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Optimal operation of large district heating networks through fast fluid-dynamic simulation / Guelpa, Elisa; Toro, Claudia; Sciacovelli, Adriano; Melli, Roberto; Sciubba, Enrico; Verda, Vittorio. - In: ENERGY. - ISSN 0360-5442. - 102:(2016), pp. 586-595. [10.1016/j.energy.2016.02.058]

*Availability:*

This version is available at: 11583/2691807 since: 2017-11-15T19:42:06Z

*Publisher:*

Elsevier Ltd

*Published*

DOI:10.1016/j.energy.2016.02.058

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Elsevier postprint/Author's Accepted Manuscript

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<http://dx.doi.org/10.1016/j.energy.2016.02.058>

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4 **Optimal operation of large district heating networks**  
5 **through fast fluid-dynamic simulation**

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19

20 **Abstract**

21 Optimization of the operating conditions of district heating networks is usually performed limiting the  
22 analysis to the primary energy related with heat production. An additional aspect that should be considered is  
23 the role played by the pumping system. Pumping may contribute to about 10% of the total primary energy  
24 consumption, especially in large networks or when small temperature levels are applied. Furthermore, the  
25 increasing share of waste heat or renewable energy sources from distributed producers requires a flexible and  
26 efficient pumping system. A further aspect which pumping strategy should face is system operation when  
27 malfunctions in the plants, pumps or pipes occur.

28 Optimization of the pumping system requires the use of detailed simulation tools, which may need  
29 significant computational resources, especially in the case of large networks. A reduced model, based on  
30 Proper Orthogonal Decomposition combined with Radial basis functions (POD-RBF model) is proposed in  
31 this paper. This approach allows maintaining high level of accuracy despite reductions of more than 80% in  
32 the computational time. This make the approach effective tool for control strategy operations. An application  
33 to a large district heating network shows that reductions of about 20% in the pumping request and effective  
34 management of failures are possible.

35 **Keywords:**

36 Proper Orthogonal Decomposition, model reduction, district heating network, pumping cost, hydraulic model

37

38 **1. Introduction**

39 District heating (DH) is considered a very efficient option for providing heating and domestic hot water to  
40 buildings, particularly when they are located in densely populated areas [1]. The main advantage of DH  
41 systems consists in the possibility of utilizing the waste heat from industries or waste-to-energy plants or the  
42 heat generated by a number of efficient/low carbon thermal plants, such as cogeneration plants, and biomass  
43 [2], solar [3] and geothermal [4] systems.

44 An important aspect to achieve high efficiency in DH is the optimization of the operating conditions the  
45 system has to face in order to comply with the household thermal request. In the literature, various papers  
46 deal with the analysis of supply temperature during daily [5] and seasonal [6] operations or with the selection  
47 of the optimal supply and return temperatures [7]. In [8] a control approach is proposed in order to increase  
48 the temperature difference across the substations with a consequent increase of overall performances. In [9],  
49 the operating conditions of a district heating system are optimized acting both on the set-point temperature of  
50 the boilers and on the water flow of the pumps; the total fuel consumption is considered as the objective  
51 function to be minimized. In [10] and [11] the opportunities to modify the thermal request profile of some  
52 users are investigated to maximize the heat production from cogeneration or renewable plants.

53 An important aspect of optimal strategy analysis refers to pumping systems. Pumping systems are used to  
54 fulfill the desired heat flux to users facing the issues related to variations in friction losses. They include a set  
55 of pumps located along the network to provide consumers with hot water from the heat generation plants.  
56 The energy consumed for pumping operations is not negligible, in particular in large district heating  
57 networks, when distances involved are long. This aspect is further stressed in the case of low temperature  
58 district heating systems, typically operating with small temperature differences between supply and return  
59 networks and large mass flow rates [12]. Moreover, pumps work continuously during the heating season,  
60 even when heat demand is low.

61 For instance, the DH system of the city of Turin, which is considered in this work as a case study, requires  
62 up to about 6 MW of power transferred to the fluid, depending on the thermal load. This means that pumping  
63 represents about 2% of the primary energy consumption at peak request and increases to about 6-8% at night.

64 This aspect is also highlighted by various papers in literature, proposing the implementation of fluid dynamic  
65 models of the network for design purpose or the analysis of the effects of the control strategy on the energy  
66 consumption. A method for district heating network dimensioning, based on the probabilistic determination  
67 of the flow rate for hot water heating was carried out in [13]. Network costs, pumping energy consumption,  
68 and power of boilers were considered. In [14] a multi-objective optimization model is performed for the best  
69 network design considering both initial investment for pipes and pumping cost for water distribution. The  
70 best pipe diameters that reduce the total cost have been evaluated. A technical-economical optimization with  
71 the aim of minimizing both the pumping energy consumption and the thermal energy losses while  
72 maximizing the yearly annual revenue is performed in [15]. In [16] a fluid-dynamic model solved with the  
73 Hardy Cross method [17] is used in order to compare hydraulic performances of distributed variable speed  
74 pumps and conventional central circulating pump. Stevanovic et al. [18] solve the fluid-dynamic model with  
75 a loop method in order to show the potential for energy savings in pumping operations; the loop method is  
76 shown to be more effective with respect to the Hardy Cross method that is affected by problems related to  
77 convergence, computational cost and limited use [19]. In [20] a fluid-dynamic model of the network based  
78 on conservation was built and a genetic algorithm used in order to minimize the energy required by the  
79 system. Most works available in literature are focused on small district heating networks. When a large  
80 district heating network is considered, the computational cost to solve a physical based model becomes very  
81 high; this excludes the use of full physical models for fast multi-scenario and fast optimization applications.  
82 In the present paper, the authors present two different model approaches for the simulation of large networks  
83 and the analysis of the optimal control strategy for the pumping system. The two models are built in order to  
84 find the set of pumping pressures that should be applied to the pumps located along the network so as to  
85 minimize the total electricity consumption for a given operating scenario. The first model is a fluid-dynamic  
86 model based on mass and momentum conservation equations which consider the network topology through a  
87 graph approach. The second method is a reduced model, which has been derived from the fluid-dynamic  
88 model. Model reduction is obtained through the combination of proper orthogonal decomposition (POD) and  
89 radial basis functions (RBF). POD is a reduction technique which is able to decrease the computational cost  
90 of full physical models without losing the most relevant information. POD is able to capture the main  
91 features of a complex problem using a smaller degree of information (eigenfunctions) than the full model.

92 This method has received much attention for the reduction of complex physical systems and it has been used  
93 in different fields of science and engineering, such as the analysis of turbulent fluid flows [21,22], unsteady  
94 thermal systems [23], image processing [24] and many other fields.

95 Both the full physical model and the POD-RBF model are used in order to find the optimal set of pumping  
96 pressures that minimize the mechanical power that should be applied to the working fluid (i.e. the efficiency  
97 of the pump and the efficiency in the overall energy supply chain from primary energy to electricity  
98 production have not been considered) to fulfill the thermal requests of the various users, once the heat  
99 production of each plant is fixed. In the following, this objective function has been indicated as pumping  
100 cost, which should be intended as a cost expressed in energy units. An analysis with different thermal loads  
101 was performed because of the peculiar characteristics of district heating networks to work for a large number  
102 of operating hours in off-design conditions. Therefore a careful analysis of optimal operating conditions,  
103 with different thermal requests, is necessary to achieve high levels of the annual efficiency. The heat flow  
104 supplied by each thermal plant is provided as an input of the model by setting the water mass flow rates  
105 exiting the various plants.

106 Results obtained with the two models are compared in terms of both minimum energy consumption and  
107 computational time for each thermal load. The POD-RBF model allows us to obtain optimal costs that differ  
108 from the cost provided by the full physical model of less than 5%. The full physical model is extremely time-  
109 consuming especially if applied to large district heating networks. The POD-RBF method is much faster than  
110 the full physical model and allows us to perform multiple simulations and optimizations using small  
111 computational resources. The POD-RBF approach is shown to be very effective for the optimal management  
112 of complex district heating systems reducing computational cost by more than 90% with respect to the full  
113 physical model. This allows the optimization process for a much larger number of scenarios. Results of the  
114 optimization are then compared with the current pumping strategy used for the district heating system of the  
115 city of Turin; the comparison shows that a change in the policy of pumping operations can reduce the energy  
116 consumption for pumping by about 20%.

117

## 118 **2. System description**

119 The Turin district heating network is the largest network in Italy. It currently connects about 55000 buildings  
120 with an annual thermal request of about 2000 GWh. The maximum thermal power is about 1.2 GW. An  
121 expansion of the system, to reach about 72 million cubic meters of buildings is already planned [25]. The  
122 water supply temperature is constant and its value is 120°C while the return temperature varies with mass  
123 flow rate in the network and thermal load; the mean value is 65 °C.

124 The complete network can be considered as composed of two parts: a transport network and a distribution  
125 network. The transport network, consists in large diameter pipes, usually larger than 200 mm, and connects  
126 the thermal plants to the thermal barycentres. Each barycentre is a subnetwork that reaches a group of  
127 buildings that are located in the same area. In the Turin network there are 182 barycentres. The ensemble of  
128 these sub-networks constitutes the distribution network. The transport network is a loop network, while the  
129 sub-networks are tree-shaped networks. Figure 1 depicts the transport pipeline network and, in detail, 3  
130 barycentres with their corresponding tree-shaped networks.

131 The model developed in this work only considers the main transport network. The total length is about 515  
132 km. Five thermal plants, which are highlighted in green in Figure 1, provide heat to the network. The main  
133 characteristics of the plants are reported in Table 1. The most usual start-up strategy of the thermal plants is  
134 the following: the two cogeneration plants in Moncalieri are started-up first (when the thermal request is  
135 below 260 MW one plant is operating, while the second one is operating when the request is below 520  
136 MW), then the cogeneration plant in Torino Nord is started up and then the storage units in Politecnico and in  
137 Torino Nord. Larger thermal requests are covered using the boilers in Politecnico, Torino Nord, Mirafiori  
138 Nord, BIT, Moncalieri. In the case some of the plants are not available or when specific constraints due to  
139 electricity production must be fulfilled, a different order can be selected.

140 Regarding pumping systems, the main pumping stations are located at the thermal plants and 9 booster pump  
141 groups are located along the network. The main pumping stations allow the desired hot mass flow rate to be  
142 pumped into the network, from the operating thermal plants to the users. Booster pumping stations are used  
143 in order to distribute the correct mass flow rate to each user, contrasting friction losses and hydraulic head.  
144 Booster pumping stations and direction of the pumped flow are indicated in Figure 1b. RP1 and RP2 include  
145 two groups of pumps, each pumping in a specific direction; RP5 includes three groups of pumps; RP3 and  
146 RP4 include only one group of pumps. The latter is not considered in the simulation because it is used in a

147 network configuration different to that examined in this work. The use of RP4 will be necessary when the  
148 network developments, which are already planned, are completed. A further utilization of this pump is  
149 possible in the case of malfunctions.

### 150 **3. Models description**

151 In order to minimize the pumping energy consumption, to provide the users with their thermal request, an  
152 optimization was performed. In the optimization, each scenario is defined by setting the total thermal load  
153 and the contribution of each plant to the thermal load, i.e. the heat production of each plant does not vary in  
154 the optimization procedure. Mass flow rates at the various plants are obtained dividing the heat production  
155 by the specific heat and the temperature difference between supply and return network.

156 There are various different settings of the pumping groups which allow combining the production of plants  
157 and the request of the users, each corresponding with a different total power consumption. The independent  
158 variables are the pressure differences in the pumping stations; therefore there are 8 independent variables,  
159 one for each pump located along the network. As previously discussed, only the booster pumping stations,  
160 not the pumps located in the plants, were considered. A maximum pressure of 17 bar has been set as a  
161 technical constraint.

162 The objective function is the energy consumption, also called the energy cost. It has been calculated as:

$$163 \quad C = \sum_p \frac{G_p \Delta p_p}{\rho} + \sum_r \frac{G_r \Delta p_r}{\rho} \quad (1)$$

164 where subscript p indicates pumping systems located in the thermal plant, which are the dependent variables  
165 in the optimization problem, and subscript r indicates the booster pumping systems, which are the  
166 independent variables. The water density in the plants was evaluated as the average value between the supply  
167 and the return temperatures. This procedure should be repeated for different thermal loads in order to build  
168 an optimal control strategy.

169 Two approaches have been used to perform the optimization: a fluid dynamic approach and a POD approach.  
170 As regards the fluid dynamic approach, a genetic algorithm[26] was applied to the model described in the  
171 next section. The algorithm starts the search for the optimal values from multiple initial points. Consequently  
172 various cases (also called individuals in literature) must be created to run the optimization. This set of cases  
173 is usually named the population. The number of individuals in the population is kept constant during the

174 optimization process, but the values of the independent variables associated with each individual are  
175 modified at each iteration. Iterations are usually called generations in GA nomenclature. To create the initial  
176 population to be used in the optimization, the non-dimensional variables are randomly selected. A  
177 population of 100 elements and a maximum number of 100 iterations are selected. 100 sets of pressure  
178 differences randomly selected constitute the first population. The genetic algorithm runs until the  
179 convergence is reached, when further changes in population members do not affect the minimal cost  
180 obtained. The convergence was reached after about 50-60 generations depending on the thermal load  
181 selected. The procedure is shown in Figure 2a. The pressure differences can vary between the values selected  
182 in order to obtain, for the most cases simulated, a maximum pressure value lower than the upper pressure  
183 limit.

184 The second optimization is performed using a POD-RBF approach. The POD-RBF model is built using a set  
185 of results from the fluid dynamic model. The set of results is called snapshot. In this work each snapshot  
186 consists in a set of mass flow rate in the branches where the pumping stations are located and the  
187 corresponding pumping cost. Once the model is built, it can be used to simulate different cases respect to the  
188 one used to build the model or used as an optimization tool. The procedure is represented in Figure 2b.

189 The fluid dynamic model is a high time consuming model because it carefully analyzes the system behavior  
190 in all the network zones, even when only information in some sections is required (in this case in the booster  
191 pumping power branches). The POD-RBF model instead provides an approximate value of the objective  
192 function, but the search for the optimum is much faster. These two methods are discussed in detail and  
193 compared in the next sections.

### 194 **3.1 Fluid-dynamic model**

195 A one dimensional model was developed to detail the thermo-fluid dynamic behavior of the main pipeline of  
196 the network (i.e. the transport network). The topology of the network has been described using a graph  
197 approach [27]. Each pipe is considered as a branch delimited by two nodes, which are identified as the inlet  
198 node and outlet node on the basis of a reference direction(velocity is positive when the fluid is flowing in the  
199 same direction as the reference direction and negative when flowing in the opposite direction). The main  
200 return pipeline network includes 685 branches and 677 nodes, with 9 loops. The fluid-dynamic model



201 considers the mass conservation equation applied to all the nodes and the momentum conservation equation  
202 to all the branches.

203 The incidence matrix  $\mathbf{A}$ , is used in order to describe the network topology by expressing the connections  
204 between nodes and branches. Matrix  $\mathbf{A}$  has as many rows as the number of nodes and as many columns as  
205 the number of branches. Its general element  $A_{ij}$  is equal to 1 or -1 if the branch  $j$  enters or exits the node  $i$  and  
206 0 otherwise. Using this matrix the mass balance equation written using matrix form is:

$$207 \quad \mathbf{A} \cdot \mathbf{G} + \mathbf{G}_{\text{ext}} = 0 \quad (2)$$

208 where  $\mathbf{G}$  is the vector that contains the mass flow rates in the branches and  $\mathbf{G}_{\text{ext}}$  the vector that contains the  
209 mass flow rates exiting the nodes outwards. The terms in  $\mathbf{G}_{\text{ext}}$  are different than zero in the case of open  
210 networks, i.e. when only a portion of the entire closed circuit is considered.

211 The steady-state momentum conservation equation in a branch for an incompressible fluid is considered,  
212 neglecting the velocity change between input and output sections and including the gravitational term in the  
213 static pressure:

$$214 \quad (p_{\text{in}} - p_{\text{out}}) = \frac{1}{2} \frac{f}{D} L \frac{G^2}{\rho S^2} + \frac{1}{2} \sum_k \beta_k \frac{G^2}{\rho S^2} - t \quad (3)$$

215 where the first and the second terms on the right-hand side terms are respectively the distributed and the  
216 localized pressure losses, while the last term is the pressure rise due to the pumps that may be located in the  
217 branch. Equation (3) can be rewritten as:

$$218 \quad G = Y(p_{\text{in}} - p_{\text{out}}) + Yt \quad (4)$$

219 where the term  $Y$  is the fluid dynamic conductance of the branch, expressed as:

$$220 \quad Y = R^{-1} = \left[ \frac{1}{2} \frac{G}{\rho S^2} \left( \frac{f}{D} L + \sum_k \beta_k \right) \right]^{-1} \quad (5)$$

221 The friction factor  $f$  has been evaluated using an explicit Haaland correlation in order to reduce the  
222 computational cost of the simulations.

223 Momentum equation can be rewritten in matrix form. This formulation is obtained using the incidence matrix in  
224 order to relate the quantities that are defined at the branches (mass flow rates and pressure variations due to  
225 friction and pumping) with pressures at the inlet and outlet nodes:

$$226 \quad \mathbf{G} = \mathbf{Y} \cdot \mathbf{A}^T \cdot \mathbf{P} + \mathbf{Y} \cdot \mathbf{t} \quad (6)$$

227 The diagonal matrix  $\mathbf{Y}$  represents the fluid dynamic conductance of branches. Because of the dependence of  
228  $\mathbf{Y}$  on mass flow rate, the obtained system of equation is non-linear. Equation (6) is finally modified by  
229 setting proper boundary conditions.

230 Mass and momentum equations are solved using a SIMPLE (semi implicit method for pressure linked  
231 equation) algorithm [28]. This is a guess and correction method: a pressure vector is first guessed and during  
232 the iterations it is corrected together with the mass flow rate vector obtained using (6). Further details on the  
233 method are available in [29]. In order to solve the system of non-linear equations a fixed point algorithm has  
234 been used.

235 The model includes both the supply and the return pipelines, which are connected in the barycentres. From  
236 fluid dynamic viewpoint, barycentres are considered as pipes with their certain friction resistances and the  
237 fittings frictions (e.g. T-junctions, curves, etc.). In a general case, the mass flow rates supplied to the  
238 barycentres,  $G_{ut}$ , differ from their requests, therefore an adjustment is necessary to model the valve  
239 controlling the barycentre mass flow rates. Therefore a variable resistance term is added to the fixed term;  
240 both resistances are expressed as equivalent lengths which affect the term  $\mathbf{Y}$  appearing in equation (4).

241 The variable resistance term is iteratively modified until an acceptable flow distribution is obtained, with all  
242 users supplied with the requested mass flow rate. To obtain the mass flow rate required from every user the  
243 value of  $L_{eq}$  in the  $n^{\text{th}}$ -iteration is calculated as follows:

$$244 \quad L_{eq}^n = L_{eq\_f} + L_{eq\_v}^{n-1} \left( \frac{G_{ut}^{n-1}}{G_{ut}} \right)^2 \quad (7)$$

245 where  $L_{eq\_f}$  is the fixed resistance and  $L_{eq\_v}$  is the variable resistance. Subscripts  $n$  and  $n-1$  refer to the current  
246 and previous iterations, respectively. The iterative procedure stops when the relative error between  $G^{n-1}$  and  
247  $G_{ut}$  is smaller than a threshold value.

248 Concerning boundary conditions, the mass flow rate supplied by each plant is fixed on the corresponding  
249 node of the supply network. Similarly, the mass flow rate returning at each plant is fixed on the  
250 corresponding node on the return network, except for the node corresponding with Moncalieri plant, where  
251 the pressure is fixed. The latter boundary condition is required for proper solution of the fluid dynamic  
252 problem, as a further condition on the mass flow rate would result in a linearly dependent equation. Pressure  
253 is imposed on the Moncalieri plant, since the master pressurizing group is located there.

## 254 3.2 Proper Orthogonal Decomposition

255 Reduced order modeling is an effective way for the development of accurate and computationally  
256 inexpensive models. A POD model can be constructed following the method of snapshots, as proposed by  
257 Sirovich [30]. A snapshot is a vector  $\mathbf{u}$  of  $N$  relevant physical quantities that identify the behaviour of the  
258 system for a particular combination of  $S$  input parameters, known as the process parameters. The latter are  
259 collected in a vector  $\mathbf{d}$ . In this work, pressure rise at the eight booster pumps, the percent thermal load (with  
260 respect to the maximum thermal load) and the contribution of each plant to the thermal load are chosen as the  
261 process parameters. For a given thermal load and contribution of the various plants, the eight values of  
262 pressure rise are the free variables that can be modified in the optimization process. It is worth remarking the  
263 fact that pressure rises in the pumps located at the thermal plants are not free variables. These should be  
264 adjusted in order to allow circulation of the mass flow rates exiting the various plants.

265 Table 2 reports the maximum pressure selected for the various booster pumps, obtained after a pre-  
266 processing stage, which has been performed in order to limit the number of random combinations of the  
267 input that are rejected because of a maximum pressure exceeding the technical limit of 17 bar.

268 The response  $\mathbf{u}$  of the system to a given set of the free variables is expressed by the mass flow rates at the  
269 booster pumps and by the total pumping power.

270 Different snapshots are obtained by varying the optimization independent variables within a predefined  
271 range. In order to avoid obtaining an ill-conditioned model, some precautionary measures have been adopted.

272 First, the input data of the model have been normalized. Furthermore the snapshots have been randomly  
273 selected considering a uniform coverage of the input ranges. The complete collection of  $M$  snapshots  
274 constituted the snapshot matrix  $\mathbf{U}$ . POD aims at approximating an arbitrary snapshot as follows:

$$275 \mathbf{u}^a = \bar{\Phi} \cdot \bar{\alpha}^a \quad (8)$$

276 where  $\bar{\alpha}^a \in \mathbb{R}^{K \times 1}$  is a reduced state variable and  $\bar{\Phi}$  is an orthogonal matrix. The latter is found solving the  
277 following eigenvalue problem [31]:

$$278 (\mathbf{U}^T \mathbf{U}) \cdot \boldsymbol{\varphi}_i = \lambda_i \boldsymbol{\varphi}_i \quad (9)$$

279 Matrix  $\bar{\Phi}$  is then built using the eigenvectors  $\boldsymbol{\varphi}_i$  corresponding to the largest eigenvalues  $\lambda_i$ , which are ranked  
280 in decreasing order. Namely,  $\bar{\Phi} = [\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \dots, \boldsymbol{\varphi}_K]$ .

281 In the present analysis, the POD method has been coupled with radial basis functions (RBF). RBF are  
282 typically applied to approximate functions known only in a finite number of points. This interpolation  
283 technique involve all known values of functions and it is particularly effective when the distribution of nodes  
284 is scattered. Specifically, the reduced state variable  $\bar{\alpha}^a$  in Eq. (8) has been expressed as a linear combination  
285 of radial basis functions of the process parameters p:

$$286 \quad \bar{\alpha} = \mathbf{B} \cdot \mathbf{g}(d) \quad (10)$$

287 where  $\mathbf{g}$  contains the radial basis functions and matrix  $\mathbf{B}$  the coefficients. Here, Euclidean norm was used as  
288 RBF:

$$289 \quad g_i(p) = \|p - p_i\| \quad i = 1, \dots, K \quad (11)$$

290 Matrix  $\mathbf{B}$  is found by enforcing that Eq. (10) is exact for each of the snapshots contained in the matrix  $\mathbf{U}$   
291 [31].

292 The evaluation of a snapshot corresponding to an arbitrary set of parameter p can be performed using Eq.  
293 (12). This is obtained by substituting Eq. (10) in Eq. (8):

$$294 \quad \mathbf{u} = \bar{\Phi} \cdot \mathbf{B} \cdot \mathbf{g}(p) \quad (12)$$

295 The entire procedure has been built in Matlab environment. To initialize the POD optimization procedure, a  
296 set of random combinations of the free variables has been collected into the initial snapshot matrix  $\mathbf{U}$  and fed  
297 as inputs to the full physical model. The corresponding values of mass flow rates in each of the eight pumps  
298 and the total pumping costs have been obtained. Snapshots and results are used to create the POD-RBF  
299 model, which is the implicit function relating the free variables to the output.

300

## 301 **4. Results and discussion**

### 302 **4.1 Full physical model validation**

303 In order to validate the fluid dynamic model in the various operating conditions, a comparison with some  
304 measured data of the Turin district heating network was carried out. The pressure differences between two  
305 nodes located at the outlet of a pump and at the inlet of the next pump located downstream were evaluated in  
306 three different portions of the network where measurements were available for an entire heating season. In  
307 Figure 3, the pressure differences are reported as a function of the mass flow rate circulating in the network.

308 The measured data reported in figure refer to the operating conditions in March, where a large variation takes  
309 place. In the figure, the results of the fluid dynamic model are also represented. In the case of the first  
310 portion, the model is able to capture the fluid dynamic behavior of the network with high accuracy. In the  
311 other sections the dispersion of data is much larger and of the same order as the pressure differences, mainly  
312 because these portions are closer to the centre of town, where a large number of sub-networks and buildings  
313 are located. In the model, the thermal request profile of the various barycentres was considered similar, i.e.  
314 with the same shape parameterized on the basis of the design request. In reality this does not occur. In  
315 addition, the model was run considering strict compliance with the control strategy, while in real operation a  
316 deviation within an acceptable range is allowed. These are the causes of the large dispersion of data.  
317 Anyhow, the average deviation is lower than 0.3 bar, therefore it is possible to state that the fluid dynamic  
318 model is able to capture the hydraulic behavior of the network.

## 319 **4.2 POD model characteristics, validation and performances**

320 Starting from the full physical model, over 15000 simulations were performed, varying the free variables  
321 randomly within the predefined ranges. These have been used to create the POD-RBF model.

322 A test of the POD model was first performed considering new random sets of the free variables, which were  
323 not included in the original set. The fluid-dynamics model is used in order to compute the pumping cost,  
324 selecting the independent variable randomly, i.e. the pumping pressure differences and the thermal load. The  
325 same data are used in order to calculate the output through the POD model. In Figure 4a the POD and Fluid  
326 dynamic models' results, in terms of pumping cost, are compared. Results evidently show that the POD  
327 based tool in almost all cases is able to reproduce the system behavior.

328 Mass flow rates obtained from a random set of data using the two models are also computed. For each  
329 simulation, the branch containing the booster pumps where the largest mass flow rate is located is analyzed  
330 in Figure 4b. The figure shows that the reduced model is able to predict the mass flow rate for all cases with  
331 small deviations.

332 The optimization has been performed for different heat loads. A comparison between the fluid dynamic  
333 model and the POD-RBF model is reported in Figure 5. Scenarios have been obtained considering the most  
334 typical start-up sequence of the thermal plants. As regards the fluid dynamic model optimization, Figure 5

335 shows that the larger the thermal load, the larger the optimal pumping cost, except for the scenario  
336 corresponding with 40% of the nominal load. The minimum cost for 40% of the nominal load is slightly  
337 larger than the minimum cost for 50% of the nominal load. This is due to the fact that when the thermal load  
338 is below 40% of the nominal load, only the Moncalieri thermal plant is operating (unless a different order is  
339 set, which can occur, for instance, in the case of network maintenance or depending on the production plans,  
340 especially related with the electricity production). When the request exceeds 40% of the nominal load, both  
341 the Moncalieri and Torino Nord thermal plants are operating. As these plants are located on opposite sides of  
342 the network, users in the North areas of the town (closer to Torino Nord plant) are reached by the water flow  
343 exiting Torino Nord plant. This allows a reduction in the pressure drops, therefore reducing the pumping cost  
344 despite an increase in the total mass flow rate flowing. When the mass flow rate further increases, the  
345 pumping cost tends to increase again.

346 The optimum pumping pressure sets obtained using the POD-RBF model were used as an input in the fluid  
347 dynamic model in order to compare the optima. Results show that the POD-RBF model is able to predict the  
348 optimal costs as a function of thermal load with average relative errors of about 5%.

349 A comparison of the computational cost requested to obtain the optimum values with the fluid-dynamic  
350 model and the POD-RBF model is reported in Figure 6. Computational costs are evaluated as the summation  
351 of the time requested to obtain the minimum cost in all the thermal load conditions that have been analyzed  
352 on a single 3.3 GHz CPU. Using the POD-RBF, the total time required for the calculation is reduced by  
353 about 95% with respect to that required by the fluid dynamic model.

## 354 **4.3 POD model for energy cost reduction**

### 355 **4.3.1 Usual start-up sequence of thermal plants**

356 In order to present the potential advantages that can be achieved using an optimized pumping strategy, a  
357 comparison between the pumping cost corresponding to the application of a pumping strategy similar to that  
358 currently adopted and the optimal strategy is reported in Figure 7. In this analysis, the usual start-up  
359 sequence of the thermal plants is considered. It is possible to notice that the use of the optimized control  
360 strategy instead of the current one allows the achievement of a significant reduction in the energy  
361 consumption for all thermal loads, particularly in the portion between 40% and 90% of the nominal thermal

362 load. The differences between results obtained with the two optimization strategies (the POD-RBF and the  
363 fluid-dynamic model) are quite negligible in comparison with the difference between optimal and current  
364 strategy, therefore only the POD-RBF results have been shown, since it is the approach that can be used in  
365 real applications.

366 To better visualize the energy cost reduction with respect to the current pumping strategy, the energy cost  
367 reductions in each thermal load is shown in Figure 8. Energy saving is between 8% and 24% and it is  
368 particularly large at high thermal load. The use of an optimized pumping strategy allows an annual reduction  
369 in primary energy consumption due to pumping of about 4.4 GWh/year (from 25.8 GWh/year in the case of  
370 the current strategy to 21.4 GWh/year in the case of the optimized strategy). This represents more than 0.5%  
371 reduction in the total primary energy consumption, which is about 842.5 GWh/year (about 768.0 GWh/year  
372 associated with heat supplied to the users, about 48.5 GWh/year due to heat losses, and 25.8 GWh/year due  
373 to pumping).

374 These results suggest that application of the POD-RBF optimization approach allows significant  
375 improvement in the overall energy performances of large district heating networks.

#### 376 **4.3.2 Different start-up sequence of the thermal plants**

377 The same POD-RBF model can be used in order to optimize the pumping strategy when different  
378 combinations of the plants is adopted in thermal production. These scenarios can be necessary in the case  
379 one of the plants is not available or if there are specific constraints on the electricity production by the  
380 cogeneration plants. When the configuration in heat production changes, also the mass flow rate distribution  
381 at the thermal plants change, therefore a different setting of the pumps is necessary, even if the thermal  
382 request of the users remains unmodified. The optimization tool should be sufficiently flexible to allow fast  
383 optimizations in variable conditions. The POD-RBF model can be used by fixing the total load, by  
384 modifying the sequence of thermal plants that are used to cover it and by limiting the maximum DH mass  
385 flow rate that is elaborated by each plant (and thus the maximum thermal load supplied by each plant).

386 Table 3 shows four different scenarios, corresponding with different plant configurations at 60% of the  
387 maximum thermal request of the users, are presented. In Figure 9, the corresponding optimal settings of the  
388 pumping group obtained using the POD-RBF model are shown.

389 Results show that the pumping cost is smaller when the two Moncalieri cogeneration plants are not used at  
390 100%. In fact in cases 1 and 2, where just the Moncalieri cogeneration group 1 is switched on the optimal  
391 cost is lower than in the cases 3 and 4, where both the cogeneration groups in Moncalieri are used. This is  
392 due to the fact that the Moncalieri power plant is located at the south end of the network, therefore when  
393 large mass flow rate are supplied by these plants, a large pumping power is necessary. When one of the  
394 Moncalieri cogeneration plants is switch off, the power spent to pump the water from the south area to the  
395 city centre (R Monc, RP1a, RP1b) is smaller, while the power to pump water from the north to the south is  
396 larger (R T.N., R Poli and RP5c). The configuration which minimizes the pumping power corresponds to a  
397 more distributed production. In case 1, in fact heat is produced in three plants, one located in the south end  
398 (Moncalieri), one in the central area (Politecnico) and one in the north end (Torino nord).

#### 399 **4.3.3 Operation in the case of malfunctioning pumping groups**

400 The POD-RBF model is also been used in order to find the optimal set of pumping pressure when a failure in  
401 a pumping station occurs and therefore that piece of equipment cannot be used. In malfunctioning scenarios,  
402 minimization of primary energy consumption may become a secondary objective. Nevertheless, the fact that  
403 a constrained optimization is performed allows one to obtain the best pumping settings which allow  
404 fulfillment of the thermal request of the users, which is instead the main objective in malfunctioning  
405 scenarios.

406 The analysis has been performed for each pumping station. Results are reported in Table 4, considering 60%  
407 of the thermal request and the usual configuration for thermal production.

408 The minimum cost is obtained when no malfunctions occur. Nevertheless in most malfunctioning cases, the  
409 optimal costs do not differ significantly with respect to the case without malfunctions, except when a failure  
410 occurs in the pump 1b. This is due to the fact that this pump is located in a crucial position for water  
411 circulation and its unavailability causes longer paths to reach the users and thus larger friction losses.

412 Possible iterative interactions between pumping system settings and plant operation can be theoretically  
413 examined using the modeling approach proposed in this paper. Such cases are meaningful in the case of  
414 possible malfunctions that may affect the hydraulic behavior of the network. In the case there are no  
415 pumping strategies that allow proper fulfillment of the thermal request, it is possible to examine scenarios  
416 where the production share among the plant is modified in order to help reducing the hydraulic issues.



417 These results show that POD-RBF model allows one to create a flexible operation tool, which allows optimal  
418 management of both normal and abnormal (malfunctioning) scenarios.

## 419 **5. Conclusion**

420 The present paper reports an optimization analysis for the minimization of the pumping cost in a large  
421 district heating network. The optimization is carried out using two different approaches. The first approach,  
422 more conventional, is based on the application of a genetic algorithm to the full physical fluid dynamic  
423 model of the network. The second approach utilizes a reduced model, obtained through radial basis function  
424 (RBF) and proper orthogonal decomposition (POD) in order to capture the main features of the physical  
425 system. This last approach requires much smaller computational time but provides more approximate results  
426 due to model reduction. The errors of the POD model in the evaluation of the objective function are quite  
427 small.

428 Fluid dynamic model and the POD-RBF model are used to find the optimal values of pumping cost. Results  
429 show that a deviation of about 2% is obtained for both optima. Therefore POD provides a good  
430 approximation of the physical behavior of the system. The difference in computational time is very large.  
431 This is a crucial feature to allow optimal operation in real networks, as the operating conditions vary  
432 significantly depending on the thermal request and the availability of both the thermal plants and the  
433 pumping groups. In the case study considered in this paper the calculation of snapshots and the optimization  
434 of the POD model requires about 4% of the time requested for the optimization using GA. This difference  
435 increases with the number of nodes that are used to represent the network topology, which means that the  
436 advantages of using such a technique increase in the case of large networks. In order to show the potential  
437 for energy saving in district heating network pumping systems, a comparison between the electricity  
438 consumption using the current control strategy and the optimized strategy was carried out. This comparison  
439 shows encouraging results which suggest the applicability of fast simulation to the optimal management of  
440 the pumping system in district heating networks. The simulation tool shows to be sufficiently flexible to  
441 allow one handling both normal operating conditions and malfunctioning conditions.

442

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505

506 **Nomenclature**

|                      |                              |
|----------------------|------------------------------|
| <b>A</b>             | incidence matrix             |
| <b>B</b>             | coefficient matrix           |
| <b>c</b>             | specific heat, J/(kg K)      |
| <b>C</b>             | energy cost, MW              |
| <b>d</b>             | input parameters vector      |
| <b>D</b>             | pipe diameter, m             |
| <b>f</b>             | distributed friction factor  |
| <b>g</b>             | radial basis function        |
| <b>G</b>             | mass flow rate, kg/s         |
| <b>K</b>             | stiffness matrix             |
| <b>L</b>             | pipe length, m               |
| <b>M</b>             | mass matrix, kg              |
| <b>p</b>             | pressure, Pa                 |
| <b>P</b>             | pressure matrix, Pa          |
| <b>S</b>             | pipe section, m <sup>2</sup> |
| <b>t</b>             | pumping pressure vector, Pa  |
| <b>T</b>             | temperature, °C              |
| <b>u</b>             | snapshot                     |
| <b>U</b>             | snapshot matrix              |
| <b>U</b>             | pipe transmittance, W/kg K   |
| <b>Y</b>             | fluid dynamic conductance    |
| <b>Greek symbols</b> |                              |
| <b>α</b>             | coefficient vector           |

---

|         |                           |
|---------|---------------------------|
| $\beta$ | localized friction factor |
|---------|---------------------------|

---

|           |               |
|-----------|---------------|
| $\varphi$ | eigenfunction |
|-----------|---------------|

---

|           |                    |
|-----------|--------------------|
| $\lambda$ | eigenvalues matrix |
|-----------|--------------------|

---

|        |                            |
|--------|----------------------------|
| $\rho$ | density, kg/m <sup>3</sup> |
|--------|----------------------------|

---

|        |                       |
|--------|-----------------------|
| $\Phi$ | eigenfunctions matrix |
|--------|-----------------------|

---

|        |                |
|--------|----------------|
| $\Phi$ | heat power, MW |
|--------|----------------|

---

**Subscripts and superscripts**

---

|     |          |
|-----|----------|
| ext | external |
|-----|----------|

---

|    |       |
|----|-------|
| in | inlet |
|----|-------|

---

|     |        |
|-----|--------|
| out | output |
|-----|--------|

---

|     |        |
|-----|--------|
| ret | return |
|-----|--------|

---

|     |        |
|-----|--------|
| sup | Supply |
|-----|--------|

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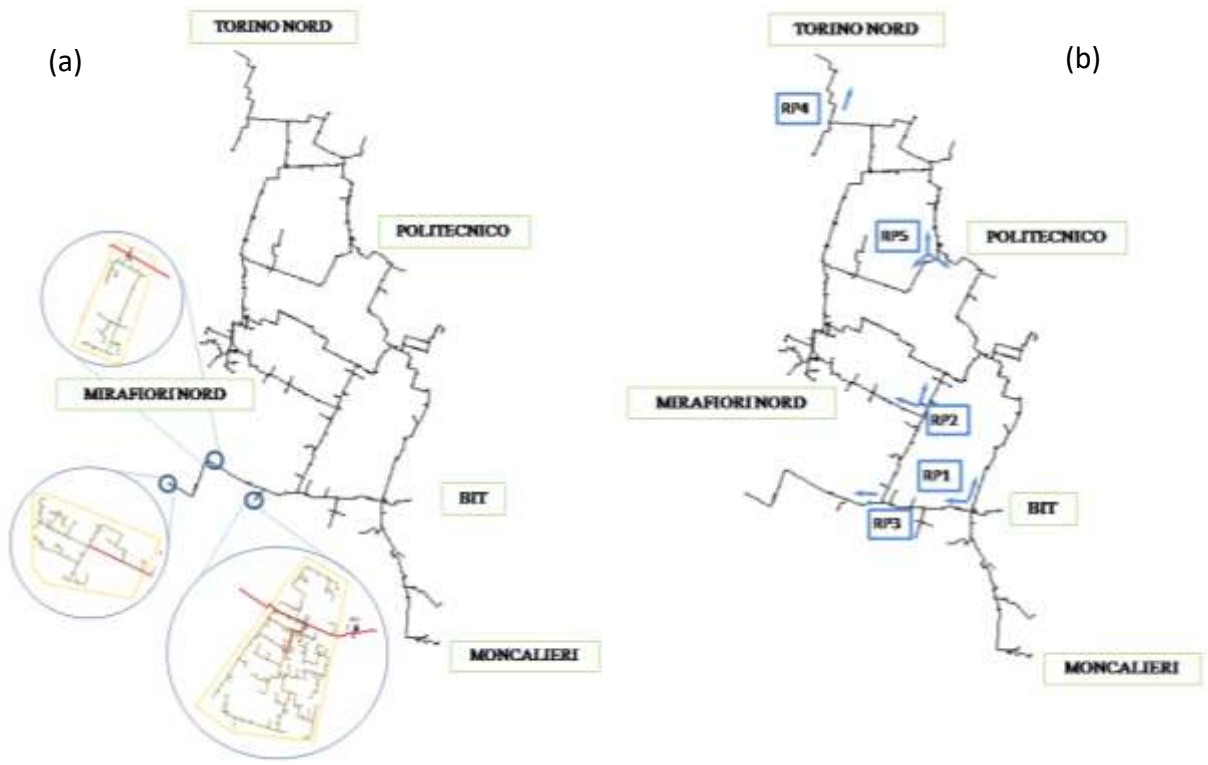
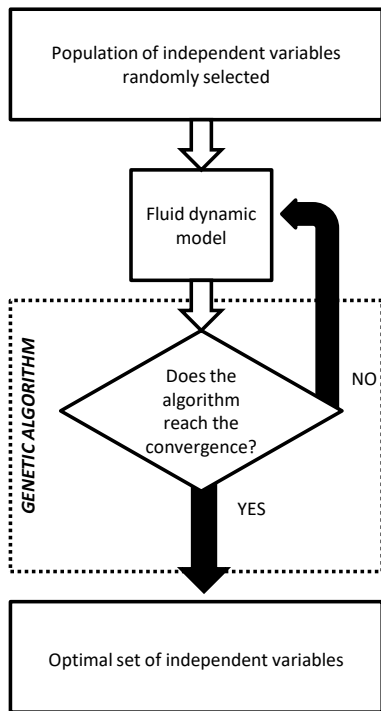
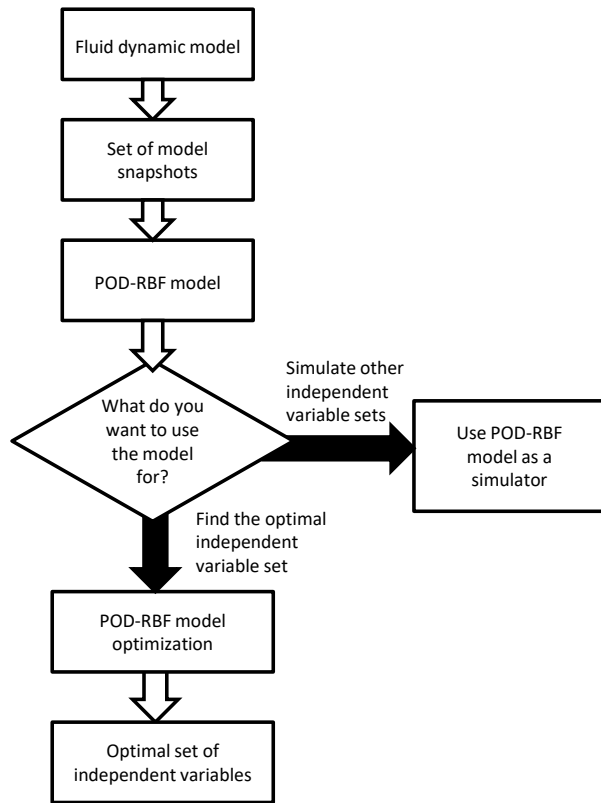


Figure 1: Schematic of Turin District Heating Network (a) In detail 3 barycentres (b) Pumping system

528  
529  
530



(a)



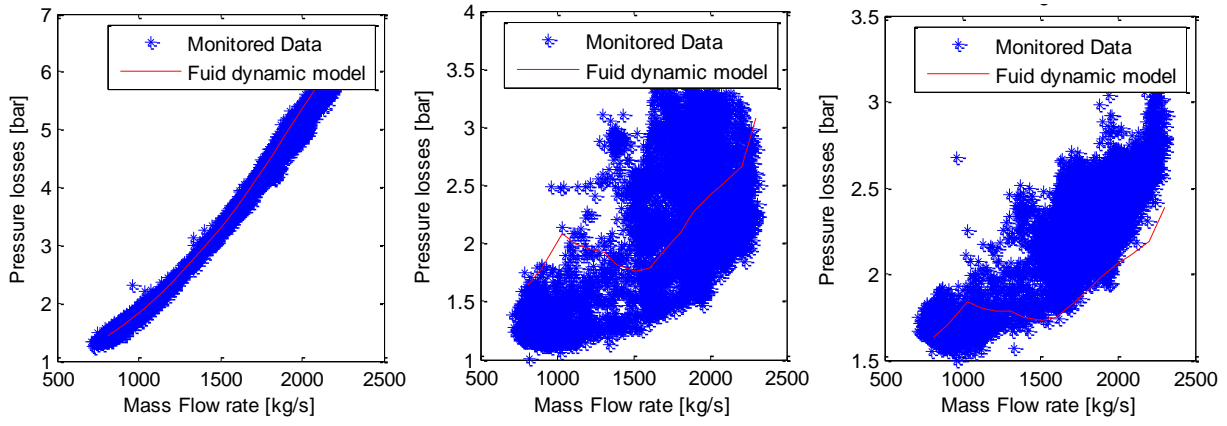
(b)

531  
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Figure 2: Schematic of the two optimization approaches (a) Fluid dynamic model (b) POD-RBF model



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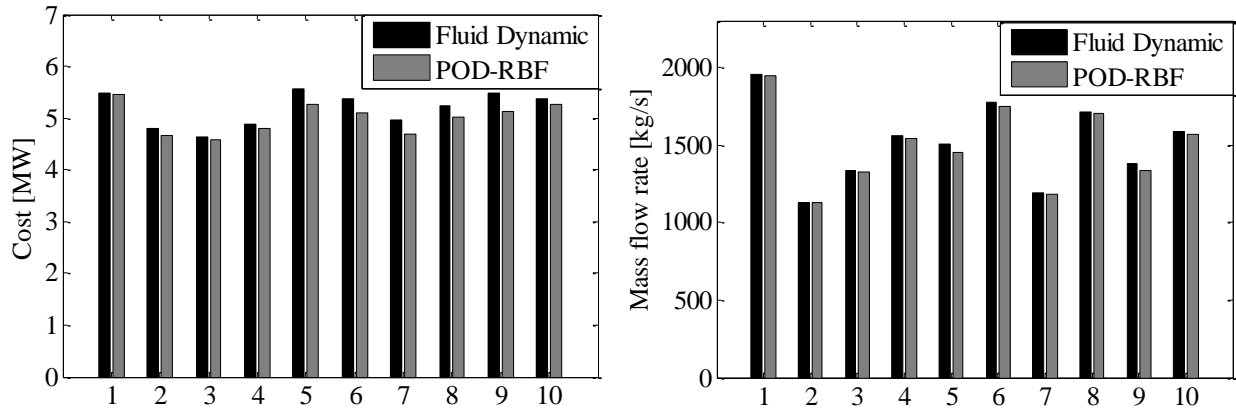


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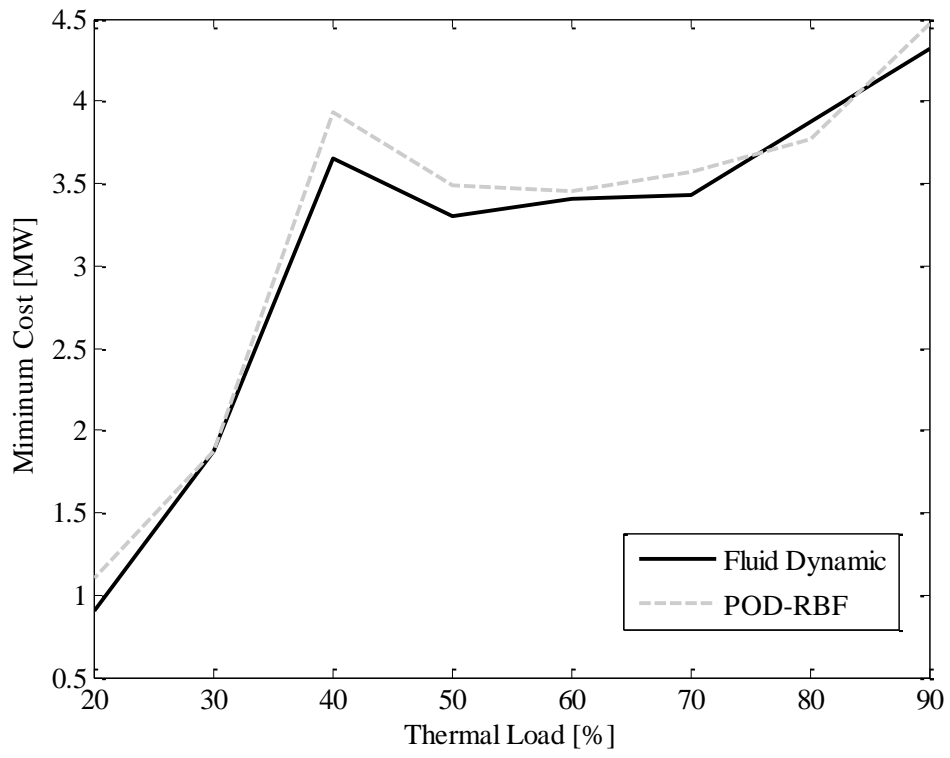
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Figure 3: Test for Fluid dynamic model simulation capability: comparison with measured data



540 Figure 4: Test for POD simulation capability with 10 random cases a) pumping costs b) mass flows rate

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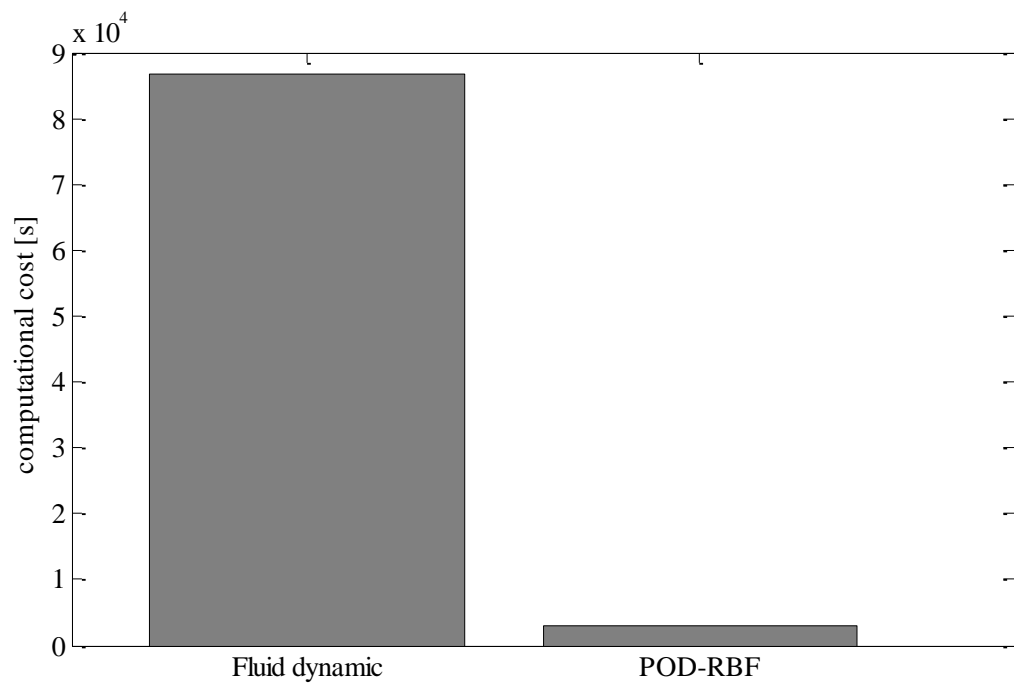


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Figure 5: Best cost comparison

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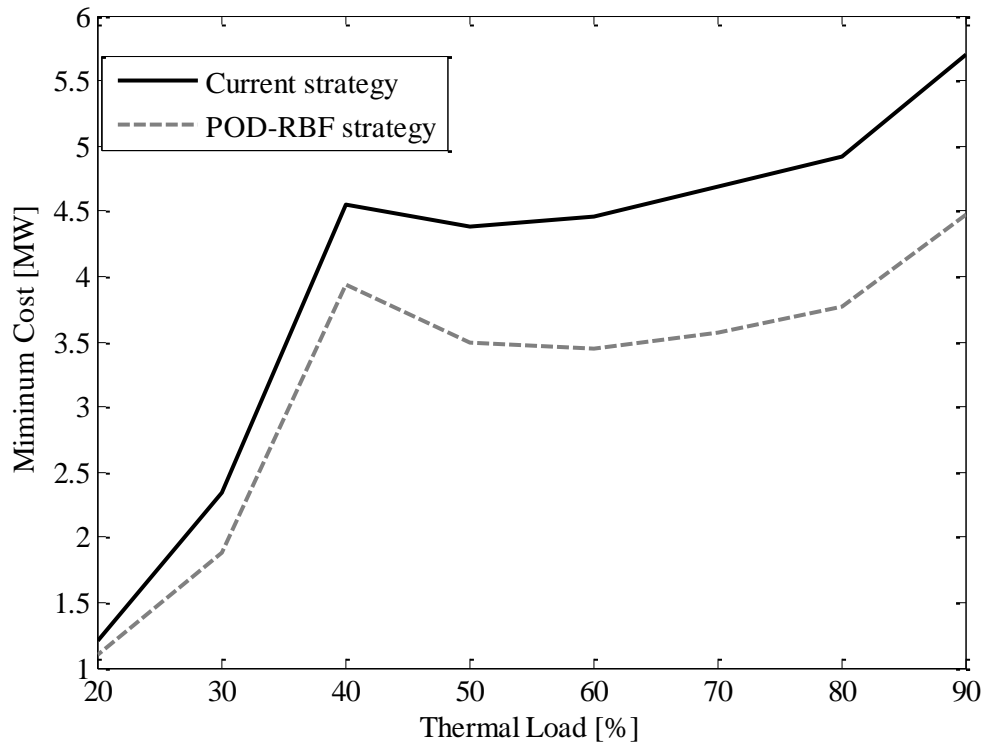


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Figure 6: Computational costs comparison

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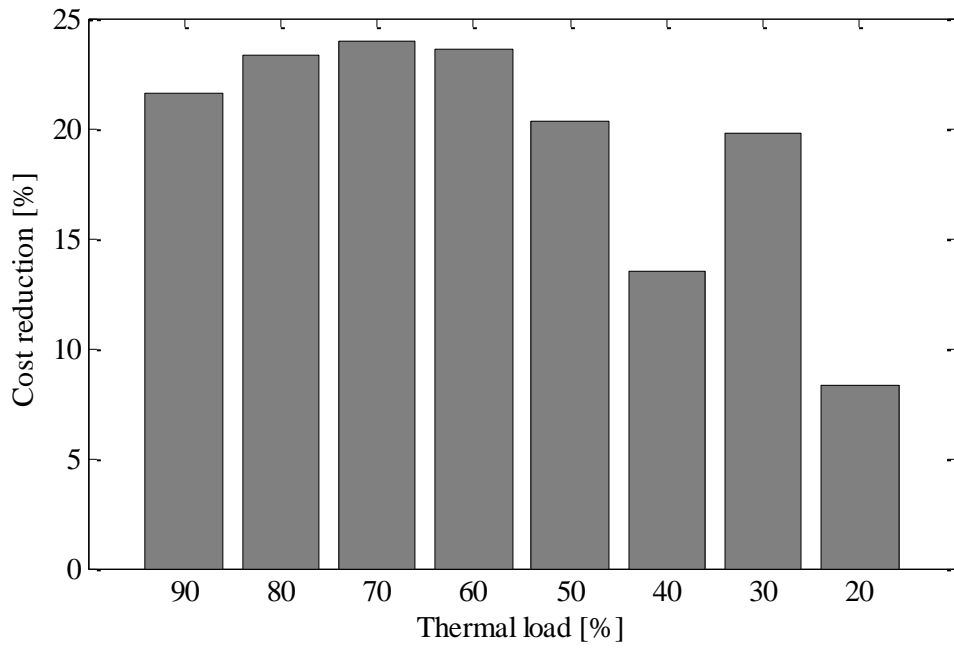


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Figure 7: Energy consumption with current and optimized pumping strategy

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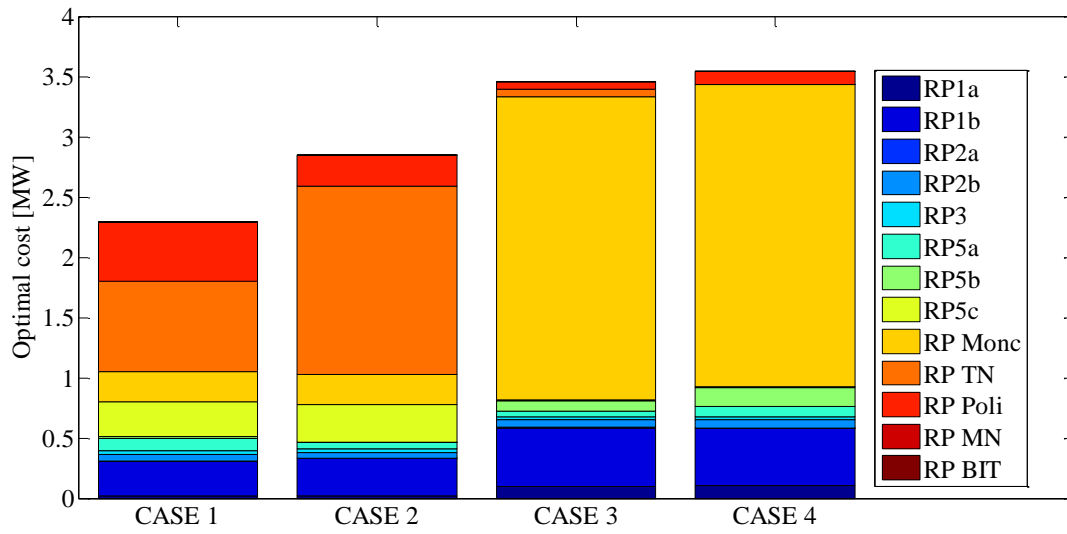


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555 Figure 8: Energy cost reduction due to use of POD-RBF method instead of current pumps control strategy

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Figure 9: Optimal pumping costs with different plants start up strategy at constant load

560

561

| <b>Plant</b>   | <b>Acronym</b> | <b>Power [MW]</b> | <b>Type</b>               |
|----------------|----------------|-------------------|---------------------------|
| Moncalieri     | Monc.          | 520               | Cogeneration (two groups) |
|                |                | 141               | Boilers                   |
| BIT            | BIT            | 255               | Boilers                   |
| Mirafiori Nord | M.N.           | 35                | Boilers                   |
| Politecnico    | Poli.          | 255               | Boilers                   |
|                |                | 60                | Storage                   |
| Torino Nord    | T.N.           | 220               | Cogeneration              |
|                |                | 340               | Boilers                   |
|                |                | 150               | Storage                   |

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Table 1. Characteristics of the thermal plants



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|      | <b>P<sub>max</sub> [bar]</b> |
|------|------------------------------|
| RP1a | 7                            |
| RP1b | 7.5                          |
| RP2a | 6.5                          |
| RP2b | 7                            |
| RP3  | 6                            |
| RP5a | 5                            |
| RP5b | 5                            |
| RP5c | 5                            |

567

568

Table 2. Maximum pressure values for the each booster pumping stations

569

570

| <b>CASE 1</b>            | <b>CASE 2</b>                           | <b>CASE 3</b>               | <b>CASE 4</b>               |
|--------------------------|---|-----------------------------|-----------------------------|
| Monc. Cog. Group<br>1    | Monc. Cog. Group<br>1                   | Monc. Cog. Group 1 and<br>2 | Monc. Cog. Group 1 and<br>2 |
| T.N. Cog.<br>Politecnico | T.N. Cog.<br>T.N. Boiler<br>Politecnico | T.N. Boiler<br>Politecnico  | Politecnico                 |

571

572

Table 3. Power plants start up strategies considered

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575

|                        | <b>Optimal Cost</b><br><b>[W]</b> |
|------------------------|-----------------------------------|
| No malfunctions        | 3.57                              |
| Malfunction in pump 1a | 3.63                              |
| Malfunction in pump 1b | 4.06                              |
| Malfunction in pump 2a | 3.58                              |
| Malfunction in pump 2b | 3.57                              |
| Malfunction in pump 3  | 3.59                              |
| Malfunction in pump 5a | 3.58                              |
| Malfunction in pump 5b | 3.60                              |
| Malfunction in pump 5c | 3.58                              |

576

577

Table 4. Minimum costs in case of malfunctions

578