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# Hausdorff-Besicovitch Dimension of Lake Nasser Coastline from Satellite Imagery

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## Abstract

The paper is proposing a method for evaluating the Hausdorff-Besicovitch dimension of natural coastlines from satellite images.

A fractal dimension is a statistical index which is describing the complexity of a given pattern embedded in given spatial dimensions. In particular, this index is providing a measure of the capacity the considered fractal pattern has to fill the space in which it is embedded [1-3].

Fractals became popular with the works of Benoit Mandelbrot, starting from his 1967 paper where he discussed the fractional dimensions [4] (for references on the fractional approach to calculus, see [5]). In [4], Mandelbrot illustrated the fractal dimensions by citing a previous work by Lewis Fry Richardson, who was discussing how a coastline's measured length could change with the length of the rigid stick used for measurements. In this manner, the fractal dimension of a coastline was linked to the number of rigid sticks, required to measure the coastline, and to the scale of the used stick [6].

Actually, using the Fry Richardson example cited by Mandelbrot, we can also imagine an “experimental method” for the investigation of the fractal dimension of natural environments, such as the abovementioned coastlines of seas and lakes, and many others. For instance, in [7], where we proposed a link between fractal dimensions and entropies, we have considered the rim of the Grand Canyon. Before discussing the experimental method, which we will illustrate in this paper by means of a part of the coastline of Lake Nasser, a definition of fractal dimension is necessary. Let us note that several formal mathematical definitions exist: here we use that of the Hausdorff-Besicovitch dimension.

Felix Hausdorff proposed a fractional dimension in 1918 as a measure of the surface roughness [8]. Since several methods to calculate this dimension had been developed by Abram Samoilovitch Besicovitch, today the Hausdorff dimension is also known as the Hausdorff-Besicovitch dimension. It is given by (in the following formula,  $N$  stands for the number of sticks used to cover the coastline and  $h$  for the scaling factor):  $D = \log(N(h)) / \log(1/h)$ , in the limit as  $h$  goes to zero. The experimental approach to this fractal dimension is similar to that proposed in [10].

For the evaluation of the fractal dimension of the natural coastline of Lake Nasser, the vast water reservoir in southern Egypt and northern Sudan, let us use the Figure 1 (the image is coming from Google Earth imagery; in a

previous article [11], we considered this lake for giving a recurrence plot of its level from altimetric data).



Figure 1: A part of Lake Nasser in Google Earth imagery.

Using GIMP, the GNU Image Manipulation Program, and in particular its Pixelize filter (<https://docs.gimp.org/2.6/en/plugin-pixelize.html>), we can render the image in the Figure 1 as a series of images composed by large color blocks. Figure 2 shows them in binary images, made of black blocks in a white background. Therefore, by means of GIMP, we have a series of binary images suitable for the experimental evaluation of the fractal dimension of the coastline. Our “sticks” are the sizes of the blocks in the Pixelize filtered images.



Figure 2: Using the Pixelize filter of GIMP we have a series of binary images that we use for evaluating the fractal dimension of the coastline.

By means of an image segmentation, for instance that proposed for some previous experiments [19-21], we can easily determine the number  $N$  of sticks we have to use to measure the length of the consider coastline of the lake, in each of the binary images of the Figure 2. In the Figure 3, we can see the corresponding Pixelize filtered coastlines. Of them, we can easily determine the lengths as  $N(h)$ , for each considered scale  $h$ .

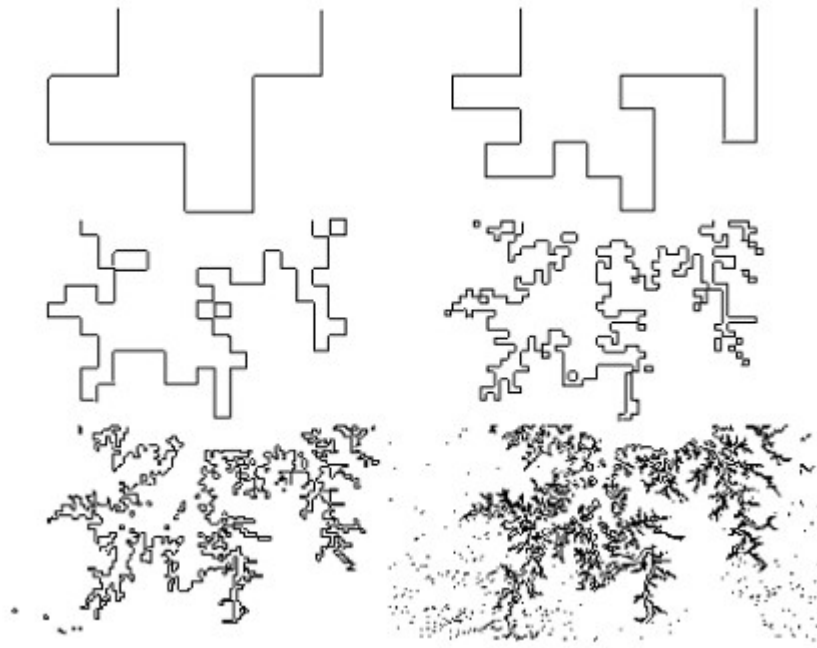


Figure 3: The Pixelize filtered coastlines considered for the evaluation of the fractal dimension, obtained by an image segmentation [20].

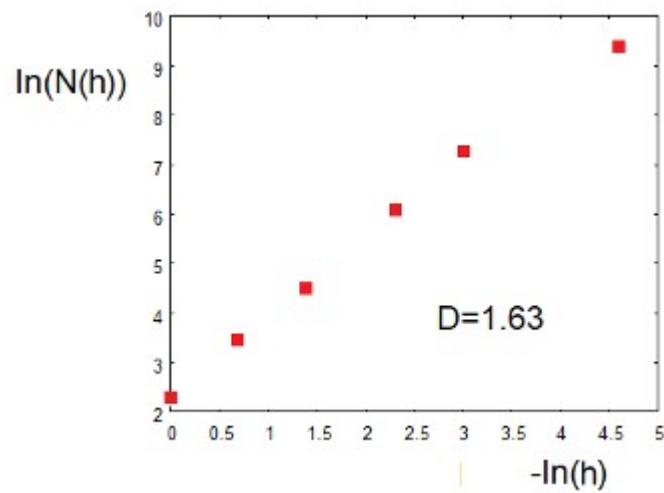


Figure 4: Data obtained from the image segmentation and used for evaluating the fractal dimension of the coastline in the Figure 1.

It is easy to plot  $\log(N(h))$ ,  $\log(1/h)$  as in the Figure 4 (natural logarithms). The slope of the best-fit line of the data in the Figure 4 approximates the Hausdorff-Besicovitch dimension [10]. In our example, the fractal dimension  $D$  of this part of the coastline is about 1.63.

It seems that this same approach can be applied to determine the fractal dimensions of other natural objects. However, further studies are necessary. Some examples are under investigation.

## References

- [1] Falconer, K. (2003). Fractal Geometry. New York: Wiley. Page 308. ISBN 978-0-470-84862-3.
- [2] Sagan, H. (1994). Space - Filling Curves. Berlin: Springer-Verlag. Page 156. ISBN 0-387-94265-3.
- [3] Vicsek, T. (1992). Fractal growth phenomena. Singapore New Jersey: World Scientific. ISBN 978-981-02-0668-0.
- [4] Mandelbrot, B.B. (1983). The fractal geometry of nature. Macmillan. ISBN 978-0-7167-1186-5.
- [5] Sparavigna, A.C. (2009). Fractional differentiation based image processing. arXiv preprint arXiv:0910.2381
- [6] Harte, D. (2001). Multifractals. London: Chapman & Hall. ISBN 978-1-58488-154-4.
- [7] Sparavigna, A.C. (2016). Entropies and fractal dimensions. Philica 2016, 559, 1–5.
- [8] Hausdorff, F. (1919). Dimension und äußeres Maß. Math. Ann. 79 157–179.
- [9] Gneiting, T., Ševčíková, H., & Percival, D. B. (2012). Estimators of fractal dimension: Assessing the roughness of time series and spatial data. Statistical Science, 247-277.
- [10] Elert, G. (1995). The Chaos Hypertextbook. 3.3 Fractal dimension. Available at <https://hypertextbook.com/chaos/>
- [11] Sparavigna, A. C. (2014). Recurrence Plots from Altimetry Data of Some Lakes in Africa, International Journal of Sciences, 3(7), 19-27. DOI: 10.18483/ijSci.534 , arXiv:1410.0850 [physics.data-an]
- [12] Sparavigna, A. C. (2017). Image Segmentation Applied to the Study of Micrographs of Cellular Solids. International Journal of Sciences, 6(02), 68-76. DOI: 10.18483/ijSci.1201
- [13] Sparavigna, A. C. (2016). A method for the segmentation of images based on thresholding and applied to vesicular textures. Philica 2016, 889. Available arXiv, <http://adsabs.harvard.edu/abs/2016arXiv161201131S>
- [14] Sparavigna, A. C. (2017). Measuring the particles in fly ash by means of an image segmentation. Philica, n.1105. Available HAL, <https://hal.archives-ouvertes.fr/hal-01579285v1>

- [15] Sparavigna, A. C. (2016). Analysis of a natural honeycomb by means of an image segmentation. Philica, n.897. Available HAL, <https://hal.archives-ouvertes.fr/hal-01416832v1>
- [16] Sparavigna, A. C. (2017). Image segmentation applied to micrographs of microcellular plastics. Philica, n.953. Available HAL, <https://hal.archives-ouvertes.fr/hal-01456692v1>
- [17] Sparavigna, A. C. (2017). Measuring the size of tubules in phloem and xylem of plants. Philica, n.1104. Available HAL, <https://hal.archives-ouvertes.fr/hal-01578826v1>
- [18] Sparavigna, A. C. (2017). Image Segmentation Applied to the Analysis of Fabric Textures. Philica, n.1157. Available HAL, <https://hal.archives-ouvertes.fr/hal-01633061v1>
- [19] Sparavigna, A. C. (2017). An image segmentation for the measurement of microstructures in ductile cast iron. PHILICA, n.1159. Available HAL, <https://hal.archives-ouvertes.fr/hal-01635665v1>
- [20] Sparavigna, A. C. (2018). Image segmentation applied to satellite imagery for monitoring water in lakes and reservoirs. PHILICA Article number 1214.
- [21] Sparavigna, A. C. (2017). Measuring the blood cells by means of an image segmentation. PHILICA, Article n.1176. Available HAL, <https://hal.archives-ouvertes.fr/hal-01654006/>

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