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# Ordinal aggregation operators to support the Engineering Characteristic prioritization in QFD

Franceschini Fiorenzo.<sup>1</sup>, Galetto Maurizio.<sup>1</sup>, Maisano Domenico.<sup>1</sup>, Mastrogiacomo Luca.<sup>1</sup>

<sup>1</sup> Politecnico di Torino, DIGEP, Corso Duca degli Abruzzi 24, 10129 – Torino, ITALY

e-mail: fiorenzo.franceschini@polito.it

## *Abstract*

Quality Function Deployment (QFD) is a consolidated management tool for supporting the design of new products/services and the relevant production/supply processes, starting from the so-called voice of the customer (VoC). QFD includes several operative phases, ranging from the VoC collection to the definition of the technical features of production/supply processes. The first phase entails the construction of the so-called House of Quality (HoQ), i.e., a planning matrix, which translates the Customer Requirements (CRs) into measurable Engineering Characteristics (ECs) of the product/service. One of the main goals of this phase is the definition of relationships between CRs and ECs, and the prioritization of these ECs, taking account of (i) their relationships with CRs and (ii) the importance of the related CRs. Given that data are collected from customers through questionnaires or interviews, both of these inputs are based on linguistic/ordinal scales. In the traditional approach, represented by the Independent Scoring Method (ISM), ordinal data are arbitrarily enriched with cardinal properties. The current scientific literature encompasses a number of alternative approaches but, even for most of them, cardinal properties are mistakenly attributed to data collected on ordinal scales.

This paper proposes a method based on a consolidated ME-MCDM (Multi Expert / Multiple Criteria Decision Making) technique, which is able to perform the EC prioritization without incurring in the aforementioned issue. This method is able to aggregate data evaluated on ordinal scales, overcoming controversial assumptions of data cardinality and avoiding any arbitrary and/or artificial “scalarization” of the data. On the other hand, its application is relatively simple and intuitional, compared to other proposed approaches alternative to the ISM, which often are conceptually complicated and difficult to implement. Furthermore, the proposed method can be effectively used when both CR importances and relationship matrix coefficients are rated on different ordinal scales and, being easily automatable, it can be effortlessly integrated into existing QFD software applications. In the paper, after a general description of the theoretical principle of the method, several application examples are presented and discussed.

**Keywords:** Quality Function Deployment, House of Quality, Customer Requirements, Engineering Characteristics, Independent Scoring Method, Ordinal scale, MCDM.

## *1 Introduction*

Quality Function Deployment (QFD) is a practical and effective tool for structuring the design activities for a new product/service and the related production/supply process, according to the real exigencies of customers [Akao, 1988; Franceschini, 2001; Zheng, Chin, 2005; Sousa-Zomer, Miguel, 2016]. Due to its practicality and effectiveness, QFD is universally recognized as a strategic approach to pursue customer

satisfaction. The large diffusion of this tool is also proved by the large amount of scientific literature produced over the years [Carnevalli and Cauchick Miguel, 2008; Cordeiro, Barbosa, Trabasso, 2016].

Many empirical studies demonstrated that the correct implementation of QFD may bring significant improvements in the development of products/services, including earlier and fewer design modifications, fewer start-up issues, improved cross-functional communications, improved product/service quality, reduced time and cost for product/service development, etc. [Biren, 1998; Chan, Wu, 2002.a; Chan, Wu, 2002.b; Lager, 2005; Zheng, Chin, 2005; Carnevalli and Cauchick Miguel, 2008].

From a procedural point of view, QFD is based on four phases, which deploy Customer Requirements (CRs) throughout a structured planning process [Akao, 1988]. Each phase is supported by a specific matrix, which establishes a relationship between variables of different nature. A schematic structure of these four phases and the relevant matrices are reported in Fig. 1 [Akao, 1988; Franceschini, 2001].

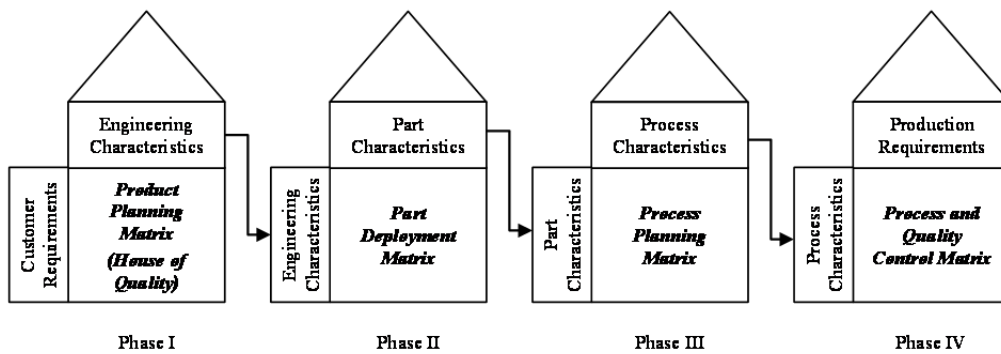


Figure 1. Scheme of the four phases of QFD. Adapted from [Lager, 2005].

Special attention is given to Phase I, characterized by the construction of the so-called Product Planning Matrix, or House of Quality (hereafter abbreviated as HoQ). The goal of this phase is turning the CRs into a set of Engineering Characteristics (ECs) and prioritizing these ECs, taking account of (i) their relationships with CRs and (ii) the importance of the related CRs. In this process, ordinal data, collected from customer questionnaires and/or interviews, are usually “promoted” to cardinal data, relying on two controversial assumptions [Roberts, 1979; Van de Poel, 2007]:

- The importance of each CR, generally expressed on an *ordinal* scale, is artificially encoded in the form of a number, expressed on a *cardinal* scale (i.e., *interval* or *ratio* scale) [Wasserman, 1993; Franceschini and Rupil, 1999; Franceschini, 2001].
- The prioritization of ECs is traditionally carried out by the *Independent Scoring Method* (ISM) [Akao, 1988; Franceschini, 2001], which requires the numerical conversion of the (qualitative) ordinal relationships between CRs and ECs into numbers.

In order to overcome these two assumptions, several alternative techniques have been proposed in the scientific literature; e.g., Multi Criteria Decision Making (MCDM) techniques, Borda’s method, techniques based on pairwise comparisons, techniques based on fuzzy logic, hybrid methods, etc. [Franceschini and Rossetto, 1995; Dym and Wood, 2002; Han et al., 2004; Wu, 2006 ; Wu, Shieh, 2006 ; Yan et al., 2013; Nahm et al. 2013; Franceschini et al., 2015; Chen and Chen, 2014; Jin et al., 2014 ; Chun-Chieh et al., 2014 ; Iqbal et al., 2015; Jianga et al., 2015, Hosseini Motlagh, Behzadian, Ignatius et al., 2015].

Even if these methods often represent effective solutions to overcome the limitations due to the poor properties of linguistic scales, in most cases, they could not help but use arbitrary conversion of data to cardinal scales or introduce weighting functions and/or criteria which subjectively depend on the Decision Maker that is performing the analysis. This is the case, for example, of all those method based on fuzzy logic, in which a membership function should be defined [Yan et al., 2013 ; Hosseini Motlagh, Behzadian, Ignatius et al., 2015].

Other methods, such as for example Borda-like ones, requires a scalarization of the rank positions, while pairwise comparisons are all based on a kind of comparison metric, which is arbitrarily defined as much [Dym and Wood, 2002;]. Other more recent methods tried to establish the CRs' importances and the relationship between CRs and ECs through quantitative analysis based on statistical approaches (Markov chain, power law models, etc.), but, even in those cases, the controversy scalarization of the collected information is disguised in the mathematical formalisms [Wu, 2006 ; Wu, Shieh, 2006].

This paper proposes an alternative method to prioritize ECs, which overcomes the aforementioned assumptions. The method is able to deal with data expressed on *ordinal* scales, with no need to "promote" them to data expressed on *interval* or *ratio* scales [Roberts, 1979]. Being inspired by a technique proposed by Yager and Filev (1994) for multi-criteria decision-making problems, the new method can be classified as a ME-MCDM (Multi Expert / Multiple Criteria Decision Making) technique.

From a technical point of view, the method (i) extends the logic of the Boolean operators *Min* and *Max* to multilevel ordinal scales and (ii) uses the importances of CRs as linguistic quantifiers for weighting the impact of the relationship coefficients [Yager and Filev, 1994]. The final result is a prioritization of the ECs, in the form of a rank-ordering.

The remainder of this paper is organized into four sections. Sect. 2 briefly recalls the basic concepts on the first phase of QFD. Sect. 3 presents a conceptual and formal description of the new method, focusing on its advantages and limitations. Some practical examples are reported and discussed in Sect. 4. Sect. 5 discusses the new method, focusing the attention on its implications, limitations and possible future developments.

## ***2 Basic concepts on QFD***

The QFD approach consists of four phases which deploy the CRs throughout the design and development process of the product/service of interest (see Fig. 1). In the first phase, CRs are related to a set of ECs of the product/service. In the second phase, ECs are associated with a set of critical part characteristics, through the so-called Part Deployment Matrix. Then, the Process Planning Matrix relates the critical part characteristics to the relevant production processes. Finally, the Process and Quality Control Matrix defines suitable quality control parameters and methods to monitor the production process. These phases should be carried out by the members of a cross-functional team of experts (i.e., the so-called QFD team).

The first phase is fundamental for the success of QFD implementation [Franceschini 2001; Tontini 2007; Li, Tang et al. 2009; Li, Tang et al. 2010], as errors at this stage can propagate throughout the subsequent phases.

With reference to Fig. 2, the construction of the HoQ can be broadly structured into ten steps; for details, see Franceschini et al. (2015).

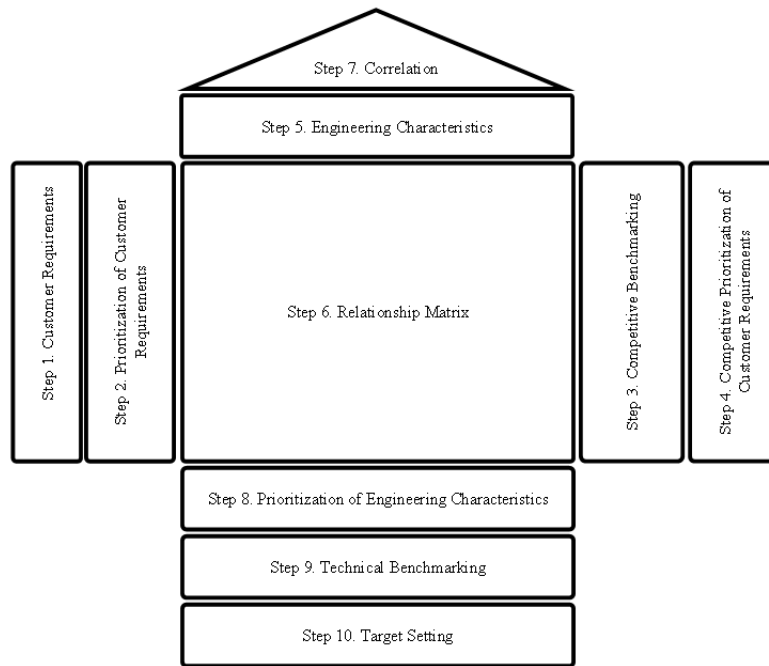


Figure 2. Main steps of House of Quality [Franceschini et al., 2015].

The focus of the present paper is on Step 8, which is aimed at prioritizing the ECs. To this purpose, several approaches are possible. The traditional method is the ISM [Akao 1988], which combines the importances of CRs and the data contained in the relationship matrix. The ISM can be subdivided in two operative steps. In the first step, the relationship matrix is turned into a cardinal matrix, according to an arbitrary convention: a typical approach is to define the ordinal relationships between CRs and ECs on four levels – i.e., absent, weak, medium and strong relationship – and encode them into four numerical coefficients, respectively 0, 1, 3 and 9. In the second step, the relative importance (or the relative weight) of each EC is evaluated through a weighted sum of the relative importances of CRs and the encoded relationship matrix coefficients, according to the following model [Akao 1988]:

$$w_j = \sum_{i=1}^n d_i \cdot r_{ij} \quad (1)$$

where:

$w_j$  is the importance of the  $j$ -th EC ( $j = 1 \dots m$ ),

$d_i$  is the importance of the  $i$ -th CR ( $i = 1 \dots n$ ),

$r_{ij}$  is the coefficient (0, 1, 3 or 9) corresponding to the relationship between the  $i$ -th CR and the  $j$ -th EC.

The cardinalization of ordinal data is not a trivial problem and it has been demonstrated that it can produce controversial results and drive to wrong decision [Franceschini and Rossetto, 1995; Franceschini and Rupil, 1999]. In fact, different numerical codifications of the ordered scale levels may lead to different rankings of ECs [Franceschini et al., 2015]. This can have a very negative impact on the use of QFD and deleterious consequences on the development of a new product both from the economical and the strategical point of view.

### 3 The proposed method

EC prioritization is aimed at selecting the ECs with a stronger impact on the most important CRs [Akao, 1988; Franceschini, 2001]. However, this prioritization should not alter the properties of the original data (i.e., CR importances and relationship matrix coefficients, both defined on ordinal scales) [Franceschini et al., 2015].

The proposed method is able to deal with ordinal data, with no need to introduce an artificial numerical conversion. As anticipated, it can be classified as a ME-MCDM (Multi Expert / Multiple Criteria Decision Making) technique [Yager, 1993].

The use of ordinal scales raises an important issue: while the distance between two elements is defined on cardinal scales (hence, sum and product operators may be applied), this is no longer true for ordinal scales [Roberts, 1979]. For this reason, the ISM and other prioritization techniques are rather questionable.

The proposed method is inspired by the work of Bellman and Zadeh (1970), lately “enriched” by Yager and Filev (1994) for the solution of MCDM problems. In the specific case of the QFD, the EC prioritization can be considered as a special decision-making problem: precisely, the CRs represent the decision *criteria* and the ECs represent the *alternatives* [Yager and Filev, 1994]; finally, the Relationship Matrix coefficients can be interpreted as *assessments* of each  $j$ -th EC ( $EC_j$ ), according to each  $i$ -th CR ( $CR_i$ ). The proposed method carries out an overall synthesis of these “assessments”, considering the CR importances as *weights* of the criteria.

Many examples of application of the method are presented and discussed in the scientific literature. The reported case studies demonstrated that it is particularly effective when the goal is to define a ranking or a prioritization of a set of elements/items by aggregating external information expressed on linguistic scales. This is the typical case of group decision problems, risk analysis or defects’ causes investigation [Park, Gwak, Kwun, 2011; Rodger, Pankaj, Gonzalez, 2014].

The approach can be organized in four steps:

- i) Definition of the scale levels for the importances associated with each  $i$ -th  $CR_i$ , ( $i = 1 \dots n$ ) and for the relationship matrix coefficients ( $r_{ij}$ ) between  $CR_i$  and  $EC_j$  ( $j = 1 \dots m$ ).

For simplicity, it is assumed that the importance associated with each CR is defined on an ordinal scale, with the same number of levels of the scale used for representing the relationship matrix coefficients. It will be shown later on that the method may be extended to scales with different number of levels.

Table 1 is a *correspondence map* between CR importances and relationship matrix coefficients, expressed on a 3-level ordinal scale ( $s = 3$ ).

Scale level	CR importance ( $d_i$ )	Importance value	Relationship matrix coefficient ( $r_{ij}$ )	Symbol
$L_1$	not (or weakly) important	1	no (or weak) relationship	(empty cell)
$L_2$	important	2	medium relationship	○
$L_3$	very important	3	strong relationship	●

Table 1. Correspondence map between CR importances and relationship matrix coefficients, expressed on a 3-levels ordinal scale ( $s = 3$ ).

ii) Data collection and construction of the relationship matrix.

iii) Implementation of the  $EC_j$  prioritization model:

$$w_j = \underset{i=1\dots n}{Min} \left\{ \underset{r_{ij}}{Max} \left[ \underset{d_i}{Neg} (d_i), r_{ij} \right] \right\} \quad (2)$$

where:

$w_j$  is the calculated importance of the  $j$ -th EC ( $j = 1 \dots m$ ),

$d_i$  is the importance of the  $i$ -th CR ( $i = 1 \dots n$ ),

$r_{ij}$  is the relationship matrix coefficient between  $CR_i$  and  $EC_j$ ,

$Min$  is the Minimum operator,

$Max$  is the Maximum operator,

$Neg(d_i)$  is the negation operator, defined as [Yager, 1993]:

$$Neg(L_k) = L_{s-k+1} \quad (3)$$

where  $L_k$  is the  $k$ -th level of the evaluation scale ( $k = 1 \dots s$ ).

It is worth noting that the resulting  $w_j$  values are defined on the same ( $s$ -level) ordinal scale, utilized for rating the CR importances and the  $r_{ij}$  coefficients.

iv) Determination of the EC prioritization, based on the weights calculated using Eq. (2). If two or more ECs have the same  $w_j$ , a more refined selection can be obtained through a further indicator:

$$T(EC_j) = Dim[A(EC_j)] \quad (4)$$

where the operator  $Dim[A(EC_j)]$  gives the number of elements contained in the set  $A(EC_j)$ , with

$$A(EC_j) = \{CR_i \mid r_{ij} > w_j\}.$$

This represents a refined investigation for estimating the dispersion in the resulting EC importance. Basically,  $T(EC_j)$  is the count of the CRs with relatively high  $r_{ij}$  coefficient (with respect to the EC importance value), related to the  $j$ -th EC. The meaning of  $T(EC_j)$  will be clarified in Sect. 4 by several practical examples.

Considering ECs with the same  $w_j$ , those with higher values of  $T(EC_j)$  can therefore be considered as the most important and the EC ordering can be refined.

In other terms, the rationale of the procedure is to consider those ECs with strong relationships with the most important CRs, as the most important ones. When two or more ECs have the same weight, a refined selection is performed using the  $T(EC_j)$  indicator.

From Eq.(2), it is possible to observe that low-importance CRs have little effect on the importance ( $w_j$ ) of a generic  $j$ -th EC. In fact, a CR with little importance entails a low importance rating  $L_k$  and therefore a high value of the negation of this value. Then, applying the  $Max$  operator, the highest value between the negation of the importance and the relationship coefficient is selected. For a given EC, all the values related to the

whole set of CRs are computed. Then, the *Min* operator extracts the smallest of these values. In this way, all the contributions from CRs with little importance are automatically cut off.

The result of the application of Eq. (2) is a balanced tradeoff between high-value relationship coefficients, related to the CRs with low importance, and low-value relationship coefficients, related to CRs with high importance.

It can be demonstrated that the model in Eq.(2) satisfies the properties of Pareto optimality, independence to irrelevant alternatives, positive association of individual scores with overall score and symmetry [Arrow and Raynaud, 1986; Yager, 1993].

An essential feature of this approach is that there is no need for numeric values and it does not force undue precision on the experts of the QFD team.

#### 4 Application examples

For the purpose of example, let us consider the design of a new model of a climbing safety harness. This example is already present in the scientific literature and may therefore represent an helpful benchmark for the application of the proposed method [Hunt, 2013; Franceschini et al., 2015].

The CRs and ECs, identified by customer interviews and a technical analysis by the QFD team, are reported in Tabs. 2 and 3 respectively.

<b>Customer Requirements (CRs)</b>		
Usability	Easy to put on	CR <sub>1</sub>
	Comfortable when hanging	CR <sub>2</sub>
	Fits over different clothes	CR <sub>3</sub>
	Accessible gear loops	CR <sub>4</sub>
Performance	Does not restrict movement	CR <sub>5</sub>
	Lightweight	CR <sub>6</sub>
	Safe	CR <sub>7</sub>
	Attractive	CR <sub>8</sub>

Table 2. CRs for the design of a new model of a climbing safety harness [Hunt, 2013].

<b>Engineering Characteristics (ECs)</b>	
Meets safety standards	EC <sub>1</sub>
Harness weight	EC <sub>2</sub>
Webbing strength	EC <sub>3</sub>
No. of clours	EC <sub>4</sub>
No. of sizes	EC <sub>5</sub>
Padding thickness	EC <sub>6</sub>
No. of gear loops	EC <sub>7</sub>

Table 3. ECs for the design of a new model of a climbing safety harness [Hunt, 2013].

Since the choice of  $s$  (i.e. the number of levels of the ordinal scale, in which CR importances  $d_i$  and  $r_{ij}$  values are defined) may impact on the results of the HoQ analysis, four distinct situations will be analyzed and discussed in the following sub-sections.



For each of these situations, the CR importances ( $d_i$ ) and the ( $r_{ij}$ ) coefficients of the relationship matrix are defined by the QFD team.

#### 4.1 Case of 3-level scale

Assuming  $s = 3$  and using the correspondence map in Tab.1, we obtain the relationship matrices reported in Figs. 3 and 4.

			Engineering Characteristics							
			EC <sub>1</sub>	EC <sub>2</sub>	EC <sub>3</sub>	EC <sub>4</sub>	EC <sub>5</sub>	EC <sub>6</sub>	EC <sub>7</sub>	
Customer Requirements		$d_i$								
	CR <sub>1</sub>	3		○			●			
	CR <sub>2</sub>	3		○			●	●		
	CR <sub>3</sub>	1				○	●			
	CR <sub>4</sub>	2								●
	CR <sub>5</sub>	3		○	●		●		○	
	CR <sub>6</sub>	2		●					○	○
	CR <sub>7</sub>	3		●		○				
CR <sub>8</sub>	1					●				

Figure 3. Relationship matrix for the design of a new model of a climbing safety harness. For details on symbols/abbreviations, see Tables 1, 2 and 3.

			Engineering Characteristics							
			EC <sub>1</sub>	EC <sub>2</sub>	EC <sub>3</sub>	EC <sub>4</sub>	EC <sub>5</sub>	EC <sub>6</sub>	EC <sub>7</sub>	
Customer Requirements		$d_i$								
	CR <sub>1</sub>	$L_3$	$L_1$	$L_2$	$L_1$	$L_1$	$L_3$	$L_1$	$L_1$	$L_1$
	CR <sub>2</sub>	$L_3$	$L_1$	$L_2$	$L_1$	$L_1$	$L_3$	$L_3$	$L_1$	$L_1$
	CR <sub>3</sub>	$L_1$	$L_1$	$L_1$	$L_2$	$L_3$	$L_1$	$L_1$	$L_1$	$L_1$
	CR <sub>4</sub>	$L_2$	$L_1$	$L_1$	$L_1$	$L_1$	$L_1$	$L_1$	$L_3$	$L_1$
	CR <sub>5</sub>	$L_3$	$L_2$	$L_2$	$L_3$	$L_1$	$L_3$	$L_2$	$L_1$	$L_1$
	CR <sub>6</sub>	$L_2$	$L_1$	$L_3$	$L_1$	$L_1$	$L_1$	$L_2$	$L_2$	$L_1$
	CR <sub>7</sub>	$L_3$	$L_3$	$L_1$	$L_2$	$L_1$	$L_1$	$L_1$	$L_1$	$L_1$
CR <sub>8</sub>	$L_1$	$L_1$	$L_1$	$L_3$	$L_1$	$L_1$	$L_1$	$L_1$	$L_2$	

Figure 4. "Transformed" relationship matrix, obtained from that in Figure 3, when using a 3-level ordinal scale for both CR importances and relationship coefficients. For details on symbols/abbreviations, see Tables 1, 2 and 3.

According to Eq. (3), the negations for the levels of a 3-point ordinal scale are:

$$Neg(L_1) = L_3, \quad Neg(L_2) = L_2, \quad Neg(L_3) = L_1.$$

Hence, the importance of EC<sub>1</sub> may be calculated using Eq. (2), as follows:

$$\begin{aligned}
w_1 &= \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg}(d_i), r_{i1} \right] \right\} = \\
&= \text{Min} \left\{ \text{Max} \left[ \text{Neg}(L_3), L_1 \right], \text{Max} \left[ \text{Neg}(L_3), L_1 \right], \text{Max} \left[ \text{Neg}(L_1), L_1 \right], \text{Max} \left[ \text{Neg}(L_2), L_1 \right], \right\} = \\
&= \text{Min} \left\{ \text{Max} \left[ \text{Neg}(L_3), L_2 \right], \text{Max} \left[ \text{Neg}(L_2), L_1 \right], \text{Max} \left[ \text{Neg}(L_3), L_3 \right], \text{Max} \left[ \text{Neg}(L_1), L_1 \right] \right\} = \\
&= \text{Min} \left\{ \text{Max} \left[ L_1, L_1 \right], \text{Max} \left[ L_1, L_1 \right], \text{Max} \left[ L_3, L_1 \right], \text{Max} \left[ L_2, L_1 \right], \right\} = \\
&= \text{Min} \left\{ \text{Max} \left[ L_1, L_2 \right], \text{Max} \left[ L_2, L_1 \right], \text{Max} \left[ L_1, L_3 \right], \text{Max} \left[ L_3, L_1 \right] \right\} = \\
&= \text{Min} \left\{ L_1, L_1, L_3, L_2, L_2, L_2, L_3, L_3 \right\} = L_1
\end{aligned}$$

The importances for the other ECs may be computed in the same way, obtaining the following results:

$$\begin{aligned}
w_2 &= \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg}(d_i), r_{i2} \right] \right\} = L_1 \\
w_3 &= \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg}(d_i), r_{i3} \right] \right\} = L_1 \\
w_4 &= \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg}(d_i), r_{i4} \right] \right\} = L_1 \\
w_5 &= \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg}(d_i), r_{i5} \right] \right\} = L_1 \\
w_6 &= \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg}(d_i), r_{i6} \right] \right\} = L_1 \\
w_7 &= \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg}(d_i), r_{i7} \right] \right\} = L_1
\end{aligned}$$

In this specific case, all the ECs obtain the same importance, hence the resulting ranking is:

$$EC_1 \approx EC_2 \approx EC_3 \approx EC_4 \approx EC_5 \approx EC_6 \approx EC_7$$

where symbol “ $\approx$ ” denotes the *indifference* relationship.

This “flattening effect” is mainly due to the low discriminating power of the method, when using scales with a small number of levels. A better discrimination of the ECs can be obtained, refining the analysis by means of the  $T(EC_j)$  indicators:

$$\begin{aligned}
T(EC_1) &= \text{Dim} \left[ A(EC_1) \right] = \text{Dim} \left[ \left\{ CR_i \mid r_{i1} > w_1 \right\} \right] = \text{Dim} \left[ \left\{ CR_5, CR_7 \right\} \right] = 2 \\
T(EC_2) &= \text{Dim} \left[ A(EC_2) \right] = \text{Dim} \left[ \left\{ CR_i \mid r_{i2} > w_2 \right\} \right] = \text{Dim} \left[ \left\{ CR_1, CR_2, CR_5, CR_6 \right\} \right] = 4 \\
T(EC_3) &= \text{Dim} \left[ A(EC_3) \right] = \text{Dim} \left[ \left\{ CR_i \mid r_{i3} > w_3 \right\} \right] = \text{Dim} \left[ \left\{ CR_5, CR_7 \right\} \right] = 2 \\
T(EC_4) &= \text{Dim} \left[ A(EC_4) \right] = \text{Dim} \left[ \left\{ CR_i \mid r_{i4} > w_4 \right\} \right] = \text{Dim} \left[ \left\{ CR_3, CR_8 \right\} \right] = 2 \\
T(EC_5) &= \text{Dim} \left[ A(EC_5) \right] = \text{Dim} \left[ \left\{ CR_i \mid r_{i5} > w_5 \right\} \right] = \text{Dim} \left[ \left\{ CR_1, CR_2, CR_3, CR_5 \right\} \right] = 4 \\
T(EC_6) &= \text{Dim} \left[ A(EC_6) \right] = \text{Dim} \left[ \left\{ CR_i \mid r_{i6} > w_6 \right\} \right] = \text{Dim} \left[ \left\{ CR_2, CR_5, CR_6 \right\} \right] = 3 \\
T(EC_7) &= \text{Dim} \left[ A(EC_7) \right] = \text{Dim} \left[ \left\{ CR_i \mid r_{i7} > w_7 \right\} \right] = \text{Dim} \left[ \left\{ CR_4, CR_6, CR_8 \right\} \right] = 3
\end{aligned}$$

The refined ranking of the ECs is:

$$EC_2 \approx EC_5 \quad EC_6 \approx EC_7 \quad EC_1 \approx EC_3 \approx EC_4,$$

where symbols “ $>$ ” and “ $\approx$ ” denote the *strict preference* and *indifference* relationship respectively.

## 4.2 Case of 10-level scale

Assuming that  $s=10$  and using the correspondence map in Table 4, we obtain the relationship matrices in Figures 5 and 6.

Scale level	CR importance ( $d_i$ )	Importance value	Relationship coefficient ( $r_{ij}$ )	Symbol
$L_1$	not important	1	no relationship	(empty cell)
$L_2$	...	2	...	◇
$L_3$	...	3	...	⋈
$L_4$	moderately important	4	medium relationship	◆
$L_5$	...	5	...	□
$L_6$	...	6	...	⊠
$L_7$	important	7	strong relationship	■
$L_8$	...	8	...	○
$L_9$	...	9	...	⊗
$L_{10}$	very important	10	very strong relationship	●

Table 4. Correspondence map between CR importances and relationship coefficients, expressed on a 10-level ordinal scale ( $s = 10$ ).

		$d_i$	Engineering Characteristics						
			EC <sub>1</sub>	EC <sub>2</sub>	EC <sub>3</sub>	EC <sub>4</sub>	EC <sub>5</sub>	EC <sub>6</sub>	EC <sub>7</sub>
Customer Requirements	CR <sub>1</sub>	9	⋈	⋈	◇		⊗	⋈	◇
	CR <sub>2</sub>	8	◇	□	◇		⊗	⊗	
	CR <sub>3</sub>	2				◆	○	◇	
	CR <sub>4</sub>	5	◇						●
	CR <sub>5</sub>	9	■	◆	●		●	■	
	CR <sub>6</sub>	7	⋈	●			◇	■	■
	CR <sub>7</sub>	10	●	⋈	⊠		◇		◇
	CR <sub>8</sub>	3				●		◇	■

Figure 5. Relationship matrix for the design of a new model of a climbing safety harness. For details on symbols/abbreviations, see Tables 1, 2 and 3..

		$d_i$	Engineering Characteristics						
			EC <sub>1</sub>	EC <sub>2</sub>	EC <sub>3</sub>	EC <sub>4</sub>	EC <sub>5</sub>	EC <sub>6</sub>	EC <sub>7</sub>
Customer Requirements	CR <sub>1</sub>	$L_9$	$L_3$	$L_3$	$L_2$	$L_1$	$L_9$	$L_3$	$L_2$
	CR <sub>2</sub>	$L_8$	$L_2$	$L_5$	$L_2$	$L_1$	$L_9$	$L_9$	$L_1$
	CR <sub>3</sub>	$L_2$	$L_1$	$L_1$	$L_1$	$L_4$	$L_8$	$L_2$	$L_1$
	CR <sub>4</sub>	$L_5$	$L_2$	$L_1$	$L_1$	$L_1$	$L_1$	$L_1$	$L_{10}$
	CR <sub>5</sub>	$L_9$	$L_7$	$L_4$	$L_{10}$	$L_1$	$L_{10}$	$L_7$	$L_1$
	CR <sub>6</sub>	$L_7$	$L_3$	$L_{10}$	$L_1$	$L_1$	$L_2$	$L_7$	$L_7$
	CR <sub>7</sub>	$L_{10}$	$L_{10}$	$L_3$	$L_6$	$L_1$	$L_2$	$L_1$	$L_2$
	CR <sub>8</sub>	$L_3$	$L_1$	$L_1$	$L_1$	$L_{10}$	$L_1$	$L_2$	$L_7$

Figure 6. “Transformed” relationship matrix, obtained from that in Figure 5, when using a 10-level ordinal scale for both CR importances and relationship coefficients. For details on symbols/abbreviations, see Tables 1, 2 and 3.

According to Eq. (3), the negations for the levels of a 10-point ordinal scale are:

$$\begin{aligned}
 \text{Neg}(L_1) &= L_{10}, & \text{Neg}(L_2) &= L_9, & \text{Neg}(L_3) &= L_8, & \text{Neg}(L_4) &= L_7, & \text{Neg}(L_5) &= L_6, \\
 \text{Neg}(L_6) &= L_5, & \text{Neg}(L_7) &= L_4, & \text{Neg}(L_8) &= L_3, & \text{Neg}(L_9) &= L_2, & \text{Neg}(L_{10}) &= L_1.
 \end{aligned}$$

According to Eq. (2), the importances related to each of the 7 ECs are:

$$\begin{aligned}
 w_1 &= \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg} (d_i), r_{i1} \right] \right\} = L_3 \\
 w_2 &= \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg} (d_i), r_{i2} \right] \right\} = L_3 \\
 w_3 &= \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg} (d_i), r_{i3} \right] \right\} = L_2 \\
 w_4 &= \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg} (d_i), r_{i4} \right] \right\} = L_1 \\
 w_5 &= \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg} (d_i), r_{i5} \right] \right\} = L_2 \\
 w_6 &= \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg} (d_i), r_{i6} \right] \right\} = L_1 \\
 w_7 &= \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg} (d_i), r_{i7} \right] \right\} = L_2
 \end{aligned}$$

The resulting ranking is:

$$EC_1 \approx EC_2 \quad EC_3 \approx EC_5 \approx EC_7 \quad EC_4 \approx EC_6$$

Comparing these results with those in the case  $s = 3$ , we note that when increasing  $s$ , the “flattening effect” tends to disappear and the discrimination power of the resulting ranking tends to increase. On the other hand, scales with too many levels may be difficult to interpret for respondents and QFD team members. For this reason, the scientific literature often suggests not to exceed 5 levels [Franceschini and Rupil, 1999; Franceschini, 2001].

Again, the  $T(EC_j)$  indicator may be calculated in order to refine the EC ordering:

$$\begin{aligned}
 T(EC_1) &= \text{Dim}[A(EC_1)] = \text{Dim}[\{\text{CR}_i \mid r_{i1} > w_1\}] = \text{Dim}[\{\text{CR}_9, \text{CR}_{10}\}] = 2 \\
 T(EC_2) &= \text{Dim}[A(EC_2)] = \text{Dim}[\{\text{CR}_i \mid r_{i2} > w_2\}] = \text{Dim}[\{\text{CR}_2, \text{CR}_7, \text{CR}_9\}] = 3 \\
 T(EC_3) &= \text{Dim}[A(EC_3)] = \text{Dim}[\{\text{CR}_i \mid r_{i3} > w_3\}] = \text{Dim}[\{\text{CR}_5, \text{CR}_7\}] = 2 \\
 T(EC_4) &= \text{Dim}[A(EC_4)] = \text{Dim}[\{\text{CR}_i \mid r_{i4} > w_4\}] = \text{Dim}[\{\text{CR}_3, \text{CR}_8\}] = 2 \\
 T(EC_5) &= \text{Dim}[A(EC_5)] = \text{Dim}[\{\text{CR}_i \mid r_{i5} > w_5\}] = \text{Dim}[\{\text{CR}_1, \text{CR}_2, \text{CR}_3, \text{CR}_5\}] = 4 \\
 T(EC_6) &= \text{Dim}[A(EC_6)] = \text{Dim}[\{\text{CR}_i \mid r_{i6} > w_6\}] = \text{Dim}[\{\text{CR}_1, \text{CR}_2, \text{CR}_3, \text{CR}_5, \text{CR}_6, \text{CR}_8\}] = 6 \\
 T(EC_7) &= \text{Dim}[A(EC_7)] = \text{Dim}[\{\text{CR}_i \mid r_{i7} > w_7\}] = \text{Dim}[\{\text{CR}_4, \text{CR}_7, \text{CR}_8\}] = 3
 \end{aligned}$$

The resulting refined

ranking is:

$$(EC_2 \quad EC_1) \quad (EC_5 \quad EC_7 \quad EC_3) \quad (EC_6 \quad EC_4)$$

Even if the relationship matrix in Figure 5 is consistent with that in Figure 3 (coefficients and CR importances in Figure 5 are obtained by splitting those in Figure 3 in a further detail), some significant rank reversals of the ECs are observed. See, for example,  $EC_1$  and  $EC_6$ .

This rank reversal is intrinsically due to the increase of the number of scale levels. It is not a peculiarity of this method, it may happen also using more “traditional” approaches, such as, for example, ISM. In fact, applying ISM to data in Figures 4 and 6 and interpreting scale levels as numbers (i.e.  $L_1 = 1, \dots, L_{10} = 10$ ), the respective results are:

$$EC_5 \quad EC_2 \quad EC_6 \quad EC_1 \approx EC_3 \quad EC_7 \quad EC_4$$

and

$$EC_5 \quad EC_1 \quad EC_6 \quad EC_2 \quad EC_3 \quad EC_7 \quad EC_4,$$

which show rank reversal for  $EC_1$  and  $EC_2$ .

### 4.3 Case of 5-level scale

This case considers the situation in which both the CR importances  $d_i$  and  $r_{ij}$  coefficients are expressed on a 5-level ordinal scale ( $s = 5$ ) (see Table 5).

This number of scale levels seems to represent a good compromise between the previous two cases. The related relationship matrices are reported in Figures 7 and 8.

Scale level	CR importance ( $d_i$ )	Importance value	Relationship coefficient ( $r_{ij}$ )	Symbol
$L_1$	not important	1	no relationship	(empty cell)
$L_2$	weakly important	2	weak relationship	□
$L_3$	moderately important	3	medium relationship	■
$L_4$	important	4	strong relationship	○
$L_5$	very important	5	very strong relationship	●

Table 5. Correspondence map between CR importances and relationship coefficients, expressed on a 5-level ordinal scale ( $s = 5$ ).

		$d_i$	Engineering Characteristics						
			$EC_1$	$EC_2$	$EC_3$	$EC_4$	$EC_5$	$EC_6$	$EC_7$
Customer Requirements	CR <sub>1</sub>	5	□	□			●	□	
	CR <sub>2</sub>	4		■			●	●	
	CR <sub>3</sub>	1				□	○		
	CR <sub>4</sub>	3							●
	CR <sub>5</sub>	5	○	□	●		●	○	
	CR <sub>6</sub>	4	□	●				○	○
	CR <sub>7</sub>	5	●	□	■				
	CR <sub>8</sub>	2				●			○

Figure 7. Relationship matrix for the design of a new model of a climbing safety harness. For details on symbols/abbreviations, see Tables 1, 2 and 3.

		$d_i$	Engineering Characteristics						
			$EC_1$	$EC_2$	$EC_3$	$EC_4$	$EC_5$	$EC_6$	$EC_7$
Customer Requirements	CR <sub>1</sub>	$L_5$	$L_2$	$L_2$	$L_1$	$L_1$	$L_5$	$L_2$	$L_1$
	CR <sub>2</sub>	$L_4$	$L_1$	$L_3$	$L_1$	$L_1$	$L_5$	$L_5$	$L_1$
	CR <sub>3</sub>	$L_1$	$L_1$	$L_1$	$L_1$	$L_2$	$L_4$	$L_1$	$L_1$
	CR <sub>4</sub>	$L_3$	$L_1$	$L_1$	$L_1$	$L_1$	$L_1$	$L_1$	$L_5$
	CR <sub>5</sub>	$L_5$	$L_4$	$L_2$	$L_5$	$L_1$	$L_5$	$L_4$	$L_1$
	CR <sub>6</sub>	$L_4$	$L_2$	$L_5$	$L_1$	$L_1$	$L_1$	$L_4$	$L_4$
	CR <sub>7</sub>	$L_5$	$L_5$	$L_2$	$L_3$	$L_1$	$L_1$	$L_1$	$L_1$
	CR <sub>8</sub>	$L_2$	$L_1$	$L_1$	$L_1$	$L_5$	$L_1$	$L_1$	$L_4$

Figure 8. “Transformed” relationship matrix, obtained from that in Figure 7, when using a 5-level ordinal scale for both CR importances and relationship coefficients. For details on symbols/abbreviations, see Tables 1, 2 and 3

According to Eq. (3), the negations of a 5-point ordinal scale are:

$$Neg(L_1) = L_5, \quad Neg(L_2) = L_4, \quad Neg(L_3) = L_3, \quad Neg(L_4) = L_2, \quad Neg(L_5) = L_1.$$

Hence, according to Eq. (2), we obtain the following EC importances:

$$w_1 = \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg}(d_i), r_{i1} \right] \right\} = L_2$$

$$w_2 = \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg}(d_i), r_{i2} \right] \right\} = L_2$$

$$w_3 = \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg}(d_i), r_{i3} \right] \right\} = L_1$$

$$w_4 = \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg}(d_i), r_{i4} \right] \right\} = L_1$$

$$w_5 = \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg}(d_i), r_{i5} \right] \right\} = L_1$$

$$w_6 = \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg}(d_i), r_{i6} \right] \right\} = L_1$$

$$w_7 = \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg}(d_i), r_{i7} \right] \right\} = L_1$$

The resulting ranking is therefore:

$$EC_1 \approx EC_2 \quad EC_3 \approx EC_4 \approx EC_5 \approx EC_6 \approx EC_7$$

Applying Eq. (4), the resulting  $T(EC_j)$  values are:

$$T(EC_1) = \text{Dim}[A(EC_1)] = \text{Dim}[\{CR_i \mid r_{i1} > w_1\}] = \text{Dim}[\{CR_5, CR_7\}] = 2$$

$$T(EC_2) = \text{Dim}[A(EC_2)] = \text{Dim}[\{CR_i \mid r_{i2} > w_2\}] = \text{Dim}[\{CR_2, CR_6\}] = 2$$

$$T(EC_3) = \text{Dim}[A(EC_3)] = \text{Dim}[\{CR_i \mid r_{i3} > w_3\}] = \text{Dim}[\{CR_5, CR_7\}] = 2$$

$$T(EC_4) = \text{Dim}[A(EC_4)] = \text{Dim}[\{CR_i \mid r_{i4} > w_4\}] = \text{Dim}[\{CR_3, CR_8\}] = 2$$

$$T(EC_5) = \text{Dim}[A(EC_5)] = \text{Dim}[\{CR_i \mid r_{i5} > w_5\}] = \text{Dim}[\{CR_1, CR_2, CR_3, CR_5\}] = 4$$

$$T(EC_6) = \text{Dim}[A(EC_6)] = \text{Dim}[\{CR_i \mid r_{i6} > w_6\}] = \text{Dim}[\{CR_1, CR_2, CR_5, CR_6\}] = 4$$

$$T(EC_7) = \text{Dim}[A(EC_7)] = \text{Dim}[\{CR_i \mid r_{i7} > w_7\}] = \text{Dim}[\{CR_4, CR_6, CR_8\}] = 3$$

The refined ranking is:

$$(EC_1 \approx EC_2) \quad (EC_5 \approx EC_6 \quad EC_7 \quad EC_3 \approx EC_4)$$

Even if the relationship matrix is consistent with those in Figures 3 and 5, a few significant rank reversals can be observed.

#### 4.4 Case of scales with a different number of levels

In typical QFD applications, CR importances and relationship coefficients may be defined on not-necessarily-identical ordinal scales. Precisely, CR importances are usually evaluated on a 5-level scale (see the first three columns of Table 5), while  $r_{ij}$  coefficients on a 4-level scale (see Table 6) [Akao, 1988; Franceschini, 2001].

Relationship coefficient ( $r_{ij}$ )	Symbol
no relationship	(empty cell)
weak relationship	$\Delta$
medium relationship	$\circ$
strong relationship	$\bullet$

Table 6. Example of relationship coefficients evaluated on a symbolic 4-level ordinal scale ( $s = 4$ ).

In this case, the aggregation method proposed in Eq. (2) cannot be applied [Yager and Filev, 1994]. However, a practical approximated solution may be obtained by merging two or more contiguous levels of the ordinal scale with the largest number of levels into one, or introducing one or more “dull” levels in the ordinal scale with the lowest number of levels; this second option is implemented in the example in Tab. 7. We remark that this approach leaves a certain discretionary power to the QFD team, in choosing the scale levels to be adjusted; however, the suggested “adjustment” does not alter the ordinal relationships between the objects represented on the initial ordinal scale(s) [Roberts, 1979].

Scale level	Relationship coefficient ( $r_{ij}$ )	Symbol
$L_1$	no relationship	(empty cell)
$L_2$ (dull)	N/A	N/A
$L_3$	weak relationship	$\Delta$
$L_4$	medium relationship	$\circ$
$L_5$	strong relationship	$\bullet$

Table 7. Example of a possible correspondence map of the relationship coefficients evaluated on a symbolic 4-level ordinal scale ( $s = 4$ ).

According to the mappings in the first three columns of Table 5 and that in Table 7, we obtain the relationship matrices reported in Figures 9 and 10.

		$d_i$	Engineering Characteristics						
			EC <sub>1</sub>	EC <sub>2</sub>	EC <sub>3</sub>	EC <sub>4</sub>	EC <sub>5</sub>	EC <sub>6</sub>	EC <sub>7</sub>
Customer Requirements	CR <sub>1</sub>	4					$\circ$		
	CR <sub>2</sub>	4		$\Delta$			$\circ$	$\circ$	
	CR <sub>3</sub>	2				$\Delta$	$\circ$		
	CR <sub>4</sub>	3							$\bullet$
	CR <sub>5</sub>	4	$\circ$	$\Delta$	$\bullet$		$\bullet$	$\circ$	
	CR <sub>6</sub>	3		$\bullet$				$\circ$	$\circ$
	CR <sub>7</sub>	5	$\bullet$		$\Delta$				
	CR <sub>8</sub>	2				$\bullet$			$\circ$

Figure 9. Relationship matrix for the design of a new model of a climbing safety harness. For details on symbols/abbreviations, see Tables 1, 2 and 3.

		$d_i$	Engineering Characteristics						
			EC <sub>1</sub>	EC <sub>2</sub>	EC <sub>3</sub>	EC <sub>4</sub>	EC <sub>5</sub>	EC <sub>6</sub>	EC <sub>7</sub>
Customer Requirements	CR <sub>1</sub>	$L_4$	$L_1$	$L_1$	$L_1$	$L_1$	$L_4$	$L_1$	$L_1$
	CR <sub>2</sub>	$L_4$	$L_1$	$L_3$	$L_1$	$L_1$	$L_4$	$L_4$	$L_1$
	CR <sub>3</sub>	$L_2$	$L_1$	$L_1$	$L_1$	$L_3$	$L_4$	$L_1$	$L_1$
	CR <sub>4</sub>	$L_3$	$L_1$	$L_1$	$L_1$	$L_1$	$L_1$	$L_1$	$L_5$
	CR <sub>5</sub>	$L_4$	$L_4$	$L_3$	$L_5$	$L_1$	$L_5$	$L_4$	$L_1$
	CR <sub>6</sub>	$L_3$	$L_1$	$L_5$	$L_1$	$L_1$	$L_1$	$L_4$	$L_4$
	CR <sub>7</sub>	$L_5$	$L_5$	$L_1$	$L_3$	$L_1$	$L_1$	$L_1$	$L_1$
	CR <sub>8</sub>	$L_2$	$L_1$	$L_1$	$L_1$	$L_5$	$L_1$	$L_1$	$L_4$

Figure 10. "Transformed" relationship matrix, obtained from that in Figure 9, when using a 5-level ordinal scale for CR importances and a 4-level one for relationship coefficients. For details on symbols/abbreviations, see Tables 1, 2 and 3.

The negations of a 5-point ordinal scale are reported in the example in Sect. 4.3.

By applying Eq. (2), we obtain the following EC importances:

$$w_1 = \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg} (d_i), r_{i1} \right] \right\} = L_2$$

$$w_2 = \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg} (d_i), r_{i2} \right] \right\} = L_1$$

$$w_3 = \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg} (d_i), r_{i3} \right] \right\} = L_2$$

$$w_4 = \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg} (d_i), r_{i4} \right] \right\} = L_1$$

$$w_5 = \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg} (d_i), r_{i5} \right] \right\} = L_1$$

$$w_6 = \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg} (d_i), r_{i6} \right] \right\} = L_1$$

$$w_7 = \text{Min}_{i=1..8} \left\{ \text{Max} \left[ \text{Neg} (d_i), r_{i7} \right] \right\} = L_1$$

The resulting ranking is:

$$EC_1 \approx EC_3 \quad EC_2 \approx EC_4 \approx EC_5 \approx EC_6 \approx EC_7$$

Using Eq. (4), the  $T(EC_j)$  indicators may be calculated as:

$$T(EC_1) = \text{Dim} [A(EC_1)] = \text{Dim} [\{CR_i \mid r_{i1} > w_1\}] = \text{Dim} [\{CR_5, CR_7\}] = 2$$

$$T(EC_2) = \text{Dim} [A(EC_2)] = \text{Dim} [\{CR_i \mid r_{i2} > w_2\}] = \text{Dim} [\{CR_2, CR_5, CR_6\}] = 3$$

$$T(EC_3) = \text{Dim} [A(EC_3)] = \text{Dim} [\{CR_i \mid r_{i3} > w_3\}] = \text{Dim} [\{CR_5, CR_7\}] = 2$$

$$T(EC_4) = \text{Dim} [A(EC_4)] = \text{Dim} [\{CR_i \mid r_{i4} > w_4\}] = \text{Dim} [\{CR_3, CR_5\}] = 2$$

$$T(EC_5) = \text{Dim} [A(EC_5)] = \text{Dim} [\{CR_i \mid r_{i5} > w_5\}] = \text{Dim} [\{CR_1, CR_2, CR_3, CR_5\}] = 4$$

$$T(EC_6) = \text{Dim} [A(EC_6)] = \text{Dim} [\{CR_i \mid r_{i6} > w_6\}] = \text{Dim} [\{CR_2, CR_5, CR_6\}] = 3$$

$$T(EC_7) = \text{Dim} [A(EC_7)] = \text{Dim} [\{CR_i \mid r_{i7} > w_7\}] = \text{Dim} [\{CR_4, CR_6, CR_8\}] = 3$$

The refined ranking is:

$$(EC_1 \approx EC_3) \quad (EC_5 \quad EC_2 \approx EC_6 \approx EC_7 \quad EC_4)$$



This result is not so different from that obtained in Sect. 4.3, although there are some variations, e.g., the significant increase in  $EC_3$ .

## 5 Conclusions

This paper introduced and discussed a new method to compute the EC prioritization in QFD. Data processing is performed consistently with the ordinal features of the scales for representing the CR importances and relationship matrix coefficients. The simplicity of this method is comparable to that of the traditional approach, i.e., the ISM.

The main novelties of the method are that:

- it is able to aggregate data evaluated on ordinal scales, overcoming controversial assumptions of data cardinality;
- it does not require any arbitrary and artificial “scalarization” of the data;
- it is also able to deal with situations in which both CR importances and relationship matrix coefficients are rated on different ordinal scales;
- it is automatable and easy integrated into QFD existing software applications.

Moreover, the proposed aggregation logic is relatively flexible since, as each case requires, it may be replaced by other aggregation logics; for example, high positions in the final ranking can be assigned only to those ECs, which are related to those CRs with maximum importance or others.

In the scientific literature, many other different methods have been proposed in order to overcome the problems related to the poor properties of the scales on which data are collected. Most of these do not completely solve this problem, including some subjective scale scalarization, or entail complex procedures, with mathematical models scarcely intuitional, which are difficult to be implemented and which can hardly be automated.

On the contrary, the rigorous respect of the scale properties, the intuitional logic and the simplicity of implementation make this approach more suitable for the practical applications.

On the other hand, the proposed method has some limitations, summarized in the following three points:

- The method may generate a “flattening effect” when applying Eq. (2) to scales with a small number of levels. This may apparently encourage the use of scales with a large number of levels (e.g., 10 or more). However, scales with too many levels may be difficult to interpret for respondents and QFD team members. The scientific literature and the examples presented in Sect. 4 suggest that using a 5-level scale can be an acceptable compromise [Franceschini and Rupil, 1999; Franceschini, 2001]. We also remark that the aforementioned “flattening effect” can also occur when the number of CRs is large.
- The importance associated with each EC is defined on a  $s$ -point ordinal scale, with the same number of levels of those used for the CR importance and the  $r_{ij}$  coefficients. As a consequence, the final ordering of the ECs cannot be expressed on more than  $s$  ordered categories.
- When both CR importances and  $r_{ij}$  coefficients have high values, the method tends to flatten the importance values ( $w_j$ ) upwards, for all the ECs. This is coherent with the aim of the method, since it indicates that several ECs are important and should not be neglected by designers. Similarly, when both

CR importances and  $r_{ij}$  coefficients have low values, the method tends to flatten all the computed EC importances downwards.

The implementation of the proposed method in modern manufacturing practices will help the design working team in defining more correct and reliable strategies for the development and the introduction onto the market of new products or services, specially focusing on the poor, but basilar information, which can be deduced from the analysis and interpretation of CRs.

Future research will be addressed to the construction of a tool able to support a Decision Maker in the selection of the most appropriate prioritization procedure basing on the properties of the data available during the design process.

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