LAGRANGE: An experiment for testing general relativity in the inner solar system

LAGRANGE: an Experiment for Testing General Relativity in the Inner Solar System

Angelo Tartaglia*, David Lucchesi†, Matteo Luca Ruggiero‡, Pavol Valko§
*Department of Applied Science and Technology,
Politecnico di Torino, Torino, Italy
Email: angelo.tartaglia@polito.it
†INAF/IASP and INFN, Rome, Italy
Email: david.lucchesi@iaps.inaf.it
‡Politecnico di Torino and INFN, Torino, Italy
Email: matteo.ruggiero@polito.it
§Slovak University of Technology in Bratislava, Bratislava, Slovakia
Email: pavol.valko@stuba.sk

***Abstract*—The paper discusses the possibility of locating four spacecraft in four out of the five Lagrange points of the Sun-Earth system and exchanging electromagnetic pulses among them. Including stations on Earth, various closed paths for the pulses are possible. Time of flight measurements would be performed. The time of flight difference between right- and left-handed circuits is proportional to the angular momentum of the Sun and the detection of the effect would reach accuracies better than 1% depending on the accuracy of the clock. The four points could also be used as "artificial pulsars" for a relativistic positioning system at the scale of the solar system. Additional interesting possibilities include detection of a galactic gravito-magnetic field, retrieving information about the interior of the Sun and refining the evaluation of the influence of the quadrupole moment of the Earth and of the Sun on the gravitational time delay.**

I. Introduction

The Lagrange points (L-points) are a feature of Newtonian gravity. They correspond to analytic solutions of the two bodies problem and localize the positions where gravitational attraction exactly counterbalances the centrifugal force on a test, negligible mass, body; furthermore the system of the L-points "rigidly" rotates with the two sources orbiting around their mass center. We present here the proposal of using the L-points of the Sun-Earth system as a physical framework for a number of measurements related to General Relativity (GR) and possible deviations thereof. The same set of L-points could furthermore be the basis for a relativistic navigation and positioning system at least at the scale of the inner Solar System. The Lagrange points are five and their traditional labelling is L_1, L_2, L_3, L_4, L_5; the geometry of the system is as sketched in Fig. 1.

Three points (L_1, L_2, L_3) are saddle points of the effective potential; in other words, the equilibrium there is unstable, however in the case of the Sun/Earth pair the instability is very mild. The remaining two points (L_4 and L_5) are real local minima so the equilibrium there is stable, though corresponding to a shallow potential well.

In General Relativity (GR) no exact solution of the two body problem exists and the presence of the analog of Lagrange points is not guaranteed. Hopefully, however, in the range of mass values typical of the Sun-Earth pair, approximate solutions for libration points (in practice L-points) do exist [1].

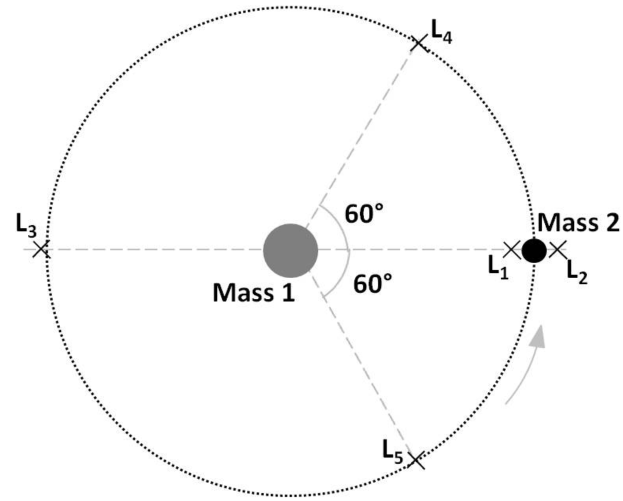

Fig. 1. Schematic view of the Lagrangian points of a two body system.

The properties of the system of the L-points makes it appropriate as basis for a physical reference frame at the scale of the inner solar system. Furthermore, considering the size of the polygon having the L's as knots, we may remark that the time of flight of electromagnetic signals going from one point to another is in the order of some 10 minutes or more; such long time may act as a multiplier for the tiny asymmetries originated by angular momentum effects predicted by GR.

We shall nickname LAGRANGE the proposal of exploiting time of flight measurements along a closed path having L-points as vertices, in order to take advantage of the asymmetric propagation produced by the angular momentum of the Sun in the case of two counter-propagating electromagnetic signals. The difference between the right- and left- time of flight (*tof*) would produce cancellation of the purely geometric component of the *tof*, so letting the above mentioned asymmetry emerge. The purpose of the experiment would be to measure the gravitomagnetic field of the Sun. We shall also consider more information in principle retrievable from the actual measurement of the asymmetry in *tof*, such as the possible presence of a galactic drag of the inertial frames (Lense-Thirring effect [3]), due to the angular momentum of the dark matter halo (if it is there).

The deployment of spacecraft in the L-points would also lend the opportunity of precise measurements of the gravitational (Shapiro) time delay. Using L_1 and L_2 and exchanging signals going back and forth between them, would permit to accurately analyze the gravitational time delay due to the mass of the interposed Earth, including the effect of the quadrupole moment of the planet and the asymmetric contribution of the terrestrial angular momentum.

Last but not least, properly equipped stations located in the L-points could act as artificial pulsars and be used for a Relativistic Positioning and navigation System (RPS) at the scale of the inner Solar System.

More possibilities will be mentioned in the Conclusion.

II. Solar Lense-Thirring drag

The Lense-Thirring effect (LT), or inertial frame dragging by a moving massive body, is a feeble effect of GR, first considered by Thirring [2] and Lense and Thirring [3] in 1918, while studying the influence of rotating masses on a test particle. LT may also be considered as a manifestation of gravito-magnetism i.e. of that typical component of the GR gravitational interaction resembling the magnetic field of moving charges.

So far, LT has been verified experimentally in a limited number of cases in the surroundings of the Earth with space based probes [4], [5]. A space based experiment is currently under way [6]. The present best accuracy reaches 5% [6].

With a different approach, the GINGER experiment is under study and preliminary test of the technology [7], [8], [9] at the Gran Sasso Laboratories in Italy. It exploits the old Sagnac effect [10]; in practice a ring laser is used to acquire the right/left asymmetry of the propagation time of light along the ring, due to the gravito-magnetic field of the Earth; what is actually measured are frequency and amplitude of a beat between two stationary counter-propagating light beams in the ring.

The use of the Sun-Earth Lagrangian frame would allow a measurement of the solar gravitomagnetic field (solar LT), exploiting the Sagnac approach, as for GINGER, but resorting to time of flight measurements rather than to interference phenomena or beat tones. For our purpose we may start from the external line element of a moving body (the observer is at rest in a non-rotating asymptotically flat reference frame); it is:

$$ds^2 = g_{00}c^2dt^2 + g_{ij}dx^idx^j + 2cg_{0i}dtdx^i \tag{1}$$

The Einstein summation convention for the indices is adopted; Latin indices range from 1 to 3.

Considering an electromagnetic signal in vacuo it is of course $ds = 0$, so that we may solve Eq. 1 for the coordinated *tof* dt. The next step is to integrate the *tof* of a signal along a path closed in space; right-handed and left-handed circuits give different results and the difference, expressed using the proper time of an observer at a finite distance from the central body, is [11]:

$$c\delta t = -2\sqrt{g_{00}}\oint \frac{g_{0i}}{g_{00}}dx^i \tag{2}$$

Explicitly, adopting a weak field approximation (appropriate almost everywhere in the universe and certainly in the Solar System) and using polar coordinates in space, for a non-rotating observer at rest with respect to the origin of the space coordinates it would be:

$$\begin{aligned} g_{00} &= 1 - 2\frac{m}{r} \\ g_{0\phi} &= 2\frac{j}{r}sin^2\theta \end{aligned} \tag{3}$$

The origin is in the center of the main body, i.e., in our case, the Sun; [1] for the ecliptic plane it is $\theta = \pi/2$. If M is the mass of the Sun, it is $m = GM/c^2 = 1475$ m. Similarly, if J is the modulus of the angular momentum of the Sun, it is $j = GJ/c^3 = 4.7144 \times 10^6$ m^2. Along the orbit of the Earth it is $m/r \sim 10^{-8}$ and $j/r^2 \sim 10^{-16}$ and the approximation order is indeed up to terms linear in j/r^2.

For practical purposes the most appropriate reference frame is a rotating one moving with the orbital velocity of the Earth. Performing the corresponding change of coordinates and preserving the first order approximation, an observer co-orbiting with the Earth (in the orbital plane, coinciding with the ecliptic plane) would find

$$c\delta t \simeq -4\oint \frac{j}{r}d\phi \tag{4}$$

Using a Lagrangian triangle (L_2, L_4, L_5) the final numerical result for the *tof* asymmetry is (see Ref. [12], [13] for details):

$$\delta t_{245} \simeq 4.30 \times 10^{-13} \text{ s} \tag{5}$$

which is well within the range of measurability, at least in terrestrial laboratory conditions. The challenge is to measure it in space.

[1] It should actually be in the barycenter of the Sun-Earth pair, but the difference should be discussed among the perturbations of the spherically symmetric system.

A. Retrievable information on the interior of the Sun

A measurement of the solar LT could also provide interesting information on the internal structure of the Sun.

Our star is of course not rotating as a rigid body. In the current scientific wisdom about the Sun there is the conviction that the differential rotation of the star triggers a near-surface layer of rotational shear, known as *tachocline*, where large-scale dipole magnetic fields are generated by dynamo action, ultimately leading to the 11-year solar cycle of sunspots [14]. The position and size of the *tachocline* is not directly observable, however, helioseismology observations have provided indirect evidence about this boundary layer between the radiative interior and the differentially rotating outer convective zone. The data mainly come from the Solar and Heliospheric Orbiter (SOHO) and the Solar Dynamics Observatory (SDO) probes [15], [16]. The estimated location of the shear layer at Sun's equator is (0.693 ± 0.002) solar radii, i.e. beneath the convection zone base, and with a width of 0.04 solar radii. Using Sun's density profile based on the Standard Solar Model [17], [18], the *tachocline* itself should contribute at the level $\sim 0.5\%$ to the total angular momentum of the Sun, i.e. to the source of the LT field. In order to verify the presence of this component, the measurement to be performed by LAGRANGE should have an accuracy of at least 1% which id undoubtedly challenging but not impossible. A periodic low frequency temporal variation of the LT field strength would then open another window for Sun interior studies.

B. Relevance of the measurement of a possible galactic gravitomagnetic field

It is worth mentioning that the LAGRANGE experiment could also reveal the possible existence of a galactic gravito-magnetic field. If the origin was in the baryonic mass distribution the corresponding effect would be totally negligible (gravito-magnetic field intensity $\sim 10^{-22}$ s^{-1}); the orders of magnitude however may change if a dark matter rotating halo is taken into account. Such a halo is commonly postulated to account for a number of observed phenomena such as the speeds of individual galaxies in galaxy clusters or the flat rotation curves of stars in galaxies, including the Milky Way. The nature of the "dark matter" in the halo is unknown and under debate, but its size has to be impressive if it is to justify the observed behaviours. Averaging over the whole visible universe the mass of dark matter is expected to be approximately five times the mass of the ordinary (visible) matter; however inside clusters and galaxies the relative abundance can be much greater, up to tens or even hundred times the baryonic mass component (see Ref. [19], [20] for the Milky Way). Whatever dark matter is, if it interacts gravitationally with visible matter, it is reasonable to conjecture that the halos rotate and consequently possess angular momentum, whence a contribution to a galactic gravito-magnetic field is in order. Recalling the proportions we mentioned, the contribution of the dark halo to galactic gravito-magnetism would even be dominant and could consequently no more be negligible. Consider then that a gravito-magnetic field could provide (through a mechanism similar to the Lorentz force of classical electromagnetism) an additional centripetal acceleration to the stars in a galaxy: the anomalous rotation velocities would depend on a combination of the simple (gravito-electric) attraction and of a Lorentz-like gravito-magnetic acceleration. In the case of the solar system, the peripheral velocity around the center of the Milky Way is $\sim$ 220 km/s; considering a realistic ordinary mass distribution model, $\sim$ 30 km/s are in excess and need some additional force to be explained [21], [22]. A galactic gravito-magnetic field intensity $\sim 10^{-16}$ s^{-1} would suffice. To verify such a possibility would be an important and relevant result for LAGRANGE.

III. Gravitational time delay

The LAGRANGE configuration lends the opportunity to refine measurements of the gravitational (Shapiro) time delay. The effect is commonly described, using a Schwarzschild metric [23], as being related to the gravito-electric field of GR [24]; the geometric influence of the deflection of the light rays (gravitational lensing) within the Solar System is safely negligible. The Shapiro time delay has been verified, so far, with an accuracy of 0.1% [25], [26], [27], [28] and the measurements have also allowed to put an upper limit to the ppN parameter γ: $\gamma - 1 \leq 2 \times 10^{-5}$ [29].

Now, using for instance a propagation line between the triangular L_4 and L_5 points the closest approach to the Sun would be 0.5 Astronomical Units (AU)[2] thus avoiding the problems encountered with the solar corona when the rays are grazing the Sun. Using instead the path between the collinear L_1 and L_2, having the Earth in between, would provide a measurement of the delay due to our planet. From the viewpoint of a co-orbiting observer the effective gravitational potential of the delaying body is:[3]

$$U \simeq -\frac{GM}{r}\left(1 - J_2\left(\frac{R}{r}\right)^2 \frac{3\left(\frac{z}{r}\right)^2 - 1}{2}\right), \qquad (6)$$

where r is the distance from the origin in the equatorial plane, z is the coordinate above that plane, R and J_2 are, respectively, the radius and quadrupole moment of the body.

Now the symmetry is no longer the simple Schwarzschild symmetry. Rather, axial symmetry and steady rotation rate is assumed. In weak field approximation the final delay along a trajectory grazing the central body is:

$$\Delta t_{prop} \simeq \frac{y_2 + y_1}{c} + \frac{2GM}{c^3} \ln\left(\frac{4y_1y_2}{b^2}\right) + \frac{2GM}{c^3}\left(\frac{R}{b}\right)^2 J_2 \pm \frac{4GJ}{c^4 b} + \ldots, \qquad (7)$$

The notation is the typical one for the description of the Shapiro delay, with a y axis passing through the delaying body

[2]An Astronomical Unit is ~ 150 million km.

[3]We considered only the main contribution, that arises from the first even zonal harmonic, with respect to the deviation from the spherical symmetry in the mass distribution of the Earth.

and parallel to the propagation line and an x axis perpendicular to y in the plane of the propagation (the equatorial plane, by default). If the rays are assumed to graze the massive body, as said, the minimal distance is called b and coincides with the radius of the source; y_1 and y_2 are the starting, respectively, arrival point along y. The first term on the right of Eq. 7 is the delay one would have in a flat space-time; the next is due to the gravito-electric field; the third calls in the quadrupole moment of the source J_2; the fourth depends on the angular momentum of the massive body. Eq. 7 is a special case of the results obtained in [30], [31]. As it is evident, the angular momentum of the source introduces an asymmetry between rays passing on the right or on the left of the main body. Doing the experiment between L_1 and L_2 would result in a better knowledge both of the quadrupole moment and of the angular momentum of the Earth. In fact, in the case of our planet, considering that the distance between L_1 and L_2 is ~ 3 million km, the propagation time would be:

$$\Delta t_{prop} \simeq (10\ \mathrm{s}) + \left(3.6 \times 10^{-10}\ \mathrm{s}\right) + \left(3.2 \times 10^{-14}\ \mathrm{s}\right) \pm \left(3 \times 10^{-17}\ \mathrm{s}\right) + \ldots, \tag{8}$$

The values are once more in the range of the possibilities for LAGRANGE.

IV. Relativistic positioning

An intrinsically relativistic positioning system (RPS) has been proposed and is described in [32], [33]. It is based on the local timing of at least four remote independent sources of electromagnetic pulses; the essence of the method is graphically presented in Fig. 2. Successive pulses correspond, in four dimensions, to a family of hypersurfaces (but they could also be periodic equal phase hypersurfaces) that covers space-time by a regular four-dimensional lattice. The world-line of an observer intersects the walls of successive cells of the lattice; the proper time interval measured by the observer between consecutive crossings provides the basic information. Counting the pulses (after identifying the various sources) and applying a simple linear algorithm it is possible to calculate the coordinates (including time) of the receiver in the fiducial reference frame [33].

In practice the null coordinates of the receiver along the light cones of the emitters are the sum of an integer part, which is obtained simply counting the successive arriving signals from each source, and a fractional part which is calculated, knowing the emission frequency, from geometrical proportions applied to sequences of proper intervals between arrivals. Once the quartet of null coordinates for a given space-time position is at hands, a straightforward projection converts them into practical coordinates in the fiducial reference frame. In the case of LAGRANGE the beacons would be located at the L-points L_1, L_2, L_4 and L_5.[4] The advantage would be that the L-frame rigidly rotates together with the Earth and the positions of the emitters in principle do not change in time, so that they constitute a sound natural fiducial reference frame; the situation is better than for all positioning systems based on satellites or other moving vehicles. In a sense, the L-points here resemble pulsars, which are regular periodic emitters occupying (almost) fixed positions in the sky. Pulsars have indeed been considered as forming a fiducial reference frame and proposed as emitters for positioning in space [34]. We may speak of the stations of LAGRANGE as "artificial pulsars".

[4] L_3 has not been considered since it is on the other side of the Sun and not visible from Earth, though it can be seen from L_4 and L_5.

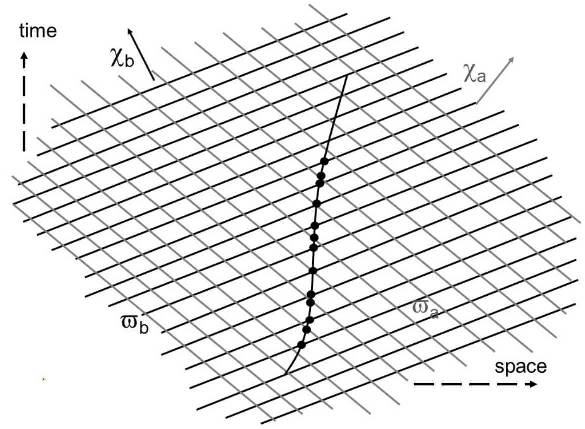

Fig. 2. Bidimensional example of the positioning method outlined in the text. The χ's are the null wavevectors of the signals coming from the two (four in the full space-time) independent sources. The ϖ's label the null wavefronts of the pulses. The wiggling line is the worldline of the receiver. The dots show some of the intersection points (reception of a signal). The relevant quantity is the length (i.e. proper time span) between successive reception events.

The distances among the stations in the L-points, range between ~ 1.5 million km (distance of L_1 and L_2 from Earth) and $\sim$ 150 million km (distance of L_4 and L_5). The size of the frame guaranties a coverage of the whole inner Solar System. The optimal configuration, in order to minimize the geometrical dilution in the positioning, would require one of the stations to be located out of the ecliptic plane, which of course is not the case; it is however true that most space missions are kept in the ecliptic plane so that, in principle, three rather than four stations would be enough and the geometric effect is generally small, excepting limited "wakes" along the lines containing a couple of emitters.

A problem is that the spacecraft carrying the emitting devices would in general not coincide with the corresponding Lagrange point, but would rather orbit around the point on stable (L_4 and L_5) or on halo or weakly unstable Lissajous orbits (L_1 and L_2). The final accuracy of the positioning would depend mainly on the accuracy with which the instantaneous position on the orbit is known or, to say better, on the stability of the path of the signals during a complete round trip.

The other limiting factor for the final result is the quality of the clock used by the receiver and the stability of the emission frequency: in principle a clock fit for a few 10^{-10} s accuracy attains also a centimeter accuracy in determining the position of the receiver (not considering other external perturbations).

V. Conclusion

We have illustrated the proposal of using the system of the Lagrange points of the Sun-Earth system for various experiments and applications.

The main proposal we have put forth is the measurement of the inertial frame dragging (Lense-Thirring effect) caused by the angular momentum of the Sun. The technique to be exploited is molded on the Sagnac effect, determining the time of flight asymmetry along a closed path whose edges are the L-points, travelled in opposite directions by electromagnetic signals. We have seen that using, for instance, L_2, L_4 and L_5, the time of flight difference would be in the order of a few 10^{-13} s, within the feasibility range of existing technologies. The direct detection of the LT effect of the Sun, besides adding a measurement of a gravito-magnetic phenomenon *per se* to the experiments made in circumterrestrial or planned in terrestrial environments, would give the possibility to extract interesting information on the interior of the Sun. We have also discussed the relevance of a possible detection of a galactic gravito-magnetism field; its presence could be evidenced by the envisioned L-points configuration at the scale of an AU. A special interest of a possible galactic LT effect is connected with the dark matter halo of the Milky Way, its consistency and, possibly, angular momentum.

The possibility to measure relativistic time delays both from the Sun and from the Earth has been discussed and the worked out numerical values show that the measurements would be within the range of possibilities offered by current technologies; the experiment would also lend the opportunity to determine the size of the contribution of the quadrupole moment, J_2, both of the Sun and of the Earth.

Passing to a practical application of the L-points system, we have presented and commented a relativistic positioning system at the scale of the full orbit of the Earth. Once more the configuration of the system, its stability in time and its being tied to the orbital motion of the Earth, lend the opportunity of building a positioning and navigation system that could profitably be used by all future space missions, at least in the inner solar system.

Of course all the above is possible provided one can know the actual position of each spacecraft with respect to its L-point and keep track of it in time. This point needs careful attention, but can profit of the experience gained by the numerous missions already deployed in L_1 and L_2, such as WMAP, the Herschel space observatory, Planck (all concluded) and now Gaia, in L_2; the Deep Space Climate Observatory, the Solar and Heliospheric Observatory (SOHO) and LISA Pathfinder, in L_1.

Once LAGRANGE would have been deployed, there are indeed many more opportunities it could offer for fundamental physics depending on the equipment one would be able to put on board the spacecraft. It is just the case to mention the possibility to detect gravitational waves (GW). The size of the experimental setup would indeed be appropriate. An option would be to exploit signals exchanged between the L-points adopting a zero-area Sagnac interferometer scheme [35]. Furthermore, considering again the size and adopting this time a wide area configuration of the light paths (as the triangle $L_2-L_4-L_5$), we should remember that GW's do carry angular momentum also. The response of the system would strongly depend on the relative orientation, but in principle a GW impinging orthogonally on the ecliptic plane, should superpose a transient asymmetry of the times of flight, on the continuous signal due to the solar (and galactic) LT-drag.

Summing up, the idea of using a set of the Lagrangian points of the Sun-Earth system (from two, to four at a time) and measuring the flight times of electromagnetic signals exchanged between spacecraft located in the L-points, turns out to be in the range of existing technologies and is appealing and would be very fruitful for fundamental physics experiments related to tests of GR and possible deviations from it, giving also information concerning the Sun, the Earth and the Milky Way. To pass from proposal to reality an undoubtedly huge effort is required to set up the missions needed to carry and locate the spacecraft at the L-points (which could be done progressively, performing different experiments gradually while the stations are launched); to properly equip them; then to control the system and perform the measurements. We think it could be rewarding to try.

References

[1] O. Perdomo, arxive: 1601.00924v1, 2016
[2] H. Thirring, *Phys. Z.*, **19**, 33, 1918
[3] J. Lense and H. Thirring, *Phys. Z.*, **19**, 156, 1918
[4] C. W. F. Everitt *et al.*, *Phys. Rev. Lett.*, **106**, 221101, 2011
[5] I. Ciufolini *et al.*, *Eur. Phys. J. Plus*, **126**, 1-19, 2011
[6] I. Ciufolini *et al.*, *Eur. Phys. J. C*, **76**, 120, 2016
[7] F. Bosi *et al.*, *Phys. Rev. D*, **84**, 122002, 2011
[8] A. Di Virgilio *et al.*, *C. R. Physique*, **15**, 866-874, 2014
[9] A. Tartaglia, A. Di Virgilio, J. Belfi, N. Beverini and M.L. Ruggiero. *Eur. Phys. J. Plus*, **132**: 73 2017
[10] M. G. Sagnac, *C. R. Acad. Sci. Paris*, **157**, 708-710, 1913
[11] M. L. Ruggiero and A. Tartaglia, *Eur. Phys. J. Plus*, **130**, 90, 2015
[12] A. Di Virgilio, *et al.*, *Int. J. Mod. Phys D*, **19**, p. 2331-2343, (2010)
[13] A. Tartaglia, D. Lucchesi, ML. Ruggiero and P Valko, arXiv: 1701.08217
[14] D. W. Hughes, R. Rosner, N. O. Weiss, *The Solar Tachocline*,Cambridge University Press, May 31, 2007
[15] *Proceedings of the SOHO 14 / GONG 2004 Workshop*, (2004), ESA SP-559, October 2004
[16] J. Christensen-Dalsgaard (2002). "Helioseismology", *Reviews of Modern Physics*, **74** (4), 10731129
[17] J. N. Bahcall, *Neutrino Astrophysics*, Cambridge University Press (1989)
[18] S. Turck-Chieze, *Journal of Physics: Conference Series* **665** (2016) 012078
[19] G.M. Eadie, W.E. Harris, (2016), *The Astrophysical Journal*, **829**, 108, doi:10.3847/0004-637X/829/2/108
[20] P.J. McMillan, (2011), *Mon. Not. R. Astron. Soc.*, **414**, 2446.
[21] F. Iocco, M. Pato, G. Bertone, (2015), *Nature Physics*, Advance Online Publication, DOI: 10.1038/NPHYS3237.
[22] Gerhard, O. (2002), *Space Science Reviews*, **100**(1), 129-138.
[23] K. Schwarzschild. Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), 1916, Seite 189-196
[24] K. S. Thorne, New Frontiers of Physics; Fairbank, J.D.; Deaver, Jr., B.S.; Everitt, C.W.F.; Michelson, P.F., Eds., 1988, pp. 573-586
[25] I. I. Shapiro, *et al.* (1971). *Physical Review Letters*, **26**, 1132
[26] J. D. Anderson, P.B. Esposito, W. Martin *et al.*, (1975). *Astrophys. J.*, **200**, 221
[27] I. I. Shapiro, *et al.* (1977). *Journal of Geophysical Research*, **82**, 4329
[28] R. D. Reasenberg, *et al.* (1979). *The Astrophysical Journal*, **234**, L219-L221

[29] B. Bertotti, L. Iess, P. Tortora, (2003). *Nature*, **425**, 374-376. doi:10.1038/nature01997
[30] M. L. Ruggiero, A. Tartaglia, (2002). Gravitomagnetic effects. *Nuovo Cim.* B, **117**, 743-768
[31] I. Ciufolini, F. Ricci, (2002). *Classical and Quantum Gravity*, **19**, 3863-3874
[32] A. Tartaglia, *Acta Futura*, issue 7, 111-124, 2013
[33] A. Tartaglia, M. Ruggiero, E. Capolongo, *Advances in Space Research*, **47**,645-653, 2011
[34] M. L. Ruggiero, E. Capolongo and A. Tartaglia, *Int. J. Mod. Phys. D*, **20**, 1025-1038, 2011
[35] K.-S.Sun, M. M. Fejer, E. Gustafson, and R. L. Byer, (1996). *Physical Review Letters*, **76**(17), 3053-3056.