Selection of Quality-Inspection Procedures for Short-Run Productions

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Fiorenzo Franceschini, Maurizio Galetto, Gianfranco Genta and Domenico A. Maisano

Department of Management and Production Engineering, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy, fiorenzo.franceschini@polito.it

ABSTRACT

Designing quality-inspection procedures may be difficult for short-run manufacturing processes, due to poor effectiveness of the classical statistical-process-control (SPC) techniques for these processes. This paper proposes a practical methodology to guide quality designers in selecting the more effective and economically convenient inspection procedures. First, the process of interest is decomposed into a number of steps, in which specific defects can occur. Next, several parameters related to inspection effectiveness and cost are combined into a probabilistic model. The more effective and economically convenient inspection procedures can finally be determined using two specific synthetic indicators. A case study concerning a short-run production of hardness testing machines is presented and discussed.

Key Words: Defect; Inspection cost; Inspection effectiveness; Probabilistic model; Quality control; Short-run manufacturing process; Single-unit production.

1. INTRODUCTION

When manufacturing complex products, typical activities are acquisition of raw materials, processing, assembly, functional testing, quality inspection, etc. (Vandebroek et al. (2016)). Quality inspections are usually aimed at checking whether specification and functional requirements are satisfied, identifying possible defects and/or product anomalies. They may be governed by strict or non-strict rules (e.g., periodical control, fixed-percentage control, etc.), following statistical or heuristic procedures and can be carried out in (at least) four different ways: (i) *simple* inspection, i.e., inspecting single items once; (ii) *fractional* inspection, i.e., the two extreme cases are those in which the fraction of the production output that is inspected is zero (no inspection) or one (100% inspection); (iii) *repeated* inspection, i.e., inspecting the same item(s) more than once; and (iv) *dynamic* or *adaptive* inspection, i.e., sequential inspection of production batches, in which inspection parameters may depend on the results of the previous inspections (Mandroli et al. (2006); Montgomery (2013)).

Typical features to be considered when designing inspection procedures are: (i) collection of the available information on the process of interest; (ii) definition of appropriate tasks and parameters; (iii) definition of the activity and responsibility of operators/inspectors; (iv) identification of the inspection cost; and (v) identification of possible inspection errors (e.g., false positives or false negatives) and relevant consequences (Tang, K., and Tang, J. (1994)).

When dealing with complex products and therefore with relatively complex/articulated manufacturing processes, it is particularly important to identify the more critical and vulnerable process features and to develop appropriate inspection strategies accordingly, defining test procedures, cases and resources (Colledani et al. (2014)).

The effectiveness of possible inspection strategies can be tightly related to the production typology and volume. In the case of mass production, Statistical Process Control (SPC) techniques are straightforwardly applied (Montgomery (2013)); on the other hand, in the case of single-unit or small-sized-lot productions (i.e., the so-called *short-runs*), most of the SPC techniques are not appropriate (Del Castillo et al. (1996); Trovato et al. (2010); Marques et al. (2015)).

This paper analyses the quality-inspection procedures for short-run and/or single-unit manufacturing processes, extending the analysis carried out in a previous work (Franceschini et al. (2016)). These processes will be decomposed into individual process *steps*, i.e., specific and recognizable transformation/assembly activities, which contribute to the realization of the end product, providing an added value. In addition, the decision-making process of the quality-control staff (i.e., inspectors) is modelled by suitable discrete event models (De Ruyter et al. (2002)).

This paper provides some guidelines for supporting the design and assessment of suitable inspection procedures, through the definition of a probabilistic model for defect prediction, trying to answer the following research question: *considering a short-run or a single-unit manufacturing process with several alternative inspection procedures, how can the more effective and economically convenient ones be selected?*

Two types of errors are associated with an inspection: (i) the error of misclassifying a good part as a defective one, which is known as type-I error (α); and (ii) the error of misclassifying a defective part as a good one, which is known as type-II error (β). Recent advancements in the automation of manufacturing systems allow reducing the inspection errors, which however cannot be completely eliminated. Furthermore, since it is not possible to automate any manufacturing system owing to budget constraints, inspector skill results to be crucial (Kang et al. (2018)). It is also worth remarking that in many production environments, quality costs are significantly affected by inspection errors, e.g., in the presence of relatively low inspection costs, high repair cost, high penalty costs or high defect probability (Ballou and Pazer (1982)). Unfortunately, these errors are often overlooked (Veatch (2000); Kakade et al. (2004)).

The construction of the probabilistic model is based on the following two phases:

I. estimating the probability of (not) detecting the defects, in each manufacturing step;

II. combining the above probabilities into a model depicting the overall effectiveness and cost of the inspection procedure.

The proposed model is supposed to have both an analytical and predictive connotation, as it allows to compare alternative inspection procedures from the perspective of effectiveness and cost, and to select the more suitable ones. For instance, it may be adopted to statistical samplings, 100% inspection, skip-lot inspection or combinations of them. Due to this flexibility, it can be particularly appropriate for short-run productions, which are generally characterized by a high level of complexity and customization. Similar approaches are adopted in the software engineering field (Rawat and Dubey (2012)); for example, probabilistic models based on Bayesian networks can be implemented for software defect prediction (Fenton et al. (2008)).

The remainder of the paper is organized into four sections. Section 2 illustrates the probabilistic model and the relevant characteristic parameters. Section 3 illustrates two practical indicators, which depict the overall effectiveness and estimated cost of an inspection procedure; the description is supported by pedagogical examples. Section 4 presents a structured case study concerning the practical application of the proposed model in the short-run production of hardness testing machines. Section 5 summarizes the original contributions of this research, focussing on its implications, limitations and possible future developments.

2. MODEL DESCRIPTION

2.1. Assumptions

Let us consider a manufacturing process in optimal setting conditions and decompose it into a number (m) of *process steps* or just *steps*, i.e., specific operations providing an added value to the end product. Each step is supposed to be arranged in the best possible way. The proposed model is based on the following simplifying assumptions:

1. For each step, there can be one-and-only-one specific defect.
2. Defects originated in different steps are uncorrelated with each other.
3. Defects and inspection errors are uncorrelated.

For realistic application contexts, the first hypothesis is not so stringent, as the totality of the possible defects within a certain step can be interpreted as a unique "macro-defect". On the other hand, the latter two hypotheses are certainly stronger. However, these are helpful for building a preliminary model. In fact, possible correlations between defects originated in different steps do not allow to decouple the corresponding steps. In case of absence of correlations, the model involves only simple probabilities, while, in presence of correlations, conditional probabilities need to be considered. Future research will aim at refining the model by relaxing uncorrelation assumption.

To clarify the last concept, a short example is proposed. Let us consider a simple process consisting of three steps: (i) thread milling, (ii) tightening of a screw in the thread and (iii) anti-rust painting. The first two steps are inherently correlated, in fact threading defects may cause tightening problems with consequent defects. Instead, defects in painting are not inherently correlated with defects in the previous two steps. Therefore, only the hypothesis of absence of correlation between the first two steps may be considered not very realistic in this case.

In each i-th process step, different kinds of quality control activities may be performed, according to the specific type of defect. For each of these activities, there is a risk of detecting a defect when it is not present (type-I error), and a risk of not detecting it when it is actually present (type-II error). Although these risks can be minimized by using sophisticated quality monitoring techniques (manual and/or automatic), they can never be eliminated.

2.2. Parameter definition

Each i-th step of the production process is modelled by a Bernoulli distribution (Montgomery (2013)), hence it can be described through three parameters:

- p_i: probability of occurrence of the defect in the i-th step (i.e., the parameter of the Bernoulli distribution);
- α_i: probability of (erroneously) detecting the defect when it is not present in the i-th step (i.e., type-I inspection error or *false positive*);
- β_i: probability of not detecting the defect when it is present in the i-th step (i.e., type-II inspection error or *false negative*).

The index i is obviously included between 1 and m, i.e. the total number of steps.

The first parameter (p_i) concerns the defectiveness or, reversing the perspective, the quality of the i-th step, while the other two parameters (α_i and β_i) concern the quality of the corresponding inspection(s). These three parameters may sometimes be difficult to estimate. Since p_i is related to the characteristics of the process and its propensity to generate defects, it can be *a priori* estimated using adequate defect-generation models (Genta et al. (2018)); alternative approaches may be based on empirical methods (e.g., use of prior experience) and/or simulations (De Ruyter et al. (2002); Sarkar and Saren (2016)). On the other hand, the estimation of α_i and β_i depends on the characteristics of the inspection procedure and the technical skills and/or experience of the inspector (Tang and Schneider (1987); Duffuaa and Khan (2005)).

2.3. Conceptual representation of the process

The graph in Figure 1 represents a generic manufacturing process with *m* steps in series. The graph in Figure 2 represents another process, consisting of two steps in parallel, followed by a third one (in series).

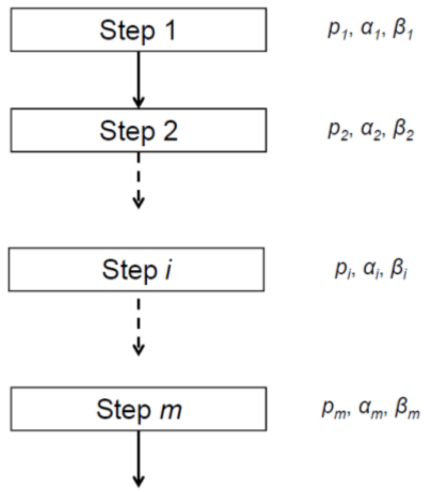


FIGURE 1. Representation of a production process with *m* steps in series.

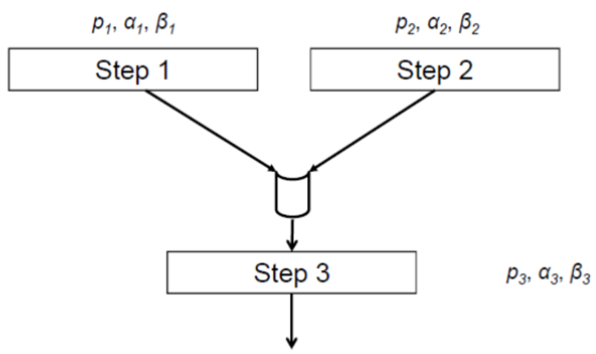


FIGURE 2. Representation of a production process with two steps in parallel, followed by a third one (in series).

More complex processes can be represented using graphs with more articulated mixed structures (in series and in parallel). Consistently with the description of Sections 2.1 and 2.2, each (i-th) step can be associated with three parameters (p_i, α_i, β_i).

2.4. Model presentation

The following probabilities can be calculated for each generic i-th step:

$$P(\text{detecting the defect in the step } i) = p_i \cdot (1-\beta_i) + (1-p_i) \cdot \alpha_i \quad (1)$$

and

$$P(\text{not detecting the defect in the step } i) = p_i \cdot \beta_i + (1-p_i) \cdot (1-\alpha_i) \quad (2)$$

where i is included between 1 and m, i.e. the total number of steps.

In the case the defect is detected, it will be *authentic*[1] with a probability $p_i \cdot (1-\beta_i)$ or *false* with a probability $(1-p_i) \cdot \alpha_i$ (see Equation (1)). On the other hand, in the case no defect is detected, there can be an inspection error (false negative), with a probability $p_i \cdot \beta_i$, or due to the real absence of any defect, with a probability $(1-p_i) \cdot (1-\alpha_i)$ (see Equation (2)). The above probabilities represent the basic elements for the construction of some indicators depicting the performance of the overall inspection procedures, which are presented in Section 3.

Considering a generic process with m steps, the above probabilities can be combined together:

$$P(\text{detecting the defects in all the } m \text{ steps}) = \prod_{i=1}^{m} \left[p_i \cdot (1-\beta_i) + (1-p_i) \cdot \alpha_i \right] \quad (3)$$

and

$$P(\text{not detecting any defect in all the } m \text{ steps}) = \prod_{i=1}^{m} \left[p_i \cdot \beta_i + (1-p_i) \cdot (1-\alpha_i) \right] \quad (4)$$

[1] i.e., a defect, which is actually present.

It is also possible to calculate the probability of detecting the defects into a specific subset of all the steps and not detecting them in the remaining steps, i.e.:

$$P(\text{detecting the defects in the steps of the set } K) = \prod_{i \in K} \left[p_i \cdot (1-\beta_i) + (1-p_i) \cdot \alpha_i \right] \cdot \prod_{i \in (M-K)} \left[p_i \cdot \beta_i + (1-p_i) \cdot (1-\alpha_i) \right] \quad (5)$$

where M is the set of all the numbers of the steps (from 1 to m) and K is its subset including the numbers of the steps for which a defect is detected. When K is empty, Equation (5) degenerates into Equation (4), while when K includes the totality of the steps, Equation (5) degenerates into Equation (3).

According to the assumptions introduced in Section 2.1 (i.e. absence of correlation between the parameters related to the different steps), the formulas in Equations (3), (4) and (5) hold for any configuration of the process (e.g., series, parallel or mixed). Furthermore, the probabilities in these equations are related to the complexity of the process, in terms of number of steps (m), quality of the process (p_i), and quality of the inspection (α_i and β_i) in each single step.

3. PROPOSED INDICATORS

Different combinations of inspection activities may be adopted for checking the conformity of the output of a specific step, e.g., visual check, dimensional verification, comparison with reference exemplars, etc. (See (2012); Bress (2017); Savio et al. (2016)). The indicators discussed in the following two subsections can be used for comparing alternative combinations of inspection activities according to their effectiveness and cost (Ng and Hui (1997); Wang et al. (2010)).

3.1. Inspection effectiveness

Let us consider m Bernoulli random variables (X_i), defined as follows:

- X_i = 0: when (i) an authentic defect is detected or (ii) no defect is present in the i-th inspection.
- X_i = 1: when an authentic defect is not detected in the i-th inspection.

According to the model proposed in Section 2.4, an authentic defect is detected with a probability $p_i \cdot (1-\beta_i)$ and not detected with a probability $p_i \cdot \beta_i$. Instead, when no defect is actually present, a defect may be detected with a probability $(1-p_i) \cdot \alpha_i$ and not detected with a probability $(1-p_i) \cdot (1-\alpha_i)$. Of course, the sum of the latter two probabilities is the probability that no defect is present, i.e. $(1-p)$. The following relationships hold:

$$
\begin{aligned}
&P(X_i = 0) = p_i \cdot (1-\beta_i) + (1-p_i) = 1 - p_i \cdot \beta_i \\
&P(X_i = 1) = p_i \cdot \beta_i
\end{aligned}
\tag{6}
$$

where i is included between 1 and m. Therefore, the mean number of authentic defects undetected in the i-th inspection is:

$$D_i = E(X_i) = p_i \cdot \beta_i \tag{7}$$

which is obviously a quantity included between 0 and 1.

Let us now consider a further random variable, which counts the total number of authentic defects that are not detected in the overall inspection procedure:

$$Y = \sum_{i=1}^{m} X_i \tag{8}$$

The expected value of the total number of authentic defects that are not detected is:

$$D = E(Y) = E\left(\sum_{i=1}^{m} X_i\right) = \sum_{i=1}^{m} E(X_i) = \sum_{i=1}^{m} D_i = \sum_{i=1}^{m} p_i \cdot \beta_i \tag{9}$$

The variable D provides an indication of the overall effectiveness of the inspections.

3.2. Inspection cost

Regarding the *i*-th step, the total inspection cost may be expressed, as a first approximation, as follows:

$$C_{tot,i} = c_i + NRC_i \cdot p_i \cdot (1-\beta_i) + URC_i \cdot (1-p_i) \cdot \alpha_i + NDC_i \cdot p_i \cdot \beta_i \qquad (10)$$

where:

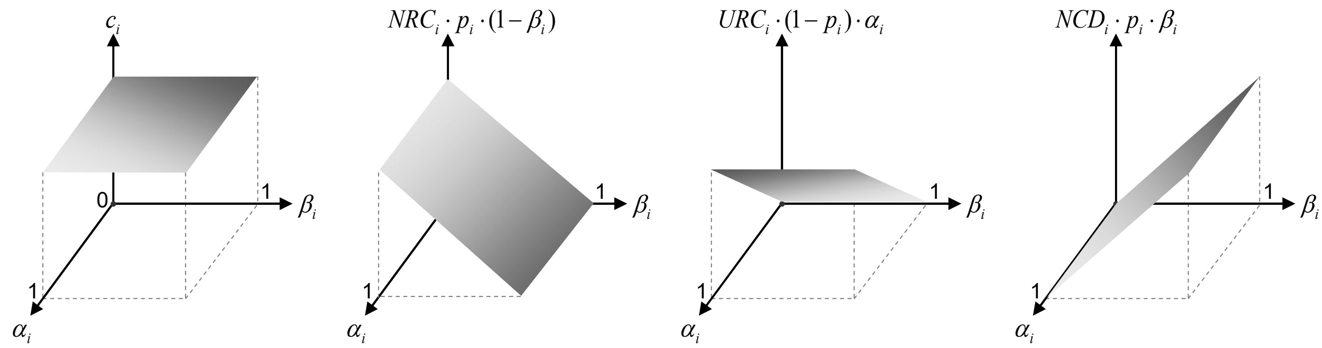
- c_i is the cost of the specific inspection activity (e.g., manual or automatic inspection activities);
- NRC_i is the necessary-repair cost, i.e., the cost for removing the defect when it is present;
- URC_i is the unnecessary-repair cost, i.e., the cost incurred when identifying false defects; e.g., although there is no repair cost, the overall process can be slowed down or interrupted, with a consequent extra cost.
- NDC_i is the cost of undetected defect, i.e., the cost related to the missing detection of defects.

Apart from the estimate of the probabilities p_i, α_i and β_i, the calculation of the total cost requires the estimate of additional cost parameters. In general, c_i and NRC_i are known costs, URC_i is likely to be relatively easy to estimate, while NDC_i is difficult to estimate since it may depend on difficult-to-quantify factors, such as image loss, after-sales repair cost, etc. It can be seen that, among the parameters in Equation (10), only c_i, α_i and β_i are related to the inspection procedure. In fact, the parameters NRC_i, URC_i and NDC_i depend on the cost concerning (in)appropriate defect repair or missing defect detection, while p_i is associated to the process propensity to generate defects. The total cost related to the manufacturing process of interest, i.e., joining the individual step-by-step costs, can be expressed as:

$$C_{tot} = \sum_{i=1}^{m} C_{tot,i} = \sum_{i=1}^{m} \left[c_i + NRC_i \cdot p_i \cdot (1-\beta_i) + URC_i \cdot (1-p_i) \cdot \alpha_i + NDC_i \cdot p_i \cdot \beta_i \right] \qquad (11)$$

The indicator C_{tot} gives a trade-off among different cost components. For each i-th step, the first cost component c_i is always present, in the case an inspection is performed, while the second component $NRC_i \cdot p_i \cdot (1-\beta_i)$ generally has an opposite behaviour with respect to the latter two components $URC_i \cdot (1-p_i) \cdot \alpha_i$ and $NDC_i \cdot p_i \cdot \beta_i$. In fact, when the defect is detected and repair is performed correctly, we certainly do not incur in the third and fourth cost components.

Let us consider the i-th step and suppose that the parameters p_i, c_i, NRC_i, URC_i, and NDC_i are known and fixed. The first cost component c_i is independent from α_i and β_i, the second and fourth component are functions of β_i, and the third component is a function of α_i, as shown in Figure 3.



1st cost component 2nd cost component 3rd cost component 4th cost component

FIGURE 3. Cost components as functions of the probabilities α_i and β_i for a generic (i-th) step of the inspection procedure.

It is worth remarking that, as the quality of the inspections is improved (i.e. α_i and β_i are likely to decrease, while c_i is likely to increase, due to the improved testing activities), then the contributions $URC_i \cdot (1-p_i) \cdot \alpha_i$ and $NDC_i \cdot p_i \cdot \beta_i$ will tend to decrease, while $NRC_i \cdot p_i \cdot (1-\beta_i)$ and c_i will tend to increase. The indicator C_{tot} may be consequently affected by compensation effects.

A more specific indicator on economic convenience of the inspection procedure C^*_{tot} may be obtained by removing the contributions $NRC_i \cdot p_i \cdot (1-\beta_i)$ from C_{tot}, as suggested in Franceschini et al. (2016). However, the indicator C_{tot} may be preferable, since it has a wider use.

In conclusion, we believe that the combined use of C_{tot} (indicator of cost) and D (indicator of effectiveness, defined in Section 3.1) enables to support the selection of the better inspection procedure(s).

3.3. Pedagogical example

Let us now focus the attention on a pedagogical example concerning the inspection activities in a production process consisting of m=5 steps; three different procedures are proposed:

- Procedure U in which two steps only (i.e., step 1 and 5) are subject to inspection;
- Procedure V in which the totality of the steps are subject to inspection.
- Procedure W in which the totality of the steps are not subject to any inspection.

The effectiveness of the three alternative procedures can be evaluated using the indicator (D) defined in Equation (9). Precisely, the mean total number of (authentic) defects, which are not detected in the three procedures are respectively:

$$\begin{aligned} D^U &= p_1 \cdot \beta_1 + p_2 + p_3 + p_4 + p_5 \cdot \beta_5 \\ D^V &= p_1 \cdot \beta_1 + p_2 \cdot \beta_2 + p_3 \cdot \beta_3 + p_4 \cdot \beta_4 + p_5 \cdot \beta_5 \\ D^W &= p_1 + p_2 + p_3 + p_4 + p_5 \end{aligned} \tag{12}$$

in which, for a generic i-th step with no inspection, the corresponding β_i was replaced with 1. Assuming that the β_i related to a generic i-th step with inspection has the same value irrespective of the inspection procedure, it follows that:

$$D^W \geq D^U \geq D^V \tag{13}$$

Not surprisingly, the procedure W is the worst one in terms of effectiveness.

From the viewpoint of inspection costs, by applying Equation (11), it is obtained:

$$\begin{aligned} C_{tot}^{U} &= c_1 + NRC_1 \cdot p_1 \cdot (1-\beta_1) + URC_1 \cdot (1-p_1) \cdot \alpha_1 + NDC_1 \cdot p_1 \cdot \beta_1 + NDC_2 \cdot p_2 + NDC_3 \cdot p_3 + \\ &+ NDC_4 \cdot p_4 + c_5 + NRC_5 \cdot p_5 \cdot (1-\beta_5) + URC_5 \cdot (1-p_5) \cdot \alpha_5 + NDC_5 \cdot p_5 \cdot \beta_5 \\ C_{tot}^{V} &= \sum_{i=1}^{5} \left[c_i + NRC_i \cdot p_i \cdot (1-\beta_i) + URC_i \cdot (1-p_i) \cdot \alpha_i + NDC_i \cdot p_i \cdot \beta_i \right] \\ C_{tot}^{W} &= \sum_{i=1}^{5} NDC_i \cdot p_i \end{aligned} \tag{14}$$

So, if a generic i-th step is not subject to inspection, then $c_i = 0$, $\alpha_i = 0$ and $\beta_i = 1$. Assuming that, for the step with inspection, the parameters (probabilities and costs) are known, the cost C_{tot} can be calculated and the alternatives inspection procedures compared with each other.

For example, in the case the c_i values tend to be higher than the NDC_i values and the p_i values are relatively lower, then the procedure W (with no inspection at all), will be likely to be more convenient than the others. Conversely, in the case the c_i values tend to be lower than the other cost components and the p_i values tend to be higher, then the procedure V (in which the totality of the steps are subject to inspection) will be likely to be more convenient than the others. Section 4 presents a case study which shows numerically these effects.

3.4. Estimation of the variability of D and C_{tot}

Equations (9) and (11) allow one to estimate the central tendency of D and C_{tot} respectively: namely their mean values can be calculated replacing the input parameters (p_i, α_i, β_i, c_i, NRC_i, URC_i and NDC_i) with the corresponding mean values. However, given that the input parameters are affected by variability, it would be appropriate to estimate the resulting variability of D and C_{tot}.

Precisely, supposing to know the variability of the input parameters in terms of variances, the variances of both D and C_{tot} may be obtained by applying the law of propagation of variances (Montgomery et al. (2010)). The variance of D may be expressed as:

$$VAR(D) = VAR\left(\sum_{i=1}^{m} D_i\right) = \sum_{i=1}^{m} VAR(D_i) = \sum_{i=1}^{m} VAR(p_i \cdot \beta_i) \tag{15}$$

in the hypothesis of absence of correlations (see Section 2.1). From Equation (15), we obtain:

$$VAR(D) = \sum_{i=1}^{m} \left[\left(\frac{\partial D_i}{\partial p_i}\right)^2 \cdot VAR(p_i) + \left(\frac{\partial D_i}{\partial \beta_i}\right)^2 \cdot VAR(\beta_i)\right] \tag{16}$$

where the partial derivatives are evaluated at the mean values of the input parameters. Therefore, it results:

$$VAR(D) = \sum_{i=1}^{m} \left[\beta_i^2 \cdot VAR(p_i) + p_i^2 \cdot VAR(\beta_i)\right] \tag{17}$$

According to Equation (17), the variance of D is the sum of the variances of the parameters p_i and β_i, weighted respectively by the squares of β_i and p_i. It can be noticed that the effect of relatively higher variances of p_i can be compensated by relatively lower β_i values, and vice versa.

Extending the reasoning to C_{tot} (see Equation (11)), the relevant variance may be expressed as:

$$VAR(C_{tot}) = VAR\left(\sum_{i=1}^{m} C_{tot,i}\right) = \sum_{i=1}^{m} VAR(C_{tot,i}) \tag{18}$$

again in the hypothesis of absence of correlations. From Equation (18) , we obtain:

$$\begin{aligned} VAR(C_{tot}) = \sum_{i=1}^{m} \Bigg[& \left(\frac{\partial C_{tot,i}}{\partial p_i}\right)^2 \cdot VAR(p_i) + \left(\frac{\partial C_{tot,i}}{\partial \alpha_i}\right)^2 \cdot VAR(\alpha_i) + \left(\frac{\partial C_{tot,i}}{\partial \beta_i}\right)^2 \cdot VAR(\beta_i) + \left(\frac{\partial C_{tot,i}}{\partial c_i}\right)^2 \cdot VAR(c_i) + \\ & + \left(\frac{\partial C_{tot,i}}{\partial NRC_i}\right)^2 \cdot VAR(NRC_i) + \left(\frac{\partial C_{tot,i}}{\partial URC_i}\right)^2 \cdot VAR(URC_i) + \left(\frac{\partial C_{tot,i}}{\partial NDC_i}\right)^2 \cdot VAR(NDC_i) \Bigg] \end{aligned} \tag{19}$$

where the derivatives are once more evaluated at the mean values of the parameters. Therefore, it results:

$$\begin{aligned} VAR(C_{tot}) = \sum_{i=1}^{m} \Big[& \left(NRC_i - NRC_i \cdot \beta_i - URC_i \cdot \alpha_i + NDC_i \cdot \beta_i\right)^2 \cdot VAR(p_i) + \\ & + \left(URC_i - URC_i \cdot p_i\right)^2 \cdot VAR(\alpha_i) + \left(-NRC_i \cdot p_i + NDC_i \cdot p_i\right)^2 \cdot VAR(\beta_i) + VAR(c_i) + \\ & + \left(p_i - p_i \cdot \beta_i\right)^2 \cdot VAR(NRC_i) + \left(\alpha_i - p_i \cdot \alpha_i\right)^2 \cdot VAR(URC_i) + \left(p_i \cdot \beta_i\right)^2 \cdot VAR(NDC_i) \Big] \end{aligned} \tag{20}$$

According to Equation (20), the variance of C_{tot} is a sum of the variances of the input parameters, weighted by polynomial combinations of p_i, α_i, β_i, NRC_i, URC_i and NDC_i. It can be noticed that the weights of the variances of the probability parameters (p_i, α_i and β_i) depend on both probability and cost parameters, while the weights of the variances of the cost parameters (c_i, NRC_i, URC_i and NDC_i) only depend on probability parameters.

Section 4 presents a case study with a numerical estimation of the variability of D and C_{tot}, according to the afore-presented models.

4. PRACTICAL CASE STUDY

4.1. Process description and modelling

Let us now consider a practical application of the proposed model and indicators in a case study of short-run production, i.e. the manufacturing process of hardness testing machines *AFFRI® LD 3000 AF* (Figure 4).

FIGURE 4. *AFFRI*® *LD 3000 AF* hardness testing machine.

The production of these machines can be considered as a short-run production. This process includes three types of activities: mechanical, electrical and software development; our attention will be focused on the first one. In particular, we will consider the production of two components of the hardness-tester head, i.e., the indenter holder and the reference support of the displacement transducer (see Figure 5 and Figure 6).

The manufacturing operations of the indenter holder involves a turning operation followed by a cylindrical grinding, while those of the reference support of the displacement transducer involves a milling operation followed by a tangential grinding. The two components are then assembled together. The whole production process of interest may be decomposed into six operations (i.e. six steps): turning, cylindrical grinding, milling, tangential grinding, mechanical assembly and sensors assembly.

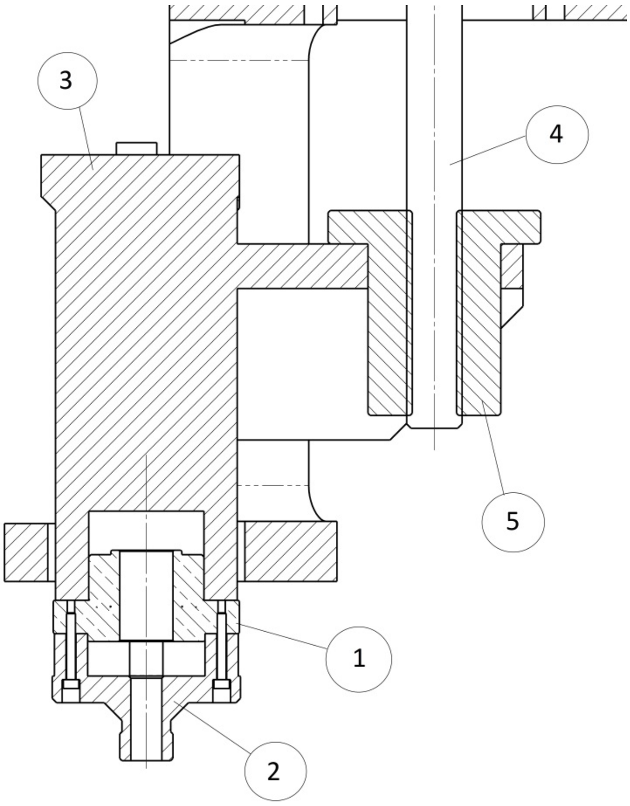
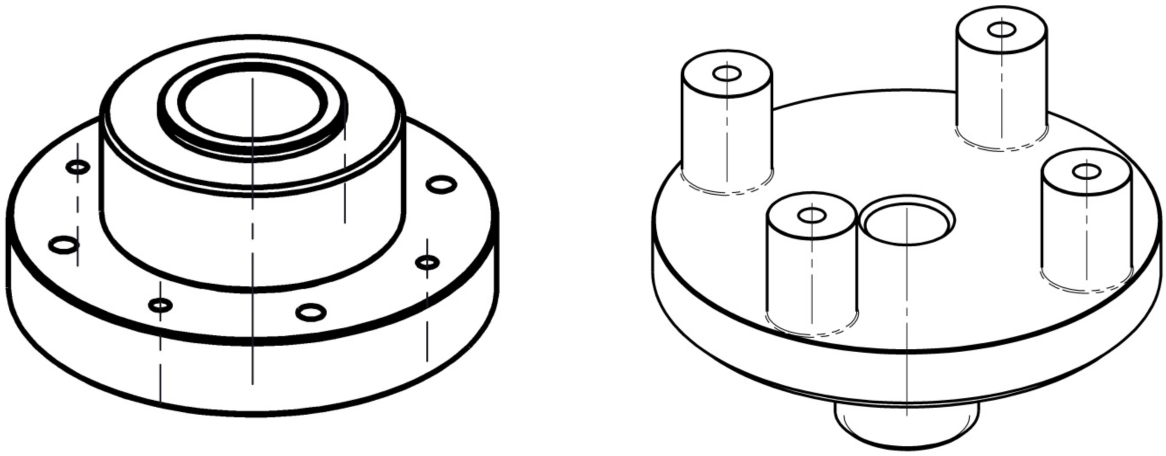


FIGURE 5. *AFFRI*® technical drawing of the head of the hardness tester, labelling the main components: (1) the indenter holder, (2) the reference support of the displacement transducer, (3) the mechanical unit of the head, (4) the threaded shaft, and (5) the nut screw.



(a) (b)

FIGURE 6. Detailed representation of (a) the indenter holder and (b) the reference support of the displacement transducer.

More precisely, there is a parallel combination of two pairs of operations – i.e., (i) turning and relevant cylindrical grinding, and (ii) milling and relevant tangential grinding – which is in turn in series with the two operations of mechanical assembly and sensors assembly. Figure 7 shows the flow chart of the manufacturing process.

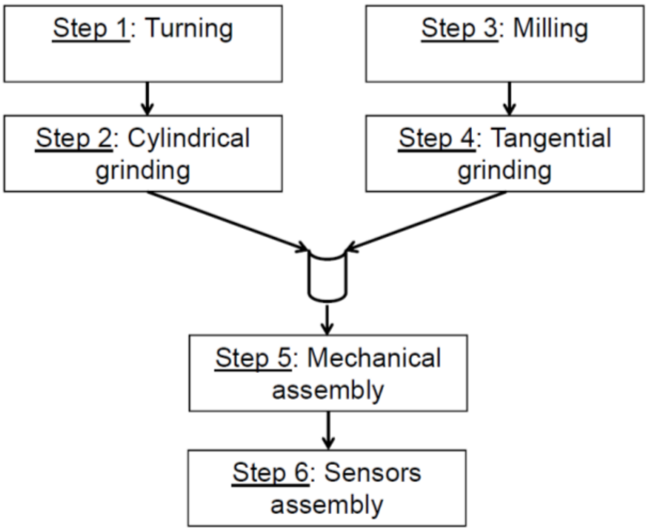


FIGURE 7. Flow chart representing the considered production process.

For the afore-illustrated process, two alternative inspection procedures, which may be adopted by the producer, are examined and compared. These procedures are denoted as IP1 (Inspection Procedure 1) and IP2 (Inspection Procedure 2).

Regarding IP1, an accurate self-inspection is performed after each of the first five steps, and a final inspection is performed by an appointed staff after the sensors assembly (see Figure 8). Precisely, the accurate self-inspections after the first four steps, consist of manual measurements, aimed at the verification of dimensional and geometrical tolerances, in addition to a visual inspection. At the end of the fifth step, a final verification of the functionality and conformity of the mechanical assembly is performed by the operators, while at the end of the sixth step, a verification of the conformity and functionality of the sensors is performed by the appointed staff.

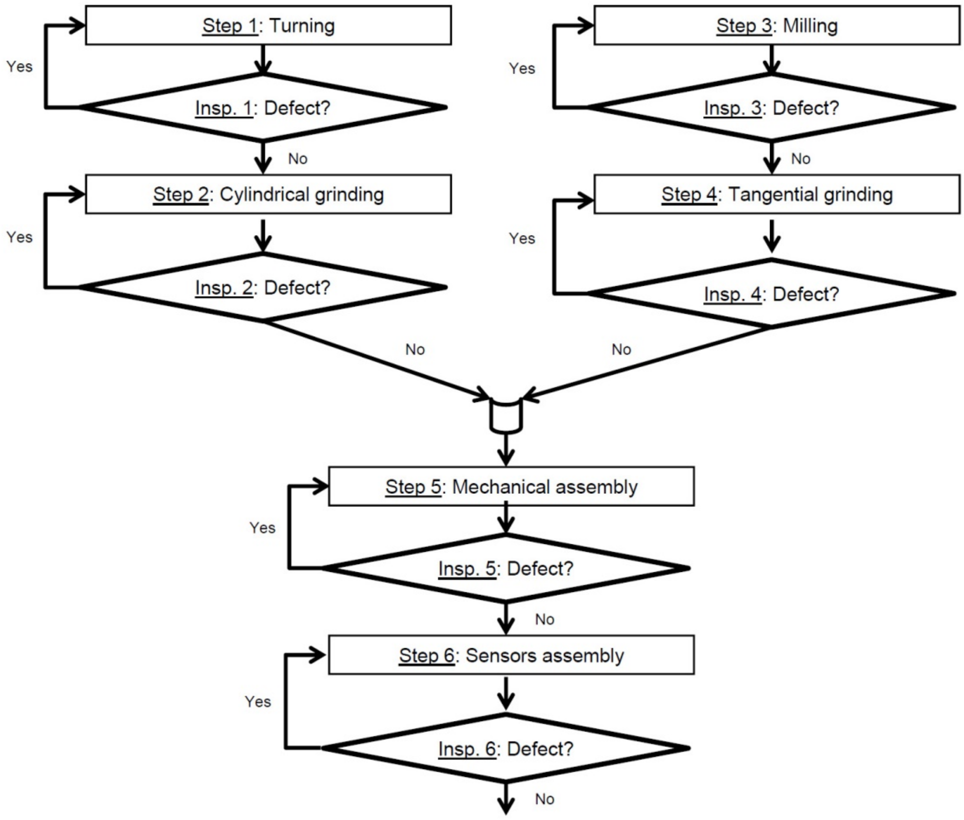


FIGURE 8. Flow chart representing the first inspection procedure (IP1) for the process of interest. Insp. 1 to 5 are accurate self-inspections, while Insp. 6 is an inspection executed by an appointed staff.

Regarding IP2, a rough self-inspection (i.e. visual inspections only) is performed after each of the first four steps, so as to detect the more critical non-conformities. Next an overall quality inspection, based on several measurements in a controlled metrological environment, is performed before assembling the components. For this reason, an additional step has been introduced at this stage (see step 4', denominated "Quality Inspection" in the process flow chart in Figure 9). Similarly to IP1, an accurate self-inspection is performed after the mechanical assembly (step 5) and then a verification of the conformity and functionality of the sensors is performed by the appointed staff (see Figure 9).

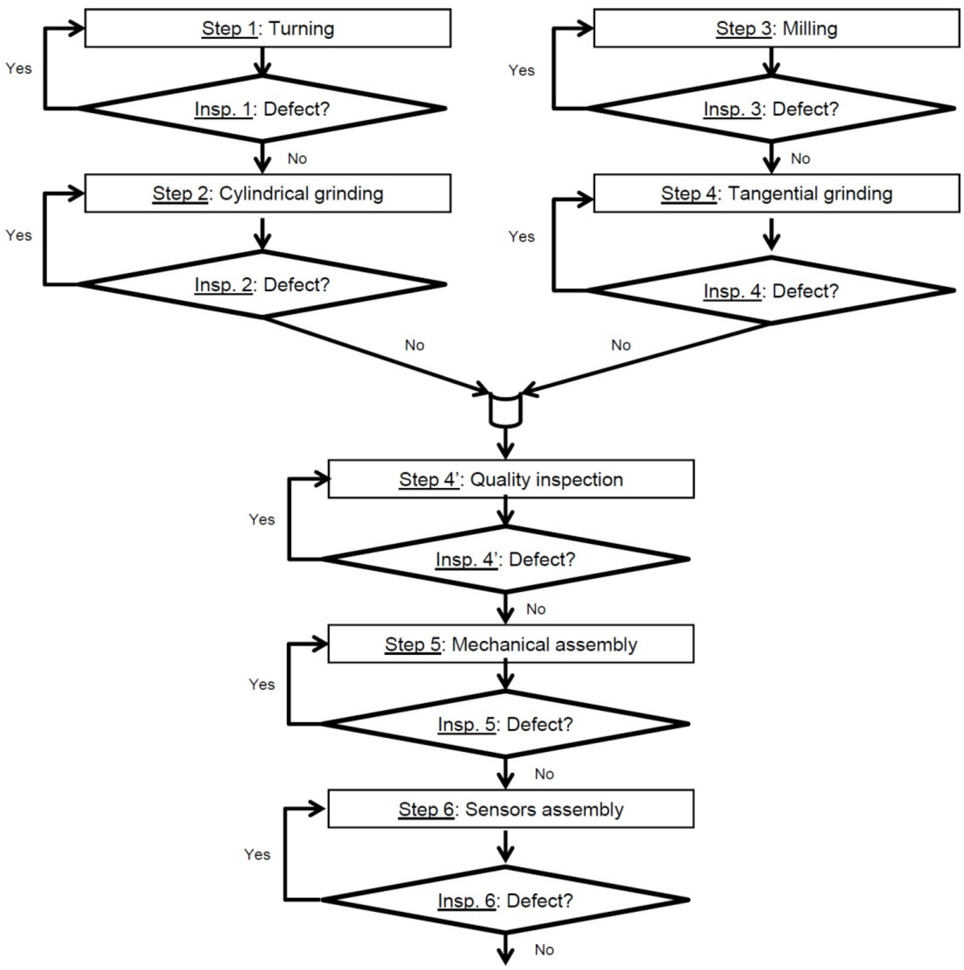


FIGURE 9. Flow chart representing the second inspection procedure (IP2) for the process of interest. Insp. 1 to 4 are rough self-inspections, Insp. 4' indicates the additional quality inspection, Insp. 5 is an accurate self-inspection, and Insp. 6 is an inspection executed by an appointed staff.

In practice, the inspection procedures may include some operations for repairing possible defects (e.g., assembly errors). For simplicity, these repair operations are omitted in the schemes in Figure 8 and 9.

4.2. Comparison of the two inspection procedures

The indicators described in Sections 3.1 and 3.2 can be applied to compare IP1 and IP2. Tables 1 and 2 report the step-by-step estimates of the probabilities (p_i, α_i and β_i) for IP1 and IP2 respectively.

TABLE 1. Estimates of probabilities p_i, α_i and β_i when implementing IP1.

Step no.	***Operation***	p_i [%]	α_i [%]	β_i [%]
1	Turning	5.0	1.0	5.0
2	Cylindrical grinding	2.0	0.5	2.5
3	Milling	5.0	1.0	5.0
4	Tangential grinding	0.2	0.5	2.5
5	Mechanical assembly	2.0	2.0	0.1
6	Sensors assembly	5.0	2.0	0.1

TABLE 2. Estimates of probabilities p_i, α_i and β_i when implementing IP2.

Step no.	***Operation***	p_i [%]	α_i [%]	β_i [%]
1	Turning	2.5	1.0	5.0
2	Cylindrical grinding	1.0	0.5	2.5
3	Milling	2.5	1.0	5.0
4	Tangential grinding	0.1	0.5	2.5
4’	Quality inspection	6.1	0.2	0.5
5	Mechanical assembly	2.0	2.0	0.1
6	Sensors assembly	5.0	2.0	0.1

For each i-th step, the p_i value and the corresponding α_i and β_i values were estimated by the operators/inspectors, based on their experience and technical knowledge of the process. For simplicity, the α_i and β_i values concerning the same steps (i.e., step 1, 2, 3, 4, 5 and 6) of the two different inspection procedures are considered identical. However, IP2 also includes step 4', with additional specific parameters. In the first four steps, the p_i values for IP2 are assumed to be one half of the homologous p_i values related to IP1, since in the rough self-inspections only a portion (assumed to be 50%) of the possible defects is inspected. The additional quality inspection at step 4' compensates for the remaining portions of p_i values of the first four steps: see Table 2, in which $p_1 + p_2 + p_3 + p_4 = p_{4'}$. The values of p_5 and p_6 are assumed to be identical for IP1 and IP2.

Tables 3 and 4 report the estimates of the cost parameters for each process step, concerning IP1 and IP2 respectively. These estimates were calculated taking into account the time required for identifying and repairing possible defects, and the labour cost of operators/inspectors. However, these values are just indicative because the real ones are confidential.

TABLE 3. Estimates of cost parameters related to IP1.

Step no.	***Operation***	***c_i* [€]**	***NRC_i* [€]**	***URC_i* [€]**	***NDC_i* [€]**
1	Turning	2.1	3.5	3.5	18.8
2	Cylindrical grinding	0.7	3.6	3.6	18.8
3	Milling	3.1	5.2	5.2	18.8
4	Tangential grinding	0.7	3.6	3.6	18.8
5	Mechanical assembly	7.0	3.5	3.5	31.4
6	Sensors assembly	6.3	2.1	2.1	25.1

TABLE 4. Estimates of cost parameters related to IP2.

Step no.	*Operation*	c_i [€]	NRC_i [€]	URC_i [€]	NDC_i [€]
1	Turning	0.2	3.5	3.5	18.8
2	Cylindrical grinding	0.2	3.6	3.6	18.8
3	Milling	0.3	5.2	5.2	18.8
4	Tangential grinding	0.2	3.6	3.6	18.8
4’	Quality inspection	7.4	22	22	132
5	Mechanical assembly	7.0	3.5	3.5	31.4
6	Sensors assembly	6.3	2.1	2.1	25.1

Table 5 reports the calculated values of D and C_{tot}, for both the inspection procedures, using respectively Equations (9) and (11), and the estimates of the probabilities and cost parameters in Tables 1 to 4.

TABLE 5. Indicators values calculated for IP1 and IP2.

Indicator	*IP1*	*IP2*
D [-]	0.00562	0.00315
C_{tot} [€]	20.94	23.64

Table 5 shows that the mean D value for IP2 is lower than that for IP1; on the contrary, the mean C_{tot} value of IP2 is higher than that of IP1. According to these results, the producer of hardness testing machines selects IP1, because it is willing to accept an increase of the number of undetected defects in order to have a reduction of the total inspection costs. Even if the mean total number of undetected defects becomes almost the double, it is still small as it refers to a short-run production. Therefore, according to a cost-benefit logic, the combined use of the two indicators enables to compare the two inspection procedures in order to select the more appropriate according to the actual requirements of the producer.

The information contained in Table 5 should be complemented with the estimated variabilities of D and C_{tot} (for both the inspection procedures). As a first approximation, the standard deviation of each (probability and cost) parameter was assumed to be 5% of the relevant value of the parameter itself. Then, the standard deviations related to D and C_{tot} were calculated by applying Equations (17) and (20) for both the inspection procedures (see Table 6).

TABLE 6. Standard deviations of the indicators calculated for the two inspection procedures.

Indicator	***IP1***	***IP2***
D [-]	0.00025	0.00013
C_{tot} [€]	0.51	0.61

Even considering the resulting variability of D and C_{tot}, IP2 remains significantly more effective, but also more expensive than IP1.

5. CONCLUSIONS

In manufacturing processes, inspection strategies are strictly related to the production typology and volume. SPC techniques are very diffused for mass productions, although difficult to manage for short-run and single-unit productions. This paper examined the latter ones, defining an overall probabilistic model for defect prediction. Furthermore, two indicators (i.e. D and C_{tot}) for estimating the expected inspection effectiveness and cost, and the relevant dispersions were defined. According to a cost-benefit logic, the combined use of the two indicators makes it possible to compare alternative inspection procedures, in order to select the more effective and economically convenient for a specific process of interest.

The proposed model and indicators may be exploited for a wide range of industrial processes. An application concerning the comparison of two alternative inspection procedures for a real-life short-run production of hardness testing machines was presented.

Some limitations of the proposed approach have to be discussed. First, some of the simplifying assumptions may be relatively stringent, such as (i) presence of a unique type of defect for each manufacturing step and (ii) absence of correlation between the parameters related to different steps. Future research will be aimed at refining/improving the proposed model by relaxing these simplifying assumptions. Furthermore, the proposed model and indicators require the estimation of various not-so-easily-quantifiable parameters (i.e. p_i, α_i, β_i, c_i, NRC_i, URC_i, NDC_i). A thorough understanding of the process of interest and the experience of operators and inspectors may contribute to overcome (at least partially) this obstacle. Also, suitable models for supporting the estimation of the probabilities p_i, α_i, β_i are under study.

6. ACKNOWLEDGMENT

The authors gratefully acknowledge *OMAG di AFFRI Davide S.r.l.* and Mr Giovanni Papaleo for the collaboration.

7. REFERENCES

Ballou, D. P., and Pazer, H. L. (1982). "The Impact of Inspector Fallibility on the Inspection Policy in Serial Production Systems". *Management Science*, 28, 4, pp. 387-399.

Bress, T. (2017). “Heuristics for Managing Trainable Binary Inspection Systems”. *Quality Engineering*, 29, 2, pp. 262-272.

Colledani, M., Tolio, T., Fischer, A., Iung, B., Lanza, G., Schmitt, R., and Váncza, J. (2014). “Design and Management of Manufacturing Systems for Production Quality”. *CIRP Annals - Manufacturing Technology*, 63, 2, pp. 773–796.

De Ruyter, A. S., Cardew-Hall, M. J., and Hodgson, P. D. (2002). “Estimating Quality Costs in an Automotive Stamping Plant through the Use of Simulation”. *International Journal of Production Research*, 40, 15, pp. 3835-3848.

Del Castillo, E., Grayson, J. M., Montgomery, D. C., and Runger, G. C. (1996). “A Review of Statistical Process Control Techniques for Short Run Manufacturing Systems”. *Communications in Statistics - Theory and Methods*, 25, 11, pp. 2723-2737.

Duffuaa, S. O, and Khan, M. (2005). “Impact of Inspection Errors on the Performance Measures of a General Repeat Inspection Plan”. *International Journal of Production Research*, 43, 23, pp. 4945-4967.

Fenton, N., Neil, M., and Marquez, D. (2008). “Using Bayesian Networks to Predict Software Defects and Reliability”. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, 222, 4, pp. 701-712.

Franceschini, F., Galetto, M., Genta, G., and Maisano, D. A. (2016). “Evaluating Quality-Inspection Effectiveness and Affordability in Short-Run Productions”. *Proceedings book of the 2nd International Conference on Quality Engineering and Management*, Guimarães (Portugal), pp. 420-432.

Genta, G., Galetto, M., Franceschini, F. (2018). “Product Complexity and Design of Inspection Strategies for Assembly Manufacturing Processes”. *International Journal of Production Research*, published online, DOI: 10.1080/00207543.2018.1430907.

Kakade, V., Valenzuela, J. F., and Smith, J. S. (2004). “An Optimization Model for Selective Inspection in Serial Manufacturing Systems”. *International Journal of Production Research*, 42, 18, pp. 3891–3909.

Kang, C. W., Ramzan, M. B., Sarkar, B., Imran, M. (2018). “Effect of Inspection Performance in Smart Manufacturing System Based on Human Quality Control System”. *The International Journal of Advanced Manufacturing Technology*, 94, 9–12, pp 4351–4364.

Mandroli, S. S., Shrivastava, A. K., and Ding, Y. (2006). "A Survey of Inspection Strategies and Sensor Distribution in Discrete-Part Manufacturing Processes". *IIE Transactions*, 38, 4, pp. 309-328.

Marques, P. A., Cardeira, C. B., Paranhos, P., Ribeiro, S., and Gouveia, H. (2015). “Selection of the Most Suitable Statistical Process Control Approach for Short Production Runs: A Decision-Model”. *International Journal of Information and Education Technology*, 5, 4, pp. 303-310.

Montgomery, D. C. (2013). *Statistical Quality Control: A Modern Introduction, 7th ed*. John Wiley & Sons, Hoboken, NJ.

Montgomery, D. C., Runger, G. C., Hubele, N. F. (2010). *Engineering Statistics, 5th ed.* John Wiley & Sons, Hoboken, NJ.

Ng, W. C., and Hui, Y. V. (1997). “Economic Design of a Complete Inspection Plan with Interactive Quality Improvement”. *European Journal of Operational Research*, 96, 1, pp. 122-129.

Rawat, M. S., and Dubey, S. K. (2012). "Software Defect Prediction Models for Quality Improvement: A Literature Study". *International Journal of Computer Science Issues*, 9, 5, pp. 288-296.

Sarkar, B., Saren, S. (2016). "Product Inspection Policy for an Imperfect Production System with Inspection Errors and Warranty Cost". *European Journal of Operational Research*, 248, 1, 2016, pp. 263-271.

Savio, E., De Chiffre, L., Carmignato, S., Meinertz, J. (2016). "Economic Benefits of Metrology in Manufacturing". *CIRP Annals - Manufacturing Technology*, 65, 1, pp. 495–498.

See, J. E. (2012). "Visual Inspection: A Review of the Literature". Sandia Report SAND2012-8590, Sandia National Laboratories, Albuquerque, NM.

Tang, K., and Schneider, H. (1987). "The Effects of Inspection Error on a Complete Inspection Plan". *IIE Transactions*, 19, 4, pp. 421-428.

Tang, K., and Tang, J. (1994). "Design of Screening Procedures: A Review". *Journal of Quality Technology*, 26, 3, pp. 209-226.

Trovato, E., Castagliola, P., Celano, G., and Fichera, S. (2010). "Economic Design of Inspection Strategies to Monitor Dispersion in Short Production Runs". *Computers & Industrial Engineering*, 59, 4, pp. 887–897.

Vandebroek, M., Lan, L., Knapen, K. (2016). "An Experimental Diagnostic Procedure to Identify the Source of Defects in Multi-Stage and Multi-Component Production Processes". *Journal of Quality Technology*, 48, 3, pp. 213-226.

Veatch, M. H. (2000). "Inspection Strategies for Multistage Production Systems with Time-Varying Quality". *International Journal of Production Research*, 38, 4, pp. 837-853.

Wang, C. H., Dohi, T., and Tsai, W. C (2010). "Coordinated Procurement/Inspection and Production Model under Inspection Errors". *Computers & Industrial Engineering*, 59, 3, pp. 473-478.