Slope diffraction coefficients for the half-planes with two face impedences

SLOPE DIFFRACTION COEFFICIENT FOR THE HALF PLANE WITH TWO FACE IMPEDANCES

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Slope diffraction correction is known for conventional reflectors, i.e. reflectors with edges having both face perfectly conducting. In this paper the canonical problem of the scattering from an half plane with two different face impedances in presence of an arbitrary illuminating field has been phrased in terms of a Wiener-Hopf matrix equation. This one has been solved and analytical expressions both for the first order diffraction coefficient and for the slope diffraction coefficient are deduced.

The problem of the scattering from an half plane with two face impedances has been widely studied (see for example (1-4)) and the first order diffraction coefficient has already been obtained. In order to obtain a slope diffraction correction the problem has been rephrased in term of line representation and Wiener-Hopf matrix equation.

Fig.1 shows the geometry of the problem; on the left and on the right side of the scatterer the following boundary conditions are assumed:

$$\underline{E}_T = \pm Z_{l,r}\, \underline{H}_T \times \hat{z} \quad , \text{ for } x>0 \text{ and } z = 0_{+,-} \qquad (1)$$

Having used twice the equivalence theorem, first for $z<0$ with $Z=Z_l$ over the all plane $z=0_-$ and then for $z>0$ with $Z=Z_r$ over the all plane $z=0_+$, the equivalent line representation of fig.1 can be adopted. Considering only magnetic field polarized along the y-axis, the line parameters are $Z_\xi = \tau/\omega\varepsilon$ and $\tau = (K_o^2 - \xi^2)^{1/2}$ while the unknown generators v_l and v_r are:

$$v_{l,r} = j\,(2\pi)^{1/2}\, \mathfrak{F}\{ (E_x(0) \mp Z_{l,r}\, H_y(0)\,)\, u(-x) \} \quad , \qquad (2)$$

where $\mathfrak{F}\{\}$ indicates the Fourier transform with kernel $\exp(j\,\xi x)$ and $u(x)$ is the unit step function.

From this line representation, a Wiener-Hopf matrix formulation of the problem can be deduced in the form:

$$G(\xi)\; \psi^a_-(\xi) = \psi^s_+(\xi) + \psi^p(\xi), \qquad (3)$$

where suffices + and - indicate quantities which are regular in the upper and lower half plane, respectively and ψ^a_- is an unknown vector related to the previously introduced generator via :

$$\psi^a_- \rightarrow (\, v_r \;,\; v_l\,)^T , \qquad (4)$$

ψ^s_+ is an unknow vector, which is related to the fields through

$$\psi^s_+ = j\,(2\pi)^{-1/2}\, \mathfrak{F}\{ (\, (E_x(0_-) - E_x(0_+)) \;,\; (H_y(0_-) - H_y(0_+))\,)^T\, u(x) \} \quad . \qquad (5)$$

$\psi^p(\xi)$ is a known vector given by

$$\psi^p(\xi) = I^p(0_-)\; (\, Z_l \;,\; 1\,)^T \quad , \qquad (6)$$

where I^p are the modal currents consistent with the primary field , that is the field radiated by the sources in the geometry obtained by removing the scatterer and assuming $Z=Z_l$ over the all plane $z=0_-$. The essential point is that the matrix G has the form:

$$G \rightarrow \frac{1}{Z_l + Z_r} \begin{bmatrix} Z_l & -Z_r \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & \dfrac{Z_r - Z_\xi}{Z_l + Z_\xi} \\ \dfrac{Z_l - Z_\xi}{Z_r + Z_\xi} & 1 \end{bmatrix} \quad , \qquad (7)$$

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from which we deduce that G can be factorized (5) in the form $G(\xi) = G_+(\xi)\, G_-(\xi)$.

It follows that Ψ^a can be easily evaluated as :

$$\Psi_-^a(\xi) = -(2\pi j)^{-1} G_-^{-1}(\xi) \int_{-\infty}^{+\infty} (\xi' - \xi)^{-1}\, G_+^{-1}(\xi')\, \Psi_-^p(\xi')\, d\xi' \qquad (8)$$

where Ψ_-^p is yielded by the decomposition $\Psi^p = \Psi_+^p + \Psi_-^p$ and the integration is extended over the real axis of ξ' ,with $\mathrm{Im}\xi' \leq 0$.
The evaluation of Ψ_-^a gives immediately the unknown generators v_l and v_r,which allow to define the scattered field everywhere.

Eq.(8) can be used to obtain both the GTD term and the slope diffraction contribution . To this purpose let $H^p(x)$,the primary magnetic field,have this form:

$$H^p(x) = (A_0 + A_1 x) \exp(j\, \xi_0\, x) , \qquad (9)$$

which can be considered as radiated by a line source as shown in fig.1 and with $\xi_0 = K_0 \cos \varphi_0$. This leads to express Ψ_-^a in the form:

$$\Psi_-^a = (-jA_0\, (\xi + \xi_0)^{-1} + A_1\, (\xi + \xi_0)^{-2})\ \ \Psi_0(-\xi_0)$$

with $\Psi_0(-\xi_0)$ a suitable constant not depending on ξ .
The evaluation of currents and voltages along the line,and consequently of the field through a saddle point contribution,leads the scattered field H^s to be expressed in the far field contribution by:

$$H_y^s(\rho,\varphi) = D(\rho,\varphi;\rho_0,\varphi_0)\, H_y^i(\rho,\varphi_0+\pi) + D_s(\rho,\varphi;\rho_0,\varphi_0)\ \partial/\partial\nu\ H_y^i\big|_{\nu=0} \qquad (11)$$

with $D = F(\xi,\xi_0)\,(\xi+\xi_0)^{-1}$, where $F(\xi,\xi_0)$ is a suitable function evaluated for $\xi = K_0 \cos\varphi$; D_s is given by:

$$D_s = -(jK_0\, \rho_0)^{-1}\ \ \partial/\partial\varphi_0\, D(\rho,\varphi;\rho_0,\varphi_0) \quad . \qquad (12)$$

In conclusion,it turns out that even in the case of arbitrary impedances surfaces, the slope diffraction contribution has "formally" the same structure obtained for the perfectly conducting case (6-8) and it can be expressed in terms of the derivative of the first order coefficient D. Fig.2 shows examples of diffraction coefficients D,both for E- and H- case.

Near the shadow boundaries the values of D and D_s are already computed through a saddle point integration but a first and a second order pole nearby must be taken into account.

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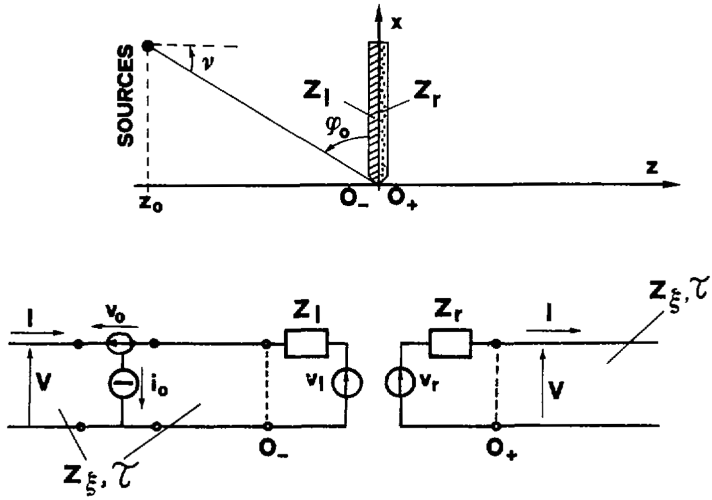

Fig. 1 . Geometry of the problem and transmission line representation.

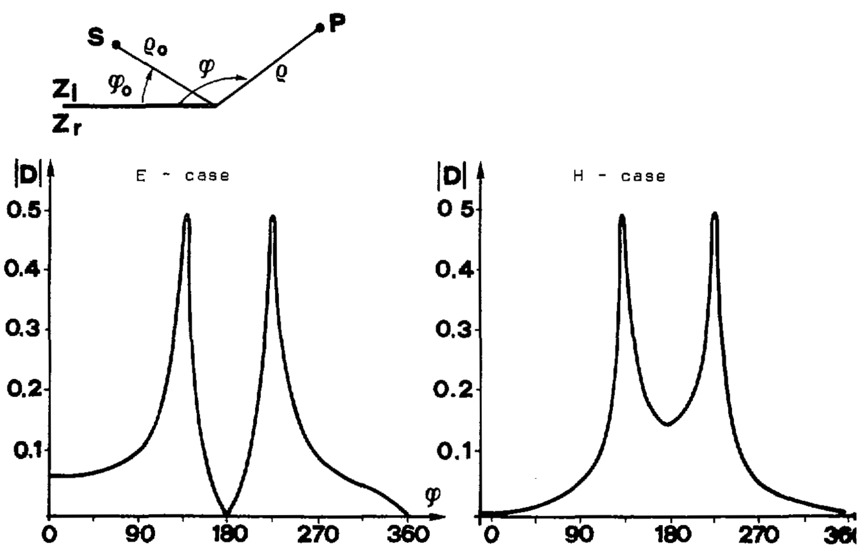

Fig.2 . Diffraction coefficients for $\varphi_o = 45^o$, $Z_l = 0$ and $K_o \varrho_o = 50$