MIMO Relay Networks: Scheduling and Outage Probability

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***Abstract*—We study a dual-hop multiple-input multiple-output (MIMO) relay network where the traffic source uses a relay node, selected among several possible ones, to serve multiple users. Considering that the nodes deployment can be described by Poisson Point Processes in a sector area, we derive the distribution of the signal-to-interference-plus-noise ratio over the two communication hops. Then, assuming that the user to be served is selected according to opportunistic, proportional fair, or selective multiuser diversity scheduling, we investigate the system outage probability, and derive a closed-form tight lower bound for it. Our analysis provides useful guidelines on the system design of MIMO relay networks. Unlike existing works, our analysis accounts for the joint impact of various multiuser scheduling schemes and random node placement.**

***Index Terms*—Relay network, MIMO channel, Multiuser scheduling, Outage probability, Stochastic geometry.**

I. INTRODUCTION

Cooperative multiple-input multiple-output (MIMO) relay networks have been investigated extensively for the last decade. The amplify-and-forward (AF) relaying has received plenty of interest in single-user settings [1], due to its implementation simplicity. In a multiuser scenario, AF multiuser relay networks (MRN) consider a relay system where the user to be served is selected according to the adopted scheduling strategy. As an example, an interesting work on MRN is [2], which aims to maximize the throughput by jointly investigating relay selection and power allocation.

Unlike most of the previous studies on MRNs, in this work we focus on an AF relay scheme where the user locations follow a random distribution, and we investigate the coupled effects of antenna diversity, spatial layout of nodes, as well as multiple scheduling strategies. In particular, we consider (i) Opportunistic Scheduling (OS) [3], a greedy approach aimed at maximizing throughput, (ii) Proportional Fair Scheduling [4]–[6] (PFS), currently exploited in Long Term Evolution (LTE) communication system as it provides an excellent tradeoff between throughput and fairness, and (iii) Selective Multiuser Diversity Scheduling (SMUD) [7], which reduces the feedback load with respect to OS [3].

In such scenario, we first provide the distribution of the SINR over each hop, and then derive a closed-form lower bound to the system outage probability for each scheduling scheme. All of our analytical derivations are numerically validated and lead to useful guidelines for the design of MRNs.

II. SYSTEM MODEL

We consider a cellular MRN communication scenario where each Base station (BS) acts as an information source and transfers data to multiple users, thanks to the help of relay nodes. More specifically, upon scheduling a transmission towards a user according to one among the OS, PFS, or SMUD schemes, the BS randomly selects a relay among the ones at its disposal. The relay then forwards the BS's signal to the scheduled user, by using the AF relay mode[1]. We limit our study to the case where no direct link exists between the BS and the user it has to serve, however all communications are subject to co-channel interference (CCI). Also, nodes operate in half-duplex mode and BSs and relays are equipped with n_s and n_r antennas, respectively, while users are equipped with n_d antennas.

We consider that BS, relays, and users are all distributed in a sector area, as shown in Figure 1. The distributions of the relays at disposal of the BS and the users associated with it, can be characterized by two independent Poisson Point Processes (PPPs), with density σ_r and σ_u, respectively. For a tagged BS s equipped with directional antennas, the possible relays are uniformly distributed in a circular sector area A_r with center s, radius equal to the transmission range R, and angle $\xi\pi$ (where $0 < \xi < 1$). The set of users associated with the tagged BS, denoted by $\mathcal{U}$, are located uniformly in an annular sector area A_u with center s, radius extending from R to L, and angle $\xi\pi$.

We then express the SNR between two generic nodes i and j as $\alpha_{ij} = P_i \frac{G_i G_j}{N_0 W} \left(\frac{4\pi f_c r_{ij}}{c}\right)^{-a}$, where G_i and G_j are the antenna gains at the two ends. P_i is the transmit power at node i, N_0 is the noise power spectral density, r_{ij} is the transmission distance, and W is the signal bandwidth. Moreover, f_c is the signal carrier frequency, a is the path loss exponent, and c is the light speed. The channel matrix between nodes i and j is denoted by $\mathbf{H}_{ij}$. Under the uncorrelated Rayleigh fading

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[1] In this work, the relay transmit power is assumed to be fixed. The analysis, however, could be extended to the case where a round robin scheduling is in place to provide better fairness and the relay adapts its transmit power so as to let the scheduled user achieve a minimum target SINR.

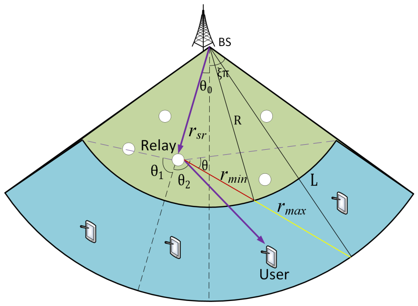

Fig. 1: The sector MRN model with randomly distributed nodes.

assumption, each element of $\mathbf{H}_{ij}$ is Gaussian-distributed with zero mean and unit variance. We consider that in all communications the Maximum Ratio Transmission (MRT) technique [8] is employed and channel state information (CSI) is perfectly known to both ends of communications.

By assuming $\mu_r = \sum_{k\in\mathcal{I}_r} \alpha_{kr}|h_{kr}|^2$ as the CCI on the relay, and $\mu_d = \sum_{\ell\in\mathcal{I}_d} \alpha_{\ell d}|h_{\ell d}|^2$ as the CCI on the user, the end-to-end SINR η_d, from the source to the user d, is given as $\eta_d = \frac{\gamma_r\gamma_d}{\gamma_r+\gamma_d+1}$, where $\gamma_r = \frac{\alpha_{sr}\lambda_r}{\mu_r+1}$ and $\gamma_d = \frac{\alpha_{rd}\lambda_d}{\mu_d+1}$ are the instantaneous SINR corresponding to the source-relay channel and relay-user channel, respectively. λ_r denotes the maximum eigenvalue of channel matrix $\mathbf{H}_{sr}$. The random variables γ_d are independent of each other, since λ_d denotes the maximum eigenvalue of channel matrix $\mathbf{H}_{rd}$ and μ_d depends on the coefficients h_{ld}, which are independently distributed over d.

III. System Outage Probability

The outage probability of the MRN under study is defined as the probability that the end-to-end SINR experienced by the scheduled user, η_d, falls below a predefined threshold, i.e.,

$$P_{\text{out}} = \mathbb{P}(\eta_d < z) = F_{\gamma_r}(z) + \int_z^\infty f_{\gamma_r}(y) F_{\gamma_d}\left(\frac{(y+1)z}{y-z}\right) \mathrm{d}y \quad (1)$$

where $f_{\gamma_r}(\cdot)$ denotes the probability density function (pdf) of γ_r and $F_{\gamma_d}(\cdot)$ represents the cumulative density function (cdf) of γ_d. We first investigate the statistical distribution of γ_d.

A. Statistical distribution of γ_d

1) OS scheme: Under OS, the source serves the user with the highest instantaneous SINR [3]. The expression of the cdf of γ_d is given by: $F_{\gamma_d}(z) = \mathbb{P}\left(\max_{u\in\mathcal{U}} \gamma_u < z\right) = \prod_{u\in\mathcal{U}} F_{\gamma_u}(z)$, where the SINR of each user, γ_u, depends on three random factors: the small-scale fading, the location of the users, and the location of the interferers.

As shown in Figure 1, we assume the selected relay has distance r_{sr} from s and an angle $\theta_r = \theta_0 + \xi\pi$. The scheduled user is contained in a region, $\mathcal{C}(r_{sr}, \theta_0)$, defined in polar coordinates as $\mathcal{C}(r_{sr}, \theta_0) = \{(r,\theta) | \theta \in [0, \theta_1 + \theta_2], r \in (r_{\min}(\theta), r_{\max}(\theta)\}$ with the relay as origin. Thus, θ_1 and θ_2 can be obtained by solving the following system of equations:

$$\begin{cases} R\sin(\theta_2 - \theta_r) = r_{sr}\sin(\pi - \theta_2) \\ R\sin(\theta_1 - (2\xi\pi - \theta_r)) = r_{sr}\sin(\pi - \theta_1) \end{cases} \quad (2)$$

Given the angle θ, the minimum distance between the relay and the scheduled user, $r_{\min}(\theta)$ can be derived by solving:

$$R^2 = r_{sr}^2 + r_{\min}(\theta)^2 - 2r_{sr}r_{\min}(\theta)\cos(\pi - \theta_2 + \theta) \quad (3)$$

while the maximum distance is given by $r_{\max}(\theta) = \min\{d_1(\theta), d_2(\theta)\}$ where $d_1(\theta)$ and $d_2(\theta)$ are obtained by solving the following system of equations:

$$\begin{cases} d_1(\theta)\sin(\theta_2 - \theta - \theta_r) = r_{sr}\sin(\theta_r) \\ L^2 = r_{sr}^2 + d_2(\theta)^2 - 2r_{sr}d_2(\theta)\cos(\pi - \theta_2 + \theta) \end{cases} \quad (4)$$

Therefore, by adopting the OS strategy, the cdf of γ_d for the relay-user hop can be obtained by rewriting [9, Eq. 12] as

$$F_{\gamma_d}(z|\theta_0, r_{sr}) = \exp\left(-\sigma_u \int_{\mathcal{C}(r_{sr},\theta_0)} \left(1 - F_{\gamma_u}(z|r)\right) \mathrm{d}s\right) \quad (5)$$

where $F_{\gamma_u}(z|r)$ denotes the cdf of the SINR of the u-th user, given the relay-user distance r, and $\mathrm{d}s = r\,\mathrm{d}r\,\mathrm{d}\theta$ is the surface element in polar coordinates. The cdf of the SINR of the relay-user hop is presented below.

Proposition 1: In the OS scheme, the cdf of γ_d for the relay-user hop can be written as

$$F_{\gamma_d}(z) = \int_{-\xi\pi}^{\xi\pi}\int_0^R f_{r_{sr}}(r_{sr}) \frac{1}{2\xi\pi} \exp\Big(-\sigma_u \int_0^{\theta_1+\theta_2}\int_{r_{\min}(\theta)}^{r_{\max}(\theta)} \cdot \left(1 - F_{\gamma_u}(z|r)\right) r\,\mathrm{d}r\,\mathrm{d}\theta\Big)\,\mathrm{d}r_{sr}\,\mathrm{d}\theta_0 \quad (6)$$

where $F_{\gamma_u}(z|r)$ can be expressed as

$$F_{\gamma_u}(z|r) = 1 - \sum_{b=1}^{t}\sum_{c=v-t}^{(t+v)b-2b^2}\sum_{w=0}^{c}\sum_{k=0}^{w} \frac{\mathrm{e}^{-bz(P_rK_{ru})^{-1}r^a}}{(bz(P_rK_{ru})^{-1})^{k-w}} \frac{\binom{w}{k}\rho_{b,c}r^{a(w-k)}}{(-1)^k w!} \frac{\mathrm{d}^k}{\mathrm{d}s^k}\Big[\exp\Big(-2\pi\sigma_s\frac{(L-R)^{-a'}}{a'a}(K_1 sbzr^a)\, {}_2F_1(1, a'; a'+1; -K_1 sbzr^a(L-R)^{-1})\Big)\Big]_{s=1} \quad (7)$$

where $t = \min\{n_r, n_d\}$, $v = \max\{n_r, n_d\}$, $K_{ru} = \frac{G_rG_u}{N_0W}(c^{-1}4\pi f_c)^{-a}$, $K_1 = G_lG_r^{-1}P_lP_r^{-1}$ and $a' = 1 - \frac{2}{a}$. The transmit power at the relay and at the user is denoted by P_r and P_u, respectively.

Proof: The conditional cdf of the SINR $F_{\gamma_u}(z|r)$ with reference to a generic user u can be expanded as

$$F_{\gamma_u}(z|r) = 1 - \int_0^\infty \mathbb{P}\left(\lambda_u > \frac{z(g+1)}{\alpha_{ru}}\right) f_{\mu_u}(g)\,\mathrm{d}g\,. \quad (8)$$

where μ_u represents the CCI on user u coming from the set of active relays in A_r, which is denoted by $\mu_u = \sum_{l\in\mathcal{I}_r} \alpha_{lu}\lambda_{lu}$. The ccdf of λ_u is given by [8], $\mathbb{P}(\lambda_u > x) = \sum_{b=1}^{t}\sum_{c=v-t}^{(t+v)b-2b^2}\sum_{w=0}^{c} \frac{(bx)^w\rho_{b,c}}{w!\mathrm{e}^{bx}}$ where $\rho_{b,c}$ can be computed by using the efficient algorithm. We then have,

$$F_{\gamma_u}(z|r) = 1 - \sum_{b=1}^{t}\sum_{c=v-t}^{(t+v)b-2b^2}\sum_{w=0}^{c}\sum_{k=0}^{w} \frac{\binom{w}{k}(bz\alpha_{ru}^{-1})^{w-k}\rho_{b,c}}{w!\mathrm{e}^{bz\alpha_{ru}^{-1}}}\int_0^\infty h^k\mathrm{e}^{-h}\tilde{f}_h(h)\,\mathrm{d}h \quad (9)$$

where we define $h = gbz\alpha_{ru}^{-1}$. The integral in the last line of (9) can be expressed by the derivative of the Laplace transform of the pdf of h,

$$\int_0^\infty h^k\mathrm{e}^{-h}\tilde{f}_h(h)\,\mathrm{d}h = (-1)^k\frac{\mathrm{d}^k}{\mathrm{d}s^k}\left[\mathcal{L}_h(s)\right]_{s=1} \quad (10)$$

The Laplace transform evaluated at s in the above equation can be expressed by following the i.i.d. distribution of λ_{lu} and its further independence from the point process $\mathcal{I}_r$,

$$\mathcal{L}_h(s) = \mathbb{E}_{\mathcal{I}_r}\Big[\prod_{l\in\mathcal{I}_r}\mathbb{E}_{\lambda_e}\big[\exp\big(-sbz\alpha_{ru}^{-1}\alpha_{lu}\lambda_e\big)\big]\Big] \quad (11)$$

In the sequel, we use the exponential distributed variable λ_e to substitute the random variables with the i.i.d. distribution of λ_{lu}. Hence, the expectation with respect to λ_e in (11) is computed as $\mathbb{E}_{\lambda_e}\big[\exp\big(-sbz\alpha_{ru}^{-1}\alpha_{lu}\lambda_e\big)\big] = \frac{1}{sbz\alpha_{ru}^{-1}\alpha_{lu}+1}$. Using the PPP probability generating function, $\mathcal{L}_h(s)$ can be evaluated as

$$\mathcal{L}_h(s) = \exp\Big(-2\pi\sigma_s\frac{(L-R)^{-a'}}{a'a}(K_1 sbzr^a) \cdot {}_2F_1(1,a';a'+1;-K_1 sbzr^a(L-R)^{-1})\Big) \quad (12)$$

where ${}_2F_1(;;)$ is the hypergeometric function, and the last step follows from [10, 3.194.2]. By combining the results of (9), (10) and (12), we obtain the thesis in (7).

∎

The derivation of the closed-form distribution of CCI is an open problem. To have a computation-friendly expression of $F_{\gamma_u}(z|r)$ in (7), we use a Gamma-distributed variable to represent the CCI, of a given user u, by leveraging the results in [11]. More specifically, we approximate μ_u with the variable $\tilde{\mu}_u$, whose distribution is $\mathbb{P}(\tilde{\mu}_u < z) = 1-\Gamma(\kappa, z/\theta)$, where κ and θ are such that: $\mathbb{E}[\tilde{\mu}_u] = \kappa\theta$ and $\mathrm{Var}[\tilde{\mu}_u] = \kappa\theta^2$. Through second-moment matching, we set $\mathbb{E}[\tilde{\mu}_u] = \mathbb{E}[\mu_u] = 2\pi\sigma_s\alpha_{lu}d_{lu}^a(L-R)^{2-a}/(a-2)$, and $\mathrm{Var}[\tilde{\mu}_u] = \mathrm{Var}[\mu_u] = 2\pi\sigma_s\alpha_{lu}^2 d_{lu}^{2a}(L-R)^{2-2a}/(a-1)$. Then, by replacing μ_u with $\tilde{\mu}_u$, $F_{\gamma_u}(z|r)$ simplifies to:

$$F_{\gamma_u}(z|r) = 1-\sum_{b=1}^{t}\sum_{c=v-t}^{(t+v)b-2b^2}\sum_{w=0}^{c}\sum_{k=0}^{w}\frac{\binom{w}{k}(bz\alpha_{ru}^{-1}r^a)^{w-k-\kappa}\rho_{b,c}}{w!\mathrm{e}^{bz\alpha_{ru}^{-1}r^a}} \frac{\Gamma(k+\kappa)}{\Gamma(\kappa)\theta^\kappa}(1+\alpha_{ru}(bz\theta r^a)^{-1})^{-k-\kappa}. \quad (13)$$

2) PFS scheme: When OS is used in a practical scenario, some users might not have the chance to be served even in a long period of time due to the poor channel. In order to provide fairness, PFS grants access to the user that experiences the best relative SINR in a period of time. Thus, the selected user d is such that $d = \arg\max_{u\in\mathcal{U}}\left\{\frac{\gamma_u}{\bar{\gamma}_u}\right\}$, where $\bar{\gamma}_u$ is the SINR of user u averaged over a given period of time. Given our signal model, we define $\omega_u = \lambda_u/(\mu_u+1)$ which are i.i.d. random variables across $\forall u \in \mathcal{U}$. Therefore, under the PFS strategy, the cdf of γ_d for the relay-user hop can be obtained by integrating the conditional cdf of γ_d over the area A_u,

$$F_{\gamma_d}(z|\theta_0, r_{sr}) = \int_{(\theta,r)\in A_u}\frac{\mathbb{P}(\gamma_d < z|\theta_0, r_{sr}, r)}{|A_u|} r\,\mathrm{d}r\,\mathrm{d}\theta \quad (14)$$

where $|A_u|$ denotes the size of the area of A_u. We have

$$\begin{aligned}\mathbb{P}(\gamma_d < z|\theta_0, r_{sr}, r) &= \mathbb{P}(\max_{u\in U}\omega_u < zr^a|\theta_0, r_{sr}, r) \\ &= \exp(-\sigma_u|A_u|(1-F_{\omega_u}(zr^a))) \quad (15)\end{aligned}$$

Since $F_{\omega_u}(zr^a)$ has the same expression as (7) in the general case, the cdf of γ_d can be obtained by combining (14) and (15).

Proposition 2: In the PFS scheme, the cdf of γ_d for the relay-user hop can be written as

$$F_{\gamma_d}(z) = \int_{-\xi\pi}^{\xi\pi}\int_0^R f_{r_{sr}}(r_{sr})\frac{1}{2\xi\pi}\int_0^{\theta_1+\theta_2}\int_{r_{\min}(\theta)}^{r_{\max}(\theta)} \frac{r}{|A_u|}\mathrm{e}^{-\sigma_u|A_u|(1-F_{\omega_u}(zr^a))}\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}r_{sr}\,\mathrm{d}\theta_0\,. \quad (16)$$

3) SMUD scheme: Both OS and PFS can be classified as full-feedback scheduling schemes, since they require all users to send the experienced channel quality back to the source. In order to reduce the system overhead, hence save user power and bandwidth, a threshold-based limited feedback scheme named SMUD has been proposed. Users send feedback on their channel quality only when their *instantaneous* SINR is higher than a given threshold, γ_{th}. Then the source selects the user to be served from a set of eligible users according to OS scheme. If all users fail to meet the SINR threshold, the scheduler reverts to a random pick among all users.

Under the SMUD scheme, the γ_u's are still i.i.d. random variables. In such scenario, when $z > \gamma_{\mathrm{th}}$, the cdf of γ_d takes the same expression as under the OS scheme (see (7)). When, instead, $z \leq \gamma_{\mathrm{th}}$, the derivation shown in [3, Eq. 10] offers the following expression for the cdf of γ_d,

$$F_{\gamma_d}(z|\theta_0, r_{sr}) = \mathrm{e}^{-\sigma_u\int_{\mathcal{C}}(1-F_{\gamma_u}(\gamma_{\mathrm{th}}|r))\,\mathrm{d}s}\cdot\frac{\int_{\mathcal{C}}F_{\gamma_u}(z|r)\,\mathrm{d}s}{\int_{\mathcal{C}}F_{\gamma_u}(\gamma_{\mathrm{th}}|r)\,\mathrm{d}s}$$

where $\mathcal{C}(r_{sr},\theta_0)$ is denoted by $\mathcal{C}$ for simplicity and $F_{\gamma_u}(\cdot|r)$ in OS scheme is provided in (7). Consequently, the cdf of γ_d in SMUD scheme is obtained by integrating over θ_0 and r_{sr}.

Proposition 3: In SMUD scheme, the cdf of γ_d for the relay-user hop is given in (6), when $z > \gamma_{\mathrm{th}}$. Instead, when $z \leq \gamma_{\mathrm{th}}$, the cdf of γ_d is expressed as

$$F_{\gamma_d}(z) = \int_{-\xi\pi}^{\xi\pi}\int_0^R \mathrm{e}^{-\sigma_u\int_0^{\theta_1+\theta_2}\int_{r_{\min}(\theta)}^{r_{\max}(\theta)}(1-F_{\gamma_u}(\gamma_{\mathrm{th}}|r))r\,\mathrm{d}r\,\mathrm{d}\theta} \cdot\frac{f_{r_{sr}}(r_{sr})}{2\xi\pi}\frac{\int_0^{\theta_1+\theta_2}\int_{r_{\min}(\theta)}^{r_{\max}(\theta)}F_{\gamma_u}(z|r)r\,\mathrm{d}r\,\mathrm{d}\theta}{\int_0^{\theta_1+\theta_2}\int_{r_{\min}(\theta)}^{r_{\max}(\theta)}F_{\gamma_u}(\gamma_{\mathrm{th}}|r)r\,\mathrm{d}r\,\mathrm{d}\theta}\,\mathrm{d}r_{sr}\,\mathrm{d}\theta_0 \quad (17)$$

B. Lower bound on the system outage probability

The randomness of the SINR of the relays depends on the fading and the location of the relays and interferers. The outage probability of the randomly selected relay can be derived as the integral with respect to the distance from the BS, i.e.,

$$F_{\gamma_r}(z) = \int_0^R F_{\gamma_r}(z|r)f_{r_{sr}}(r)\,\mathrm{d}r \quad (18)$$

where $F_{\gamma_r}(z|r)$ takes an expression similar to (7) by substituting the corresponding parameters. When CCI μ_r is approximated with a Gamma variable, $F_{\gamma_r}(z|r)$ can be written as

$$F_{\gamma_r}(z|r) = 1-\sum_{b=1}^{p}\sum_{c=q-p}^{(p+q)b-2b^2}\sum_{w=0}^{c}\sum_{k=0}^{w}\frac{\binom{w}{k}(bz\alpha_{sr}^{-1}r^a)^{w-k-\kappa'}}{w!\mathrm{e}^{bz\alpha_{sr}^{-1}r^a}} \cdot\frac{\rho_{b,c}\Gamma(k+\kappa')}{\Gamma(\kappa')\theta'^{\kappa}}(1+\alpha_{sr}(bz\theta' r^a)^{-1})^{-k-\kappa'}\,. \quad (19)$$

where κ and θ are parameters that can be obtained through second-moment matching.

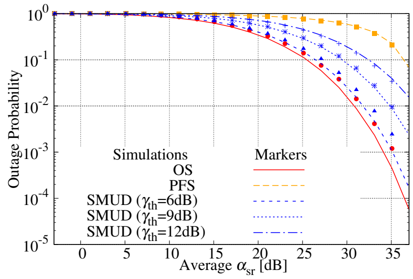

Fig. 2: Outage probability of the system, under the three scheduling schemes. Analytical results are compared against simulations.

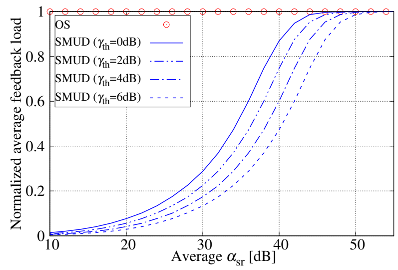

Fig. 3: Normalized feedback load for the OS and the SMUD scheme with different threshold values.

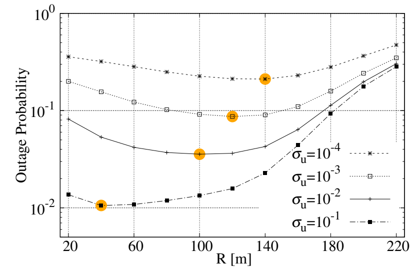

Fig. 4: Outage probability (lower bound) under the PFS scheme, for different values of R and user density.

Although the system outage probability can be obtained by solving the integral in (1), such integral cannot be computed in closed form. Thus we present an upper-bound to η_d, which leads to a lower-bound of the system outage probability,

$$\eta_d = \frac{\gamma_r \gamma_d}{\gamma_r + \gamma_d + 1} < \frac{\gamma_r \gamma_d}{\gamma_r + \gamma_d} \leq \min\{\gamma_r, \gamma_d\} = \gamma^\star . \quad (20)$$

Then, due to the independence between γ_r and γ_d, the closed-form lower-bound of the system outage probability is

$$P_{\text{out}}^{\text{LB}}(z) \quad = \quad F_{\gamma_r}(z) + F_{\gamma_d}(z) - F_{\gamma_r}(z) F_{\gamma_d}(z) . \quad (21)$$

IV. Performance results

We now validate our analysis against Monte Carlo simulations. We consider a realistic LTE network scenario with the following settings. All nodes transmit at $f_c = 2.6\,$GHz and the signal bandwidth is $W = 20\,$MHz. The noise power spectral density is set to $N_0 = -174\,$dBm/Hz and the path-loss exponent is assumed to be $a = 3.5$. We assume a practical scenario where $R = 60\,$m, $L = 300\,$m, unless otherwise stated. The source, the relay and the destination are equipped with $n_s = 2$, $n_r = 2$, and $n_d = 2$ antennas, respectively. The density of the users and relays is set to $\sigma_u = 2 \times 10^{-4}$ and $\sigma_r = 8 \times 10^{-3}$, and the density of the interferers of relay and users are set to 2×10^{-5}. The outage threshold is set to 5 dB.

The results in Figures 2 and 3, respectively, validate the closed-form expressions of the lower bound of the outage probability, and compare the normalized feedback load under the different scheduling schemes. With the increase of γ_{th}, the outage performance of SMUD degrades since more users are selected randomly, and the load due to the user feedback decreases substantially since fewer users are above threshold. It is interesting to note that for γ_{th} less than or equal to 6 dB, SMUD performs similarly to OS in terms of outage probability, while generating much lower control overhead. Figure 4 underlines the impact of the value of R on the system outage performance under the PFS scheme with fixed L. The optimum value of R that yields the lowest outage probability is marked in orange. Upon changing the value of the user density σ_u, the outage performance varies accordingly. For small values of R, the relay-user link becomes the bottleneck, hence the performance improves as the relay is located closer to the user. However, for R beyond a certain value, the opposite situation occurs: the source-relay link becomes the bottleneck and the outage probability gets worse as R increases. Given the radius of the sector, there is an optimal separation between relays and users for which the outage probability is minimum. The optimal R varies depending on the user density: as σ_u grows, the optimal value of R decreases. This is because the probability that at least one user experiences high SINR grows as σ_u increases, thus larger distances between relay and users can be tolerated. Thus, the source-relay link tends to become the bottleneck, and a relay closer to the source is preferable.

V. Conclusion

We studied multiuser MIMO relay networks where the source delivers data to a scheduled randomly placed user through a randomly picked relay node. The network may adopt different user scheduling schemes, namely, OS, PFS, and SMUD to select the served user. We derived the distribution of the SINR over each communication hop, as well as of the end-to-end SINR experienced by a user. We validated our analysis through Monte Carlo simulations. Furthermore, our results highlight the impact on the system performance of the placement of the relay nodes between source and users.

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