Extended Topological Metrics for the Analysis of Power Grid Vulnerability

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Ettore Bompard, Enrico Pons, and Di Wu

Abstract—**Vulnerability analysis in power systems is a key issue in modern society and many efforts have contributed to the analysis. Recently, complex networks metrics, applied to assess the topological vulnerability of networked systems, have been used in power grids, such as the betweenness centrality. These metrics may be useful for analyzing the topological vulnerability of power systems because of a close link between their topological structure and physical behavior. However, a pure topological approach fails to capture the electrical specificity of power grids. For this reason, an extended topological method has been proposed by incorporating several electrical features, such as electrical distance, power transfer distribution, and line flow limits, into the pure topological metrics. Starting from the purely topological concept of complex networks, this paper defines an extended betweenness centrality which considers the characteristics of power grids and can measure the local importance of the elements in power grids. The line extended betweenness is compared with the topological betweenness and with the averaged power flow on each line over various operational states in the Italian power grid. The results show that the extended betweenness is superior to topological betweenness in the identification of critical components in power grids and at the same time could be a complementary tool to efficiently enhance vulnerability analysis based on electrical engineering methods.**

Index Terms—**Betweenness, complex networks, electrical betweenness, vulnerability.**

NOMENCLATURE

The symbols and abbreviations used in this paper are listed as follows.

Y	Transmission network, Y = {L, B}.
L	Set of lines, L = {..., l_{ij}, ...}, dim{L} = N_L, $i, j \in$ B.
L^V	Set of lines connecting bus v, L^V = {..., l_{vi}, ..., l_{jv}, ...}, $i, j, v \in$ B.
B	Set of buses, B = {..., i, ..., j, ...}, dim{B} = N_B, B = G∪D∪T.
W	Weights associated with the lines W = {..., w_{ij}, ...}, dim{W} = N_L, $i, j \in$ B.
G	Set of buses connected with generators, G⊆ B = {..., g, ...}, dim{G} = N_G.
D	Set of buses connected with loads, D ⊆ B = {..., d, ...}, dim{D} = N_D.
T	Set of transmission buses, T⊆B, dim{T} = N_T.
$\boldsymbol{F}$	L x N matrix of power transmission distribution factors.
d_{ij}	Geodesic distance between vertices i and j.
$\sigma_{ij}(v)$	Number of the geodesic paths between vertices i and j that pass through a vertex v ($i \neq j \neq v$).
$\sigma_{ij}(l)$	Number of the geodesic paths between vertices i and j that includes the edge l.
σ_{ij}	Total number of the geodesic paths connecting vertices i and j.
f_{lj}	Change of the power on line l corresponding to a unit change of power injection at bus j and withdrawal at the reference bus.
f_l^{gd}	Change of the power on line l for injection at generation bus g and withdrawal at load bus d.
$P_l^{\max}$	Power flow limit of line l.
$C_{g,}^d$	Power transmission capacity from buses g to d.
$T(v)$	Electrical betweenness of bus v.
$T(l)$	Electrical betweenness of line l.
$T^p(l)$	Positive electrical betweenness of line l.
$T^n(l)$	Negative electrical betweenness of line l.
Z_g^d	Equivalent impedance between buses g and d.
U_g^d	Voltage drop when a unit of current is injected at generator g and withdrawn at load d.
I_g	Current is injected at generator g.
z_{gd}	g-row, d-column entry of the impedance matrix of a power grid.
E_Y	Efficiency of transmission network Y.
A_Y	Net-ability of transmission network Y.

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I. INTRODUCTION

VULNERABILITY analysis is necessary for transmission system operators to identify the vulnerable components whose protection or backup will result in a more robust system against accidents or malicious threats.

In general, the physical behavior of power systems is determined by two aspects: topological structure and operational state. Hence, the vulnerability analysis of power systems can also be classified as the conventional vulnerability analysis and the structural vulnerability analysis [1]. The conventional vulnerability analysis based on complete operational data and topological information, as well as standard engineering models in power systems [2], [3], is challenged in large-scale power systems since the topological and operational states become more and more complicated with the increase of size of the system and with the introduction of the electricity market. On the other hand, there exists a close relationship between the topological structure and the physical behavior in power systems because the structural change could alter operational conditions of a power system and then change its physical behavior. Although structural vulnerability analysis cannot substitute for the conventional vulnerability analysis, it may be a complementary tool for the conventional vulnerability analysis. Moreover, the structural vulnerability analysis is also useful to understand the global properties of power grids affecting their local behaviors [4].

Complex network methodology is a popular method to analyze and comprehend power grids from a topological point of view [5]–[10]. Recent works demonstrate that electrical power grids have not only the characteristics of the small-world networks [6]–[8], but also the features of the scale-free networks [9], [10]. In the methodology, topological properties can be analyzed using different metrics, local or global. Local metrics benefit from computational speed but give only a local measure, while global metrics can measure the overall performance of a network but suffer from the computational time. Among these metrics, betweenness centrality [11] as a local measure plays a key role in identifying the criticality of components (vertices and edges) [12], [13]; on the other hand, efficiency [14] is one of the most widely used global metrics: it cannot only indicate the importance of components in a network but also enables us to assess its performance [15]–[18]. However, these existing metrics that are used to analyze structural vulnerability of power grids from a pure topological perspective fail to capture some basic and important features of power grids. Consequently, these purely topological metrics could result in misleading research results [19], which may be far from real physical behaviors of power grids.

To overcome this problem, the extended topological method was proposed by introducing some electrical engineering specificity into the complex networks method. For instance, efficiency is redefined as net-ability [20] by incorporating electrical distance, power transmission capacity, and bus classification. In this paper, we redefine the topological betweenness centrality as an extended betweenness centrality that takes account of power transmission capacity, power transfer distribution, and bus classification in order to identify the criticality of components in power grids.

The rest of this paper is organized as follows. Nomenclature lists the symbols that will be used in this paper, Section II gives the definition of the topological betweenness centrality, the extended betweenness is introduced in Section III, the numerical analysis is presented in Section V, and the conclusion is summarized in Section VI.

II. Pure Topological Betweenness

In complex networks, the networked systems such as the power grid can be abstracted as a directed and weighted graph $\mathtt{Y} = \{\mathtt{B}, \mathtt{L}, \mathtt{W}\}$ to analyze the inherent structure features, where B is the set of vertices (or nodes) and $\mathtt{L}$ is the set of edges (or links) with an associate set of weights $\mathtt{W}$. Each vertex can be identified by its code i; the edge is identified by l_{ij} that represents a connection going from vertex i to vertex j and that is associated with a weight w_{ij}.

A walk from vertex i to vertex j is a sequence of vertices and edges that begins with i and end with j while a path is a walk in which no vertex is visited more than once. A geodesic path (i.e., shortest path) is the path which has the minimal number of edges between two vertices. The geodesic distance d_{ij} is the number of edges in geodesic path between vertices i and j. In a network, the importance of a vertex can be measured by the betweenness of the vertex, which is defined in [11]

$$B(v) = \sum_{i}^{N} \sum_{j}^{N} \frac{\sigma_{ij}(v)}{\sigma_{ij}} \quad i \neq j \neq v \in \mathsf{B} \tag{1}$$

where $\sigma_{ij}(v)$ is the number of the geodesic paths between vertices i and j that pass through a vertex v $(i \neq j \neq v)$. σ_{ij} denotes the total number of the geodesic paths connecting vertex i and vertex j. N is the number of vertex in network. Similarly, the edge betweenness can also be defined as follows [11]:

$$B(l) = \sum_{i}^{N} \sum_{j}^{N} \frac{\sigma_{ij}(l)}{\sigma_{ij}} \quad i \neq j \in \mathsf{B} \quad l \in \mathsf{L} \tag{2}$$

where $\sigma_{ij}(l)$ is the number of the geodesic paths between vertices i and j that includes the edge l.

III. Extended Betweenness

The complex network theory has been successfully applied in the analysis of technological networks, such as the World Wide Web. However, the pure topological concepts and metrics disregard the real physical properties and the operative constraints of power grids so that the straight application of the topological perspective fails in capturing their specificity. In complex networks, each vertex, which may be a source or a sink, has equal function when some physical quantities are transmitted over the network. However, in power grids, buses are distinguished depending on their functions as generation buses [$\mathtt{G}$, $\dim(\mathtt{G}) = N_{\mathrm{G}}$], load buses [$\mathtt{D}$, $\dim(\mathtt{D}) = N_{\mathrm{D}}$], and transmission buses [$\mathtt{T}$, $\dim(\mathtt{T}) = N_{\mathrm{T}}$]. Furthermore, power is only transmitted from generation buses to load buses and each transmission line provides its own contribution involving basically all the buses and lines of the system.

In the linearized model of power systems, the contribution of each transmission line to power transmission can be computed by the power transfer distribution factors (PTDFs). PTDF reflects the sensitivity of the power flowing on each line for a power injection/withdrawal at a couple of buses. PTDF can be represented by a LN matrix $\boldsymbol{F}$ in which each element f_{lj} expresses the change of power on each line l for a unit change of power injection at bus j and withdrawal at the reference bus; f_l^{gd} is the change of the power on line l ($l \in \mathtt{L}$)

for injection at generation bus g and withdrawal at load bus d, and f_l^{gd} can be computed as follows:

$$f_l^{gd} = f_{lg} - f_{ld} \quad l \in \mathsf{L} \tag{3}$$

where f_{lg} and f_{ld} are, respectively, the lth row gth column and lth row dth column of $\boldsymbol{F}$.

In order to maintain stability and security in the operation of power grids, each transmission line l has its own transmission limit $P_l^{\max}$ which is a physical parameter of line l, unrelated to operational conditions and electrical transactions in the power grid. As the line flow limit plays a major role in the power transmission between generation buses and load buses, we define the power transmission capacity C_g^d in (4) to consider the impact of the parameter on the structural analysis

$$C_g^d = \text{Min}\left\{\frac{P_1^{\max}}{\left|f_1^{gd}\right|}, \ldots, \frac{P_l^{\max}}{\left|f_l^{gd}\right|}, \ldots \frac{P_{N_\mathsf{L}}^{\max}}{\left|f_{N_\mathsf{L}}^{gd}\right|}\right\} \tag{4}$$

where C_g^d represents the maximum power which can be injected at bus g and withdrawn at bus d, while the power on each transmission line is smaller than or equal to its own line flow limit.

According to the above-mentioned specific features of power grids, the electrical betweenness of bus v can be redefined as follows:

$$T(v) = \frac{1}{2}\sum_{g\in\mathsf{G}}\sum_{d(g\neq d)\in\mathsf{D}} C_g^d \sum_{l\in\mathsf{L}^v}\left|f_l^{gd}\right| \quad v \neq g \neq d \in \mathsf{B} \tag{5}$$

where $\sum_{l\in\mathsf{L}^v}\left|f_l^{gd}\right|$ is the sum of the PTDF of all the lines connecting bus v when power is injected at bus g and withdrawn at bus d; $1/2 \cdot C_g^d \sum_{l\in\mathsf{L}^v}\left|f_l^{gd}\right|$ represents the transmission power taken by bus v when the power is transmitted from generation bus g to load bus d; $\mathtt{G}$ and $\mathtt{D}$, respectively, are the sets of generation buses and load buses; $\mathtt{L}^V$ is the set of lines connecting bus v. Equation (5) computes the extended betweenness of bus v. The bus v is neither a specified generator bus g nor a specified load bus d. Meanwhile, generator g and load d are not the same, too.

Similarly, the electrical betweenness of line l can be redefined as follows:

$$T(l) = \max[T^p(l), |T^n(l)|] \quad l \in \mathsf{L} \tag{6}$$

where $T^p(l)$ and $T^n(l)$ represent, respectively, the positive electrical betweenness and the negative electrical betweenness of the line l

$$T^p(l) = \sum_{g\in\mathsf{G}}\sum_{d(g\neq d)\in\mathsf{D}} C_g^d f_l^{gd} \text{if } f_l^{gd} > 0$$
$$T^n(l) = \sum_{g\in\mathsf{G}}\sum_{d(g\neq d)\in\mathsf{D}} C_g^d f_l^{gd} \text{if } f_l^{gd} < 0$$

where $C_g^d f_l^{gd}$ represents the power flowing on line l when the power is transmitted from generator bus g to load bus d.

It is worthy of noticing that the computational complexity of the extended betweenness is lower than that required for the original metrics defined in (1) and (2). For example, the computational complexity may be $O(N_\text{L}N_\text{B}^2)$ [21] to compute the original betweenness of all vertices in a power grid. On the other hand, the complexity could be $O(N_\text{G}N_\text{D})$ for the extended betweenness and in general, the number of generators and loads are smaller than the number of buses and transmission lines in a real power grid. Taking the Italian power grid as an example, there are 641 lines and 521 buses, including 158 generators and 205 loads in the real power grid. Therefore, the extended betweenness is superior to the original betweenness in the computational complexity when measuring the importance of buses or lines in real power grids.

IV. Extended Metric for Net Efficiency: Net Ability

Efficiency was proposed to measure the overall performance of a network [14] and to locate the critical components of the networked infrastructure systems [15]–[18]. The efficiency E_Y of a network $\mathtt{Y}$ defined in (7) quantifies the overall performance of the network $\mathtt{Y}$ as the mean geodesic distance over all pairs of vertices in the network

$$E_\mathsf{Y} = \frac{1}{N_\mathsf{B}(N_\mathsf{B} - 1)}\sum_{i\neq j\in\mathsf{B}}\frac{1}{d_{ij}} \tag{7}$$

where N_B is the total number of vertices in a network, d_{ij} is the geodesic distance between vertices i and j.

The general goal of a power transmission network is the feasible and economic power transmission from generation buses to load buses. Feasibility refers to technical issues (losses, voltage drop, stability, and so on). Economy is related to other aspects (transmission costs, market efficiency, and so on). Therefore, we extend then efficiency as the new concept of net-ability which measures the network ability to perform properly the function of a power grid under normal operating conditions. Since performing the function properly depends on the maximum (real or apparent) line flow limits (transfer arbitrary amounts of power) and on the impedance of the lines (economic and technical convenience), we define net-ability A_Y of a power grid $\mathtt{Y}$ in [20] which quantifies the performance of power grid $\mathtt{Y}$ as the mean "electrical" distance over all pairs of generators and loads in the power grid

$$A_\mathsf{Y} = \frac{1}{N_\mathsf{G}N_\mathsf{D}}\sum_{g\in\mathsf{G}}\sum_{d(d\neq g)\in\mathsf{D}} C_g^d \frac{1}{\left|Z_g^d\right|} \tag{8}$$

where N_G is the total number of generators in a power grid, N_D is the total number of loads in a power grid, C_g^d is the power transmission capacity defined in (4), $|Z_g^d|$ is the electrical distance between a pair of generator g and load d. The detailed discussions on the electrical distance are presented in the Appendix.

Besides, the unit for net-ability is MW/Ω which indicates with one unit of cost (Ω) how many benefits (power transmission) can be achieved through the considered network from any generator to any load. This meaning is consistent with the concept of efficiency in [15]–[17].

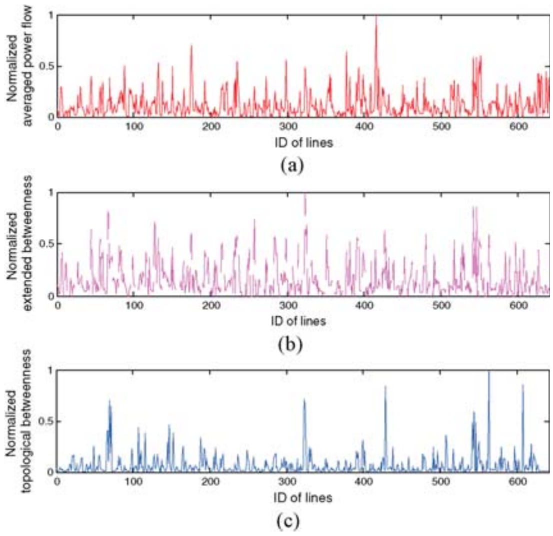

Fig. 1. Criticality of lines measured by various scenarios in Italian power grid. (a) Criticality of lines through normalized averaged power flow. (b) Criticality of lines through normalized extended betweenness. (c) Criticality of lines through normalized topological betweenness.

V. Numerical Studies

In this section, we compare the line extended betweenness with the line topological betweenness in the Italian power grid. The Italian power grid is a real power grid that is composed of 641 lines and 521 buses, including 158 generator buses and 205 load buses. Meanwhile, we also compare the two types of line betweenness with the power flow on each line averaged over various operational states in the real power grid. The averaged power flow is chosen as a standard contingency metric to rank lines because the two types of line betweenness quantify the flow on each line in a power grid from the structural point of view. Moreover, the larger averaged power flow on a line means the line could be more critical since in most cases the more critical line failure could lead to more overloaded line failures in the remaining lines due to more power flow redistribution from the faulted line to the remaining lines. Therefore, the averaged power flow on each line can be seen as a contingency ranking metric to be compared to two types of line betweenness. Although some researchers have directly compared the power flow on each line with the line topological betweenness in order to investigate the correlation between structure and function of power systems [22], the power flow on each line is sensitive to operative conditions. Hence, we average the power flow on each line over 100 evaluations. In each evaluation, the load randomly changes in the interval $[0,\, 2P_d]$ (P_d is the base load in Italian power grid, $d \in \mathtt{D}$), and the power flow on each line is determined by DC power flow computation.

Fig. 1(a) illustrates the averaged power flow on each line of the Italian power grid. The averaged power flow is normalized by its maximum. The extended betweenness and topological betweenness on each line are also normalized by its each own maximum, as shown in Fig. 1(b) and (c), respectively.

At the beginning, we compare the line extended betweenness with the line topological betweenness and averaged power flow on lines by investigating the effect of random and intentional attacks on critical lines on Italian power grid by means of remaining load in the power grid. The criticality of a line can be evaluated in terms of the value of the extended betweenness, topological betweenness, or averaged power flow, respectively: a bigger value in these metrics means a more critical line. We rank lines in a descending order of the extended betweenness, the topological betweenness, and the averaged power flow, respectively. The remaining load is the residual amount of load in the power grid after each removal of ranked lines. The remaining load is indirectly computed by optimal dispatch and load shedding based on DC power flow. Similar to the evaluation of the averaged power flow, assessing remaining load under attacks is related to the operational states of power grids as well. Therefore, the simulated results are averaged among 50 evaluations both for random attacks and for intentional attacks. In each evaluation, the load also randomly changes in the interval $[0,\, 2P_d]$. For random attacks, 50 lines are randomly selected and then removed successively from the Italian power grid in each simulation. For deliberate attacks, the top 50 most critical lines ranked by the above-mentioned three measures are successively removed from the Italian power grid. The results for random and deliberate attacks are compared in Fig. 2(a)–(c), where the whiskers represent the standard deviation over 50 evaluations.

In these figures, we show the decrease of remaining load in the Italian power grid as a function of the number of removed critical lines when the power grid is attacked either randomly or deliberately. As we can see from these figures, the Italian power grid is sensitive to intentional attacks in terms of the extended betweenness and averaged power flow on lines but relatively robust to random failures since the remaining load drops more quickly when the lines are removed according to the ranking suggested. However, when the ranked lines are removed according to topological betweenness, it is not always true that the remaining load drops more quickly than the removal of random lines. This implies that topological betweenness is unable to effectively identify critical lines.

We further compare the line extended betweenness with topological betweenness and averaged power flow on each line in the case where only critical lines are deliberately removed as shown in Fig. 2(d). It can be observed that the power grid is more vulnerable when attacking lines which are ranked by the averaged power flow on each line rather than the two types of line betweenness since the remaining load in the power grid deceases faster. On the other hand, the line extended betweenness is better than the topological betweenness to locate the critical lines because the remaining load drops faster under the attack of critical lines identified by the extended betweenness. Similar results can be found in Fig. 3, where the network performance under various scenarios of attacks is evaluated by net-ability.

When the averaged power flow on each line as a contingency ranking metric identifies the criticality of lines in a power grid, both structure and operational states are considered in the identification. However, the extended betweenness identifies the criticality of lines from a structural point of view, though the extended betweenness introduces electrical specificity

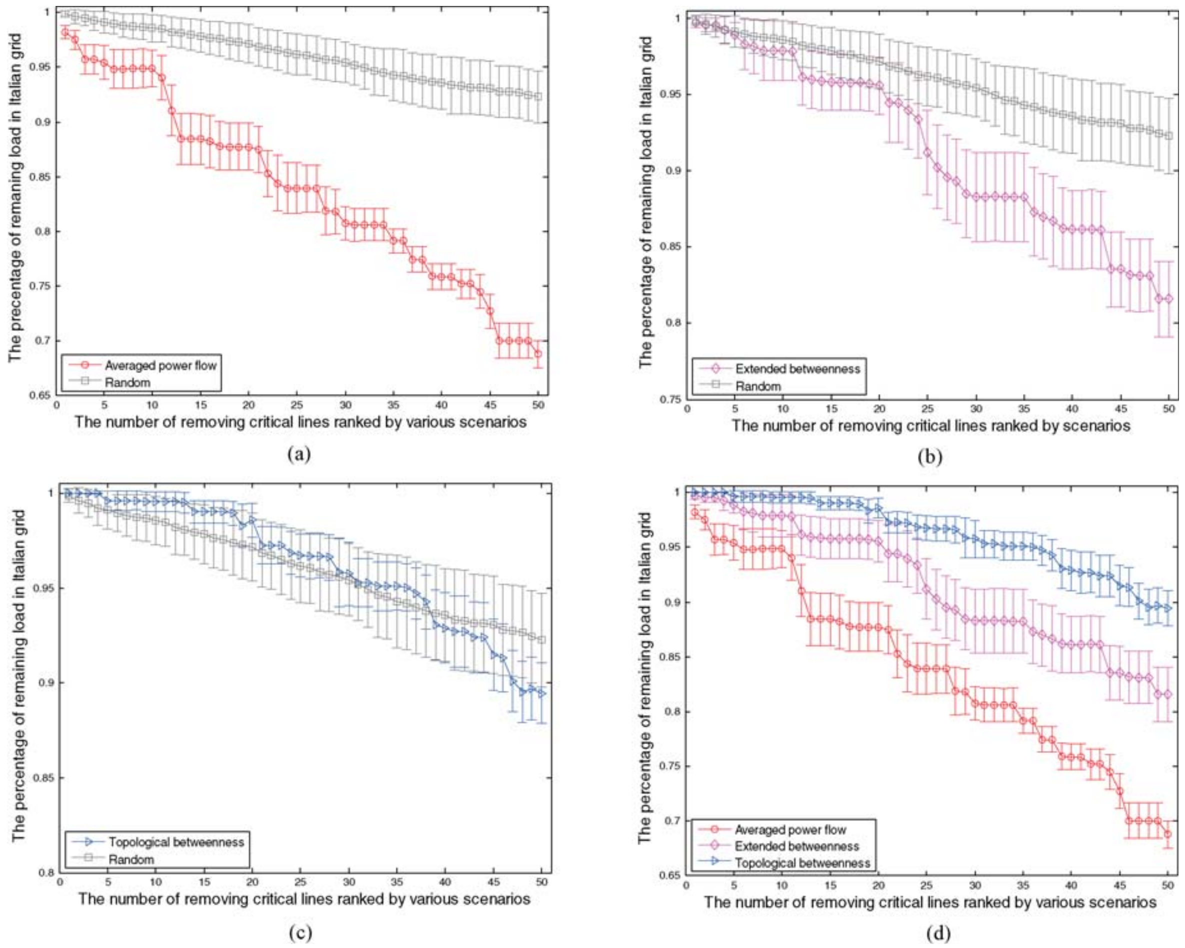

Fig. 2. Remaining load in the Italian power grid after removing 50 most critical lines in various scenarios. (a) Random attack versus power flow criterion attack. (b) Random attack versus extended betweenness criterion attack. (c) Random attack versus topological betweenness criterion attack. (d) Comparison between the three criteria.

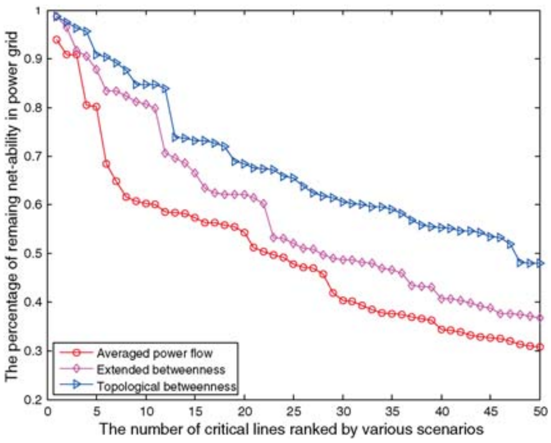

Fig. 3. Remaining net-ability in the Italian power grid after removing 50 most critical lines in various scenarios.

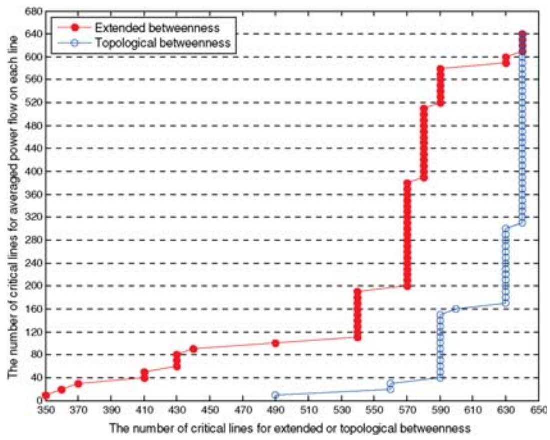

Fig. 4. Correlation of the number of critical lines among various scenarios.

into topological betweenness. Therefore, the identification for averaged power flow is superior to the extended betweenness. Even so, when we use the averaged power flow on each line as a contingency ranking metric to locate critical lines whose failures could cause serious consequence at a majority of operational conditions, we have to average evaluations over a multitude of various operational states. The computation is time-consuming, especially in large-scale power grids.

To enhance the computational efficiency, it could be useful to find the correlation between the extended betweenness and the averaged power flow on each line. As for the Italian power grid, the correlation is reported in Fig. 4 which shows a number of the first critical lines for the averaged power flow on each line can be found in a subset of critical lines ranked by the extended betweenness or topological betweenness. For instance, we can spot out the first 50 most critical lines for the averaged power flow on each line in a subset of the first 410 critical lines ranked by the extended betweenness. This implies that after the subset of critical lines for extended betweenness is first spotted out, it could be efficient and effective that the averaged power flow on each line is applied to the subset of critical lines to evaluate the criticality again. Though the dimension of the subset might be large, the dimension of the subset smaller than the total number of lines in the Italian power grid and the computational simplicity of the extended betweenness make the extended betweenness possible to be a complementary tool for contingency ranking metrics like the averaged power flow on each line to analyze vulnerability of power systems.

VI. Conclusion

An extended topological method was proposed to overcome the shortcomings that complex networks methodology is directly applied to analyzing power grids from a topological point of view. In this paper, we proposed the extended betweenness by introducing some electrical engineering specificity into the topological betweenness. Taking line extended betweenness as an example, we showed that it is superior to topological betweenness in the identification of critical lines in power grids. Although the extended betweenness is still not as good as electrical engineering contingency ranking metric to accurately measure the criticality of components, thanks to its computational simplicity it may hopefully become a complementary tool to improve the efficiency of the vulnerability analysis based on electrical engineering methods.

Appendix

The distance in complex network method is quantified as geodesic distance since it is assumed that physical quantity is transmitted along the geodesic path. However, in a power grid, the current or power is transmitted from generators and loads not only along geodesic paths but also the remaining paths. Hence, the geodesic distance should be replaced with an electrical distance when power grids are analyzed from the structural point of view by means of the complex network method. In this paper, the electrical distance is defined as the magnitude of equivalent impedance $|Z_g^d|$ between generator g and load d in (9). The equation indicates $|Z_g^d|$ is the magnitude of voltage drop $|U_g^d|$ between generator g and load d when a unit of current is injected at generator g and withdrawn at load d (i.e., $I_g = 1$), and the equivalent impedance $|Z_g^d|$ can be computed in terms of elements in the impedance matrix of a power grid (see the details in [20])

$$\left|Z_g^d\right| = \frac{\left|U_g^d\right|}{I_g} = \left|U_g^d\right| \Rightarrow \left|Z_g^d\right| = \left|(z_{gg} - z_{gd}) - (z_{gd} - z_{dd})\right| \tag{9}$$

where z_{gd} denotes the g-row, d-column entry of the impedance matrix of a power grid; z_{gd} is generally a complex number composed of resistance and reactance.

It is worth noticing that the electrical distance $|Z_g^d|$ is different from the electrical distance in [10], where the electrical distance between a pair of buses i and j is defined as the magnitude of nondiagonal elements $|z_{ij}|$ in the impedance matrix. The magnitude of element $|z_{ij}|$ represents the magnitude of the voltage drop between bus i and specified reference bus (rather than bus j) when a unit of current or power is injected in bus j and withdrawn at the reference bus (rather than bus j).

Besides, the electrical distance $|Z_g^d|$ among all pairs of generators and loads in a power grid can be represented as a matrix whose dimension is $N_{\mathrm{G}} \times N_{\mathrm{D}}$. The elements of the matrix are nonnegative. The symmetry of the matrix depends on its elements and the number of generators N_{G} and the number of loads N_{D} in a power grid. The matrix is possibly asymmetric because its entry $|Z_i^j|$ is probably not equal to the entry $|Z_j^i|$ and N_{G} is also possibly smaller than N_{D} in a power grid. Besides, the entry in the matrix is always not to satisfy the triangle inequality

$$\left|Z_i^j\right| + \left|Z_j^k\right| \geq \left|Z_i^k\right|. \tag{10}$$

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