

# A consistency study of cohesive zone models for mixed-mode debonding problems

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Cohesive zone models (CZMs) are commonly used to describe mixed-mode failure and debonding processes at material interfaces or within quasi-brittle materials. These models consist in non-linear relationships between tractions and relative displacements across the crack. Although a large number of CZMs have been proposed, and despite the extensive related literature, little attention has been devoted to ensuring the consistency of these models for mixed-mode conditions, primarily in a thermodynamical sense. For many of these models, traction-separation laws have been directly postulated in an *ad hoc* manner, which may lead to unphysical dissipation behavior.

A consistency check was performed by van den Bosch et al. [1] for the exponential model by Xu and Needleman [2]. Thereby, an adjusted non-potential-based exponential model was also proposed to correct the unphysical behavioral features of the original model in the description of mixed-mode decohesion. Although non-potential-based models have been adopted for many practical applications, this class of models is not guaranteed to be thermodynamically consistent [3]. Beside thermodynamical consistency, an important requirement of CZMs is to allow for different values of the fracture energy in the normal and tangential directions, as measured experimentally.

In the first part of this contribution, two widely used mixed-mode CZMs [4,5] are examined. The consistency of their predictions in both stress and energy terms is checked. A parametric analysis on the effect of the coupling parameters on stress distributions and energy dissipation is performed in order to evaluate physical inconsistencies such as local abnormalities in the coupled elastic or softening mechanical response of the interface and incomplete dissipation of the fracture energy during decohesion. The path-dependence of the mixed-mode debonding work of separation ( $W$ ) and of the failure domains are additionally evaluated.  $W$  is given by

$$W = \int_{\Gamma} p_N(g_N, g_T) dg_N + \int_{\Gamma} p_T(g_N, g_T) dg_T \quad (1)$$

where  $\Gamma$  is the separation path,  $p_N$  and  $p_T$  are the normal and tangential tractions,  $g_N$  and  $g_T$  are the normal and tangential relative displacements across the crack. The first term in Eq. 1 is the work done by the normal tractions ( $W_N$ ), while the second term is the work done by the tangential tractions ( $W_T$ ).

Analytical predictions are also compared with results from numerical finite element models, where the interface is described with zero-thickness contact elements. A node-to-segment strategy as employed in [6] is here adopted, which incorporates decohesion and contact within a unified framework. Three case studies are analyzed for the numerical prediction of mixed-mode interface debonding: a simple patch test, a bimaterial peel test under mixed-mode loading conditions, and the standard mixed-mode bending test (MMB). Figure 1 illustrates sample analytical and numerical results in terms of  $W$ ,  $W_T$  and  $W_N$ , obtained for a patch test with the models in [4,5]. It is evident that the total work of separation does not vary monotonically, which reveals an energetic inconsistency.

In the second part of the paper, a new thermodynamically consistent mixed-mode CZM is proposed based on a modification of the model in [1]. Based on a predefined Helmholtz energy, the interface model is derived by applying the Coleman and Noll procedure, in accordance with the second law of thermodynamics, whereby the inelastic nature of the decohesion process is accounted for by means of damage variables. The model accounts monolithically for loading and unloading conditions, as well as for decohesion and contact. Its performance is demonstrated through suitable examples.

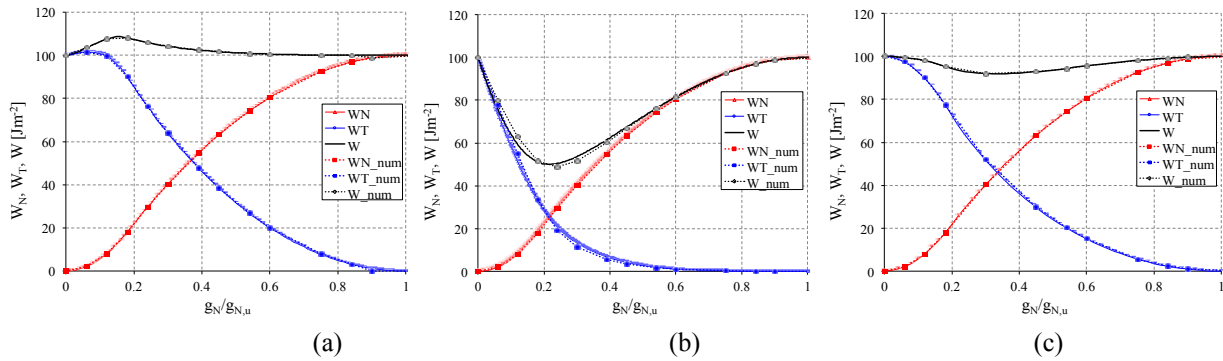


Figure 1: Work of separation for bilinear models by Högberg (a), and by Camanho et al. for a P-L criterion (b), or a B-K criterion (c).  $\phi_N = \phi_T = 100$  N/m.

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## References

- [1] M.J. van den Bosch, P.J.G. Schreurs, M.G.D. Geers. An improved description of the exponential Xu and Needleman cohesive zone law for mixed-mode decohesion, *Eng. Fract. Mech.* 73 (2006) 1220-1234.
- [2] X.P. Xu, A. Needleman. Void nucleation by inclusion debonding in a crystal matrix, *Model. Simul. Mater. Sc.* 1 (1993) 111-132.
- [3] J. Mosler, I. Scheider. A thermodynamically and variationally consistent class of damage-type cohesive models. *J. Mech. Phys. Solids.* 59 (2011) 1647-1668.
- [4] J.L. Högberg. Mixed mode cohesive law, *Int. J. Fract.* 141 (2006) 549-559.
- [5] P.P. Camanho, C.G. Dávila, M.F. De Moura. Numerical simulation of mixed-mode progressive delamination in composite materials, *J. Compos. Mater.* 37(16) (2003) 1415-1438.
- [6] P. Wriggers, G. Zavarise, T.I. Zohdi. A computational study of interfacial debonding damage in fibrous composite materials, *Comput. Mater. Sci.* 12 (1998) 39-56.