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On a generalized sum of the Mersenne Numbers

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Abstract Here we discuss the Mersenne numbers to give an example of a generalized sum. Using this sum, a recurrence relation is given.

Keywords Generalized sums, Mersenne numbers.

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Of generalized sums of numbers, we have given some examples in previous works [1-3]. Here we propose the study of the Mersenne Numbers, using the same approach. About these numbers, a large literature exists (see for instance that given in [4]). The form of the numbers is that of a power of two minus 1. Among them we find the Mersenne primes. The numbers are named after Marin Mersenne (1588 – 1648), a French Minim friar, who studied them in the early 17th century.
Mersenne numbers are:

$$M_n=2^n-1$$

Let us consider them to give an example of generalized sum. We can start from the following calculus:

$$. \quad M_{m+n}=2^{m+n}-1$$

$$M_{m+n}=2^{m+n}-1=2^m 2^n-1-2^m+2^m-2^n+2^n-1+1=2^m(2^n-1)-1+2^m-2^n+1+2^n-1$$

$$M_{m+n}=(2^m-1)(2^n-1)+2^m-1+2^n-1$$

Therefore, we can write the following generalized sum:

$$M_{m+n}=M_m\oplus M_n=(2^m-1)(2^n-1)+(2^m-1)+(2^n-1)$$

or:

$$(1) \qquad M_{m+n}=M_m\oplus M_n=M_m+M_n+M_m M_n$$

This is a generalized sum that we find in the case of the multiplicative groups (for the use of multiplicative groups in statistics and statistical mechanics see [5,6]).
Using (1), for the Mersenne numbers we can imagine the following recursive relation:

$$M_{n+1}=M_n \oplus M_1 = M_n + M_1 + M_n M_1$$

That is:

$$2^{n+1}-1=(2^n-1)+(2^1-1)+(2^n-1)(2^1-1)=2^n+2^{n+1}-2^n-2+1=2^{n+1}-1$$

The sum (1) is associative, so that:

$$M_m \oplus M_n \oplus M_l = M_m + M_n + M_l + M_m M_n + M_n M_l + M_m M_l + M_m M_n M_l$$

We cannot have a group of the Mersenne numbers, without considering also the opposites of them, so that:

$$0=M_n \oplus Opposite(M_n)$$

Therefore:

$$Opposite(M_n)=-\frac{M_n}{M_n+1}=M_{-n}$$

Explicitly:

$$Opposite(2^n-1)=-\frac{(2^n-1)}{(2^n-1)+1}=\frac{(-2^n+1)}{2^n}=2^{-n}-1$$

These numbers are the Mersenne numbers with a negative exponent. So we have:

$$M_{nnn}=M_n \oplus M_{-n}=M_n+M_{-n}+M_n M_{-n}$$

$$0=2^0-1=(2^n-1)+(2^{-n}-1)+(2^n-1)(2^{-n}-1)=2^n+2^{-n}-2+2^n 2^{-n}-2^{-n}-2^n+1=0$$

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