A perspective overview of topological approaches for vulnerability analysis of power transmission grids

Title

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Abstract

Vulnerability analysis is a key issue in power systems since power transmission grids play a crucial role as a critical infrastructure. The power grid structure (number of nodes and lines, their connections, and their physical properties and operational constraints) is one of the main factors to assure power system security. Complex network theory as a promising topological approach for the structural vulnerability analysis has been widely used in many different fields. Recently, many complex network metrics have been proposed to assess the topological vulnerability of power transmission grids. However, these approaches are purely topological and fail in capturing the specific features of power systems. In this paper, an extended topological approach which incorporates electrical features such as flow path, line flow limits, etc., is presented. Three new metrics, net-ability, electrical betweenness and entropy degree are

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provided and used to assess structural vulnerability in power transmission grids.

Keywords
Power transmission grids; structural vulnerability analysis; topological approach; net-ability; electrical betweenness; entropy degree.

Text

1. Introduction
Complex networks (CN) science has received considerable attention recently which has been used in many different fields such as: biology, chemistry, social sciences, computer networks, etc (S. Boccaletti et al., 2006). A lot of researches including basic characteristics, statistical global graph properties, small-world property, scale-free property, degree distribution, betweenness distribution and vulnerability analysis, have been performed to power grids since they are infrastructures in our society. It is noticed that there is a strong link between the topological structure and operation performance in power systems because the structural change could alter operational condition of a power system and thus change its operation performance. As a result, there is an increasing interest in analysing structural vulnerability of power grids by means of CN methodology.
However, most part of these researches are purely topological because they are based on the graphical representation of a network as nodes and connection patterns which neglects the specific engineering features of power grids. Firstly, from the perspective of electrical engineering, the metric of distance in CN theory should have a more practical meaning. For power systems, this conception can be described from economic or technological point of view such as transmission loss or line impedance, etc. Secondly, in traditional CN methodology, all elements are treated identically. Actually, in power transmission, there are different kinds of nodes like generation buses, transmission buses and load buses. Thirdly, in pure topological approach, edges are described in an un-weighted way. While in power systems, transmission lines definitely have line flow limits which restrict the ability of the line for power. This feature is critical for the networks to perform their essential functions and cannot be neglected in vulnerability assessment. Finally, in previous researches, the physical quantity between two vertices is supposed to be transmitted through the

shortest path. For power system, power transmission from a generator bus to a load bus will involve most lines (E. Bompard et al., 2009; 2010). From the discussion above we can see that combining some electrical engineering features with CN theory will be more appropriate to analyse the structural vulnerability of power grids. Based on this consideration, in this paper, three extended metrics of entropy degree, electrical betweenness and net-ability are proposed to replace the traditional metrics of degree, betweenness and efficiency to access the vulnerability of power grids.

The rest of this paper is organized as follows: In section 2, we review and compare the scientific studies conducted using CN analysis techniques concerning power systems. In section 3, the extended topological measures are introduced. Open issues and conclusions are discussed in section 4.

2. Overview of the CN application in power systems

2.1 General approach review

Complex networks methodology can be conceptualized as the intersection between the graph theory and statistical mechanics (L. D. Costa, et al., 2007). Specifically, a complex system is firstly abstracted as a network with a set of edges (or lines) connecting a set of vertices (or nodes). Then, structural properties of the abstracted network and its evolution process can be studied by a set of informative indices based only on the geometric features of the system that do not change over time.

When applying to power systems, the electrical power grid as a weighted and directed network identified by a set $\mathbf{Y} = \{\mathbf{B}, \mathbf{L}, \mathbf{W}\}$ where $\mathbf{B}$ (dim $\{\mathbf{B}\} = \boldsymbol{N}_{\mathbf{B}}$) is the set of vertices (or nodes), $\mathbf{L}$ (dim $\{\mathbf{L}\} = \boldsymbol{N}_{\mathbf{L}}$) is the set of edges (or links) and $\mathbf{W}$ is set of line weights. Vertices are identified by index i. Edges are identified by l_{ij}, which represents a connection between vertex i and vertex j. And the weight element w_{ij} in the set $\mathbf{W}$ is associated with each line l_{ij}.

After abstracting the power grid as a graph, an interesting first way to categorize it is analyzed. An important metric that characterizes a vertex of a graph is the *degree* which means the number of vertexes the node connects with. Moreover, it is not essential to just have the specific information regarding the node degree of only a certain node. The overall characteristics of a graph considering its statistical measure like *node degree probability distribution*. The degree and the node degree distribution will give some information about the static feature of the

network. However, there isn't any information about the paths that can be followed in the graph to move from one node to another. The number of edges in a path connecting vertices *i* and *j* is called the length of the path, while the *shortest path* between these two vertices is one of the paths connecting them with minimum path length. To describe the importance of a node with respect to the shortest paths in the graph, *betweenness* is proposed which is defined as the number of shortest paths between any other nodes that traverses it. Except as the local measurement, it is also useful to have a high level glimpse of the betweenness property for a whole network which can be restored to a statistic measure: *betweenness distribution*.
A general approach about how to apply CN theory to power systems has been overviewed. The more specific methodology review will be addressed in following section.

2.2 Overview of the CN applied to power systems

Structure property analysis

Power grids have been widely acknowledged as a typical type of complex networks (A. L. Barabasi et al., 1999; T. Xu et al, 2004; V. Rosato et al., 2007), and many works have applied concepts and measures of complex networks to analysing the structural vulnerabilities from a topological perspective. According to the models of complex networks, they can be topologically classified as small-world, scale-free and random networks. Therefore, the first question to analyse power grid is what type of power grid is. The first reference comes from Watts and Strogatz (D. J. Watts and S. H. Strogatz, 1998) who analysed the graph of the United States western power grid. It was deduced that the western power grid seemed to be a small-world network. After that, Barabasi and Albert in 1999 (A. L. Barabasi and R. Albert, 1999) firstly published that degree distribution of a power grid was supposed to be scale-free following a power law distribution function, but few of the subsequent later references would support this finding. Exponential cumulative degree function was detected in Californian power grid (L. A. N. Amaral, 2000) and the whole United States grid (R. Albert, 2004). The topological features of the UCTE (Union for the Co-ordination of Transport of Electricity) power grid and its individual nation grids are analysed and results showed these national transmission power grids' topologies are similar in terms of mean degree and degree distribution, which could suggest similar topological constraints, mostly associated with technological considerations and spatial limitations (M. Rosas-Casals, 2007). Besides, the topologies of the

North American eastern and western electric grids were analysed to estimate their reliability based on the Barabasi–Albert network model. The results were compared to the values of power system reliability indices previously obtained from some standard power engineering methods, which suggested that scale-free network models are applicable to estimate aggregate electric grid reliability (D. P. Chassin and C. Posse, 2005).

Structural robustness analysis

The vulnerability analysis of network is the main motivation for the studies involving CN analysis and power grids. The first power grid whose robustness was analysed was the North American power grid (R. Albert, et al., 2004). The authors removed vertices randomly and in decreasing order of their degrees for both generation vertices and transmission vertices, and monitored the connectivity loss which measured the decrease of the ability of distribution substations to receive power from the generators. The loss of generating substations does not significantly alter the overall connectivity of the grid owing to a high level of redundancy at the generating substations. However, the grid is sensitive to the loss of transmission nodes. Even the removal of a single transmission node can cause a slight connectivity loss. Especially, the connectivity loss is substantially higher when intentionally attacking higher degree or high load transmission hubs. They concluded that the transmission highly connected hubs guarantee the connectivity of the power grid but meanwhile they are also its largest liability in case of power breakdowns. The first reference to European power grids was made by Crucitti et al. The authors studied and compared the topological properties of the Spanish, Italian and French power grids, finding those components whose removals seriously affected the structure of these graphs (P. Crucitti, et al., 2005). Since the proposed improvements also treat power grid as a simple graph and no physical features are taken into consideration, we think that the power grid vulnerability results obtained with this approach could be different from the real situation. . Rosato *et al.* studied the topological properties of high-voltage power grid in Italy (380 kV), France (400 kV) and the Spain (400 kV) (V. Rosato et al., 2007). An assessment of the vulnerability of the networks has been implemented by analysing the level of damage caused by a controlled removal of links. Such topological studies could be useful to make vulnerability assessment and to design specific action to reduce topological weaknesses. Since the grids are the same as used in the former case, most of the results are consistent. Robustness of the whole European power grid is studied in ((V. Rosato, et al., 2007; R. Solé, et al., 2008), where also includes the resilience against

to the failures and attacks of every national power grid. The authors found that European power grid composed of the thirty three EU power grids could broadly be classified into two separate groups, fragile and robust. It is noticed that cascading failures have frequently occurred throughout electrical power grids of various countries. The cascading failures firstly were analysed in electrical power grid of the western United States (A. Motter and Y. Lai, 2002). The degree distribution in this network appeared exponential and was thus relatively homogeneous. The distribution of loads, however, was more skewed than what displayed by semi-random networks with the same distribution of links. This implied, to a certain extent, that the power grid may have structures not being captured by existing complex network models. As a result, global cascades are supposed to be triggered more probably by load-based intentional attacks than by random or degree-based removal of vertices. The attack on a single vertex with large load may make the largest connected component decrease to less than a half of its initial size, though the network is highly tolerant. In North American Power Grid, the cascade phenomenon was also modelled (R. Kinney, et al., 2005). It was observed that the loss of a single substation can lead to a 25% loss of transmission efficiency caused by an overload cascade in the network. A systematically study of the damage caused by the loss of vertices suggested that 40% of the disrupted transmission substations may lead to cascading failures. While the loss of a single vertex can exacerbate primary substantial damage, the subsequent removals only make the situation worse. Crucitti et.al applied cascading failure model into the Italian power grid where they neglected the details of the electromagnetic processes and only focused on the topological properties of the grid (P. Crucitti et al., 2004). The objective of this study was to demonstrate that the structure of an electric power grid may provide important information about the vulnerability of the system under cascading failures. The power grid has 341 vertices (substations) and 517 edges (transmission lines). Different kinds of vertices have been distinguished. Although the degree distribution is not very different through the network, it still exhibits a high heterogeneity in the vertex load. Most of the vertices are only responsible for a small load, but a few other vertices have to carry an extremely high load. Large scale blackouts can be triggered by the failure of vertices with high loads. Perhaps it is due to the fact that some highly connected vertices may be not necessarily involved in a high number of paths. However, the used model is quite simplified for a real electric power grid, so that this result may be not very credible since the definition of degree and load here are not very meaningful for power grids. Jiang-wei, et al (J. W. Wang and L. L. Rong,

2009) proposed a cascading failure model based on degree centrality to analyse the Western United States power grid. A counterintuitive result is found that the attack on the vertices with the lowest loads is more harmful than the attack on the ones with the highest loads. Simonsen *et al.* (I. Simonsen et al., 2008) study cascading failures in power grids using a dynamical flow model based on simple conservation and distribution laws. Within the framework, it is studied that the role of the transient dynamics of the redistribution of loads towards the steady state after the failure of network edges. It is found that considering only flow of loads in the steady state gives a best case estimate of the robustness; the worst case of robustness can be determined by the instantaneous dynamic overload failure model. Bakke *et al* (J. O. H. Bakke et al., 2006) analysed the power blackout of Norwegian high-voltage power grid using a model with Kirchhoff equations and the same line conductance. The results showed that the size distribution of power blackouts in Norwegian power grid seems to follow a power law probability distribution.

From static to dynamic

The works reviewed so far are mainly about the static properties such as the categorization of power networks and vulnerability assessment of the components (buses and lines) in power systems. Recently published papers extend these static analyses to dynamic ones. For example, a Kuramoto oscillator model is introduced as a phase model to analog the synchronous generator in order to analyze the synchronization stability property of the coupled generators in the whole power networks. The Kuramoto oscillator is motivated by the behavior of systems of chemical and biological oscillators, and it is also adopted as the synchronization model in the complex network.

In Bullo and Dorfler' papers (F. Dorfle and F. Bullo, 2013; F. Dorfler, M. Chertkov and F. Bullo 2013), Kron reduction of graphs was introduced to eliminate the load buses of the power network and Kuramoto oscillator like model is used to model the synchronous machine, then the whole power grid is a coupled Kuramoto oscillator like network. The explicit necessary and sufficient condition on the critical coupling strength to achieve synchronization is studied. Similar results are also addressed by M. Rohden et al. (M. Rohden, A. Sorge, M. Timme, 2012) and S. Lozano et al. (S. Lozano, L. Buzna, and A. Diaz-Guilera, 2012). In H. Sakaguchi's paper (H. Sakaguchi and T. Matsuo, 2012), this Kuramoto like model has been used to analyze the cascading failure in power grid. In the meantime, some other dynamic features are also taken into consideration: a dynamical flow model is used by Helibing et al. to study the cascading

failure in a power grid (I. Simonsen et al., 2008). Restrepo et al (D. B. Larrenmore et al., 2011) proposed instead a general theoretical approach to study the effects of network topology on dynamic range. All these works extend the complex network theory from steady-state analysis to dynamic, which significantly improves the studies about CN theory application in power systems.

3. Extended topological measures

3.1 Introduction of extended topological measures

The initial research works on complex networks developed many common concepts and measures which are supposed to be applicable to different types of networks. When the complex networks methodology is directly applied to some fields that neglect of the specific features of these networks, analyzing results is unavoidably a deviation from reality. Therefore, electrical properties should be taken into account when extending the purely topological approach of complex networks.
Firstly, we consider the equivalent impedance to define the extended electrical distance. Assume a unit current is injected at bus *i* and withdrawn at bus *j* (i.e., I_i=1 and I_j=-1) while no current is injected or withdrawn at other buses, then equivalent impedance can be calculated as (E. Bompard, R. Napoli and F. Xue, 2009):

$$\begin{aligned} Z_i^j &= \frac{U_i^j}{I_i} = U_i^j = U_i - U_j \\ &= (Z_{ii} - Z_{ij}) - (Z_{ij} - Z_{jj}) = Z_{ii} - 2Z_{ij} + Z_{jj} \end{aligned} \tag{1}$$

where z_{ij} is the *i*th, *j*th element of the impedance matrix. Then, on the one hand, in order to maintain the stability and security operation of a power grid, each transmission line l_{ij} has its own transmission limit $P_{l_{ij}}^{max}$; on the other hand, the power transmission between generation buses and load buses will involve all lines between two buses. Therefore, to evaluate the feature mentioned above quantitatively, we define the power transmission capacity C_g^d as the power injected at bus g when the first line in all lines connecting generation bus g and load bus d reaches its limit (E. Bompard, R. Napoli and F. Xue, 2009):

$$C_g^d = \min\left[\frac{P_{l_{ij}}^{\max}}{\left|f_{l_{ij}}^{gd}\right|}\right] \tag{2}$$

where $f_{l_{ij}}^{gd}$ is the power on line l_{ij} ($l_{ij} \in L$) for a unit of power injected at generation bus g ($g \in G$) and withdrawal at load bus d ($d \in D$), and $f_{l_{ij}}^{gd}$ can be computed as follows:

$$f_{l_{ij}}^{gd} = f_{l_{ij}}^{g} - f_{l_{ij}}^{d} \qquad (3)$$

where $f_{l_{ij}}^{g}$ and $f_{l_{ij}}^{d}$ are respectively the l_{ij} –th, row g -th column and the l_{ij} -th, row d -th column of Power Transmission Distribution Factors (PTDF) matrix (A. Fradi et al., 2001).

Based on the previous introduction of new conception combining electrical features of power systems and traditional CN measurements, three new extended metrics are proposed:

3.2 Entropy degree

Degree, k_i, is a basic metric that can measure the criticality of the vertex i in the network through their connectivity and a vertex with higher degree means that the vertex is more important than others (Newman, M.E.J., 2010). A good definition of degree should consider all the following factors altogether: 1), the strength of connections in terms of weights of edges; 2), the number of edges connected with the vertex; 3), and the distribution of weights among edges.

To consider all the three mentioned factors, we resort to the concept of entropy to redefine entropic degree of a vertex i (E. Bompard et al., 2009; 2010):

$$e_i^w = \left[1 - \sum_{j=1}^{N} p_{ij} \bullet \log(p_{ij})\right] \sum_{j=1}^{N} w_{ij} \qquad (4)$$

where p_{ij} is the normalized weight of the edge l_{ij} connecting vertices i and j:

$$p_{ij} = \frac{w_{ij}}{\sum_{j=1}^{N} w_{ij}} \qquad (5)$$

where weight w_{ij} is defined as the line flow limit on line l_{ij} because the electrical parameter can reflect the strength that two buses connected.

For better understanding of our defined entropy degree, figure 1 gives a simple example to illustrate the difference between traditional degree (both un-weighted and weighted) and entropy degree.

Figure 1 Different distribution of weight

In figure 1, the traditional un-weighted degree and weighted degree obtained:

$k_i^A = k_i^B = 2$;

$s_i^A = s_i^B = 1$.

The entropy degree obtained:

$e_i^A = 1.3$; $e_i^B = 1.14$.

It is noticed that, for case A, both edges have the same extent of importance for the node. On the other hand, for case B, one edge is more important than the other since it takes 90% of the connection. Obviously, under a failure of the most important line, case B is more vulnerable than case A. Using traditional un-weighted degree and weighted degree, the values for both nodes are unique which can't point out the difference existed in the weights distribution among edges. While using entropy degree, this difference can be distinguished accurately.

3.3 Electrical betweenness

Betweenness is the number of geodesic paths, connecting whichever pair of vertices, passing through a given vertex or edge in a network, which can be considered as another measure of the criticality of a vertex or an edge through their centrality beside degree (Newman, M.E.J., 2010). Since the definition of betweenness neglects the electrical features of power system, the electrical betweenness is redefined. Electrical betweenness of a line is defined as (E. Bompard, E. Pons and D. Wu, 2012):

$$B_e(l_{ij}) = Max(B_e^p(l_{ij}), \left|B_e^n(l_{ij})\right|) \qquad (6)$$

Where

$$B_e^p(l_{ij}) = \sum_{g\in G}\sum_{d\in D} C_g^d f_{l_{ij}}^{gd} \text{ if } f_{l_{ij}}^{gd} > 0 \qquad (7)$$

$$B_e^n(l_{ij}) = \sum_{g\in G}\sum_{d\in D} C_g^d f_{l_{ij}}^{gd} \text{ if } f_{l_{ij}}^{gd} < 0 \qquad (8)$$

The input power of a bus *v* should be equal to output power of the bus, so the extended betweenness of a bus *v* is the half of sum of power flowing through the lines connecting the bus.

$$B_e(v) = \frac{1}{2}\sum_{g\in G}\sum_{d\in D} C_g^d \sum_{l_{ij}\in L^V} f_{l_{ij}}^{gd} \quad (9)$$

where L^v is the set of lines connecting a bus v.

In the following figure the electrical betweenness is compared with the topological betweenness in IEEE 300-bus network, which consists of 409 lines and 300 buses, including 69 generator buses and 199 load buses. The comparison is made by calculating the un-served energy after the network is attacked by removing the components (buses or lines). The un-served energy is computed by an engineering vulnerability assessment model (VSM) (J. Salmeron K. Wood, and R. Baldick, 2004) through optimal dispatch and load shedding.

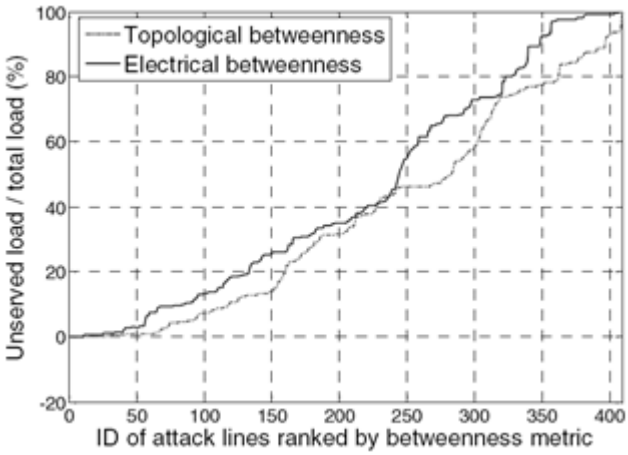

Figure 2 The average change of un-served load after line attack for strategy 1 & 2

Firstly, the electrical betweenness metric is used to rank lines in descending order criticality to attack the network. In each evaluation, after successive removal of the lines according to the order, the change of un-served energy is computed by VSM. Similarly, topological betweenness metric is also applied to generate the removing order of lines. Figure 2 shows the average change of un-served load for these two strategies. From Figure 2, the amount of un-served load after each line attack according to electrical betweenness is more than that depending on topological one. It implies that the criticality of lines ranked by electrical betweenness is more effective than the topological betweenness.

3.4 Net-ability

Efficiency is defined as the average of distance between vertices to measure the global performance of a network (V.Latora and M.Marchiori, 2001). In order to analyse the performance of a power grid in consideration of their engineering features, the shortest path length distance should be replaced with electrical distance while the whole performance of a power grid should be averaged by all pairs of generators and loads rather than all pairs of nodes since the power is transferred only from generators to loads in a power grid. Besides, the power transmission capacity can also be considered. Therefore, a new metric named net-ability is defined as a global metric to measure the performance of whole network (E. Bompard et al., 2009; 2010):

$$A(y) = \frac{1}{N_G N_D} \sum_{g \in G} \sum_{d \in D} \frac{C_g^d}{Z_g^d} \qquad (10)$$

where N_G and N_D respectively are the number of generation buses and load buses in a power grid; Z_g^d is the equivalent impedance for injection at generation bus g and withdraw at load bus d.

Our proposed extended metrics have been also applied to test systems with different scales and configuration (E. Bompard et al., 2009; 2010; 2012). Especially in (E. Bompard et al., 2012) a large scale interconnected power grid of UCTE with IPS/UPS (IPS/UPS has 713 buses and 943 lines while there are 1254 buses and 1944 lines in simplified UCTE power grid) is created and used for vulnerability analysis from topological point of view. The results demonstrate that the extended approach captures important features of power grids that are ignored by purely topological approaches, and is more effective at identifying critical components in a network which is very useful when performing a security analysis.

4. Discussion and Conclusion

In this communication, an overview and comparison about the scientific studies using CN theory application in power systems, especially concerning the vulnerability analysis of power grids, is given. Based on that, an extended topological method is proposed in which electrical specificity is introduced into CN methodology to analyse and understand power systems as a complex system from a topological perspective and to analyse the vulnerability of power grids. Three extended metrics: entropy degree, electrical betweenness and net-ability, are introduced as new

measurements to access the criticality of the composing elements (buses and branches) in the network. It is confirmed that extended metrics are superior to purely topological metrics in analysing the vulnerability of components in power grids from both a global and a local point of view. However, there are also still some open questions and some possible research paths by means of extended topological thoughts to further comprehend the complex system.

Firstly, as discussed in this paper, power grid is a flow-based network in which the physical quantity (electric power) transmission between two vertices will involve most lines. However, currently, the linear model is considered and DC power flow is calculated to indicate this feature. A natural thought is to introduce the AC power flow computation and dynamic security assessment into our proposed approach to analyse the cascading failure in power grids.

Secondly, although a large number of researches have been performed to the power grids by means of CN methodology, when prompted to approach reality, difficulties arise. Because the hypothesis in current researches is that structure will influence dynamics and vice versa. But how can we demonstrate this hypothesis? Therefore, a linkage between topological measures and real dynamic outputs should be analysed and constructed.

Finally, a large scale power grid itself is comprised of various layers in terms of various voltage levels such as transmission network and distribution network. In our research, power grids are considered as a single layer network (i.e., all layers in power grids are considered together) while overlook the role each layer played and its interaction. It could be possible that each layer has different impacts on structural property, static and dynamic robustness of a large scale power grid.

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