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Original
Simulation of buck converters via numerical inverse Laplace transform / Trinchero, Riccardo; Stievano, IGOR SIMONE; Canavero, Flavio. - ELETTRONICO. - (2017), pp. 1-4. (Intervento presentato al convegno 2017 IEEE 21st Workshop on Signal and Power Integrity (SPI) tenutosi a Lake Maggiore, Baveno (I) nel 7-10 May, 2017) [10.1109/SaPIW.2017.7944017].

Availability:
This version is available at: 11583/2674399 since: 2017-06-09T16:49:16Z

Publisher:
IEEE

Published
DOI:10.1109/SaPIW.2017.7944017

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12 October 2024

Simulation of Buck Converters via Numerical Inverse Laplace Transform

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***Abstract*—This paper focuses on the application of the theory of periodically switched circuits and systems to the steady-state and transient analysis of switching power converters. The proposed methodology is based on the derivation of an augmented companion network from topological inspection only. The above interpretation turns out to be time-invariant, thus allowing the frequency-domain analysis of the circuit by means of standard tools for the circuit analysis. Two alternative methods are used to compute the transient and the steady-state waveforms, namely the inverse discrete Fourier transform (IDFT) and the numerical inverse Laplace transform (NILT). The strength and the accuracy of the proposed approach is demonstrated on a buck converter via the prediction of its noisy absorbed current.**

***Index Terms*—Frequency-domain analysis, transient analysis,numerical inverse Laplace transforms, linear time-varying circuits, switching converters.**

I. Introduction

Nowadays, switching power converters are massively used to supply energy to electrical and electronic equipments in different domains (e.g., integrated circuits, motherboard, satellite communications, automotive applications) [1]. In spite of a number of key advantages, switching converters are considered as one of the main source of conducted emissions (CE) in a power distribution network. Due to the commutations of the internal switches, the converters are characterized by noisy absorbed currents which exhibit a complex dynamical behavior with a rich spectrum content. Within the electromagnetic compatibility (EMC) scenario, the spectral level of the absorbed currents must be kept under control during the design phase, to comply with the stringent EMC regulations [2].

During the last decades a number of different techniques and strategies for the frequency-domain analysis of switching circuits have been proposed with the aim of providing a robust and accurate simulation framework [3]–[7]. Operating in frequency-domain has a number of important advantages with respect to the standard time-domain analysis: (i) it directly provides the steady-state responses of the circuit without waiting for the evolution of a possible long initial transient; (ii) it avoids any issue related to the proper choice of the integration time step and waveform windowing; (iii) it allows to include frequency dependent elements such as lossy transmission line in the frequency-domain simulation framework without additional efforts.

The main differences among the available methodologies reside in the definition of the kernels needed to analyze the circuit in frequency-domain. Recently, an effective frequency-domain alternative has been proposed, where a switching circuit is interpreted in terms of an augmented linear time-invariant (LTI) network built from the information of the circuit topology only [8]–[11]. This solution allows to compute the steady-state response of the circuit by means of a frequency-domain analysis carried out using standard tools for circuit analysis, as SPICE [11]. It is important to remark that the above interpretation can be considered as a linear version of the harmonic balance technique widely adopted for the steady-state analysis of a non-linear network [12].

The aim of this work is to extend the aforementioned augmented frequency-domain modeling framework to include the transient behavior of the network variables. This can be accomplished via a pure frequency-domain approach as the superposition of the circuit responses to a series of harmonic excitations via either the inverse discrete Fourier transform (IDFT) or the numerical inverse Laplace transform (NILT) [13]. The strength and the accuracy of the proposed approach is demonstrated on a open-loop buck converter.

II. Illustrative Example: Buck Converter

The switching buck converter of Fig. 1 is used hereafter in this paper as an illustrative example for the proposed formulation. The converter operates in continuous mode and the MOS is driven by a periodical square-wave with a switching frequency $f_c = 250\,\text{kHz}$ and duty cycle $D = 50\,\%$.

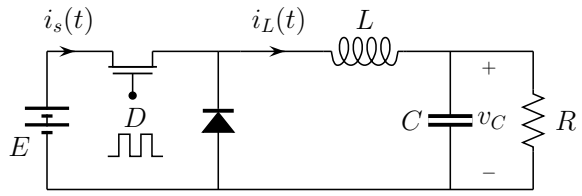

Fig. 1. Buck converter with its relevant electrical parameters and variables. The circuit elements take the following values: $E = 5\,\text{V}$, $L = 50\,\mu\text{H}$, $C = 44.1\,\mu\text{F}$, $R = 5\,\Omega$ [15].

According to the classification in [14], when the two non-linear elements of the network are replaced by two switches with a well-defined periodic behavior, the converter can be considered as a periodical linear time-varying system.

III. Steady-State analysis of Switching Converters via the Augmented Representation

This Section briefly introduces the main concepts behind the frequency-domain steady-state analysis of a switching circuit based on an augmented time-invariant representation [8]–[11]. The proposed formulation is derived from the Fourier expansion of the steady-state response of a generic variable of a switching circuit to a monochromatic excitation at angular frequency ω_0 [16]. As an example the absorbed steady-state current $i_s(t)$ of the converter in Fig. 1 writes,

$$i_s(t) \approx \sum_{n=-N}^{+N} I_{s,n} \exp(j(n\omega_c + \omega_0)t), \tag{1}$$

where $\omega_c = 2\pi f_c$ is the switching angular frequency related to the periodic behavior of the switches, $I_{s,n}$ are the coefficients of the Fourier series and N is the number of the positive harmonics considered in the truncated expansion.

Following the procedure in [8]–[11], the Fourier expansion in (1) suggests an augmented representation of the variables of the switching converter, where all the node voltages and the branch currents of the original circuit are replaced by $(2N+1)$ new nodes and branches representing the harmonic coefficients of the augmented representation.

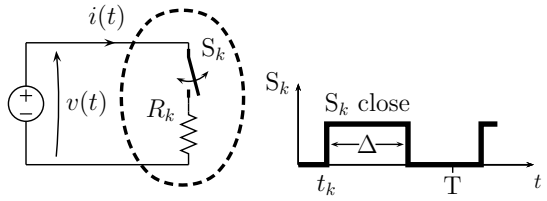

Fig. 2. Generic switch with a periodic time-domain behavior.

A. Augmented circuit interpretation

According to [11], the v-i characteristic of a generic switch with periodic commutations as the one shown in Fig. 2 involves a fully coupled relation between the spectra of the voltage $V(\omega)$ and current $I(\omega)$, via following admittance operator:

$$\begin{bmatrix} I_{-N} \\ \vdots \\ I_0 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{0,k} & Y_{-1,k} & \dots & \dots & Y_{-2N,k} \\ Y_{1,k} & Y_{0,k} & \dots & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & Y_{-1,k} \\ Y_{2N,k} & \dots & \dots & Y_{1,k} & Y_{0,k} \end{bmatrix} \begin{bmatrix} V_{-N} \\ \vdots \\ V_0 \\ \vdots \\ V_N \end{bmatrix} \tag{2}$$

where V_n and I_n represent the coefficients of the staedy-state representation in (1) of the voltage and current of the switch.

The entries $Y_{n,k}$ of the above $(2N+1)\times(2N+1)$ augmented admittance matrix $\mathbf{Y}_k$ of a generic k-th switch are known terms, which are univocally defined by the coefficients of the Fourier expansion of the time-domain behavior of the switch.

Similarly, the corresponding augmented representations of the time-invariant elements of the circuit turn out to be diagonal impedance or admittance matrices, e.g., $I_n = j(n\omega_c + \omega_0)CV_n$ for a capacitor.

Based on the above interpretation, all the elements of a switching converter like the one of Fig. 1 can be suitably replaced by their augmented replica, leading to above mentioned linear time-invariant circuit which becomes $(2N+1)$ times larger then the original one.

B. Frequency-domain steady-state solution

The above augmented circuit can be solved directly in the frequency-domain via standard tools for circuit analysis (e.g., the modified nodal analysis (MNA) [17] or SPICE), leading to the solution of the augmented nodal quantities that correspond to the harmonics of the corresponding steady-state voltage and current responses of the switching circuit.

The proposed approach has been applied to the converter of Fig. 1. Figure 3 compares the steady-state behavior of the current i_s estimated via the proposed augmented formulation and an expansion order $N = 120$, with the result of a time-domain Simulink simulation within the Matlab environment. The augmented time-invariant approximation allows to accurately predict the steady-state behavior of the absorbed current of the converter via a single frequency simulation with a remarkable simulation time (less than 1 s).

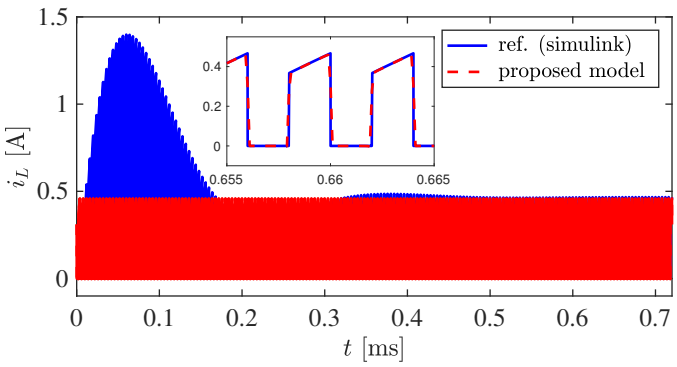

Fig. 3. Steady-state behavior of the input current $i_s(t)$ of the buck converter of Fig. 1. The results of the proposed augmented frequency-domain approach (dashed red curve) are compered with the results of a standard time-domain simulation (blue curve).

It is important to remark that the augmented replica of the circuit provides the steady-state behavior of the circuit variables by neglecting the transient portion of their responses. This idea represents an accurate, robust and fast tool for the CE analysis.

IV. Transient Analysis of Switching Converters via the Augmented Representation

This Section focuses on the transient analysis of the open loop buck converter of Fig. 1 via the augmented formulation discussed in the previous Section and two different approaches: the IDFT and NILT, respectively. In this preliminary work, the main limitation is that the proposed approach does

not allow to account for the effect of a control system on the transient behavior of the converter.

A. IDFT

For the sake of simplicity, the discussion starts by recalling the well-known relation between the transient response of a variable of a LTI network and its corresponding transfer function and excitation in frequency-domain. As an example the absorbed current $i_s(t)$ of the circuit in Fig. 1 for a fixed position of the switches writes:

$$i_{s,LTI}(t) = \int_{-\infty}^{+\infty} H(j\omega)E(j\omega)\exp(j\omega t)d\omega \tag{3}$$

where $E(j\omega)$ is the Fourier transform of the circuit excitation $e(t)$ and $H(j\omega) = I_s(j\omega)/E(j\omega)$ is a standard transfer function of an LTI network.

In general, the above integral does not guarantee an analytical closed-form solution, and therefore it has to be discretized and implemented in a numerical code by means of the IDFT which writes,

$$\tilde{i}_{s,LTI}(t) = \sum_{m=-M}^{+M} H(jm\omega_s)E(jm\omega_s)\exp(jm\omega_s t) \tag{4}$$

where $(2M+1)$ is the number of frequency samples $E(jm\omega_s)$ accounted in the expansion and $\omega_s = 2\pi f_s$ is the angular sampling frequency.

Equations (3) and (4) can be generalized for the case of a switching circuit. As an example, the time-domain response of the current $i_s(t)$ of the buck converter during its working condition can be directly and univocally calculated by means of the mixed-domain transfer function $H(t; j\Omega)$ defined in [14], via the following relation:

$$i_s(t) = \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} H_n(j\Omega)\exp(j(\Omega+n\omega_c)t)E(j\Omega)d\Omega, \tag{5}$$

where the terms $H_n(j\Omega)$ are estimated from the responses of the augmented circuit to a single tone excitation at angular frequency Ω.

Similar to (4), the above integral is replaced by a sum collecting $(2M+1)$ discrete samples of the spectrum of the excitation $E(j\Omega)$, leading to the following sum of IDFT,

$$\tilde{i}_s(t) = \sum_{m=-M}^{+M} \sum_{n=-N}^{+N} H_n(jm\omega_s)\exp(j(m\omega_s+n\omega_c)t)E(jm\omega_s). \tag{6}$$

Due to the discrete approximation of the spectrum $E(j\omega)$, the current $\tilde{i}_s(t)$ in (6) turns out to be a periodic representation of the real current behavior $i_s(t)$,

$$\tilde{i}_s(t) = \sum_{n=-\infty}^{+\infty} i_s(t - nT_s), \tag{7}$$

where the period $T_s = 1/f_s$ depends on the sampling angular frequency $\omega_s = 2\pi f_s$.

The time-domain periodization in (7) unavoidably leads to aliasing errors and demands for suitable counter-actions for improving the accuracy of the predicted responses. As an example, the Heaviside unit-step function $u(t)$, that is possibly used to represent the activation of dc excitation E in the schematic of Fig. 1 (i.e., $e(t) = 5u(t)$), has to be approximated by a periodic square-wave signal $\tilde{u}(t) \approx \sum_n \Pi_T(t - nT_s)$ which can be represented via its Fourier series with a discrete spectrum. The two parameters T and T_s can be defined by the empirical rule $T_w \leq T \leq T_s$, where T_w is the observation time of the transient simulation (maximum time considered in the simulation).

In circuits with low-frequency dominant poles, such as the buck converter in Fig. 1, the aliasing error can compromise the accuracy of the transient results, as highlighted in Fig. 4. However, this issue can be easily overcome by increasing the period T_s of the signal $\tilde{u}(t)$, with a possible detrimental impact on the simulation time.

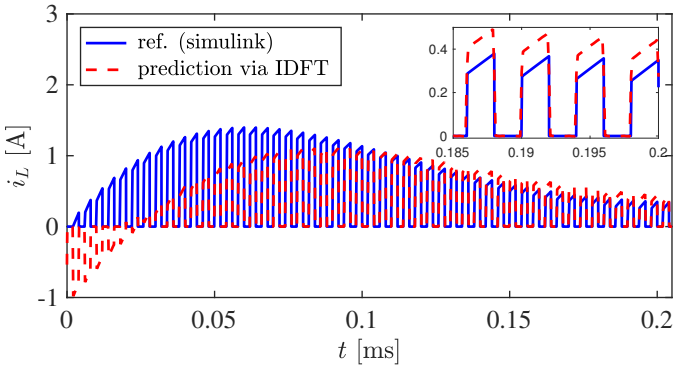

Fig. 4. Comparison between the results of a transient simulation of the absorbed current $i_s(t)$ at the input port of the buck converter in Fig. 1 provided by the IDFT approach with $T_w = 0.205$ ms, $T = 0.22$ ms and $T_s = 0.28$ (dashed red curve) and the results of a standard time-domain simulation (blue curve).

B. NILT

In order to partially overcome the aliasing effect, an alternative approach for the transient analysis based on the NILT has been applied to the transient analysis of the buck converter, along with the augmented representation in the Laplace domain. The key idea is to consider the product of the circuit variables with a dumped exponential term $\exp(-ct)$ with $c \geq 0$ to ensure their convergence to zero when time goes to infinity.

As an example, the absorbed current $i_s(t)$ can be written as $i_{s,c}(t) = i_s(t)\exp(-ct)u(t)$. The above interpretation corresponds to calculate the unilateral Laplace transform of the current $i_s(t)$, since:

$$\begin{aligned} I_s(s) &= \mathscr{L}\{i_s(t)\}(s) = \int_0^{+\infty} i_s(t)\exp(-st) \\ &= \mathscr{F}\{i_s(t)\exp(-ct)\}(c+j\omega) \end{aligned} \tag{8}$$

where $s = c + j\omega$ is the complex frequency.

In order to minimize the aliasing, the damping term c defining the region of convergence of the Laplace transform can be obtained from the following empirical expression:

$$c = \frac{2\ln(M)}{T_w} \tag{9}$$

where M is the number of the positive frequency samples and T_w is the observation time.

The transient analysis can be obtained from the transfer function $H(t; j\Omega)$ by replacing the frequency-domain variable $j\Omega$ with complex frequency $s = c + j\Omega$, leading to its corresponding representation in the Laplace domain $H(t; s)$. According to [13] and the previous definitions, the inverse Laplace transform and therefore the transient responses of the switching circuit variables can be obtained numerically from the DIFT via the following simple relation,

$$i_s(t) = \exp(ct)\mathscr{F}^{-1}\{I_{s,c}(c + j\omega)\}. \tag{10}$$

The proposed approach based on the NILT has been applied to compute the transient behavior of the absorbed current $i_s(t)$ of the buck converter in Fig. 1, again by considering a voltage excitation $e(t) = 5u(t)$. Figure 5 compares the simulation results obtained via a standard time-domain simulation in Simulink and the predicted response obtained via the augmented formulation and the NILT with $N = 120$ and $M = 120$. The above comparison highlights the excellent accuracy of the predictions obtained via the proposed simulation strategy, with a reasonable simulation time on the order of 10 s, which is even less than the time required to compute the time-domain response of the same switching circuit via a classical SPICE or Simulink simulations.

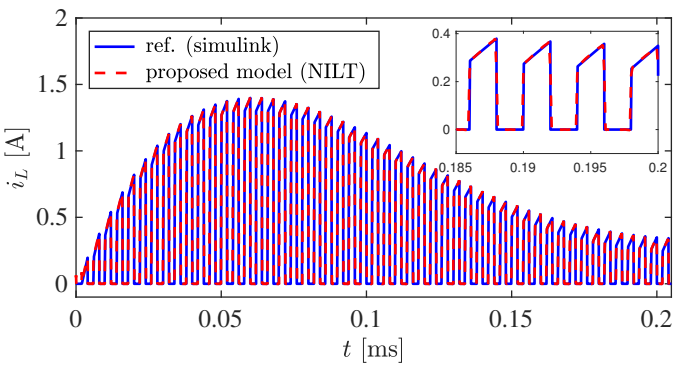

Fig. 5. Transient simulation of the absorbed current $i_s(t)$ at the input port of the buck converter in Fig. 1. The results of the proposed augmented s-domain approach (dashed red curve) are compared with the results of a standard time-domain simulation (blue curve).

It is important to remark that the above Laplace framework also allows to compute the steady-state circuit responses directly by setting $c = 0$.

V. Conclusions

This paper collected some preliminary and stimulating results on the application of a frequency-domain technique to the steady-state and transient analysis of a switching converter. The propose approach is based on an augmented representation of the switching circuit, suitably derived from the topology of the network in which all the switching elements are replaced by their linear time-invariant approximation. The circuit response is computed using either the inverse discrete Fourier transform or the numerical inverse Laplace transform, being the latter a better solution for this class of circuits, leading to a remarkable accuracy and efficiency. The strength and the accuracy of the method has been demonstrated through the prediction of the absorbed current of a switching buck converter.

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