Entire-Domain Spectral Basis Functions for the Efficient Design of Metasurface Antennas of Circular Shape

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F. Verní and G. Vecchi
Dipartimento di Elettronica e Telecomunicazioni
Politecnico di Torino
Torino, Italy
francesco.verni@polito.it, giuseppe.vecchi@polito.it

M. Righero
Advanced Computing and Electromagnetics
Istituto Superiore Mario Boella
Torino, TO, Italy
righero@ismb.it

***Abstract*—This paper offers an efficient numerical approach to optimizing the design of large circular planar Metasurfaces (MTSs) antennas based on specially varying impedance profiles. Div-conforming spectral basis functions on the whole antenna are defined. The goal is to reduce the number of unknowns, to lead to a well-conditioned system, and to speed up the multiple solutions needed by an automatic optimization code. We consider the overall numerical complexity of the scheme and show its advantages in optimization applications.**

I. INTRODUCTION

Metasurfaces (MTSs) are electrically-thin metamaterial layers designed to exhibit unusual reflective/refractive properties or guided wave characteristics. Metasurface antennas are electrically large antennas composed of a dense periodic texture of small elements printed on a grounded slab. The typical design starts from an analytic determination of the surface impedance, e.g. in [1]. Hence, metasurface antennas can be initially modelled as a continuous slowly varying electric sheet impedance. It has been shown in [2] that numerical simulation of metasurfaces with transparent Impedance Boundary Condition (IBC) Electric Field Integral Equation (EFIE) yields a stable discretization and accurate results. The goal of this work is to develop a full-wave fast-solver (Method of Moments based on Fast Fourier Transform [3]) for the design of metasurface antennas, with a special emphasis on modulated metasurfaces, i.e. aperiodic cell arrangements. Optimization is necessary in the design phase, mandating that the field distribution is known for several impedance profiles for a specific geometry, e.g. Fig.1a. Such solver has to consider larger structures and span a wider space of patterns, due to the speed up of the overall performances. A spectral approach to canonic shape metasurfaces with IBC-EFIE can be adopted [6]. The electrical dimension of a metasurface antenna is often several wavelengths; therefore the number of unknowns with conventional RWG basis is very large. Moreover, the solution is a guided wave over a dielectric, which means the cell size in the mesh discretization is proportional to λ_r in place of λ_0, which results in more RWGs. The condition number of the system always increases with number of unknowns. In the following article, spectral basis functions are presented which are defined over the whole antenna, reduce the number of unknowns, regularize the system and speed up repeated solutions.

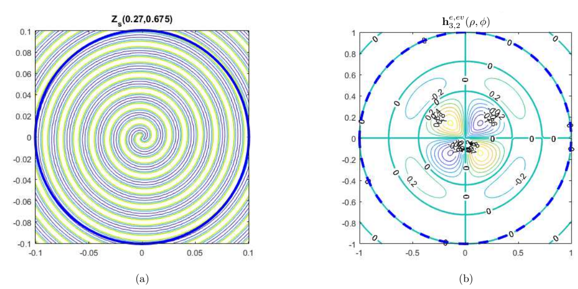

Fig. 1. (a) Contour plot of a spiral sheet impedance over a circular domain (blue line) modulated with $(X_s, M) = (0.27, 0.675)$. (b) Contour plot of e-mode with even angular dependence and $(m, n) = (2, 3)$.

II. ENTIRE DOMAIN FUNCTIONS: CWG MODES

From the cross section (Ω) of the h-field of a Circular Wave Guide (CWG) of radius R, orthogonal eigen-modes $\mathbf{h}^e_{mn}$ and $\mathbf{h}^h_{mn}$ over the entire-domain Ω are established. Applying the separation of variables along $\hat{\rho}$ and $\hat{\phi}$, we can write the vector-valued functions as:

$$\mathbf{h}^{e,i}_{mn}(\rho,\phi) = c^{e,i}_{mn}\left[h^{e\rho}_{mn}(\rho)h^{e\rho,i}_{m}(\phi)\hat{\rho} + h^{e\phi}_{mn}(\rho)h^{e\phi,i}_{m}(\phi)\hat{\phi}\right]$$
$$\mathbf{h}^{h,i}_{m,n}(\rho,\phi) = c^{h,i}_{mn}\left[h^{h\rho}_{mn}(\rho)h^{h\rho,i}_{m}(\phi)\hat{\rho} + h^{h\phi}_{mn}(\rho)h^{h\phi,i}_{m}(\phi)\hat{\phi}\right]$$

where the superscripts 'e' and 'h' stand for e-modes (TM) and h-modes (TE) respectively. The superscript 'i' accounts for the symmetry type of the angular (ϕ) dependence, which can be even or odd. Further details about the $\mathbf{h}^{e,i}_{mn}(\rho,\phi), \mathbf{h}^{h,i}_{m,n}(\rho,\phi)$ can be found in [4]. If correctly used, the CWG modes can be written as div-conforming basis (as imposed by the presence of the EFIO), which can ideally describe the entire solution space, e.g. in [5] with cavity-modes.

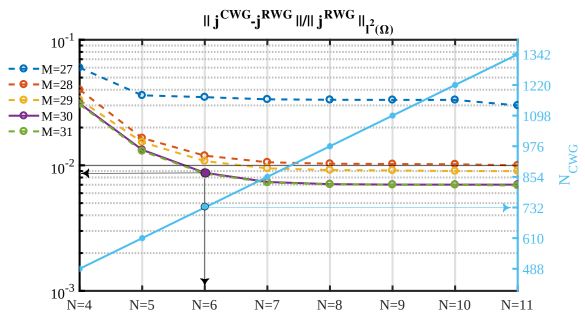

Fig. 2. The right-plot shows the $l_2(\Omega)$-error between CWGs and RWGs with respect to (m, n). In the left-plot (cyan) is shown the number of CWGs with respect to (m, n)

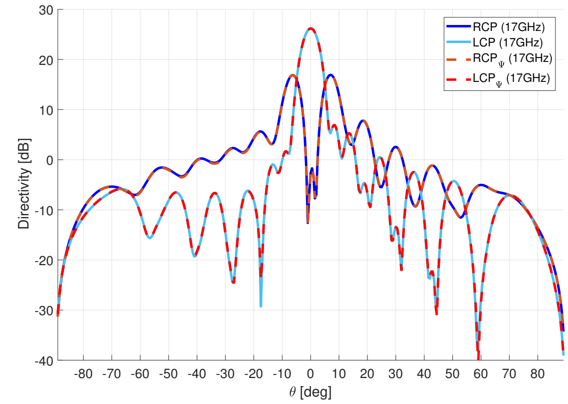

Fig. 3. Co-polar and cross-polar components of the field radiated by the RWGs (solid) and the CWGs (dash), when $(M, N) = (30, 6)$

1) Practical Use of Spectral Basis: We can express the Spectral Basis functions $\psi_k(\mathbf{r})$ both as linear combination of RWGs, $\mathbf{f}_l(\mathbf{r})$, and even/odd CWG modes, $\mathbf{h}_{mn}^{e/h}(\rho, \phi)$. This leads to a spectral representation of the solution as:

$$\mathbf{j}^{\mathrm{CWG}}(\mathbf{r}) = \sum_{l=1}^{L} I_l \mathbf{f}_l(\mathbf{r}) = \sum_{k=1}^{K} b_k \psi_k(\mathbf{r}) \tag{1}$$

where $(K \ll L)$ is equal to $(2M + 1) \times 2N$. Based on the spectral content of impedance profile, which drives also the spectrum of the solution space, a-priori spectral-truncation can be assumed. The regularizing effect of spectral basis aides in convergence, yielding a stable well-conditioned system. It has been observed also that the condition number does not depend anymore from the characteristics of the dielectric slab, as e.g. happens for RWGs. Hence, the spectral basis representation drastically compresses the system; if the reduction is sufficiently large the reduced matrix can be stored. It is worth noting that, with a compressed system, multiple solutions can be computed rapidly for various impedance profiles. On the other hand, the computational cost of the compression is $O(K^2L^2)$. Thus, we must determine whether or not such a method is appropriate based on the number of optimization steps. This cost can be reduced with fast solvers, e.g. [3], to $O(L^2)$. Typically after a few steps we observe a significant advantage. Numerical results are presented in the next sections.

III. Numerical Results: Holographic Antenna

A planar circularly polarized leaky-wave (LW) antenna, of radius 5.7λ, excited by a single-point feed at $f = 17$ GHz, is presented. The grounded dielectric slab has thickness $h_d = 1.524$ mm and permittivity $\epsilon_r = 3.66$. Fig.1a shows the spiral-shape modulated surface impedance profile:

$$X(\rho, \phi) = X_s \left[1 + M \sin(\beta_{sw}\rho - \phi)\right]. \tag{2}$$

A vertical probe excites a cylindrical surface wave (SW) on the impedance surface, and the latter converts it into a circularly polarized LW. The reader is referred to [1] for further details. The Fig.2 shows the l_2-error between the reference RWG solution $\mathbf{j}^{\mathrm{RWG}}$ and the CWG solution (1) for fixed typical values of X_s and M. The currents are calculated, respectively, with 195253 RWGs and 732 CWG basis functions. When considering radiated field, the error observed is strongly reduced by the far-field filtering effect. In Fig.3, we observe perfect agreement between the fields radiated by the reference current $\mathbf{j}^{\mathrm{RWG}}$ and the ones radiated by the approximate current $\mathbf{j}^{\mathrm{CWG}}$. An error of 10^{-7} in the far-field validate the use of circular waveguide modes as entire domain basis function to approximate the solution.

IV. Conclusions

Optimization is necessary in the design phase of MTS antennas. The cost function is related to the the field distribution, which therefore has to be known for several impedance profiles. Expressing the unknown surface currents using div-conforming entire domain basis functions, obtained from orthogonal eigen-modes, allows a large reduction in the number of unknowns while maintaining the accuracy of the solution. This reduction is mandatory to handle the large systems we have in practical applications. The method described can be generalized to geometries of arbitrary shape.

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