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Modelling framework of pedestrian-footbridge interaction in vertical direction / Venuti, Fiammetta; Racic, Vitomir; Corbetta, Alessandro. - In: PROEDIA ENGINEERING. - ISSN 1877-7058. - ELETTRONICO. - 199:(2017), pp. 2901-2906. [10.1016/j.proeng.2017.09.588]

*Availability:*

This version is available at: 11583/2680346 since: 2017-09-15T11:41:27Z

*Publisher:*

Elsevier

*Published*

DOI:10.1016/j.proeng.2017.09.588

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X International Conference on Structural Dynamics, EURODYN 2017

# Modelling framework of pedestrian-footbridge interaction in vertical direction

Fiammetta Venuti<sup>a,\*</sup>, Vitomir Racic<sup>b</sup>, Alessandro Corbetta<sup>c</sup><sup>a</sup>*Politecnico di Torino, Department of Structural, Building and Geotechnical Engineering, Corso Duca degli Abruzzi 24, I-10129, Torino, Italy*<sup>b</sup>*Politecnico di Milano, Department of Civil and Environmental Engineering, Piazza Leonardo da Vinci 32, I-20133, Milano, Italy*<sup>c</sup>*Eindhoven University of Technology, Department of Applied Physics, P.O. Box 513, 5600 MB Eindhoven, The Netherlands*

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## Abstract

This study presents a modelling framework of human-structure interaction in the vertical direction, which integrates the three following key issues: crowd dynamics, pedestrian-structure interaction (PSI) and inter-subject and intra-subject variability of pedestrian walking loads. The framework comprises two main models: a microscopic model of crowd dynamics and a coupled dynamic model of the PSI. The latter is composed of three sub-models: a SDOF system having the dynamic properties of the empty structure, a SDOF system for each pedestrian and a stochastic force model. Each pedestrian SDOF moves along the footbridge following the trajectory and at the velocity simulated by the crowd model and is accompanied by a stochastic walking force time-history that accounts for inter- and intra-subject variability. Performance of the suggested modelling framework is studied through simulations of the vibration responses of four virtual footbridges due to different traffic scenarios.

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Peer-review under responsibility of the organizing committee of EURODYN 2017.

**Keywords:** human-structure interaction, human-induced vertical vibrations, footbridges, stochastic walking force

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## 1. Introduction

Modelling effect of multiple pedestrians on a lively structure should account for the following three aspects:

*Inter- and intra- subject variability of walking loading* [1], so called “ground reaction forces” (GRFs). It is nowadays accepted that traditional deterministic and perfectly periodic models of GRFs based on the Fourier approach are inadequate to describe reliably the actual random nature of walking excitation and can lead to considerable under- or over-estimation of the structural response [1–3].

*Human-structure interaction* (HSI), i.e. changes of dynamic properties of the empty structure due to the presence of human bodies. The HSI models proposed so far are of two different types. The first is so called “biomechanical model”, where the pedestrian is described as an inverted pendulum [4] moving along the bridge. The other type couples a single-degree-of-freedom (SDOF) model of a structure and a moving SDOF representing a pedestrian walking at a constant speed and pace rate [5,6]. The latter has gained wider popularity and attempts have recently been made

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\* Corresponding author. Tel.: +39-011-090-4844 ; fax: +39-011-090-4999.

E-mail address: [fiammetta.venuti@polito.it](mailto:fiammetta.venuti@polito.it)

to experimentally identify the dynamic properties of the human body during walking [7,8]. The loading can be expressed through biomechanic forces, that represents the excitation of the coupled system considering interaction [5,9], or through an external harmonic force attached to the base of the pedestrian SDOF and applied to the structure only [6]. In the first case, calibration of the force parameters still remains an open issue.

*Crowd dynamics*, i.e. walking trajectories and gait patterns of the pedestrians under the influence of the surrounding people and environment. Crowd models are usually divided into macroscopic models, which are based on the analogy between a flow of pedestrian crowd and a continuous flow of a fluid, and microscopic models, which describe time varying positions and velocities of each individual [10]. With respect to the macroscopic approach, microscopic models can account explicitly for the inter-subject variability of pedestrians.

This work presents a modelling framework that attempts to address all three key aspects described above. This framework could ideally be applied to any kind of lively structure with any kind of pedestrian traffic. However, in the present study the framework is demonstrated on straight footbridges without obstacles along the deck and occupied by unidirectional traffic.

## 2. Modelling framework

The scheme in Figure 1 outlines the proposed modelling framework. It involves two different physical sub-systems, i.e. the pedestrians and the structure. The system “Pedestrians” is mathematically described by three sub-models: (C) a crowd model, (P) a model of each individual pedestrian and (F) a force model of individual GRFs. The system “Structure” is modelled as a SDOF system (S). The pedestrian-structure interaction (PSI) is described by the three sub-models P, F and S analogously to the approach proposed by Caprani et al. [6]. In the case of human-induced vertical vibrations, the equations governing the crowd dynamics can be decoupled from those simulating vibration response. Therefore, the coupling is only between P and S models. The framework is elaborated in [11]. Some key features are outlined in the following paragraphs.

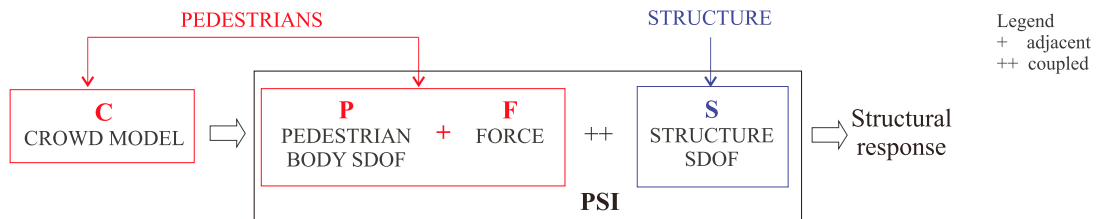


Fig. 1. Outline of the modelling framework

The crowd dynamics is described by a microscopic model, which describes the position  $\mathbf{x}$  and velocity  $\mathbf{v}$  of each pedestrian according to the following principles [10]: i) each pedestrian attempts to walk towards her/his target at so called *desired velocity*; ii) while walking, the desired velocity is modified by the interactions with neighbouring pedestrians and environment; iii) interactions between pedestrians are anisotropic in space and are restricted to a so called *sensory region*, corresponding to the pedestrian visual field; iv) interactions can be both repulsive and attractive. Here, only repulsive interactions are considered, since they prevail in case of crowded situations.

Let us consider a crowd of  $N$  pedestrians, walking on a footbridge deck of dimensions  $L \times B$ , which lies in the horizontal plane  $x - z$  (Figure 2a). The velocity of the  $i$ -th pedestrian  $\mathbf{v}_{p,i} = \{v_{xp,i}, v_{zp,i}\}$ , located in position  $\mathbf{x}_{p,i} = \{x_{p,i}, z_{p,i}\}$ , is given by the sum of the desired velocity  $\mathbf{v}_{d,i}$  and a social velocity  $\mathbf{v}_{s,i}$  that accounts for the interaction with the surrounding pedestrians:

$$\mathbf{v}_{p,i} = \mathbf{v}_{d,i} + \sum_{\substack{j=1 \\ j \neq i}}^N \mathbf{v}_{s,i}(\mathbf{x}_{p,i}, \mathbf{x}_{p,j}). \quad (1)$$

The desired velocity is expressed as the vector sum of a *free desired velocity*  $\mathbf{v}_{d,i}^f$  and *wall-repulsive velocity*  $\mathbf{v}_{d,i}^w$ . The latter is directed perpendicular to the wall/obstacles and reads as:

$$\mathbf{v}_{d,i}^w = \alpha \left[ \frac{1}{(d_w(\mathbf{x}_{p,i}) - d_0)^\beta} - \frac{1}{(d_{w,0} - d_0)^\beta} \right] \mathbf{n}_w, \quad (2)$$

where  $\mathbf{n}_w = \{0, \pm 1\}$  is the unit vector directed inwards, i.e. towards the bridge longitudinal axis  $x$ ,  $d_w$  is the distance between the pedestrian and the wall,  $d_0$  is a half the lateral width of the human body,  $d_{w,0}$  is the maximum distance from the wall at which the repulsion takes place, and  $\alpha$  and  $\beta$  are the parameters that characterize the repulsion.

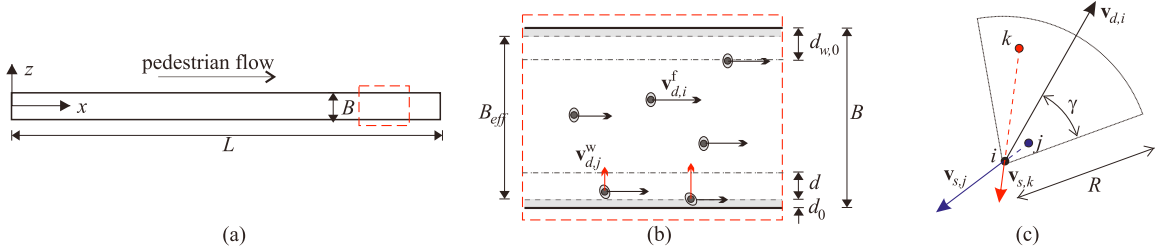


Fig. 2. Scheme of (a) the footbridge deck, (b) the desired velocity vectors and (c) sensory region

The social velocity reads:

$$\mathbf{v}_{s,i} = -c \left[ \frac{\mathbf{x}_{p,i} - \mathbf{x}_{p,j}}{|\mathbf{x}_{p,i} - \mathbf{x}_{p,j}|} \left( \frac{1}{|\mathbf{x}_{p,i} - \mathbf{x}_{p,j}|} - \frac{1}{R} \right) \right] \cdot h(\mathbf{x}_{p,i}, \mathbf{x}_{p,j}), \quad (3)$$

where the positive scalar  $c$  controls the intensity of the repulsive interaction and the function  $h$  limits the interaction within the sensory region. In this study, the sensory region is modelled as a circular sector area with radius  $R$  and angle  $2\gamma \in [0, \pi]$  (Figure 2b). Eq.s (2) and (3) can generate unnaturally high values of the velocity when a pedestrian is very close to the wall or to another pedestrian. Therefore, velocities generated through the crowd model are limited to  $2.5 \text{ ms}^{-1}$ . Table 1 summarises the adopted values of the crowd model parameters (cf. [11]).

Table 1. Suggested values of the parameters in Eq.s (2) and (3)

Parameter	$d_0$ [m]	$R$ [m]	$\gamma$ [°]	$v_m$ [ms <sup>-1</sup> ]	$v_{\text{std}}$ [ms <sup>-1</sup> ]	$\alpha$ [ms <sup>-1</sup> m <sup><math>\beta</math></sup> ]	$\beta$ [-]	$d$ [m]	$c$ [m <sup>2</sup> s <sup>-1</sup> ]
Value	0.225	2	85	1.34	0.24	20	5	0.35	0.1

The PSI is described by a dynamic system that couples a SDOF representing a structural vibration mode of interest (S) and  $N$  SDOFs (P) with adjoining vertical walking GRFs (F) representing  $N$  individual pedestrians (Figure 3). In the modal domain, the dynamics of the coupled system can be written in matrix form as:

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{F}, \quad (4)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$  and  $\mathbf{K}$  are the mass, damping and stiffness matrices, while  $\mathbf{y}$  and  $\mathbf{F}$  the displacement and force vectors.

Values of the dynamic properties of the human body  $m_{p,i}$ ,  $c_{p,i}$  and  $k_{p,i}$  are randomly assigned to each pedestrian according to the following statistics:  $m_{p,i}$  has Normal distribution with mean 75 kg and std 15 kg, while  $c_{p,i}$  and  $k_{p,i}$  follow uniform distribution within range  $[0, 400] \text{ Nsm}^{-1}$  and  $[2000, 13000] \text{ Nm}^{-1}$ , respectively (refer to [11] for a discussion about the adopted values).

Pedestrian GRFs (F) are modelled using the stochastic generator of vertical walking force signals by Racic and Brownjohn [12], which enables the introduction of both intra- and inter-subject variability. The key input parameters of the model are mean pace rate  $f_{p,\text{mean}}$  during footbridge crossing and durations of successive footfalls  $\Delta t$ . Both parameters are derived from pedestrian position  $\mathbf{x}_{p,i}(t)$  and walking velocity  $\mathbf{v}_{p,i}(t)$  and adopting the following relationship between pace rate and velocity:  $f_p = 2.93v_p - 1.59v_p^2 + 0.35v_p^3$ .



similar to the trend of  $a_{r,peak}$  observed in Figure 4b. Experimentally estimated values of the effective damping found in the literature are also reported in Figure 5b for comparison [14,15], showing a good agreement.

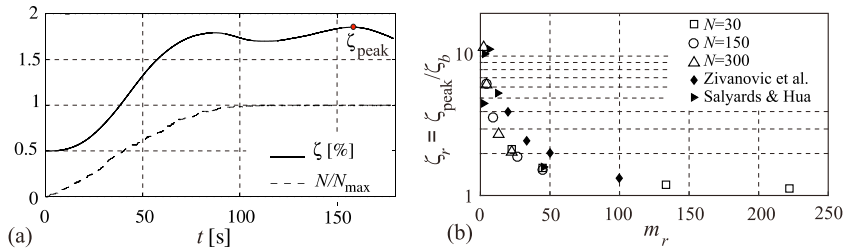


Fig. 5. (a) Example of time history of  $\zeta$  and  $N$  on the footbridge for  $N = 150$  and  $m_b = 50$  tons; (b)  $\zeta_r$  against  $m_r$

The adopted PSI model allows studying acceleration of the pedestrian SDOFs due to the bridge vibration. PDFs of peak values are shown in Figure 6 for each virtual footbridge and  $N$ . In all cases, the PDFs display a logarithmic trend, with the most frequent peak values at the lower range of acceleration. Literature relevant to “vibration” of pedestrian body mass is very rare, making difficult the verification of these findings. For instance, Qin et al [16] simulated one pedestrian crossing a footbridge ( $m_b \approx 1000$  kg,  $f_b = 2.375$  Hz) with a different PSI model and obtained a peak acceleration of the pedestrian centre of mass of around  $10 \text{ ms}^{-2}$ , which is within the range obtained in the present study.

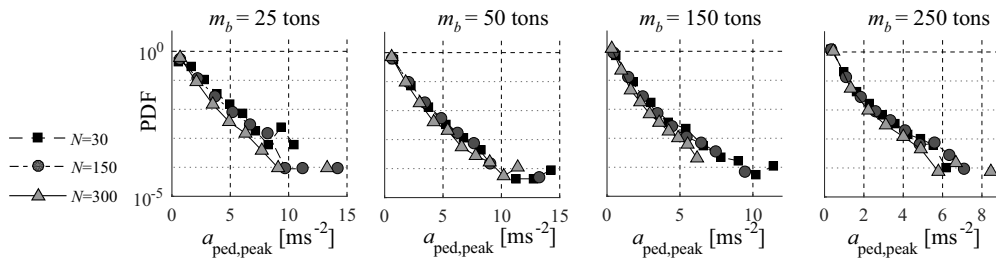


Fig. 6. PDFs of peak accelerations of pedestrian SDOFs corresponding to different values of  $N$  and  $m_b$

Finally, the acceleration response in the CPFS case is analysed in a detail considering the case of  $N=150$  pedestrians and  $m_b=50$  tons. Figure 7a shows the empirical PDF of the acceleration considering all the 50 simulated time histories, together with the fitted Normal distribution. It can be observed that the empirical PDF does not closely follow the Normal distribution. As a result, the peak per cycle accelerations do not follow the Rayleigh distribution (Figure 7b). As in Zivanovic [13], the Weibull distribution provides the best fit to the empirical PDF in the present study. The fitted Weibull distribution can be used to estimate the likelihood of exceeding any given acceleration limit. The most likely peak acceleration value (extreme peak)  $A_{E,peak}$  and the peak with a 5% probability of exceedance  $A_{E,95}$  during the return period  $T_r$  can be estimated through the following equations:

$$A_{E,peak} = \lambda \left[ -\ln \left( \frac{1}{n} \right) \right]^{1/\kappa} ; A_{E,95} = \lambda \left[ -\ln \left( 1 - 0.95^{1/n} \right) \right]^{1/\kappa} . \quad (5)$$

where  $\lambda$  and  $\kappa$  are respectively the scale and shape parameters of the Weibull distribution,  $n = T_r \cdot f_m$  is the number of peaks in the return period and  $f_m$  is the maximum frequency of oscillation.

#### 4. Conclusions

Performance of the proposed modelling framework are evaluated through simulations of the vibration response of virtual footbridges under different traffic scenarios. Comparison between the results corresponding to cases with and

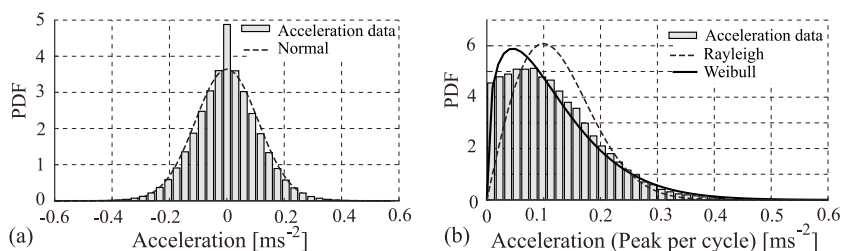


Fig. 7. PDFs of the instantaneous (a) and the peak per cycle (b) acceleration

without considering PSI allowed estimating the effective damping of the pedestrian-structure system. The obtained results are in line with the findings from other published studies that took completely different approaches to modelling PSI. The acceleration data were further studied using a statistical approach. Considering the inherent randomness in crowd dynamics, human bodies and the loading, such an approach is better suited for vibration serviceability assessment of pedestrian structures than a single acceleration value featuring in the relevant design guidelines. In conclusion, the proposed modelling framework provides a solid foundation for its improved versions in the future, as soon as more reliable models for crowd dynamics, pedestrian bodies and walking forces are made available.

## Acknowledgements

Project PRIN 2015-2018 “Identificazione e diagnostica di sistemi strutturali complessi”.

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