Collective behaviour of linear perturbation waves observed through the energy density spectrum

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Collective behaviour of linear perturbation waves observed through the energy density spectrum

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Abstract. We consider the collective behaviour of small three-dimensional transient perturbations in sheared flows. In particular, we observe their varied life history through the temporal evolution of the amplification factor. The spectrum of wave vectors considered fills the range from the size of the external flow scale to the size of the very short dissipative waves. We observe that the amplification factor distribution is scale-invariant.

In the condition we analyze, the system is subject to all the physical processes included in the linearized Navier-Stokes equations. With the exception of the nonlinear interaction, these features are the same as those characterizing the turbulent state. The linearized perturbative system offers a great variety of different transient behaviours associated to the parameter combination present in the initial conditions. For the energy spectrum computed by freezing each wave at the instant where its asymptotic condition is met, we ask whether this system is able to show a power-law scaling analogous to the Kolmogorov argument. At the moment, for at least two typical shear flows, the bluff-body wake and the plane Poiseuille flow, the answer is yes.

1. Introduction

A fundamental notion in the phenomenology of turbulence (in the sense of Kolmogorov 1941) is that a power-law scaling with an exponent close to -5/3 is observed for the energy spectrum over a quite large range of a few decades of wavenumber. This interval is called the inertial range since, at these wavenumbers, the dynamics of the Navier-Stokes equations is dominated by the inertia terms (Kolmogorov, 1941; Frisch, 1995). It is a common criterium for the successful production of a fully developed homogeneous turbulent field to verify that the energy spectrum has such a scaling in the inertial range (Sreenivasan & Antonia, 1997).

We propose an experimental approach, based on the numerical determination of a large number of perturbations, to approximate the general perturbation solution of a Navier-Stokes field for two typical shear flows, the plane Poiseuille flow and the bluff-body wake. The set of small three-dimensional perturbations constitutes a system of multiple spatial and temporal scales which are subject to all the processes included in the perturbative Navier-Stokes equations: linearized convective transport, molecular diffusion, linearized vortical stretching. Leaving aside the nonlinear interaction among the different scales, these features are the same as those found in the turbulent state.

The answer to two key questions is the main goal of the present work: (i) Does a power-law scaling for the energy spectrum exist for an intermediate range of wavenumbers or frequency even in the linear dynamics of the perturbative Navier-Stokes equations? (ii) And, if so, how does it compare, in terms of decay exponent and width of range where it applies, to the well-known -5/3 Kolmogorov law for homogeneous fully developed turbulence?

In order to answer such questions, we study how the energy spectrum resulting from the analysis of a large set of solutions of the linearized perturbative Navier-Stokes equations behaves (Scarsoglio & Tordella, 2010) and we compare it with the energy spectrum of homogeneous fully developed turbulence.

In Section 2 the initial-value problem formulation is introduced and a collection of transient behaviour is showed. In Section 3 the self-similarity of the perturbative system is discussed and relevant energy spectra results are reported. In Section 4 some concluding remarks are given.

2. Initial-value problem

The energy spectrum behaviour of the perturbed system is studied using the initial-value problem formulation (Criminale & Drazin, 1990). We consider two different typical shear flows, i.e. the wake behind a circular cylinder and the plane Poiseuille channel flow (see Fig. 1). The bluff-body wake is approximated at an intermediate ($x_0 = 10$) and far longitudinal station ($x_0 = 50$), through a two-dimensional analytical expansion solution (Tordella & Belan, 2003) of the Navier-Stokes equations. The plane Poiseuille flow is, instead, taken as a parallel flow (Criminale *et al.*, 1997).

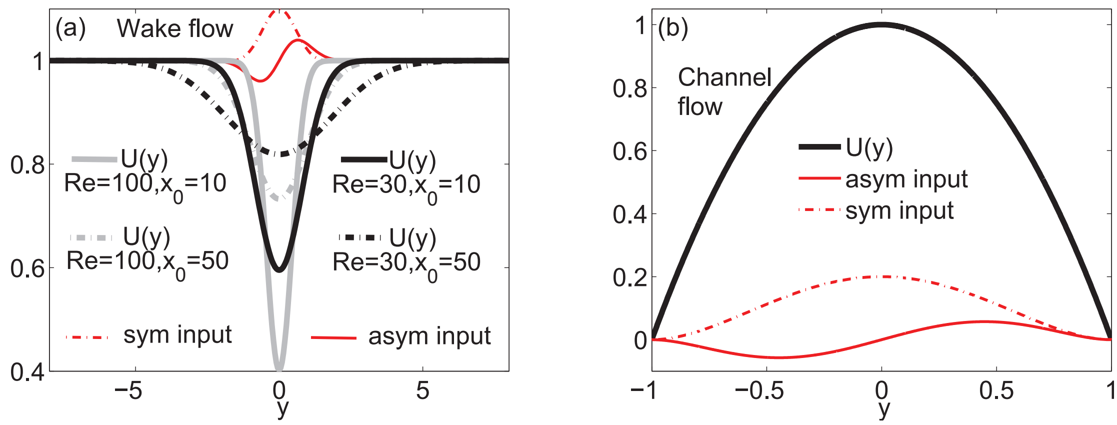

Figure 1. (a) Wake flow $U(y; Re, x_0)$ at different downstream stations ($x_0 = 10$ and $x_0 = 50$) and at different Reynolds numbers ($Re = 30$ and $Re = 100$). (b) Plane Poiseuille channel flow $U(y)$. Initial conditions in terms of $\hat{v}(y, t = 0)$ are represented by thin curves: symmetric (dotted) and asymmetric (solid) inputs.

The Reynolds number, Re, is defined through a typical velocity (the free stream velocity, U_f, and the centerline velocity, U_0, for the 2D wake and the plane Poiseuille flow, respectively), a characteristic length scale (the body diameter, D, and the channel half-width, h, for the 2D wake and the plane Poiseuille flow, respectively), and the kinematic viscosity, ν. The Reynolds number values are set in order to consider stable and unstable configurations for the bluff-body wake ($Re = 30$ and $Re = 100$, respectively) as well as for the plane Poiseuille flow ($Re = 500$ and $Re = 10000$, respectively).

The viscous perturbative equations are written in terms of the vorticity and the transversal velocity and then transformed through a Laplace-Fourier decomposition (Scarsoglio *et al.*, 2009, 2010) in the plane (x, z) which is normal to the base flow plane (x, y). We define k as the polar wavenumber, $\alpha_r = kcos(\phi)$ as the wavenumber in x direction, $\gamma = ksin(\phi)$ as the wavenumber

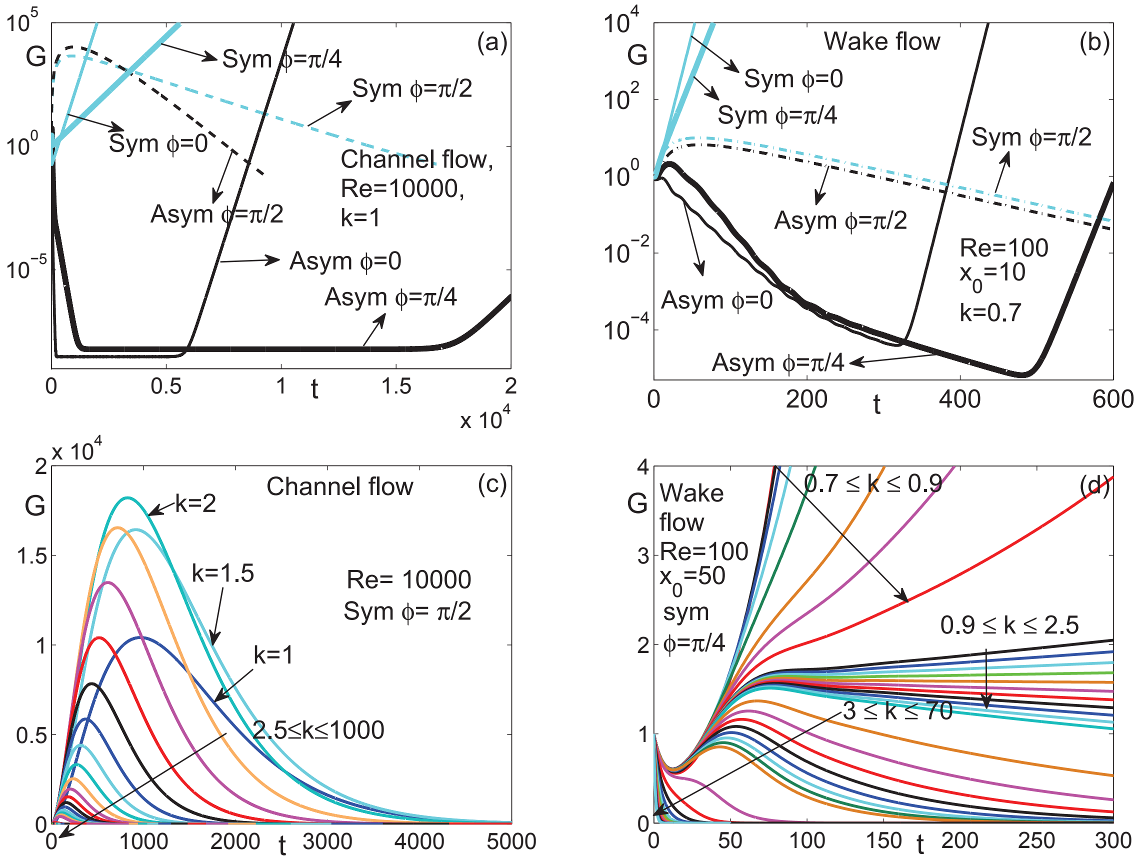

Figure 2. Relevant behaviours of the linear transient dynamics: amplification factor, $G(t)$, as a function of time. (a-b) Channel and wake flow at unstable configurations. Asymmetric longitudinal and oblique waves (dark solid curves) show temporal modulations before minima of energy are reached, then they are asymptotically amplified. The corresponding symmetric perturbations (light solid curves) are instead immediately amplified. Three-dimensional symmetric and asymmetric waves (thin dotted curves) are slowly damped in time. (c) Plane Poiseuille channel flow at $Re = 10000$, $\phi = \pi/2$, symmetric inputs, $k \in [1, 1000]$. High maxima of energy are reached before the three-dimensional perturbations are asymptotically damped.(d) Wake flow at $Re = 100$, $x_0 = 50$, symmetric initial condition, $\phi = \pi/4$, $k \in [0.7, 70]$. Unstable behaviour occurs by decreasing the wavenumber, k.

in z direction, and ϕ as the angle of obliquity with respect to the physical plane. The measure of the perturbation growth can be defined through the disturbance kinetic energy density in the plane (α, γ):

$$\begin{aligned} e(t;\alpha,\gamma) &= \int_{-y_d}^{+y_d} (|\hat{u}|^2 + |\hat{v}|^2 + |\hat{w}|^2) dy = \\ &= \frac{1}{|\alpha^2+\gamma^2|} \int_{-y_d}^{+y_d} \left(\left| \frac{\partial \hat{v}}{\partial y} \right|^2 + |\alpha^2+\gamma^2||\hat{v}|^2 + |\hat{\omega}_y|^2 \right) dy, \end{aligned} \quad (1)$$

where $\hat{u}$, $\hat{v}$ and $\hat{w}$ are the components of the perturbation velocity, $\hat{\omega}_y$ is the transversal vorticity, while $2y_d$ is the extension of the spatial numerical domain. The amplification factor $G(t)$ can be introduced in terms of the normalized energy density, $G(t;\alpha,\gamma) = e(t;\alpha,\gamma)/e(t=0;\alpha,\gamma)$. We account for symmetric and asymmetric initial conditions (see thin curves in Fig. 1) in terms of the transversal velocity $\hat{v}$, while the transversal vorticity $\hat{\omega}_y$ is initially equal to zero. Three different angles of obliquity ($\phi = 0, \pi/4, \pi/2$) are analyzed for the perturbative waves.

The variety of the transient linear dynamics observed in recent exploratory analyses (Scarsoglio *et al.*, 2009; Marais *et al.*, 2011; Reddy & Henningson, 1993; Schmid, 2007) - i.e. emergence of different temporal scales, maxima of energy followed by an asymptotic damping, minima of energy beyond which an ultimate slow amplification occurs (see some relevant behaviours in Fig. 2) - suggests the idea to investigate the ensemble behaviour of many of these perturbations, considered all together even in the linear dynamics, to understand analogies and/or differences with the turbulent state.

3. Energy spectra

The appearance of different temporal scales associated to the different perturbation wavelengths (see Fig. 3a) suggests that a self-similarity approach should be adopted to describe the temporal evolution (Barenblatt, 1996). A continuous instantaneous normalization can be used by defining $t^* = t/\tau$, with $\tau = G(t)/|dG/dt|$. For the wake case, in Fig. 3, the amplification factor, G, is reported as a function of both t and t^* for a group of perturbations with $k \in [0.45, 500]$. It should be noted that, depending on whether the perturbations are stable or unstable, two subsets of waves ($k \in [0.45, 2]$ and $k \in [2.5, 500]$) showing self-similarity features can be observed (see Fig. 3b). Assuming that for each of these ranges the amplification factor distribution is scale-invariant, then $G(\lambda t) = \lambda^h G(t)$, with h unique. It can be observed, that $G(t^*) = G\left(\frac{t}{G(t)/|dG/dt|}\right) \approx \frac{G(t)}{\tau} = |dG/dt|$, so that $\lambda = 1/\tau$ and $h = 1$.

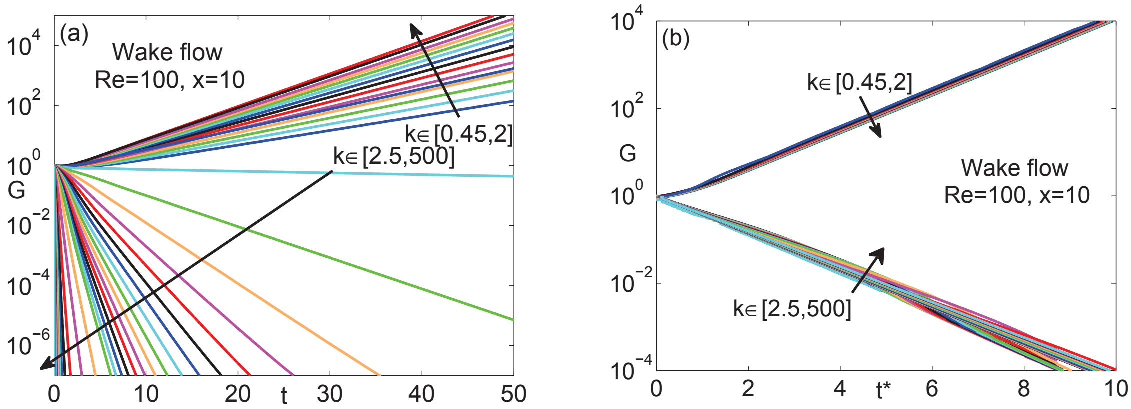

Figure 3. Temporal scaling. (a) G as function of t; (b) G as function of the normalized variable $t^* = t/\tau$, with $\tau = G(t)/|dG/dt|$. Wake flow at $Re = 100$ and $x_0 = 10$. $\phi = 0$, symmetric inputs.

The energy spectrum is evaluated as the wavenumber distribution of the perturbation kinetic energy density, $G(k)$, in asymptotic condition. That is when the exponential behaviour is stabilized. We base the definition of this temporal asymptotic limit on the temporal variation of the normalized energy, G. We thus assume the asymptotic condition is reached when $d(|dG/dt|/G)/dt = d\lambda/dt \to 0$ is satisfied for stable and unstable waves. The normalized energy density G in the asymptotic state is shown - as function of the polar wavenumber k - in parts (a), (b), (c), (d) of Figure 4, for the bluff-body wake (parts (a) and (b)) and for the channel flow (parts (c) and (d)).

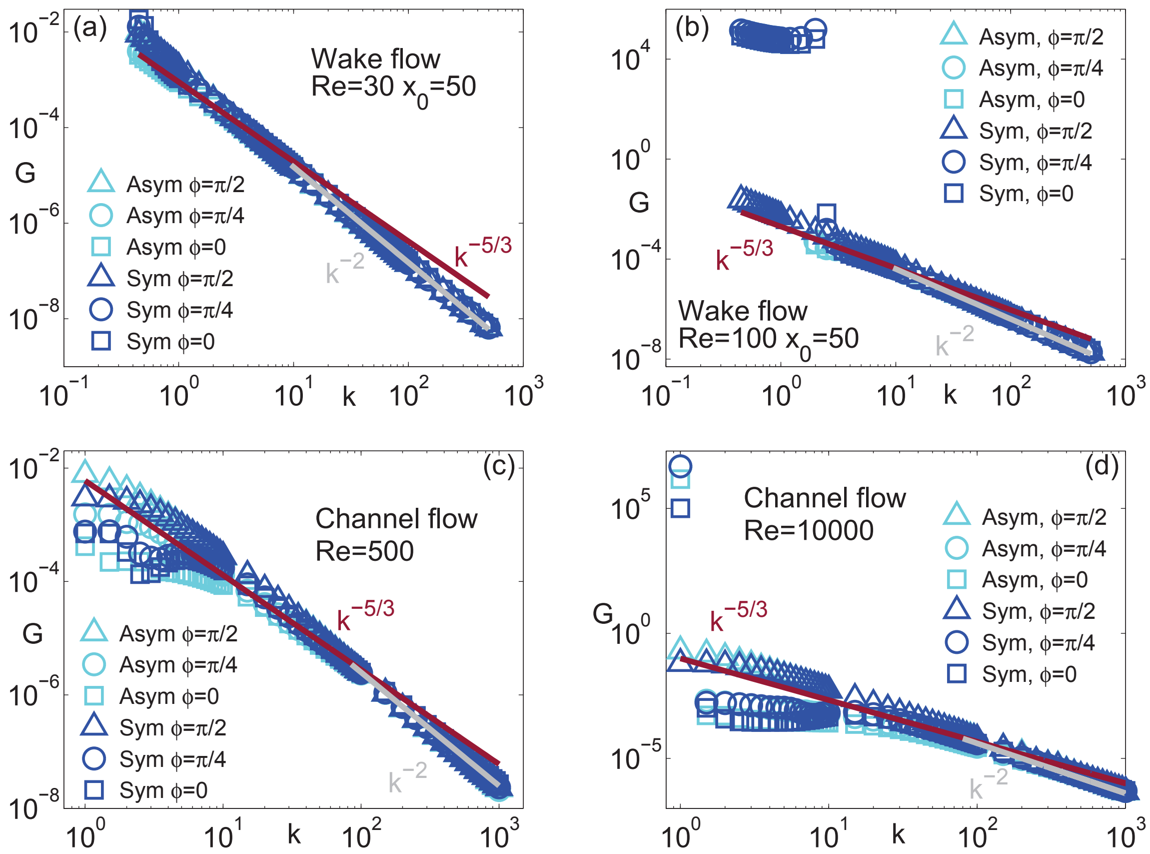

Figure 4. Energy spectrum G of symmetric (dark symbols) and asymmetric (light symbols) perturbations ($\square$: $\phi = 0$, $\bigcirc$: $\phi = \pi/4$, $\triangle$: $\phi = \pi/2$). (a)-(b) Bluff-body wake at $Re = 30$, $x_0 = 50$ (stable) and $Re = 100$, $x_0 = 10$ (unstable), respectively. (c)-(d) Plane Poiseuille flow at $Re = 500$ (stable) and $Re = 10000$, respectively. Light and dark curves: -2 and $-5/3$ slopes, respectively.

For both stable and unstable configurations, there exists an intermediate range of about a decade ($k \in [2, 20]$ and $k \in [15, 150]$ for the bluff-body wake and the plane Poiseuille flow, respectively) where longitudinal and oblique perturbations present a power-law decay which is close to -5/3 (dark curves), while purely three-dimensional waves have a decay of about -2 (light curves). For larger wavenumbers ($k > 20$ and $k > 150$ for the bluff-body wake and the plane Poiseuille flow, respectively), all perturbations show a power-law decay very close to -2 (light curves). The transition from -5/3 to -2 power-law scalings smoothly occurs inside the self-similar range at about a wavenumber of order 10 for the wake and 10^2 for the plane channel. For the longer waves ($k < 1 - 2$ and $k < 10$ for the bluff-body wake and the plane Poiseuille flow, respectively), results do not seem to reveal any characteristic behaviour. The energy spectrum strongly depends, here, on initial and boundary conditions as well as on the shape and wavelength of perturbations. It seems that as soon as the dissipative influence becomes less important, but still perturbations are not too long, the energy spectrum is able to show a decay rate which is close to the one observed in a fully developed turbulent field, where nonlinearities are considered as a dominant aspect of the dynamics.

4. Concluding remarks

We have numerically computed a large collection of three-dimensional small perturbations for the bluff-body wake and the plane Poiseuille flow in stable and unstable configurations. We consider the time instant where all the waves meet the asymptotic exponential condition and build the energy spectrum. Whether the waves are aligned with the base (bounded or unbounded) flow or not, the energy of the intermediate range of wavenumbers in the spectrum decays with an exponent close to $(-5/3)$. That is we observe a situation close to the spectrum of the velocity fluctuation of fully developed turbulent flows, where the nonlinear interaction is dominant. It seems possible to conclude that the spectral power-law scaling of intermediate/inertial waves (with an exponent close to $-5/3$) is a general dynamical property of the Navier-Stokes solutions which encompasses the nonlinear interaction.

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