be interesting to study the global exam and the relation between the different parts.

2.3 The Level Zero Core Curriculum and the Assumed Knowledge

To understand better the choice of the topics covered, we describe briefly the Italian high school system and the passage from high school to university.

All the students who have obtained a degree at the end of a 5-year course in high school (Maturità) can enter the university.

Obviously, the Politecnico is chosen mostly by students with a stronger scientific or technical background. The majority of the freshman of the Engineering Faculty of

Politecnico have attended the Liceo Scientifico (a lyceum with emphasis on mathematics and physics, but also with a vast curriculum in humanities) or one of the several technical high schools (Istituti tecnici). These schools prepare technicians (generally at a good level, compared with the short time of study) in fields like electronics, computer sciences, aeronautics, mechanics and so on.

The background of these students includes elementary calculus and covers completely the Level Zero Core Curriculum and generally goes beyond it (especially in some technical schools).

A smaller number of students has attended different types of schools, like the Liceo Classico (lyceum with a great emphasis on humanities and a limited mathematical curriculum), or schools for 'Geometra' (surveyor), or schools for accounting, for elementary school teachers and other schools. The background of these students does not include calculus; it usually covers other topics of the Level Zero Core Curriculum, with the possible exception of trigonometry.

It must be noted that the same topics appear in the curriculum of these schools and in the curriculum of the Liceo Scientifico. However, they are not covered in the same depth; some examples are analytic geometry of the conic sections, graphing of functions and inequalities.

The necessity of giving the same opportunities to all students at the entrance examinations leads us to drop calculus from the assumed background. The examinations are based on subjects listed in the paragraphs on real numbers and algebra, trigonometry and geometry of the Level Zero Core Curriculum.

Some notions of combinatorics and elementary probability are included, at least in the most recent high school curricula, and so they are part of the examinations.

3. A General Analysis of the Results

3.1 Grouping the Questions

The 40 questions concerning mathematics in the two examinations have been divided into the following eight groups:

Arithmetic	5 questions
Algebra	6 questions
Analytic geometry	2 questions
Exponentials and logarithms	3 questions
Trigonometry	5 questions
Combinatorics and probability	2 questions
Euclidean geometry	11 questions
Problem comprehension	4 questions

The combinatorics and probability group and the analytic geometry group include only two questions and are less significant than the others, which include from three up to 11 questions.

A small remark about the 'problem comprehension': in this unit are included questions that do not require any special knowledge, except very elementary arithmetic. Rather, they require a careful understanding of the (sometimes involved) question. For this reason they seem more significant for determining the reasoning capability of the student than of his/her ability of using some mathematical tools; so they are not included in other groups, where the knowledge of a mathematical skill is emphasized.

Two questions are not included in this scheme, since they are 'technical' questions, but they cover different fields. In one of these the student is asked to identify the correct statement among five mathematical relations where logarithms, algebraic equations, numerical inequalities and trigonometry appear. In the other question, the way of solving the problem can lead to analytic geometry, graphing of functions or two algebraic techniques.

3.2 Confidence Index and Knowledge Index

The first element we want to analyze is the familiarity of the student with the various topics. Due to the grading policy, a blank answer is better than a wrong one, so it is natural to assume that lack of familiarity in a subject induces the student to skip a related question. For this reason, we consider the percentages of omitted answers and we introduce the 'confidence index' relative to a given unit as follows. If p_i is the percentage of given answers for the ith question, the confidence index is defined as

$$I_C = \frac{1}{n} \sum_{i=1}^{n} p_i$$

where n is the total number of questions of the unit considered.

If I_C is close to one this shows good confidence, while I_C close to zero in an indication of diffidence. The results are shown in Fig. 1.

Considering now only the given answers, we examine the percentage q_i of the right ones. We introduce a 'knowledge index' I_K of a group by

$$I_K = \frac{1}{n} \sum_{i=1}^{n} q_i$$

The results are shown in Fig. 2.

From the analysis of these data, we see a correlation between confidence and knowledge on a subject. For instance, analytic geometry, arithmetic and algebra and the ones that have the best results, while units like logarithms and exponentials and Euclidean geometry are usually disliked and not very well known by the students.

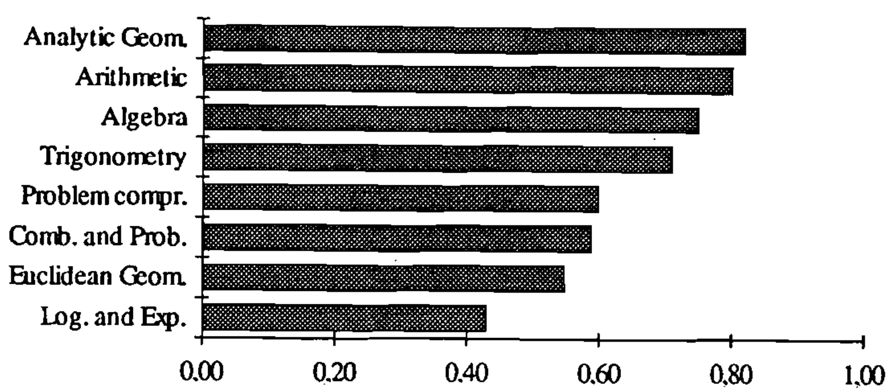

FIG. 1. The confidence index.

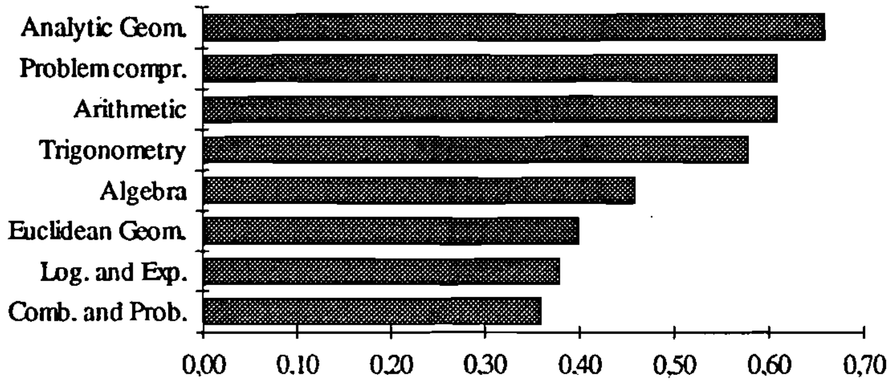

FIG. 2. The knowledge index.

The only exception concerns the 'problem comprehension' section where a low confidence index matches good results.

3.3 *Significance of the Answers*

The total number of questions asked during the examination is quite high and the time is very short. It is plausible that a large number of students are not able to organize the available time and spend more time on the first problems, neglecting completely the last part of the series. If this happens, the resuls of the last part of each series are less significant and they cannot be used to draw conclusions.

In order to check the 'reliability as a function of time', we plot the percentage of missed answers for each question. In the 1991 exam, there was a big block, the last six questions, that had a considerable number of blanks. They were all related to Euclidean geometry. The negative final conclusion about the knowledge of this subject must be modified a little.

On the contrary, the 1992 examination showed a more regular pattern and all the questions (with the possible exception of the last two) seemed to be significant (see Fig. 3).

4. A More Detailed Analysis of Some Results

4.1 *Trigonometry*

In the trigonometry unit, we find five questions:

- T1 is very elementary and requires only the capability of recognizing the sine and tangent of an angle on the unit circle.
- T2 concerns again the basic definitions and the sines and cosines of specific angles (such as 30°, 45°, ...).
- T3 reads: "From $\sin x = 1/2$ it is possible to conclude that:
  (A) $\sin x/2 = 1/4$
  (B) $\sin 2x = 1$
  (C) $\cos x = 1/2$
  (D) $\text{tg } 2x = 1/2$
  (E) $\cos 2x = 1/2$."
- T4 is about the properties of the composition of the cosine and the sine functions.

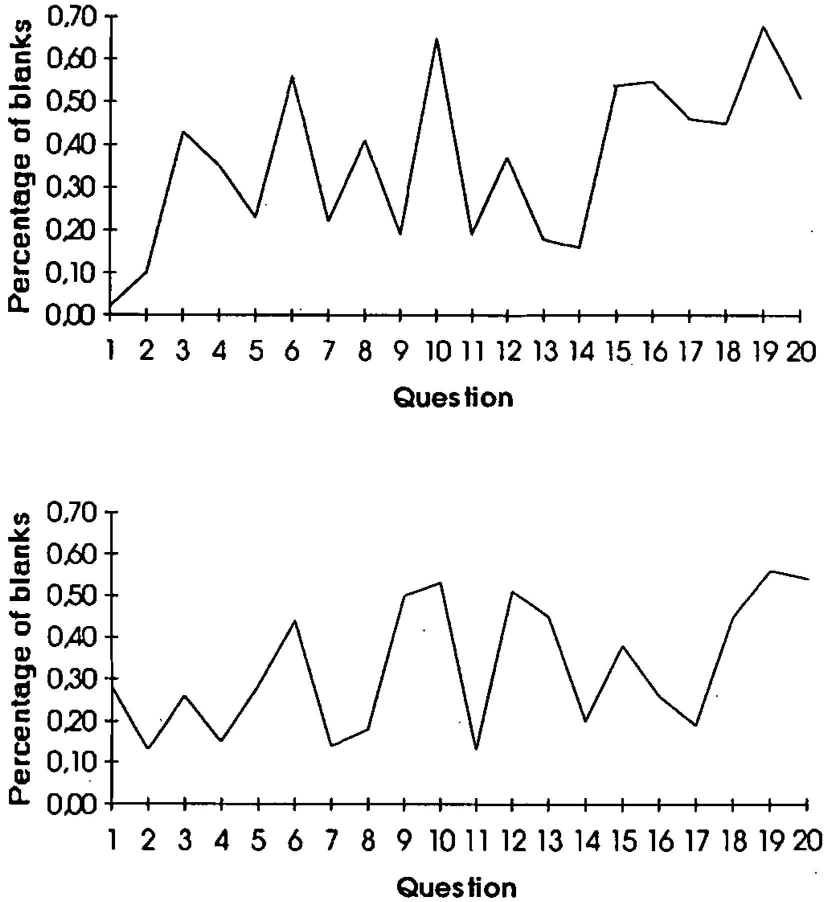

FIG. 3. Percentage of missed answers.

- T5 reads: "How many solutions has the equation tg $(2x-5\pi) = -10^4$:
  (A) no solution
  (B) two solutions
  (C) four solutions
  (D) seven solutions
  (E) an infinite number of solutions."

The results are as follows:

Question	Correct	Omitted	Wrong
T1	68	15	17
T2	59	13	28
T3	46	22	31
T4	22	41	36
T5	20	53	27

From the previous data relative to T1, T2 and T3, we can immediately see that at least 15% of the students never studied trigonometry. Moreover, an impressive percentage (about 25%) give wrong answers to these basic questions.

As some difficulties arise (as in T4 and in T5), the number of missing and wrong answers increases in a 'non-linear way'.

4.2 Algebra

In the algebra unit, we have six questions:

- A1 reads: "Let n be a positive integer. The expression $(2^n+2^{n+1})^2$ is equal to:

  (A) 9.4^n
  (B) 2^{4n+2}
  (C) 4^{4n+2}
  (D) 2^{2n^2+2n}
  (E) 9.2^{n^2}."

- A2 is about the algebra of radicals.
- A3 is a problem about numbers which can be solved by simple system of two equations.
- A4 reads: "The following system in the real unknown x and y

  $$\begin{cases} xy-x=0 \\ x^2+xy=-2 \end{cases}$$

  has:
  (A) no solutions
  (B) one solution
  (C) two solutions
  (D) a finite number of solutions, larger than two
  (E) an infinite number of solutions."

- A5 asks to deduce an inequality starting from an algebraic relation.
- A6 reads: "How many ordered pairs (m, n) of positive integers m and n satisfy the condition:

  $(m+n)^2=(m-n)^2+64$?

  (A) none
  (B) five
  (C) six
  (D) ten
  (E) infinite."

The results are summarized as follows:

Question	Correct	Omitted	Wrong
A1	23	35	42
A2	44	23	32
A3	40	13	47
A4	39	19	42
A5	47	18	34
A6	16	43	40

We note that the relative position in the confidence and knowledge index tables is the same for trigonometry, while I_C for algebra is much bigger than I_K. In effect a long time is devoted to algebra in the high school curriculum. Despite this, positive results are always below 50%.

The choice of the answers proposed in A1 gives a good idea of the posible mistakes in the use of powers in algebra. It is interesting to look at the distribution of the answers: (A) 23.3%; (B) 14.2%; (C) 10.9%; (D) 9.6%; (E) 6.7%.

A4 is a very simple algebraic system; the students show good confidence for this kind of problem. However, the wrong answers are more than the correct ones. We notice that it is possible to get the correct answer (A) following a wrong method. Indeed, they can consider for the first equation only the $x = 0$ solution, which leads to an impossible second equation.

Despite all this, about 23% chose (C) and 12% chose (D). Perhaps students automatically use ideas like "a second degree equation has two solutions" and "a fourth degree system has four solutions".

We think that the negative results of A6 can be explained, at least partially, by the use of notation. Since the problem involves integers, the student thinks that it can be solved by trial and error. This leads to the wrong answer (A), because it is difficult to find immediately a couple of integers satisfying the relation. It is worth noting that about 65% of the students did not try to simplify the equality.

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[1] Angelini, E. & Beccari, C. *et al.* (1992) Il Test Orientativo-attitudinale per l'Iscrizione alle Facoltà di Architettura e di Ingegneria del Politecnico di Torino, Anno Accademico 1991–92. CIDEM del Politecnico di Torino, Torino.

Appendix: The Level Zero Core Curriculum

Real Numbers and Algebra

1.1.1 Basic rules of arithmetics applied to the manipulation of real numbers.
1.1.2 Indices, logarithms, rational powers and real roots.
1.1.3 Manipulation of algebraic expressions including completion of squares and partial fractions.
1.1.4 Graphs of elementary functions including simple polynomial and rational functions. Roots of equations.
1.1.5 Solutions of systems of linear algebraic equations.
1.1.6 Graphical and algebraic treatment of inequalities.
1.1.7 Logarithmic and exponential functions and their graphs.
1.1.8 The binomial theorem with integer exponents.

Trigonometry

1.2.1 Trigonometric functions, angular measure, graphs and identities.
1.2.2 Application to right-angled triangles.
1.2.3 Sine and cosine rules.
1.2.4 Solution of simple trigonometric equations.

Geometry

1.3.1 Euclidean geometry of the triangle, simple polygons, the circle and elementary solids.

1.3.2 Geometry of the straight line.
1.3.3 The circle in Cartesian coordinates.
1.3.4 The conic sections, i.e. ellipse, parabola and hyperbola in canonical form and their parametric representation.

Differential Calculus of One Variable

1.4.1 Concept of the derivative and its interpretation as a rate of change and slope of a curve.
1.4.2 Derivatives of standard functions, polynomial, exponential, logarithmic and trigonometric.
1.4.3 Rules of differentiation—product, quotient and function of function.
1.4.4 Elementary treatment of maximum, minimum and points of inflexion.

Integral Calculus of One Variable

1.5.1 The concept of integration as the limit of a sum.
1.5.2 Integration as the reverse process of differentiation.
1.5.3 The integrals of standard functions.