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Abstract

In this paper, we introduce the Generalized Bin Packing Problem with bin-dependent item profits (GBPPI), a variant of the Generalized Bin Packing Problem. In GBPPI, various bin types are available with their own capacities and costs. A set of compulsory and non-compulsory items are also given, with volume and bin-dependent profits. The aim of GBPPI is to determine an assignment of items to bins such that the overall net cost is minimized.

The importance of GBPPI is confirmed by a number of applications. The introduction of bin-dependent item profits enables the application of GBPPI to cross-country and multi-modal transportation problems at strategic and tactical levels as well as in last-mile logistic environments.

Having provided a Mixed Integer Programming formulation of the problem, we introduce efficient heuristics that can effectively address GBPPI for instances involving up to 1000 items and problems with a mixed objective function. Extensive computational tests demonstrate the accuracy of the proposed heuristics. Finally, we present a case study of a well-known international courier operating in northern Italy. The problem approached by the international courier is GBPPI. In this case study, our methodology outperforms the policies of the company.

Keywords: Bin Packing problems, Generalized Bin Packing Problem, metaheuristic, last-mile logistics.

1. Introduction

The transportation services market is estimated to be worth approximately 3 trillion euros worldwide with a gross value added (GVA) of 600 billion in the EU-28 at basic prices, corresponding to a weight of approximately 5% of the total GVA [26]. In addition to their classical uses at the operational level, packing problems have recently appeared as tools for modeling strategic and tactical decisions taken in the transportation and supply chain sectors [18, 52]. This has led to new problems, such as the variable size and cost bin packing [17], the multi-handler knapsack problem under uncertainty [53], and the generalized bin packing problem [6].

In this paper, we introduce Generalized Bin Packing Problem with bin-dependent item profits (GBPPI), which is a variant of Generalized Bin Packing Problem (GBPP). The aim of GBPP is to minimize the difference between the total cost of the bins used and the total profit from the loading of the items. In GBPPI profits accrued from the items are bin-dependent, i.e., they vary according to the bin in which an item is loaded.

The contribution of this paper is threefold. First, we consider a typical setting of strategic and tactical managerial decisions for large dispatching and logistics companies related to the subcontracting of their deliveries to third-party logistics (3PLs). This is a sensitive problem for these companies, where subcontracting is used by managers to

introduce flexibility in their business models [1, 19]. On the other hand, the increase in freight volumes (and the consequent increase in subcontracting) in conjunction with an extreme customization of the delivery services (and the related costs) force managers to carefully consider the cost and quality of service at the contract level. Flexibility, cost reduction and high standard of service quality are conflictual objectives. Existing models for Last Mile delivery only partially consider these objectives while addressing the combination of several subcontractors [52, 53]. We model this managerial problem through GBPPI. The introduction of bin-dependent item profits allows the use of GBPPI in cross-country and multi-modal transportation, where contracts between a firm and 3PL vectors are stipulated through a trade-off [18, 58]. This trade-off can be effectively modeled through the introduction of bin-dependent item profits. In these settings, heuristic approaches are consistently exploited to solve complex transportation problems [27, 12]. In these problems, 3PLs play a fundamental role, as reported in a recent survey [1] and in [40], where a case study was presented with the aim of fostering challenging strategies among 3PLs.

The second contribution of our paper is to apply GBPPI to model a specific case study arising in the context of Smart City and involving a well-known international courier operating in northern Italy. Every day, the courier collects its customers demands to pick-up and deliver freight. Customers pay the courier, who is responsible for the entire transportation process. The courier refers to a set of transportation companies (TCs) and selects those that have the capacity to perform the pick-ups and deliveries using their own trucks. Therefore, on a daily scale, the courier must decide which TCs to select for day service, the number of trucks to be used by the selected TCs, and the assignment of pick-ups and deliveries to trucks. The final decision concerning this daily choice of courier must be reached within a limited time (approximately 10 minutes) to ensure the prompt assignment of freight to vehicles chosen from the selected TCs and, therefore, an efficient service.

Third, with regard to packing problems literature, the introduction of the bin-dependent profits provides a more flexible packing formulation that will enable to tackle the majority of real-world performance indicators and to address problems with mixed objective functions. This new formulation consists of a more general objective function consisting of two terms with different signs: 1) the total cost of the bins used to be minimized; and 2) the total bin-dependent profit of the selected items to be maximized. We provide an integer-programming model for GBPPI. Unfortunately, the mere use of commercial solvers is not sufficient to find good solutions of this model in reasonable computational time for real-world scenarios. Thus, some efficient heuristics are introduced. In particular, we derive two constructive heuristics and two metaheuristics.

The remainder of this paper is organized as follows: in Section 2, we introduce the problem and review important literature on packing problems on which GBPPI is based. In Section 3, the GBPPI model is described, and in Section 4 heuristics for addressing GBPPI are introduced. Instance sets and computational results are given in Section 5. In Section 6, we present a case study and related results. Conclusions are provided in Section 7.

2. Problem setting and literature review

Each instance of GBPPI consists of a set of items and a set of bins. Items can be compulsory (i.e., mandatory to load) or non-compulsory, and are characterized by volume and profit, which depend on the available bins. Bins are characterized by the capacity and cost of use, and are classified by type. All bins belonging to the same bin type have the same capacity and cost. The aim of GBPPI is to load compulsory items and profitable non-compulsory items into appropriate bins in order to minimize net cost. This is the difference between the total cost of the bins used and the total profit accruing from the loading of the items. Unlike GBPP, profits from items are bin-dependent, i.e., they vary according to the type of the bin into which an item is loaded.

The proposed problem reveals an interesting parallel with another problem addressed during ESICUP 2015 [25], sponsored by the automobile manufacturer Renault. Although the two problems are similar in the common goal to find an optimal assignment of items to bins, they are different in terms of problem structure and planning horizon. More details about the comparison of the two problems are given in Appendix A.

The oldest problem is Bin Packing Problem (BPP), which consists of a set of items to be loaded into bins of equal size such that the number of bins used is minimum. This problem was first introduced in the 1970s [59, 32, 38, 37]. An online variant of BPP was studied by Seiden [56], whereas Rhee and Talagrand [54] and Peng and Zhang [48] studied the problem in a stochastic setting. Bounds for BPP can be found in a number of works [42, 60, 29, 16, 15]. BPP has also been addressed by means of column generation [61] and polynomial-time approximation schemes [20, 39].

Bin Packing with Rejection (BPR) is a variant of BPP where items can also be rejected. If an item is rejected, a penalty is paid. Therefore, the goal is to minimize the number of bins used and the penalties for the rejected items. A number of studies on BPR can be found in the literature [21, 10, 22].

Variable Sized Bin Packing Problem (VSBPP) is a generalization of BPP, which was proposed in the 1980s by Friesen and Langston [31], and involves the introduction of bin types. Murgolo [45] and Epstein and Levin [23] provided efficient approximation schemes for it. Lower bounds have been proposed by Monaci [44], Crainic et al. [17], Haouari and Serairi [34], Mohamed et al. [43], and Balogh et al. [9]. Monaci [44] and Haouari and Serairi [34] studied exact algorithms, whereas heuristic approaches were adopted in [33, 35, 41]. The online variant has been discussed in a number of works [55, 57, 62, 24, 11], whereas the stochastic variant has recently been studied by Fazi et al. [28] and Crainic et al. [18].

Generalized Bin Packing Problem (GBPP) is a generalization of previous bin packing problems, involving the introduction of item profits as well as compulsory and non-compulsory items. Heuristics, and exact and approximate algorithms can be found in [6, 8]. Online and stochastic variants have been discussed in [7, 50]. Finally, approximation issues have been studied in [5].

3. The GBPPI model

In this section, we propose a model for GBPPI. We define the following:

- $\mathcal{I}$: set of items
- $n = |\mathcal{I}|$: number of items
- $\mathcal{I}^{\mathrm{C}} \subseteq \mathcal{I}$: set of compulsory items
- $\mathcal{I}^{\mathrm{NC}} \subseteq \mathcal{I}$: set of non-compulsory items. Clearly, $\mathcal{I}^{\mathrm{C}}$ and $\mathcal{I}^{\mathrm{NC}}$ are a partition of set I, i.e., $\mathcal{I}^{\mathrm{C}} \cup \mathcal{I}^{\mathrm{NC}} = \mathcal{I}$ and $\mathcal{I}^{\mathrm{C}} \cap \mathcal{I}^{\mathrm{NC}} = \emptyset$
- $\mathcal{J}$: set of bins
- $m = |\mathcal{J}|$: number of bins
- $\mathcal{T}$: set of bin types
- $\sigma : \mathcal{J} \longrightarrow \mathcal{T}$: indicator function, where given bin $j \in \mathcal{J}$, reveals its type $t \in \mathcal{T}$, i.e., $\sigma(j) = t$ iif bin $j \in \mathcal{J}$ belongs to type $t \in \mathcal{T}$
- p_{it}: profit generated by item $i \in \mathcal{I}$ when accommodated into a bin of type $t \in \mathcal{T}$
- w_i: volume of item $i \in \mathcal{I}$
- C_t: cost of a bin of type $t \in \mathcal{T}$
- W_t: capacity of a bin of type $t \in \mathcal{T}$
- L_t: minimum number of bins to be used of type $t \in \mathcal{T}$
- U_t: maximum number of bins to be used of type $t \in \mathcal{T}$
- $U \leq \sum_{t \in \mathcal{T}} U_t$: maximum number of bins to be used
- $\mathcal{S} \subseteq \mathcal{J}$: set of bins used in a solution of GBPPI
- $W_{res}(b) \ : \ b \in \mathcal{S}$: residual volume of a bin for a solution of GBPPI. This is given by the capacity of bin b minus the sum of the volumes of the items loaded in b.

An optimal solution of an instance of GBPPI must satisfy the following requirements:

- The overall cost given by the difference between the cost of the bins used and the profits incurred by the loaded items is minimized.
- All compulsory items must be accommodated into some bins.
- The sum of the volumes of the items loaded into a bin cannot exceed the capacity of that bin.
- At least L_t and at most U_t bins of type $t \in \mathcal{T}$ can be used.
- At most U bins can be used in total.

In order to provide a model for GBPPI, we need to introduce the following binary variables:

$$x_{ij} = \begin{cases} 1 & \text{if item } i \in \mathcal{I} \text{ is accommodated into bin } j \in \mathcal{J} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$y_j = \begin{cases} 1 & \text{if bin } j \in \mathcal{J} \text{ is used} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

A model for GBPPI can then be formulated as follows:

$$\min \quad \sum_{j\in\mathcal{J}} C_{\sigma(j)} y_j - \sum_{j\in\mathcal{J}} \sum_{i\in\mathcal{I}} p_{i\,\sigma(j)} x_{ij} \tag{3}$$

$$\text{s. t.} \quad \sum_{i\in\mathcal{I}} w_i x_{ij} \le W_{\sigma(j)} y_j \qquad j \in \mathcal{J} \tag{4}$$

$$\sum_{j\in\mathcal{J}} x_{ij} = 1 \qquad i \in \mathcal{I}^{\mathrm{C}} \tag{5}$$

$$\sum_{j\in\mathcal{J}} x_{ij} \le 1 \qquad i \in \mathcal{I}^{\mathrm{NC}} \tag{6}$$

$$\sum_{j\in\mathcal{J}:\sigma(j)=t} y_j \le U_t \qquad t \in \mathcal{T} \tag{7}$$

$$\sum_{j\in\mathcal{J}:\sigma(j)=t} y_j \ge L_t \qquad t \in \mathcal{T} \tag{8}$$

$$\sum_{j\in\mathcal{J}} y_j \le U \tag{9}$$

$$y_j \in \{0,1\} \qquad j \in \mathcal{J} \tag{10}$$

$$x_{ij} \in \{0,1\} \qquad i \in \mathcal{I}, \quad j \in \mathcal{J} \tag{11}$$

The objective function (3) ensures that the solution minimizes the overall cost, given as the cost due to bin use minus the profit from the loading of the items into the bins. Constraints (4) are the so-called capacity constraints that ensure that the sum of the volumes of the items loaded into a bin does not exceed bin capacity. Constraints (5) ensure that all compulsory items are loaded, whereas constraints (6) state that non-compulsory items may or may not be accommodated. Constraints (7)–(8) are bin usage constraints. Finally, constraints (10)–(11) force the variables involved to be binary.

Model (3)–(11) inherits all the variables and constraints from the model for GBPP described in [6] except for the objective function. Although the change in the objective function due to the introduction of bin-dependent item profits might seem trivial at first glance, in Appendix B we show that this change strongly worsens the problem. In Appendix B, we present a detailed comparison in further detail between GBPP and GBPPI using a commercial solver on the corresponding models. The results showed that the solver provides smaller gaps when applied to GBPP in the same computational time. Moreover, as we show in Section 4, the introduction of bin-dependent item profits implies a relevant generalization of the constructive heuristics used to tackle both problems.

Finally, we wish to point out that the presence of bin usage constraints (7)–(9) and a limited number m of bins might lead to infeasible solutions. As shown in [6], in order to ensure that the problem is feasible, a dummy bin with a large capacity and high cost is added to the problem.

4. Heuristics

In this section, we present efficient heuristics for addressing GBPPI. As mentioned in the introduction and will be shown in Appendix B, the use of commercial solvers is not sufficient to efficiently address the problem, in particular if fast solutions with a computational time of at most 10 minutes are required (i.e., the average time for reaching decisions at the operational level). For this reason, we developed a series of heuristics useful for addressing GBPPI. The common principle of our heuristics is to address problems where the sign of the objective function can be either positive, null, or negative. In previous bin-packing problems the goal was to minimize a single objective: the number of bins used, the wasted space, etc. This implied an objective function which is always non-negative. Vice versa, in generalized bin packing problems like the GBPP or the GBPPI, we deal with an objective function which terms can have different signs. In fact, minimizing the net cost implies the optimization of two contributes: the minimization of the costs (which signs are non negative) and the maximization of the profits (which signs are non positive). As it will be shown in this section, our heuristics take this broadening of the objective function into account. Thus, they are also suitable to address those problems with a mixed target in the objective function. The proposed heuristics are:

- one constructive heuristic named Best profitable (BP)
- one constructive heuristic named Best Assignment (BA),
- one metaheuristic named Greedy Randomized Adaptive Search Procedure (GRASP) [30],
- a parallel metaheuristic named Model-Based Metaheuristic (MBM).

These heuristics provide a flexible trade-off between quality of solution and computational time, in light of the imposed maximum computational time of 10 minutes.

4.1. The constructive heuristics

The proposed constructive heuristics are a variant of Best Fit Decreasing (BFD) introduced by Johnson et al. [38] to address BPP. As already discussed, this generalization is necessary to address GBPPI and problems with a mixed objective function. Our constructive heuristics are called Best profitable (BP) and Best Assignment (BA), and operate with a list of available bins *SBL* and one of sorted items *SIL*. The major variant implemented in order to address GBPPI is the broadening of the definition of the *best* bin. Let $\mathcal{S} \subseteq \mathcal{J}$ be the set of bins used in a solution of a bin packing problem, and let $W_{res}(b)$ be the residual volume of bin $b \in \mathcal{S}$. In previous versions of the bin-packing problem, the best bin for an item was defined as the one that can accommodate the item such that the residual space is minimized. In GBPPI, instead of considering the minimum residual space, we compute a figure of merit consisting of a weighted sum that takes into account both item profit and bin volume. Again, this choice is motivated by the fact that in GBPPI, we need to consider two factors. In classical bin packing problems the aim is to minimize residual space in order to reduce the number of bins used or the cost of bin usage. The adoption of this approach in GBPPI does not always guarantee effective outcomes because item profits in such cases rely heavily on bins. Our weighted figure of merit, defined as $\alpha \cdot p_{i\,\sigma(j)} - (1-\alpha) \cdot W_{res}(j)$, simultaneously addresses these two loading policies. The term

$p_{i\sigma(j)}$ maximizes item profit, whereas the term $W_{res}(j)$ minimizes residual space. $\alpha \in [0, 1]$ is a coefficient that is varied during the execution of GRASP (cf. Section 4.2), and allows both loading policies to be spanned.

A further generalization is that this definition of the best bin is applied to a subset of $N \ll |SIL|$ items, rather than a single item. When we consider item i in list *SIL*, we take into account the sublist $SIL' = \{i, i + 1, \ldots, i + N - 1\}$. For each item, we compute the best bin; at the end of this process, we select the best item $i^* \in SIL'$ and the best bin b that maximizes the aforementioned figure of merit. Our computational experience confirmed that this "medium-term" memory improves the performance of the heuristics. The behavioral difference between BP and BA heuristics can be observed when we cannot load item $i \in SIL$ into any of the already bins used in $\mathcal{S}$. In this case, we try to select a new bin where to accommodate item i. The BP heuristic considers item i with the remaining succeeding items in SIL, and selects the bin that minimizes the difference between bin cost and the sum of profits from items that can be loaded into that bin. Similarly, the BA heuristic selects the bin that maximizes profit for item i. Both heuristics perform a post-optimization procedure that tries to move items to cheaper bins and to remove non-profitable bins. It is clear that the solutions provided by the BP and BA heuristics depend on the two parameters α and N. As we show in Section 4.2, the values of these parameters are varied using the GRASP procedure, which employs BP and BA as sub-heuristics.

4.2. The GRASP

Greedy Randomized Adaptive Search Procedure (GRASP) algorithms consist of a multi-start procedure to find a good initial solution and of a loop where, at each iteration, a new solution is generated by means of a simpler heuristic. In our GRASP for GBPPI, we employ the Best profitable and Best Assignment heuristics, which were already described in Subsection 4.1.

At each iteration of the main loop, we try to generate a different and improved solution by varying the order of the items in the list *SIL*. This is performed by associating a *score* with each item. A score update procedure randomly assigns a different score value to each item.

Finally, a long-term reinitialization procedure is executed each time a solution does not improve in consecutive iterations. Its purpose is to explore a different area of the feasible set by changing parameters α and N of the constructive heuristics. The proposed method incorporates some ideas from [49]. The pseudo-code of our GRASP is proposed in Algorithm 1.

The algorithm presented in this section satisfies the terminology of the term GRASP, namely Greedy Randomized Adaptive Search Procedure. It is a greedy algorithm because it is based on the BP and BA procedures that can be classified as greedy algorithms. It is also a randomized algorithm because randomization is present in the assignment of scores. Finally, it is also an adaptive-search algorithm because the long-term reinitialization procedure is helpful to explore new regions of the solution space.

The GRASP metaheuristic can be easily parallelized. Let $\mathcal{P}$ be the set of available threads in a parallel computation. It is enough to execute one GRASP metaheuristic for each thread $p \in \mathcal{P}$ and with a different seed for the

Algorithm 1 The GRASP

1: IS: initial solution provided by the MULTI-START INITIALIZATION procedure
2: $BS := IS$
3: $numConsecutive$: number of consecutive non-improving solutions
4: $numConsecutive := 0$
5: **while** time limit has not been reached **do**
6: sort the items
7: perform either the BP or the BA constructive heuristic
8: store the resulting solution as CS
9: **if** $CS < BS$ **then**
10: $BS := CS$
11: $numConsecutive := 0$
12: **else**
13: $numConsecutive := numConsecutive + 1$
14: **end if**
15: SCORE UPDATE procedure
16: **if** $numConsecutive = MAXCONSECUTIVE$ **then**
17: LONG-TERM REINITIALIZATION procedure
18: $numConsecutive := 0$
19: **end if**
20: **end while**

random-number generator. Let $BP(p)$ be the final best solution provided by the GRASP executed by thread $p \in \mathcal{P}$, then the overall best solution BS will be

$$BS = \min_{p \in \mathcal{P}} BS(p).$$

4.2.1. Multi-start initialization

The purpose of our multi-start initialization procedure is twofold: to feed the main loop of GRASP with a good initial solution, and automatically calibrate the parameters used by the constructive heuristics. The three parameters involved in GRASP are α, N, and $N_{\max}$, where $N_{\max}$ is the maximum of parameter N. The first two parameters are used in constructive heuristics, as previously discussed, whereas the latter is only updated in the long-term reinitialization procedure.

In multi-start initialization, we execute $M = \min\{25, n\}$ Best profitable heuristics 100 times, with N varying from 1 to M, and α initially set to 0.25. The value of N corresponding to the best solution found during the M iterations is set as the maximum dimension $N_{\max}$.

4.2.2. Score update

Item scores are randomly extracted from a discrete uniform distribution in the range $[0, 4n]$. The motivation for this choice is that working with integer values rather than real values implies a faster resolution of the sorting procedure. We prefer to distribute the item scores in a range proportional to the number of items, and then use integer scores rather than concentrating the scores with real values within a smaller range.

4.2.3. Long-term reinitialization

An index is incremented modulo 5, and α is set to the corresponding position in set $\{0.25, 0.4, 0.5, 0.6, 0.75\}$. Moreover, the parameter N of the group of items used in constructive heuristics is incremented. N is set to 1 again after it reaches the maximum dimension allowed $N_{\max}$.

4.3. The Model-Based Metaheuristic

We present here a parallel metaheuristic for GBPPI, where a set of computer threads $\mathcal{P}$ run simultaneously. It consists of a loop, where at each iteration each thread solves a subproblem using model (3)–(11). The resolution of subproblems with model (3)–(11) allows us to take into account two targets at the same time, namely the minimization of the cost and the maximization of the profits. Thus, this metaheuristic is suitable to address problems with a similar structure.

In each subproblem, a small set of bins is randomly selected from the incumbent solution and the set of available bins. In order to further improve the given solution, we merge couples of partial solutions that do not have any bins and items in common. The best solution is updated if the new solution is an improvement over the incumbent one. This process continues until an overall time limit is reached.

According to the taxonomy of parallel methods proposed by Crainic and Toulouse [14], this approach can be classified as 1C/RS/MPSS, where the following hold:

- 1C: One Control, i.e., one master thread controls all the remaining threads.
- RS: Rigid Synchronization, i.e., at each iteration, we wait for all threads to complete their computations.
- MPSS: Multiple points same strategy, i.e., each thread solves a different subproblem, but using the same strategy.

We chose to use this kind of parallelism because it can easily be implemented in our metaheuristic. As we show in the next section, although the metaheuristic can be exploited without parallel computation, this approach is strongly recommended when GBPPI is employed as a subproblem of a larger problem, and significantly improves the performance of the metaheuristic. The main steps of the MBM metaheuristic are shown in Algorithm 2.

Algorithm 2 The MBM metaheuristic

1: $\mathcal{P}$: set of threads
2: IS: Initial solution provided to the MBM metaheuristic
3: BS: Best solution
4: $BS := IS$
5: **while** time limit has not been reached **do**
6: **for all** $p \in \mathcal{P}$ **do**
7: Randomly select a subset $b(p)$ of bins from the set of bins $\mathcal{S}$ making up solution BS.
8: Solve a GBPPI subproblem with a solver with a time limit of 1 s, the bins $b(p)$ plus selected available bins, the items loaded into bins $b(p)$, and the items not loaded in solution BS.
9: **end for**
10: Merge partial solutions provided by each thread and store the new current solution in CS.
11: **if** $CS < BS$ **then**
12: $BS := CS$
13: **end if**
14: **end while**

5. Computational results

As there are no instances for GBPPI in the literature, we have created new instance sets, starting from instances of GBPP (cf. [6]). At the same time, instances of GBPP are based on Monaci's VSBPP instances [44]. To make this article self contained, we report here the details of both VSBPP and GBPP instances.

Monaci's instances for the VSBPP consist of a group of 300 instances where the items are all compulsory. Table 1 lists the features of these instances in terms of items and bin types.

FEATURES	VALUES
# of items	25
	50
	100
	200
	500
Item volume	I1: [1, 100]
	I2: [20, 100]
	I3: [50, 100]
Bin types	A: 100, 120, 150
	B: 60, 80, 100, 120, 150

Table 1: Description of Monaci's instances for VSBPP.

Ten instances were generated for each combination of features in Table 1, for a total of 5×3×2×10 = 300 instances. For each instance, the minimum and the maximum number of available bins for bin type $t \in \mathcal{T}$ is respectively set to

$$L_t = 0, \qquad \forall\, t \in \mathcal{T}$$
$$U_t = \left\lceil \frac{\sum_{i \in \mathcal{I}} w_i}{W_t} \right\rceil, \qquad \forall\, t \in \mathcal{T}.$$

GBPP instances consist of four classes of instances numbered from 0 to 3. Class 0 coincides with the instances in Monaci [44] for VSBPP. Instances of classes 1 and 2 are identical to those belonging to class 0 but the profits. In fact, in class 0 the items are all compulsory. Vice versa, in classes 1 and 2 the items are all non compulsory. The profit p_i of item $i \in \mathcal{I}$ belonging to an instance in class 1 or 2 is given by

$$p_i \in \lceil \mathcal{U}(0.5, 3) w_i \rceil \qquad \text{class 1,}$$
$$p_i \in \lceil \mathcal{U}(0.5, 4) w_i \rceil \qquad \text{class 2}$$

respectively, where $\mathcal{U}$ denotes uniform distribution. Finally, Class 3 is a 500-item class, with 60 instances selected from classes 0–2 with 0%, 25%, 50%, 75%, and 100% of compulsory items. GBPP instances are available on line [47].

Beginning with a selection of instances of GBPP, we created 600 instances for GBPPI while maintaining the same values, but with item profits computed as

$$p_{it} = p_i + \theta_t$$

with

$$\theta_t = \max_{l \in L} \left\{ 0.4 \frac{C_{\min} C_{\max}}{W_t} \vartheta \right\}, \tag{12}$$

where ϑ can either be extracted from a uniform distribution $\mathcal{U}(0, 1)$ or from a Gumbel distribution $\mathcal{G}(0, 1)$. Values in (12) ensure that profit values approximate realistic settings. For the sake of brevity, we refer to instances of GBPPI with item profits extracted from a Gumbel distribution according to (12) as Gumbel instances, and instances of GBPPI with item profits extracted from a uniform distribution according to (12) as uniform instances. GBPPI instances are also available on line [46].

Computational resources were provided by HPC@POLITO [36], on a system with Opteron 2.3 GHz processors and 124 GB of RAM. The metaheuristic was implemented in C++, with eight threads and with the following time limits: 1s for the GRASP, 1s for each subproblem solved with the solver, and 120s for the metaheuristic. We used the commercial solver CPLEX 12.6 [13] to solve each subproblem and for the comparison between the GBPP and the GBPPI (see Appendix B)

In Table 2, we list the percentage gaps between the algorithms proposed in this paper and the best solution found by the solver in one hour. The results are reported according to the type of instance (Gumbel and uniform), the number of items (from 25 to 200), and the number of threads (one and eight). We also wanted to estimate the benefits of a parallel computation in comparison with a single-thread approach. The columns in Table 2 describe the following: col. 1) is the type of distribution for item profits; col. 2) shows the number of items; col. 3) shows the percentage gap in the best constructive heuristic with respect to the solver with one thread; cols. 4) – 5) are the percentage gaps of GRASP and the MBM with one thread with respect to the solver with one thread; and cols. 6) – 7) are the percentage gaps of GRASP and the MBM with eight threads with respect to the solver with eight threads. The best constructive heuristic was given by the minimum of the BEST PROFITABLE and BEST ASSIGNMENT heuristics.

The aim of column 3 is to show the performances of these constructive heuristics, which were executed with $\alpha = 0.5$ in order to emphasize both components in the figure of merit, and with the items sorted by non-increasing mean profits over volume. The best constructive heuristic was compared to the solver with one thread only because the execution of BEST PROFITABLE and the BEST ASSIGNMENT heuristic does not require parallelism. The computational time of the constructive heuristic was practically zero, but this immediate execution was paid for in terms of the highest percentage gaps. In fact, the overall gap was 12.76%. Although this value was high, the BEST PROFITABLE and BEST ASSIGNMENT heuristics found their utility in the execution of GRASP and MBM, which yielded much lower percentage gaps. Moreover, from Table 2 we can also observe the benefits of introducing parallelism. The percentage gap of GRASP was almost halved when switching from one to eight threads. This gap reduced from 4.08% to 2.82%, which is a considerably improved result if we consider that the computational time of the GRASP was approximately 1 second. The percentage gaps of the MBM were satisfactory. Again, parallelism improved quality of solution. In fact, gaps reduced from -0.25% to -0.47% when the number of threads increased from 1 to 8.

DISTRIBUTION	ITEMS	CONSTR.	1 THREAD		8 THREADS	
			GRASP	MHEUR	GRASP	MHEUR
GUMBEL	25	13.07	0.97	0.10	0.86	0.02
	50	15.05	3.94	0.16	2.69	0.14
	100	14.76	5.69	0.39	4.66	0.31
	200	13.77	6.89	0.12	4.56	-0.02
	500	9.98	2.99	-1.95	1.03	-2.94
	OVERALL	**13.32**	**4.10**	**-0.24**	**2.76**	**-0.50**
UNIFORM	25	14.28	1.03	0.05	0.97	0.04
	50	14.06	3.71	0.24	2.89	0.17
	100	12.56	5.93	0.41	4.52	0.39
	200	11.86	7.24	0.19	5.01	0.16
	500	8.24	2.44	-2.19	1.04	-2.97
	OVERALL	**12.20**	**4.07**	**-0.26**	**2.89**	**-0.44**
OVERALL		**12.76**	**4.08**	**-0.25**	**2.82**	**-0.47**

Table 2: Percentage gaps in the proposed algorithms reported according to item profit distribution (column 1), number of items (column 2), and parallelism (columns 4–7).

Table 3 presents the average computational times of the last best solution found by the MBM. The columns in Table 3 indicate the following: col. 1) represents the type of distribution for the item profits, col. 2) shows the number of items, cols. 3) and 4) list the average computational times needed to find the best solution for the MBM with one and eight threads, respectively.

DISTRIBUTION	ITEMS	1 THREAD	8 THREADS
GUMBEL	25	2.94	2.48
	50	31.16	13.56
	100	64.97	47.30
	200	92.88	70.87
	500	99.03	104.50
	OVERALL	**58.20**	**47.74**
UNIFORM	25	5.74	1.84
	50	29.00	14.31
	100	57.25	45.05
	200	93.70	74.67
	500	99.03	109.43
	OVERALL	**56.94**	**49.06**
OVERALL		**57.57**	**48.40**

Table 3: Best computational times for the MBM reported according to item profit distribution (column 1), number of items (column 2), and parallelism (columns 3–4).

The analysis of Table 3 reveals that the average time tended to increase in line with instance size. Moreover, the overall average time was approximately one minute.

Finally, we compared the MBM with the branch-and-price by Baldi et al. [8] on the same instances as the classical GBPP. We computed the percentage gaps of the MBM with respect to the branch-and-price results. The results are presented in Table 4 according to the number of items. In particular, columns 2, 4, 6, and 8 indicate the percentage gaps of the branch-and-price with respect to its best lower bound computed at the root node (cf. [8] for further details). Columns 3, 5, 7, and 9 present the percentage gaps of the MBM with respect to the best objective function provided by the branch-and-price. The instances in Class 3 were defined for 500 items only. The overall percentage gap of the MBM compared to the branch-and-price was approximately 0.22%. Nevertheless, we observed that for 34 instances, better results were found than those provided by the branch-and-price in Baldi et al. [8], and within a considerably smaller computational time. The time limit of the branch-and-price was one hour, while that of the MBM was two minutes. Moreover, for a third of the instances of GBPP, the Model-Based Metaheuristic could find the same results generated by the branch-and-price.

ITEMS	CLASS 0		CLASS 1		CLASS 2		CLASS 3	
	BEP	**MHEUR**	**BEP**	**MHEUR**	**BEP**	**MHEUR**	**BEP**	**MHEUR**
25	0.00	0.01	0.00	0.02	0.00	0.00	***	***
50	0.00	0.19	0.00	0.14	0.01	0.12	***	***
100	0.02	0.32	0.03	0.26	0.01	0.15	***	***
200	0.06	0.33	0.02	0.37	0.03	0.28	***	***
500	0.12	0.38	0.12	0.34	0.11	0.26	0.57	0.59
OVERALL	**0.04**	**0.25**	**0.03**	**0.23**	**0.03**	**0.16**	**0.57**	**0.59**

Table 4: Comparisons between the branch and price and the MBM. Columns 2, 4, 6, and 8 show the percentage gap of the branch and price with respect to the best lower bound. Columns 3, 5, 7, and 9 show the percentage gap of the MBM with respect to the best objective function provided by the branch and price. *** The instances in Class 3 were defined for 500 items only.

6. Smart City case study

As described in the Introduction, the case study referred to the planning of pick-up and delivery operations of an international courier. The decisions of the courier arose from a competitive tender between the courier and transportation companies (TCs). In this competitive tender, each TC proposes a transportation cost for each truck used. This cost covered expenses such as drivers, fuel, and depreciation in vehicle value. Moreover, a TC received a bonus for each successful pick-up or delivery. The courier selected TCs according to the proposed transportation costs and TC reliability. The courier migh have checked the quality of each shipment through truck inspection and feedback from customers. Drivers who did not load the freight into the truck correctly, or customers who filed a complaint with the courier regarding a delayed or missed delivery, or for receiving a damaged product, clearly reduced the reliability of a TC. Conversely, prompt pick-ups and deliveries and positive feedback from customers increased the reliability of a TC. The higher TC reliability, the higher customer satisfaction. Therefore, the profit for the courier in shipping a freight was not only proportional to the economic value of the freight, but also to the TC reliability.

In our case study, the set of items $\mathcal{I}$ represented the freight that the courier needed to pick-up and deliver. The different bin types represented the available TCs, the bins were the trucks, and U_t is the cardinality of the fleet of the TC $t \in \mathcal{T}$. Item revenues consisted of the sum of two terms, ρ_i and $\chi_{\sigma(j)}$, where ρ_i was the economic value of shipping item $i \in \mathcal{I}$, and $\chi_{\sigma(j)}$ was the gain due to the reliability of TC $j \in \mathcal{J}$. A bonus $c_{\sigma(j)}$ was awarded to the TC for each shipment with a truck of type $\sigma(j)$. Therefore, item profits became $p_{i\sigma(j)} = \rho_i + \chi_{\sigma(j)} - c_{\sigma(j)}$.

In the case study, we compare our MBM solutions with those of the courier based on its business policy. We analyzed 30 instances (i.e., 30 competitive tenders) with 1,000 daily pick-ups and deliveries (i.e., the items), 10 TCs (i.e., the bin types), and up to 100 trucks (i.e., the bins) per TC, available in a distribution area located in northern Italy.

In Table 5, we present the percentage gaps of our MBM compared to the business policy and the solver (this time with a time limit of two hours) over the 30 competitive tenders. Our MBM always found better results than those provided by the business policy and the solver.

INSTANCE	COURIER GAP	SOLVER GAP	INSTANCE	COURIER GAP	SOLVER GAP
Instance 1	-4.10	-1.22	Instance 16	-3.19	-1.47
Instance 2	-3.54	-0.98	Instance 17	-3.14	-1.04
Instance 3	-3.89	-1.21	Instance 18	-3.17	-1.15
Instance 4	-3.85	-1.86	Instance 19	-3.18	-0.69
Instance 5	-3.80	-1.95	Instance 20	-3.70	-1.28
Instance 6	-3.79	-1.12	Instance 21	-3.05	-2.15
Instance 7	-3.82	-1.35	Instance 22	-3.05	-0.91
Instance 8	-3.56	-1.18	Instance 23	-2.80	-2.25
Instance 9	-3.84	-1.05	Instance 24	-3.14	-2.57
Instance 10	-4.24	-0.92	Instance 25	-3.12	-2.57
Instance 11	-3.45	-1.66	Instance 26	-2.88	-2.29
Instance 12	-2.90	-0.52	Instance 27	-2.86	-2.62
Instance 13	-3.24	-1.49	Instance 28	-3.10	-2.04
Instance 14	-3.22	-1.27	Instance 29	-3.07	-1.83
Instance 15	-3.33	-0.26	Instance 30	-2.89	-0.79
			OVERALL	**-3.36**	**-1.46**

Table 5: Percentage gaps of the MBM compared with the business policy and the solver. Columns 1 and 4 present the instance number of the case study, columns 2 and 5 the percentage gaps of the business policy compared to the MBM, and columns 3 and 6 the percentage gaps of the solver compared to the MBM.

A more interesting outcome results from the analysis of the instances from an economic and managerial point of view. The size of each instance was representative of the daily parcels of a parcel delivery company in a medium-sized city. If we compare the solution with one based on expert opinion (tactical decisions based on expert opinion and day-to-day decisions optimized by means of specific optimization tools), the assignments given by GBPPI achieve a constant reduction of the overall cost by between 3 and 4%. In terms of the economic impact of costs, this amounts to 120,000–180,000 euros for a city the size of Turin (the interval depends on different scenarios of annual numbers of parcels). Moreover, this cost will increase in the near future, due to increase in B2C flows resulting from e-commerce, mass customization, and decentralized production. This leads to problems that are not solvable based simply on expert opinion. The increase in volumes in the B2C market goes hand in hand with a reduction in the marginal unitary revenues of the delivery companies due to contracts with large e-commerce companies and a high dispersion

of deliveries [2, 3, 4].

7. Conclusions

In this paper, we introduced a new packing problem named Generalized Bin Packing Problem with bin-dependent item profits (GBPPI). We have shown that GBPPI can be applied at both tactical and operational levels. At the tactical level, GBPPI models cross-country and multi-modal transportation settings. At an operational level, GBPPI describes the problem of a courier selecting the appropriate number and type of vehicles from a set of available transportation companies. We have also demonstrated that the introduction of bin-dependent item profits is not trivial in terms of problem resolution. We presented a number of heuristics to efficiently address the problem within limited computational time. We have also presented extensive computational results and a case study of a well-known international courier operating in northern Italy.

GBPPI can also be a starting point for future research. In fact, GBPPI can be exploited as a subproblem in the resolution of the stochastic variant of GBPP, namely Stochastic Generalized Bin Packing Problem. Stochastic problems are affected by uncertainty. The Progressive Hedging Algorithm is an iterative technique for addressing these problems [19]. At each iteration, random variables describing uncertain attributes are fixed to particular values. In this way, each iteration of the Progressive Hedging Algorithm applied to Stochastic Generalized Bin Packing Problem implies the solution of a deterministic GBPPI subproblem. This solution may successfully be performed through the heuristics proposed in this article.

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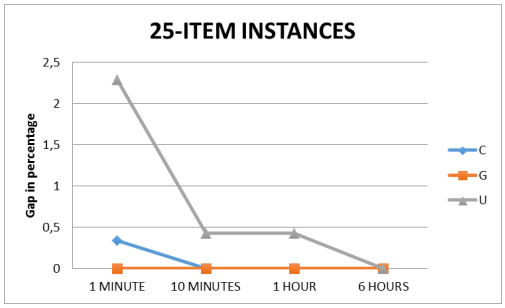
Appendix A. Comparisons between GBPPI and the ESICUP 2015 problem

The ESICUP 2015 context dealt with a container-loading problem to be considered by Renault for the daily loading of containers, and to "estimate every week the number of containers needed for the following weeks." There is an interesting number of analogies between GBPPI and the ESICUP 2015 problem. Both the problems deal with assignment of items to bins. Moreover, the items are not necessarily compulsory. In fact, the least profitable items can be put aside and be packed in the next shipment. Despite these analogies, we note that the two problems are quite different. In particular, the ESICUP 2015 problem is a three-dimensional bin packing problem. For each bin, items can be packed into stacks. Stacks consist of layers, and layers consist of rows. Particular rotations of the items are allowed, and both capacity and weight constraints need to be satisfied. Another big difference can be noticed in the objective functions of the two problems. GBPPI minimizes the overall net cost. The ESICUP 2015 problems presents a lexicographic objective function, which main objective is the minimization of the wasted space. Finally, GBPPI is a one-dimensional problem, which is characterized by the introduction of item profits that depend on the bins. This dependency, which is not present in the ESICUP 2015 problem, also allows the use of GBPPI at a tactical level, as discussed in Section 1.

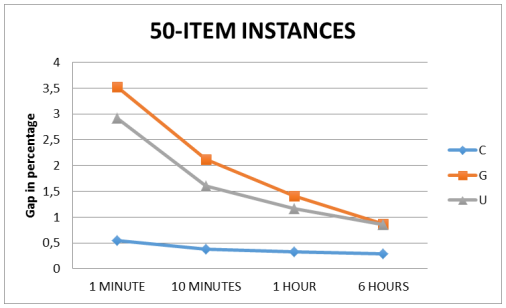
Appendix B. Comparisons between GBPP and GBPPI

We compare here the classical version of GBPP with the new GBPPI. This comparison allows us to reveal two important considerations. First, as GBPPI is a further generalization of the classical GBPP, higher computational effort is required for it. Second, for this reason, it is important to design a heuristic that is accurate and fast at the same time, in order to be able to exploit the benefits of GBPPI in a city logistics setting, as discussed in the Introduction. In city logistics decisions usually must be reached within a short time.

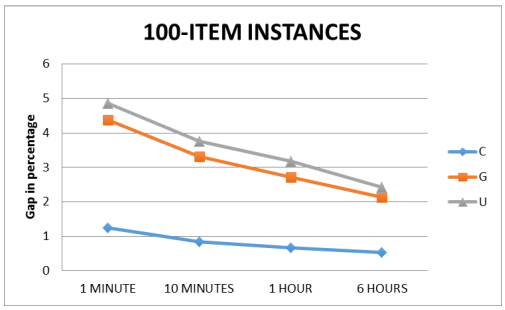
To make the comparison, we selected 30 instances of GBPP along with the corresponding Gumbel and uniform instances of GBPPI, as explained in the text. Thus, the number of instances in this instance set is 90. The number of items range from 25 to 500. We solved each instance with the solver, eight threads and a time limit of six hours. Furthermore, we monitored the percentage gap provided by the solver after one minute, 10 minutes, one hour, and six hours of computation. The results of these initial tests are summarized in Figure B.1, where we show the gaps in mean percentage between test instances, grouped by the number of items and type of instances. In particular, C denotes the original instances of GBPP, G stands for the Gumbel instances of GBPPI, and U represents the uniform instances of GBPPI.



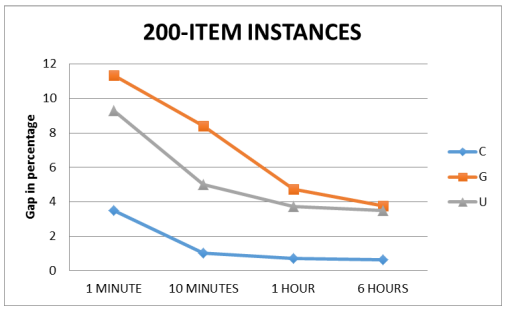
a) Comparisons with 25-item instances



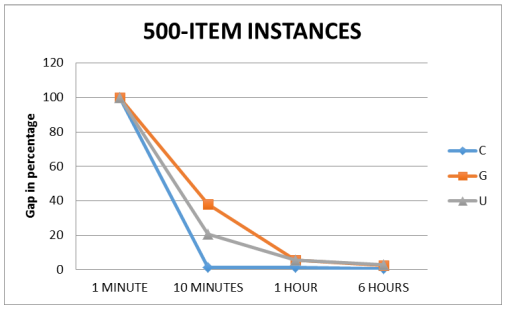
b) Comparisons with 50-item instances



c) Comparisons with 100-item instances



d) Comparisons with 200-item instances



e) Comparisons with 500-item instances

Figure B.1: Comparisons between the classical GBPP and the new GBPPI. The *x*-axis represents time and the *y*-axis gaps in percentage. C denotes classical GBPP instances, G indicates Gumbel-GBPPI instances, and U denotes uniform-GBPPI instances.

From Figure B.1, we can see that the gaps provided by the solver for the GBPPI instances (both Gumbel and uniform) are clearly higher than those of the original GBPP. For example, the gap in mean percentage of the 500-item-GBPP instances is 0.43%, whereas the gap in mean percentage of the corresponding GBPPI instances is 2.50% for the Gumbel instances and 2.72% for the uniform instances. This suggests that the introduction of the dependence of item profits on bins involves greater computational effort.

It is worth remarking on the trend in gaps with respect to the range of the minutes. This interval is crucial in city-logistics settings, where decisions need to be made quickly [51]. It was not possible to report the percentage gaps for the 500-item instances after one minute of computation because the solver was unable to determine an initial solution (i.e., the percentage gap was arbitrarily large). The percentage gaps provided by the solver after 10 minutes for the GBPPI instances were acceptable for the 25-item instances only. As the number of items increased to 50, the gaps increased to 2.12% for Gumbel instances and to 1.60% for uniform instances. The percentage gaps tended to increase as the number of items increased. This trend became considerably more evident for the 500-item instances, with the GBPP instances having a mean percentage gap of 1.43 and the GBPPI having gaps of 38.19% and 20.57%, respectively, for the Gumbel and uniform instances. These gaps were clearly prohibitive. Moreover, for some instances the solver was unable to find an initial solution after 10 minutes of computation. All these considerations indicate that the solver alone is not appropriate to address GBPPI in a city-logistics environment where decisions are made quickly and the number of items is greater than 500.