Isogeometric contact and debonding analyses using T-splines

R. Dimitri[1], L. De Lorenzis[2], P. Wriggers[3], G. Zavarise[1]

[1] *Dipartimento di Ingegneria dell'Innovazione, Università del Salento, Italy,* *rossana.dimitri@unisalento.it , giorgio.zavarise@unisalento.it*
[2] *Institut für Angewandte Mechanik, Technische Universität Braunschweig, Germany, l.delorenzis@tu-braunschweig.de*
[3] *Institut für Kontinuumsmechanik, Leibniz Universität Hannover, Germany, wriggers@ikm.uni-hannover.de*

In this work T-spline-based isogeometric analysis (IGA) is applied to frictionless contact and debonding problems between deformable bodies, in the context of large deformations. The key feature of IGA is the exact description of the geometry (i.e. exactly equal to the geometry generated by CAD) with a tailorable degree of continuity at the element boundaries. Moreover, this takes place in addition to the advantageous features of variation diminishing, convex hull properties, and non-negativeness of the basis functions [1]. As shown by the first results in the literature [2-5], contact formulations based on non-uniform rational B-splines (NURBS) provide a robust description of large deformation contact between deformable bodies, which is both effective and accurate for different interpolation orders. A multivariate NURBS discretization, however, does not provide a natural possibility for local mesh refinement due to its rigid tensor product structure. NURBS-based design deficiencies can be overcome by using T-splines, which allow for local refinement through the introduction of T-junctions and extraordinary points [6].

NURBS and T-Spline discretizations are here incorporated into an existing finite element framework by using Bézier extraction, i.e. a linear operator which maps the Bernstein polynomial basis on Bézier elements to the global NURBS or T-spline basis. A recently released commercial T-spline plugin for Rhino3d is used to build the analysis models adopted in this study.

In such context the continuum is discretized with cubic T-splines and NURBS, and a Gauss-point-to-surface (GPTS) formulation is combined with the penalty method to treat the contact constraints in the discretized setting [7]. Some numerical examples demonstrate the potential of T-spline IGA to solve challenging contact problems in 2D and 3D. More specifically, the Hertz problem is used as benchmark to compare the performance of cubic T-spline discretizations with NURBS of equal order from the standpoint of spatial convergence, characterized by uniform (N_u) and non-uniform (N_{nu}) patterns. The convergence study shows a very similar order of convergence, due to the equal polynomial degree and contact formulation, and to the absence of error estimation criteria in performing the local T-spline refinement. However, the T-spline error curve is shown to lie below all the NURBS curves, thus demonstrating the superior accuracy of T-splines for a given number of degrees of freedom (DOFs) (Figure 1).

The purely geometric enforcement of the non-penetration condition in compression is then generalized to encompass both contact and mode-I debonding of interfaces which is here approached through cohesive zone (CZ) modelling [8]. Based on CZ models, non-linear relationships between tractions and relative displacements are assumed. These relationships dictate both the work of separation per unit fracture surface and the peak stress that has to be reached for the crack formation. Depending on the contact status, an automatic switching procedure is used to choose between cohesive and contact models.

Results for the double cantilever beam (DCB) test and for the bi-material peel test with varying resolutions of the process zone, and number of Gauss points used for the enforcement of the contact constraints, are presented and compared. The superior accuracy of T-splines interpolations with respect to the NURBS and Lagrange ones for a given number of DOFs is verified. Figure 2 shows the main results obtained for a DCB problem with DOFs=1698, and a CZ law with cohesive strength p_{Nmax}=6 MPa, fracture energy G_{IC}=0.1 N/mm, and ratio between the ultimate and maximum opening displacements g_{Nu}/g_{Nmax}=12.5.

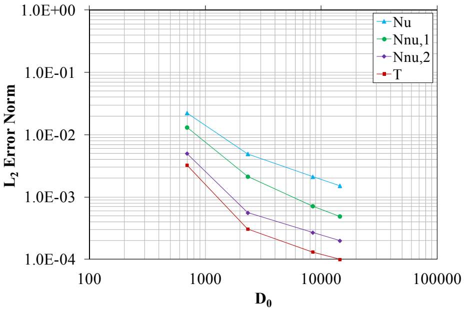

Figure 1: L_2 error norm of the contact pressure. $\varepsilon_N=10^3$.

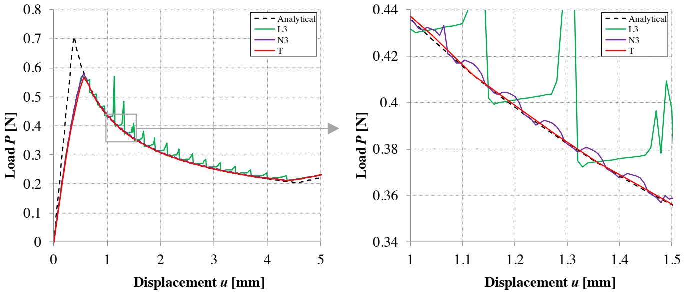

Figure 2: Load-displacement response for a DCB problem.

Acknowledgements

The authors at the Università del Salento and at the Technische Universität Braunschweig have received funding for this research from the European Research Council under the European Union's Seventh Framework Programme (FP7/2007-2013), ERC Starting Researcher Grant "INTERFACES", Grant agreement n° 279439.

References

[1] T.J.R. Hughes, J.A. Cottrell, and Y. Bazilevs. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. *Computer Methods in Applied Mechanics and Engineering*, 194: 4135–4195, 2005.

[2] L. De Lorenzis, I. Temizer, P. Wriggers and G. Zavarise. A large deformation frictional contact formulation using NURBS-based isogeometric analysis. *International Journal for Numerical Methods in Engineering*, 87(13): 1278−1300, 2011.

[3] L. De Lorenzis, P. Wriggers, and G. Zavarise. A mortar formulation for 3D large deformation contact using NURBS-based isogeometric analysis and the augmented Lagrangian method. *Computational Mechanics*, 49(1): 1-20, 2012.

[4] I. Temizer, P. Wriggers, and T.J.R. Hughes. Contact treatment in isogeometric analysis with NURBS. *Computer Methods in Applied Mechanics and Engineering*, 200: 1100-1112, 2011.

[5] I. Temizer, P. Wriggers, and T.J.R. Hughes. Three-dimensional mortar-based frictional contact treatment in isogeometric analysis with NURBS. *Computer Methods in Applied Mechanics and Engineering*, 209: 115-128, 2012.

[6] M.A. Scott, X. Li, T.W. Sederberg, and T.J.R. Hughes. Local refinement of analysis-suitable T-splines. *Computer Methods in Applied Mechanics and Engineering*, 213–216: 206–222, 2012.

[7] R. Dimitri, L. De Lorenzis, M. Scott, P.Wriggers, R.L. Taylor, and G. Zavarise. Isogeometric large deformation frictionless contact using T-splines. *Computer Methods in Applied Mechanics and Engineering*, 269: 394-414, 2014.

[8] R. Dimitri, L. De Lorenzis, P. Wriggers, and G. Zavarise. NURBS- and T-spline-based isogeometric cohesive zone modeling of interface debonding, submitted.