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A COMPARATIVE ASSESSMENT OF SYNTHETIC INDICES TO MEASURE MULTIMODALITY BEHAVIOURS

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A comparative assessment of synthetic indices to measure multimodality behaviours

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The study of how people jointly use different travel means is one of the key issues in contemporary transport research. However, measuring multimodality behaviours presents some intricacies that deserve more attention in order to come up with an instrument that is effective both on a modelling and on a policy viewpoint. The present work considers some methods that have been proposed in different disciplinary ambits to measure diversity and assesses to what extent they are useful to measure multimodality. A broad set of indices is then analysed, ranging from welfare economics (Gini, Dalton and Atkinson indices) to information theory and ecology (entropy, Herfindahl index). Theoretical investigations and empirical experiments on the properties of such indices show that there is not a measure of multimodality that consistently outperforms all the others in any circumstance. On the other hand, it emerged that some methods are clearly preferable for specific problem instances, as discussed in the conclusions.

Keywords: multimodality behaviours, inequality index, variability measures, transportation modes, sensitivity analysis

1. Introduction

Multimodality is commonly defined as the use of more than one transport mode to complete a trip. Transport policy makers are actively promoting multimodal trip making in several countries (European Commission 2011), for example through the development of interchange points (park and ride, mobility hubs etc.) in order to increase the efficiency and sustainability of transport system. The best ambits of use of each travel means are in fact different, according for example to the travel length or to the number of travellers sharing the same path. Therefore, encouraging the combined use of different modes is probably both preferable and more effective than almost exclusively promoting any of these, including public transport.

However, current travel analysis methods seem not totally fit to inform such policies. Previous researches showed that both classical and state of the art models present shortcomings when representing or predicting multimodal trips (van Eck et al. 2014). Even merely observing such trips can be problematic when using traditional travel survey instruments (Clifton and Muhs 2012). On the other hand, a truly multimodal transport planning process would require a paradigm shift of conventional practices (Litman 2014), that seems far from being accomplished in many countries, despite the recommendations

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of strategic documents such as the latest European Commission White Paper on transport (European Commission 2011).

Interestingly enough, the concept of multimodality can be applied not only to trips but also to travellers, therefore defining multimodality as the use of more than one travel means by an individual in a given time period (Buehler and Hamre 2015a,b). While it is important to advance the state of the art of trip-based and activity-based transport models to better analyse multimodal trips, considering multimodality at the more aggregated level of the individual could nevertheless help to advance our knowledge on a number of open issues in transport research. Observed multimodality and modality styles could be employed as exogenous variables to improve behavioural choice models while avoiding the use of attitudinal data, which are typically more difficult to collect and analyse (Kuhnimhof 2009; Vij, Carrel, and Walker 2013). Following such approach, Diana (2010) has for example shown that multimodal habits positively affect the willingness to use a non-existing transport service involving innovative concepts such as car sharing or ride sharing, while Chlond (2012) investigates the relationship between multimodality and greenhouse gas emission reductions. There is also an obvious connection with the study of travel habits, that has been the object of intensive research in past decades (e.g., Goodwin 1977; Schlich and Axhausen 2003; Friedrichsmeier, Matthies, and Klöckner 2013).

This paper will therefore focus on the problem of measuring multimodality at the individual level. The purpose is to study to which extent a synthetic multimodality index can represent a series of measured intensities of use of different travel means in a way that is useful to transport researcher and practicioners, by comparing different methods and analysing benefits and drawbacks of each one. Clarifying this issue could lead to better understanding the open questions investigated in the above reviewed seminal studies concerning the relationship between multimodality behaviours, travel choices and environmental impacts.

In the next section, a discussion on how to measure traveller-related multimodality in a useful way for policy makers is developed. This will form the basis for the identification of the properties that a multimodality index should have according to the specific problem under investigation or policy context. On a mathematical viewpoint, even if such research question (namely, how to best represent variability with a single index) is relatively new in the transport sector, it has been already deeply investigated in other disciplinary ambits, from which interesting lessons can be learned. Therefore, in Section 3, a broad overview about how the concept of variability and inequality is tackled in fields as biology and economics is proposed, and how such results can be applied by analogy to the study of transport systems is discussed. The most interesting indices from a travel research perspective are presented and factorised in Section 4 and 5, respectively. Then, the behaviour of these indices is analysed through sensitivity analyses in order to highlight their limits and their qualities when they are adapted to measure multimodality (Section 6). Their best ambits of use are accordingly presented in the conclusions of the present work.

2. How to Measure Individual Multimodality?

Previous recent research has focused on measuring multimodality at the individual level (Nobis 2007; Kuhnimhof et al. 2012; Buehler and Hamre 2015a,b), and on modelling multimodality levels as a function of a variety of individual characteristics (Heinen and Chatterjee 2015). The latter study is also reviewing quantitative results of observed multimodality behaviours in different works, where multimodality is generally measured by

considering the fraction of travellers that use a given number of travel means. According to these works, 51% of Germans for example used only one travel means among car, bycicle and public transport in 2002 (Nobis 2007), while Buehler and Hamre (2015a,b) categorise U.S. travellers into monomodal car users, multimodal car users and exclusive users of walk, bike and public transport. Multimodality in such cases is then captured through some descriptive statistics on the number of travel means, without any consideration on the intensity of use of each mode. On the contrary, we later show that a multimodality index would need to encompass both aspects in a unique measure.

At the outset, quantitatively studying multimodality seems a rather straightforward task. Concerning data requirements, it is grounded on the analysis of sequential or repeated behaviours, implying the need for a longitudinal travel survey dataset (or at least some answers to retrospective questions on modal usages frequencies over a time period). In principle, the researcher could work on such data to individuate the set of travel means being used by an individual or a group, to measure the intensity of use of each one through some appropriate units (number of trips, distance covered or travel time) and to compare and synthesise these different mode-specific measures. Yet translating such steps into a detailed methodology is indeed a tricky task, involving some choices that could not lead to univocal outcomes. A more operational definition of multimodality is needed when referring this concept to individuals. To the best of our knowledge, such investigation has never been attempted in the open literature, where individual multimodality is measured as if this concept were already unanbiguously defined.

The first issue deals with the measurement unit. Considering travel times or distances rather than number of trips could lead to different results, since travel means have widely different speeds and are often used for kinds of trips taken with different frequencies (e.g. recreational versus systematic trips). The best measurement unit to use could depend on the problem under consideration: for example, if the subjective viewpoint is to be considered, then travel times or trip frequencies seem more appropriate, whereas studies on the relationship between individual multimodality and environmental impacts could rather consider travel distances. In any case, the choice of the best measurement unit comes after the investigation of the mathematical properties of the multimodality measure, and therefore is not further discussed in this paper. In the following, we refer to any of the above mentioned measurement units through the generic locution 'intensity of use' of a means.

The second issue is the need to consider both the variability of the set of travel means and the variability in the intensities of use in a balanced way. For example, let us assume that one individual both drives a car and rides a bus five times a week, and another one drives a car every day, while riding a bus and using a bike once a week. One might wonder which of the two is showing a greater multimodal behaviour. It is easy to see that the answer depends on how the measures of different modal usages are compared: the former person uses less means (factor decreasing multimodality) but without the predominance of any means (factor increasing multimodality). Again, according to the practical instance, a synthetic index that is sensitive more to one or another aspect could be desirable. We later show how this issue has been tackled in a related field, namely the study of biological diversity, and which lessons can be drawn when measuring multimodality behaviours. In short, it is possible to define some mathematical properties that such indices should have when there is a change in one or several of the following aspects:

- differences of intensities of use of different means;
- absolute values of intensities of use of one or more means;
- number of considered means;
- classification of means (e.g. considering all public transport modes together or

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separately).

We believe that the two above issues are the most important ones when looking for a good measure of individual multimodality. Nevertheless, there are at least two additional ones that would be worth considering in subsequent phases of the research[1], and that present some aspects that are common with the study of the choice set in mode choice models:

- Related to the variability of the set of travel means: which is the most appropriate aggregation level for modes? Should similarities across travel means play a role? For example, should all forms of public transport be jointly or separately considered? More in general, which is the proper classification of travel modes to consider? Previous research has shown that commonly used classification schemes based on technology (road, rail etc.) are not the best option in the context of behavioural mode choice models (Flamm 2003; Diana, Song, and Wittkowski 2009). Heinen and Chatterjee (2015) test two different aggregation levels of travel modes to measure multimodality (three and eight means). In their work, models explaining the observed multimodality give different results according to the used indicator.
- Should different combinations of modes be differently considered, rather than merely counting how many modes were used? For example, driving to a 'park and ride' lot and then boarding the train seem a 'more multimodal' behaviour than walking to the train station, even if two travel means are used in both cases. To the best of our knowledge, this latter issue is still unexplored in the literature.

To summarise, the scientific literature reviewed so far shows that analysing individual multimodality behaviours is an emerging topic in travel research, with an increasing number of works progressively focusing on different issues. Yet the fundamental question of how multimodality should be measured has never been systematically addressed, even if the above discussion has pointed to some potential problems or ambiguities. The present paper will not exhaustively deal with all these aspects. Rather, it will show how related work that has been carried out in different disciplinary fields can shed some light on the issues at stake. We particularly focus on the problem of balancing variety of travel means and intensities of use and on the variability of the set of available travel means across individuals.

3. A Cross-disciplinary Review of Variability Measures

Assessing multimodality can be seen as a particular instance of a more general issue, namely the measure of diversity and heterogeneity, that is studied in widely different research ambits. As a consequence, a variety of methods has been developed more or less in parallel in different disciplines. It is therefore important to look at how the problem is tackled outside the transport engineering field and to understand if the assumptions of the related studies are valid also in our framework. So, we briefly discuss the most relevant approaches from our viewpoint, by looking at the work made in different fields and how we can draw lessons concerning the measurement of diversity. The outcome is to identify some of the most promising methods to be analysed in the following sections.

From elementary statistics, the most straightforward method would be to consider the variance, or the coefficient of variation if the analyst wishes to make the measure independent from the mean intensity of travel across different modes. The lower the variance

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(or the coefficient of variation) in such intensities of use, the higher the multimodality would be, at least when the number of potentially used modes is fixed. However, let us consider two individuals using four different modes with intensities equal to [10 10 2 0] and [11 9 1 1]: the average intensity and its variance are the same, but we would say that the second one is 'more multimodal', since s/he uses more travel modes and the difference in the levels of use of the first two (11 versus 9) seems not so significant. From this example, we infer two desirable properties of a multimodality measure, that are also related to the discussion in the previous section. First, it should increase when we increase the number of travel means being used. Second, differences in the levels of use should have a decreasing negative impact on multimodality as the overall mean intensity of use is increasing (i.e. levelling the difference between 2 and 0 should cause a larger increase of the multimodality index than levelling the difference between 11 and 9). Stated in another way, we can say that two individuals A and B using the same set of modes can be considered as equally multimodal even if A is overall travelling more than B and the variance of modal intensities is the same, provided that B travels more than A by the modes s/he overall uses less. This idea can be connected by analogy with the inequality aversion concept in social sciences and can play a role in defining a multimodality index, as we discuss in the following.

It is in any case possible to resort on the methodological work already done in the measurement of diversity to clarify the characteristics that a multimodality measure should have. One of the most active research fields in this area is dealing with Econometrics. There, the central interest is the study of inequalities on one hand, for example related to the distribution of incomes, on the other the concentration of economic actors in a market (competitive market versus monopoly). Inequality and concentration are actually two analytically related concepts (Rosenbluth 1955) and, as such, they are studied with similar methods. Cowell (2011) presents an in-depth analysis of the main inequality measures, that can be derived either from statistics (such as variance) or empirical evidence (Gini index) but also from theoretical considerations linked to social welfare functions (Dalton and Atkinson indices) or from information theory (Theil entropy, Herfindahl index).

We later analyse some of these indices in greater depth, but it is now of interest to establish an analogy between income inequality and multimodality, where individuals and their incomes respectively map into travel means and their intensities of use. In our search for a multimodality measure we are not necessarily privileging the perspective of social inequality studies; however we are not aware of a review work of the breadth of Cowell's in other fields, so we start having a closer look at this and later extend our investigation to other research areas. This approach seems to us preferable rather than directly deriving desired properties and axioms of a multimodality index from the discussion in the preceding section, since on one hand such desired properties are often depending on the practical problem under consideration and on the other we think it is important to build on the body of knowledge that has been accumulated over the years.

By applying the analogy mentioned in the preceding paragraph, we can therefore reformulate the five properties that Cowell (2011) indicates as desirable in an inequality index as follows:

(1) *Weak principle of transfers*: consider two travel means whose intensities of use are I and $I + \delta$, where $\delta > 0$. If the intensity of the most used mode decreases and that of the least used increases by the same quantity $\Delta I < 2\delta$, then the multimodality index should increase.

(2) *Strong principle of transfers*: let us define a distance measure $d = h(I_1/I_{tot}) - h(I_2/I_{tot})$ for modes 1 and 2, with $I_1 < I_2$, where I_{tot} is the sum of all intensities and h is a decreasing function such as $h(I) = (1 - I^{\beta})/\beta$, with β a parameter.

If the intensity of the most used mode I_2 decreases and the one of the least used I_1 increases, the variation of the index depends only on the variation of d. The ratios I_i/I_{tot} are the 'intensity shares' of mode i: the larger the share, the more predominant is the use of that mode compared to others. The function h is introduced to decrease the distance, and therefore the effect on the index, when the modal transfer is taking place between two means that are progressively more predominant, even if the difference in their relative intensity shares is constant. We later elaborate a small example to better illustrate the practical meaning of this propriety.

(3) *Scale independence*: if the frequency of use of each mode changes by the same proportion, the multimodality index should remain the same.

(4) *Population size independence*: the multimodality index should remain the same for any replication of the modes with their corresponding intensities of use. The choice set of the modes represents our population and 'replicating a mode' can be seen in our context simply as an increase in the population size due to the consideration of an additional number of modes with the same intensities of use of those already in the choice set.

(5) *Decomposability*: multimodality rankings of alternative distributions of intensities of use in the whole set of travel means should match the multimodality rankings of the corresponding distributions of intensities within any of the subgroups in which the whole set of travel means can be composed.

It is interesting to note that not all those properties are still desirable when dealing with multimodality. Property (1) can be seen as a basic requirement of a multimodality index, as noted above when analysing the variance. Coming to property (2), let us consider a traveller using four different means with intensities [12 10 3 1]. The strong principle of transfers with $\beta = 1$ would imply that the related multimodality index does not change if one unit of intensity is shifted from the first to the second mode or from the third to the fourth. However, when examining the variance we already noted that multimodality values should increase when we level off intensities of less used means. It is easy to see that this happens when $\beta < 1$, therefore property (2) is desirable only when bounding the values of the parameter in this way.

Concerning properties (3) and (4), it does not seem reasonable to affirm that if an individual has twice the intensity of use of all means as another one, the two have the same multimodal behaviours, nor that a traveller having used two modes with an intensity of use of [1 5] is as much multimodal as another one having used four different means with intensity equal to [1 1 5 5]. Related to the latter example, the granularity in the definition of different modes could play a role. Consider for example the case where the first traveller is using car and bus, and the second one bus, tramway, underground and train: the latter traveler is probably less multimodal. Apart from such extreme situations, we can in any case conclude that properties (3) and (4) seem not desirable when studying multimodality, whereas the acceptability of the second one depends on the distance function being considered. Finally, property (5) could be relevant only for particular studies, for example when studying multimodality only within different public transport means as opposed to a more general multimodality involving all public and private modes.

Since inequality measures have been designed to satisfy most of the above properties, we conclude that none of the above mentioned ones perfectly matches our framework, even if they can obviously be used also in the transport sector to study concentration and inequality issues. However, in this case the original ambit of use of the measures is maintained and the above introduced analogy to study multimodality is not applied. For example, the Herfindahl index (or some variants) is one of the most used to study

concentration effects in transport services (Alderighi et al. 2007; Huber 2009; Le and Ieda 2010; Suau-Sanchez and Burghouwt 2011). On the other hand, equity issues related to transport have been often studied through the Gini index (Maruyama and Sumalee 2007; Karlström and Franklin 2009; Delbosc and Currie 2011).

To the best of our knowledge, only four works have reinterpreted such indices to study multimodality, in a vein similar to ours. Susilo and Axhausen (2014) assess the stability in individuals' choices of their daily activity-travel-location combinations by using the Herfindahl index. Heinen and Chatterjee (2015) test four measures of multimodality: two Herfindahl indices with different definitions of the mode set, the difference in proportion of use between the most used and the second most used mode, and the number of modes used. They analysed data from the 2010 British National Travel Survey and found that the mean values of these four measures are respectively 0.66 (8 modes), 0.67 (5 modes), 0.60 and 2.19. The closest antecedent of our work is however Diana and Mokhtarian (2008, 2009), who derive two multimodality indices from the Shannon entropy formula and use them to profile travellers on the basis of objective, subjective and relative desired travel amounts through different transport modes. These two papers do not report descriptive statistics on the proposed indices.

The above review of the properties of the indices in Economics has shown that such framework is not completely fit for our problem. However, additional insights can be gained by looking at similar work that has been done in other disciplinary fields. Another relevant research domain is, for example, the study of biological diversity. Diversity and heterogeneity are seen in this case as manifold concepts, involving issues such as species richness, evenness and dominance (on the analogy: the number of travel means, their relative intensities of use and the intensity of use of the most common mode). A lot of measures have been proposed that are more sensitive to any of these issues (Magurran 2004; Tuomisto 2012), including the Shannon index, that encompasses both richness and evenness, and the Simpson index (having the same formulation of the above Herfindahl index) that stresses more on dominance. The analogy between biodiversity and multimodality seems very informative for our problem: also the multimodality concept encompasses both richness and evenness. Therefore, the ideal multimodality index should be dependent on both aspects, as mentioned in the preceding section and again when assessing variance as a candidate measure. On the other hand, evenness alone is more in line with inequality studies, and related indices should have most of the above reviewed properties (Smith and Wilson 1996).

Other more complex or specific approaches to measure variability in the transport sector have been proposed. Previous specialised methods related to multimodality specifically developed in the mobility research domain include the work of Kuhnimhof (2009): however, the proposed index does not seem to respect the above weak principle of transfers, that is surely an attractive feature of any multimodality measure. Variability in travel choices, beyond modal usages, is of central interest in contemporary transport planning techniques such as activity-based models and, therefore, it has been extensively studied. While Pas and Koppelman (1987) investigate the intrapersonal variance of trip rates, other authors jointly consider several different trip characteristics through repetition and similarity indices (Huff and Hanson 1986; Schlich et al. 2004), time-space prisms (Kitamura et al. 2006) or graphically representing the variability (Jones and Clarke 1988). Other studies consider travel variability as a pattern recognition problem, that can be analysed with techniques from molecular biology such as sequence alignments (Joh et al. 2002). However, such methods consider variability across several different variables, thus adopting a point of view different from that considered here.

4. Candidate Multimodality Indices

Many variability, inequality and diversity measures are available in the literature, yet the above review has shown that none of them is perfectly suitable as a multimodality index, essentially because of the fact that the sought properties are different. Therefore, in the following we focus on a selection of the most common measures, followed by their mathematical characterisation to further assess them.

4.1. *Gini Index*

The Gini index, also called Gini ratio or Gini coefficient, is a summary statistic of the Lorenz curve and is usually used as a measure of inequality in a population. It considers the differences among values of a frequency distribution.

We translate the usual formulation of the index, which is based on the income value of a population, in the context of multimodality. So, if we consider f_i as the intensity of use of i-th mode and the data ordered by increasing size of elements, the index can be formulated as follows (Damgaard and Weiner 2000; Dixon et al. 1987, 1988):

$$\text{GI} = \frac{2}{n} \frac{\sum_{i=1}^{n} i \cdot f_i}{\sum_{i=1}^{n} f_i} - \frac{n+1}{n},$$

(1)

where n is total number of modes in the choice set.

The Gini coefficient ranges from a minimum value of zero to a maximum of one. In an economics context these two values represent the hypothetical situations of, respectively, a pure equal distribution of the richness of the population and the maximal concentration of the whole income in only one person hands. In our view, the former one corresponds to an equal usage of all modes, while the latter refers to an infinite population of modes in which all of them except one are not used (perfect monomodality). In an example where two travellers, namely Traveller A and Traveller B, have modes intensities of use [10 10 3 0] and [14 8 1 1] respectively, this index takes the value of 0.40 in the former case and 0.48 in the latter.

4.2. *Herfindahl Index*

This index, known also as Herfindahl-Hirschman index, is a typical measure of market concentration since it depends on the number of firms and their size related to the market. Sizes are expressed by fractions, leading to an index ranging from zero to one, where extreme values represent, respectively, a perfectly competitive market with a high number of small firms and a monopoly. So, in our framework, the value of the index is closer to zero when a lot of different travel means are used and no means is very intensively used, whereas the value increases when the use of a smaller number of modes tends to dominate. The index can be defined as follows (Rosenbluth 1955):

$$\text{HH} = \frac{1}{n}(CV^2 + 1),$$

(2)

where CV is the coefficient of variation:

$$CV = \frac{\left[\dfrac{1}{n}\sum_{i=1}^{n}(f_i - \overline{f})^2\right]^{1/2}}{\overline{f}} \qquad (3)$$

so that the final formulation becomes

$$\mathrm{HH} = \frac{1}{n}\left[\frac{\dfrac{1}{n}\sum_{i=1}^{n}(f_i - \overline{f})^2}{\left(\dfrac{1}{n}\sum_{i=1}^{n}f_i\right)^2} + 1\right] = \frac{1}{n}\left[\frac{n\sum_{i=1}^{n}(f_i - \overline{f})^2}{\left(\sum_{i=1}^{n}f_i\right)^2} + 1\right], \qquad (4)$$

where f_i is the intensity of use of i-th mode, $\overline{f}$ is the mean value of the intensities of the all n modes considered. Considering the same numerical example introduced at the end of Section 4.1, this index has the same value to that found for GI (0.40) for Traveller A and is equal to 0.46 for Traveller B.

Unlike the previously reviewed studies in Economics and Biology, where individuals with zero income are few or species with zero individuals are not relevant, in our framework it is important to distinguish between the set of available modes and the set of effectively used modes. Therefore, we propose a variant of the above equation (4) which takes into account only the m elements different from zero, while the coefficient of variation and the variance are computed over all n modes. In this way, we define the HH_m index as follows:

$$\mathrm{HH}_m = \frac{1}{m}\left[\frac{n\sum_{i=1}^{n}(f_i - \overline{f})^2}{\left(\sum_{i=1}^{n}f_i\right)^2} + 1\right]. \qquad (5)$$

While in our example this variant of HH does not influence the index value for Traveller B (again equal to 0.46), HH_m is increasing to 0.53 for Traveller A, due to the fact that only the used modes (three instead of four) are considered. Most remarkably, the two indices give inconsistent indications, since Traveller B now becomes the one with higher multimodality behaviour (lower index value).

4.3. *Multimodality Indices from the Shannon Entropy*

In information theory, the amount of information supplied by an experiment is given by the amount of uncertainty that is associated with the experiment itself, the latter being computed through the Shannon entropy formula. Diana and Mokhtarian (2008, 2009) reinterpret this concept by considering an hypothetical mode choice experiment, where the uncertainty of the outcome is proportional to past multimodality behaviours of the

traveller, thus defining a multimodality index OM_PI given by:

$$\text{OM\_PI} = \sum_{i=1}^{n} \left[\frac{f_i}{\sum_{j=1}^{n} f_j} \log_n \left(\frac{\sum_{j=1}^{n} f_j}{f_i} \right) \right] = \sum_{i=1}^{n} \left[\frac{f_i}{\sum_{j=1}^{n} f_j} \ln \left(\frac{\sum_{j=1}^{n} f_j}{f_i} \right) \frac{1}{\ln n} \right], \qquad (6)$$

with, as before, f_i intensity of use of i-th mode and n the total number of means. For the two travellers in our example, this index takes the value of 0.29 for Traveller A and of 0.32 for Traveller B.

Diana and Mokhtarian (2008) propose then to take into account also the mean mobility level of individuals, by introducing a value M, defined as the absolute maximum reported frequency of utilisation of any mode across all observations in the sample (we refer the interested reader to that paper for a discussion on the rationale behind such approach). In this way, the potential maximum total frequency across all considered modes is given by nM and it is possible to define a Mobility-level-sensitive multimodality index, OM_MI, that in biological diversity terms is more sensitive to richness than to evenness compared to OM_PI:

$$\text{OM\_MI} = \sum_{i=1}^{n} \left\{ \frac{f_i}{nM} \left[1 + \ln \left(\frac{M}{f_i} \right) \right] \right\}. \qquad (7)$$

Both indices range from zero (exclusive use of only one means) to one (equal use of all means). Since the other indices decrease their values as multimodality increases, in the following we take their complement values to one. In our example, OM_MI is the same for both the travellers (0.47). Therefore, the two travellers have the same multimodality level according to OM_MI, unlike all measures considered so far.

4.4. *Theil Index*

As the previous indices, also the Theil index comes from the information theory concept of entropy. To obtain it, we subtract the OM_PI from its maximum value, that refers to intensities of use of all modes equal to $\overline{f}$, leading to the following formulation:

$$\text{TH} = \frac{1}{n} \sum_{i=1}^{n} \frac{f_i}{\overline{f}} \ln \left(\frac{f_i}{\overline{f}} \right). \qquad (8)$$

This index is also used to measure economic inequality: the minimum value reached is zero while the maximum comes to be $\ln n$. The former refers to a situation of same income for all individuals, i.e. there is a sort of 'maximum disorder' in the income of the population, while the latter occurs when all the richness belongs to only one person, i.e. the 'maximum order' holds. In our vision, the lowest value of the index comes if the traveller uses all means with the same frequency. The highest one, instead, is referred to a situation of only one mode used among all the n possible choices. The computation of TH for the travellers of our example gives 0.40 Traveller A and 0.44 for Traveller B.

4.5. Dalton Index

Cowell (2011) proposes an additional inequality measure related to a parameter ε, which operationalises the above mentioned inequality aversion concept from welfare economics. In our framework, such parameter represents the decreasing influence of more intensely used modes to determine the degree of multimodality of a traveller. So, larger values of ε make the index more sensitive to the intensities of use of less frequently used modes. Given n different modes with f_i intensity of use of each one, the definition of the Dalton inequality index is:

$$\mathrm{DAL} = 1 - \frac{\frac{1}{n}\sum_{i=1}^{n}(f_i^{1-\varepsilon} - 1)}{\overline{f}^{1-\varepsilon} - 1} = 1 - \frac{\frac{1}{n}\sum_{i=1}^{n} f_i^{1-\varepsilon} - 1}{\overline{f}^{1-\varepsilon} - 1}. \tag{9}$$

As for the Herfindahl index, we will also study a different formulation, taking into account only the m values different from zeros in the computation of the mean and in the values of the numerator, so that the index becomes:

$$\mathrm{DAL}_m = 1 - \frac{\frac{1}{n}\sum_{i=1}^{m}\left(f_i^{1-\varepsilon} - 1\right)}{\left(\frac{1}{m}\sum_{i=1}^{m} f_i\right)^{1-\varepsilon} - 1}. \tag{10}$$

Both indices cannot be computed when the mean intensity of use of all modes is equal to one. In this case, it is sufficient to change the measurement unit of f to make the index work (e.g. considering monthly rather than weekly frequencies, or travel time in hours rather than minutes). The minimum value is zero for both variants and refers to an equal use of all means, whereas when only one mode is used with intensity f we obtain as maximum values $\mathrm{DAL} = (1 - n^{-\varepsilon})/[1 - (f/n)^{\varepsilon-1}]$ and $\mathrm{DAL}_m = (n-1)/n$ respectively. Unlike the previous indices, the value obtained for the index DAL depends on f. In our example, both the indices give the same value for Traveller B (0.21), while Traveller A has DAL equal to 0.27 and a slightly higher DAL_m (0.29). Traveller B is now considered more multimodal, like for index HH_m and unlike the majority of indices.

4.6. Atkinson Index

A more commonplace inequality measure, yet quite similar to Dalton one (Cowell 2011), is the Atkinson index that, for n different modes of intensities f_i and mean $\overline{f}$ becomes:

$$\mathrm{ATK} = 1 - \left[\frac{1}{n}\sum_{i=1}^{n}\left(f_i/\overline{f}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{11}$$

Unlike the previous one, it can be seen that this is scale independent. When all the means are used with the same intensity the Atkinson index is again equal to zero, whereas the maximum value representing perfect monomodality is $\mathrm{ATK} = 1 - n^{-\varepsilon/(1-\varepsilon)}$. In the example provided, the index takes on the values of 0.29 for Traveller A and 0.23 for Traveller B.

5. Factorisation of the Indices

The numerical example that was presented along with the different indices in the previous section has shown that we have inconsistent indications on who is showing higher multimodality behaviours. A more systematic investigation of the behaviour of the above indices is therefore needed, aiming at checking to which extent they have the properties discussed in Section 3. This goal can be more easily achieved by rewriting the different indices as a function of a common set of parameters, that allow for a more immediate assessment of those numerical properties that are of interest here.

Therefore, we reformulate the different indices by considering that they all summarise an n-dimensional vector of intensities. We sort intensities inside the vector by increasing order and we have $m \leq n$ modes with intensity of use greater than zero. We further define:

- v as the lowest intensity inside the vector strictly greater than zero;
- c as the number of modes with intensity equal to v;
- B as the number of different levels of intensities greater than v;
- b_j as the number of modes showing a given level of intensity, given $j = 1, \ldots, B$.

The following equality holds:

$$m = c + \sum_{j=1}^{B} b_j. \tag{12}$$

Finally, we define a_j as the ratio between the j-th level of intensity and the lowest intensity. For example, if the vector of intensities for a given traveller is $[0\ 2\ 9\ 9\ 15]$, we have $n = 5$, $m = 4$, $v = 2$, $c = 1$, $B = 2$, $\mathbf{b} = [2, 1]$ and $\mathbf{a} = [9/2, 15/2]$. Through some algebraic manipulation, that are available from the Authors upon request, the above introduced indices can then be reformulated as follows:

$$\text{GI} = \frac{1}{n} \frac{\left[2c(n-m) + c^2 + c + (2(n-m+c)+1)\sum_{j=1}^{B} a_j b_j + \sum_{j=1}^{B} a_j b_j \left(b_j + 2\underbrace{\sum_{k=1}^{j-1} b_k}_{(A)} \right) \right]}{c + \sum_{j=1}^{B} (a_j\, b_j)} - \frac{n+1}{n}, \tag{13}$$

where the component (A) becomes 0 if $j = 1$.

$$\text{HH} = \frac{1}{n} \left\{ \frac{n \left[c + \sum_{j=1}^{B} a_j^2 b_j - \frac{1}{n} \left(c + \sum_{j=1}^{B} a_j b_j \right)^2 \right]}{\left(c + \sum_{j=1}^{B} a_j b_j \right)^2} + 1 \right\}. \tag{14}$$

$$\text{HH}_m = \frac{1}{m} \left\{ \frac{n \left[c + \sum_{j=1}^{B} a_j^2 b_j - \frac{1}{n} \left(c + \sum_{j=1}^{B} a_j b_j \right)^2 \right]}{\left(c + \sum_{j=1}^{B} a_j b_j \right)^2} + 1 \right\}. \tag{15}$$

$$\text{OM\_PI} = \frac{1}{\left(c + \sum_{j=1}^{B} a_j \, b_j \right) \ln n} \left[\left(c + \sum_{j=1}^{B} a_j \, b_j \right) \ln \left(c + \sum_{j=1}^{B} a_j \, b_j \right) - \sum_{j=1}^{B} a_j \, b_j \ln a_j \right]. \tag{16}$$

$$\text{OM\_MI} = \frac{v}{nM} \left\{ \left[1 + \ln \left(\frac{M}{v} \right) \right] \left(c + \sum_{j=1}^{B} a_j b_j \right) - \sum_{j=1}^{B} a_j b_j \ln a_j \right\}. \tag{17}$$

$$\text{TH} = \ln n + \frac{\sum_{j=1}^{B} a_j b_j \ln a_j - \left(c + \sum_{j=1}^{B} a_j b_j \right) \ln \left(c + \sum_{j=1}^{B} a_j b_j \right)}{c + \sum_{j=1}^{B} a_j b_j}. \tag{18}$$

$$\text{DAL} = 1 - \frac{\frac{1}{n} \left[\left(c + \sum_{j=1}^{B} a_j^{1-\varepsilon} b_j \right) v^{1-\varepsilon} - \left(c + \sum_{j=1}^{B} b_j + n - m \right) \right]}{\left[\frac{1}{n} \left(c + \sum_{j=1}^{B} a_j b_j \right) v \right]^{1-\varepsilon} - 1}. \tag{19}$$

$$\text{DAL}_m = 1 - \frac{\frac{1}{n} \left[\left(c + \sum_{j=1}^{B} a_j^{1-\varepsilon} b_j \right) v^{1-\varepsilon} - \left(c + \sum_{j=1}^{B} b_j \right) \right]}{\left[\frac{1}{m} \left(c + \sum_{j=1}^{B} a_j b_j \right) v \right]^{1-\varepsilon} - 1}. \tag{20}$$

$$\text{ATK} = 1 - \frac{n^{\frac{\varepsilon}{\varepsilon-1}} \left(c + \sum_{j=1}^{B} a_j^{1-\varepsilon} b_j \right)^{\frac{1}{1-\varepsilon}}}{c + \sum_{j=1}^{B} a_j b_j}. \tag{21}$$

If B=0, then all summations in the preceding formulas are equal to zero as well.

Through such factorisation it is easier to check if these indices have some of the five properties that were introduced in the first section. In particular, changing scale (property (3)) simply implies altering v while leaving unchanged the other factors. Hence, we conclude that all indices but OM_MI, DAL and DAL$_m$ are scale invariant, since only these three depend on v. In particular, OM_MI increases when v is increasing given its sensitivity to 'species richness', which is not a desirable feature when $m < n$: in this case, a scale change implies in fact an increasing disparity between modes that are used and modes that are not used. When some modes are not used, DAL and DAL$_m$ appear as the only indices that show a desirable behaviour concerning this issue.

Now, let us consider property (4). Replicating the vector of intensities to change the population size essentially means multiplying by the same term the parameters n, m, c and b while leaving a and B unchanged. It follows that HH, HH$_m$, OM_PI and OM_MI are not replication invariant and, therefore, are better candidates as multimodality indices when the choice set size is not constant across individuals, since we already noted in Section 3 that property (4) is in general not desirable. The choice set size could change, for example, when some means are not available to some subjects (e.g. car driving for those without a licence). In such cases, when using indices different from the above mentioned four, it is advisable to consider the same modal choice set for all individuals, therefore assigning an intensity equal to zero also to unavailable means, even if this would blur the difference between unavailable modes and available but not used modes.

The above preliminary assessment of the indices has already shown that there is not a clear winner among the competing formulations. However the two properties reviewed so far, namely scale and population size independence, are not equally important in all situations. For example, when the set of modes under consideration is the same for all travellers, population size is not relevant. To deepen our knowledge on the behaviour of the different candidate multimodality indices, we run three sensitivity analyses that allow to study also other properties. An additional preliminary sensitivity analysis on an empirical dataset will also provide insights on a side issue related to Dalton and Atkinson indices, namely how to set the value of their parameter ε in our framework.

6. Sensitivity Analyses

6.1. *Setting the parameter ε*

Welfare economics studies generally estimate the inequality aversion parameter ε, that appears in Dalton and Atkinson indices, on the basis of empirical observations. Employed methodologies include the consideration of income distributions at country level when assuming a 'natural rate of subjective inequality' Lambert, Millimet, and Slottje (2003), or surveys directly eliciting personal opinions on inequality Pirttilä and Uusitalo (2010). However, ε values are not representing any human judgement in our case: we therefore take a more pragmatic approach, where they are fixed in such a way that the resulting
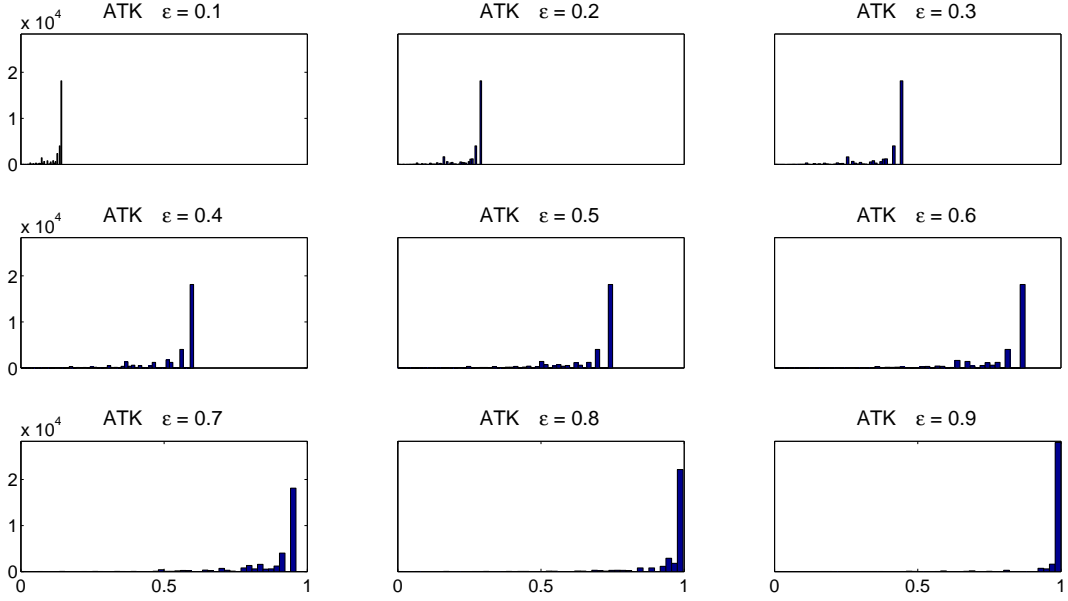
Figure 1. Distribution of values computed for the index ATK with different values of aversion parameter ε.

multimodality indices present some desirable properties.

To this effect, we consider the 'Aspects of daily life' survey, that was carried out in 2012 by the Italian National Statistical Institute (ISTAT), and we derive the annual frequencies of use of four travel means for a representative sample of 38,751 individuals of the Italian population aged 18 or more which use at least one means of transport. Values of Dalton and Atkinson indices for each individual in the population are then computed, with values of ε ranging from 0.1 to 0.9. The consideration of such range of values can be justified as follows. If a value equal to 1 is assigned, the exponent $1 - \varepsilon$, which appears both in Dalton and in Atkinson indices, would become 0. If $\varepsilon > 1$ the exponent would be negative, so the trend of the index would change. In fact, those means with intensity of use equal to zero would produce a component going to infinite and, so, the indices ATK and DAL could be computed only when all n modes are taken into account. Finally, the parameter is supposed to be not negative since it should represent an 'inequality aversion' rather than an 'inequality propensity', i.e. when differences in intensities are decreasing multimodality values should increase even if the overall travel levels are not increasing as well.

Figures 1 and 2 respectively show the nine distributions of ATK and DAL_m values when ε is varying within the above defined range (results for DAL are not reported to save on space). Intensities of use in the ISTAT dataset can take only some pre-defined values, related to the categories of the responses (namely: 'every day', 'some times a week', 'some times a month', 'some times a year' and 'never'), as it is customary in this kind of questions and surveys. The corresponding annual trip rates that we considered are the following: 365, 150, 20, 5 and 0. As a consequence, the values of the indices are not varying continuously among respondents, they are based on censored variables and normality tests for their distributions are failing. However, the distribution of ATK index shows a concentration towards the extremes 0 and 1 when the parameter ε is 0.1 and 0.9 respectively (Figure 1, respectively top left and bottom right chart), whereas values are more evenly distributed when ε is around 0.5, as shown by the three central charts of Figure 1. This value of the parameter is therefore maximising the sensitivity of the index and will be retained in the following simulations.
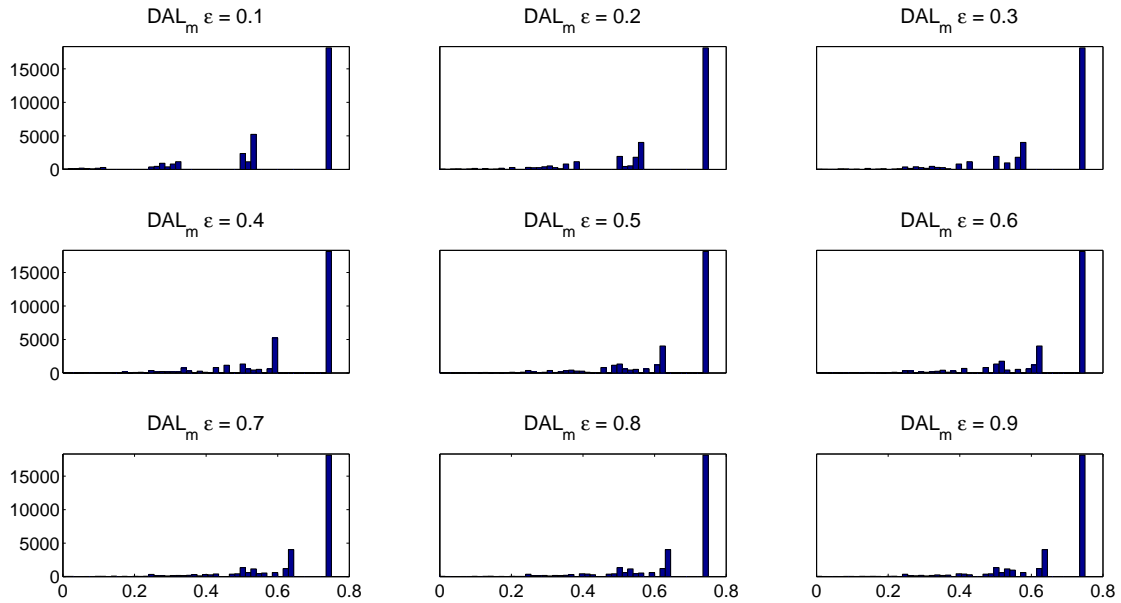
Figure 2. Distribution of values computed for the index DAL_m with different values of aversion parameter ε.

On the other hand, four clusters of values of the index DAL_m generate when $\varepsilon = 0.1$. The top left chart of Figure 2 shows them rather clearly, while such clusters tend to dissolve when ε increases. In particular, two clusters in the right side of the charts can still be distinguished for $\varepsilon < 0.4$ along with a third one. Each cluster is characterised by the number of means being used, that are in fact four in this numerical example: therefore, for low values of ε multimodality levels measured through DAL_m are essentially depending only on n and are rather insensitive to the actual intensities of use of the means. This is not desirable. To conclude, in the remainder of the paper we will set the value of this parameter ε to 0.5 also for the Dalton index.

6.2. *Bi-level Analysis*

Unlike the preceding subsection where real data have been used, it is now more convenient to proceed through simulation, since it is possible to apply the above defined factorisation of the indices to systematically study the values they take under different conditions. Let us start from the easiest case, in which the intensity of use of each means within the 'modal basket' can be either zero or a fixed value v greater than zero. Therefore, $B = 0$ and the sensitivity analysis of the above defined indices as a function of n and m is given in the following Figures 3 and 4 (where $v = 2$). Since our goal is to analyse the qualitative behaviour of the indices rather than their effective values, a min-max normalisation is done to fix the extremes to zero and one. Moreover, DAL was not plotted, since the considered values make the denominator of equation (9) equal to zero: a change of scale would be needed.

As anticipated, it can be seen that the Herfindahl index HH is not changing with n, given its above mentioned focus on 'species dominance'. In other words, modes within the choice set that are not used do not influence the value of these two indices for this particular instance. Additionally, we already mentioned that HH is scale invariant and not sensitive to 'richness' (i.e. the overall intensity of use of different means): both are not desirable features as above discussed. This index has therefore some unattractive properties and we consider in the following its variant HH_m, that does not present those
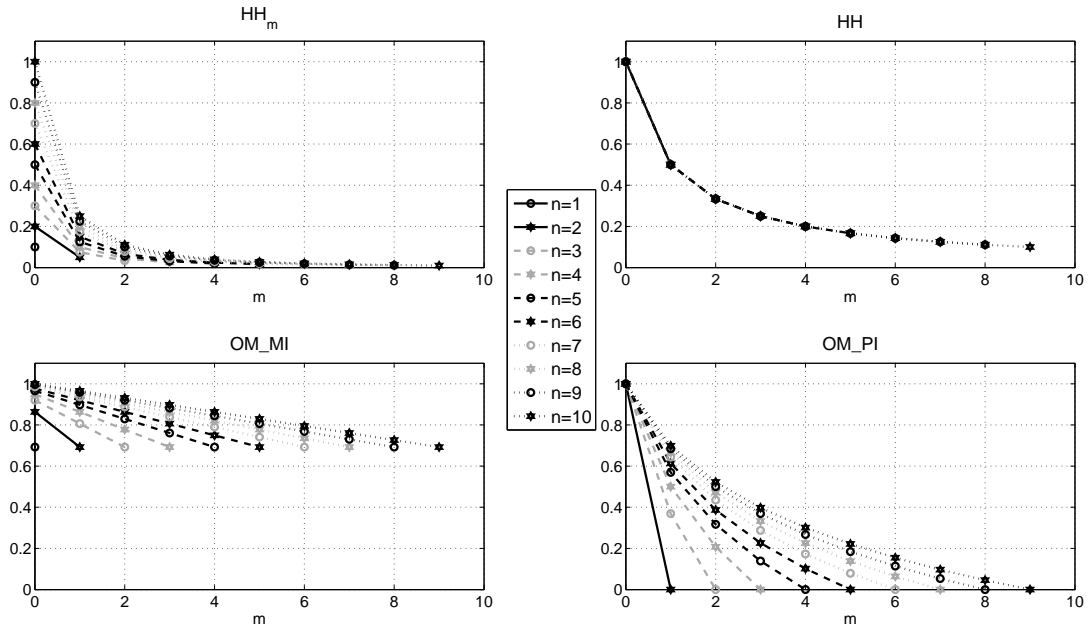
16

Figure 3. Variation of the indices HH_m, HH, OM_MI and OM_PI after min-max normalisation. All points of a given index with the same value of n are joined by lines.
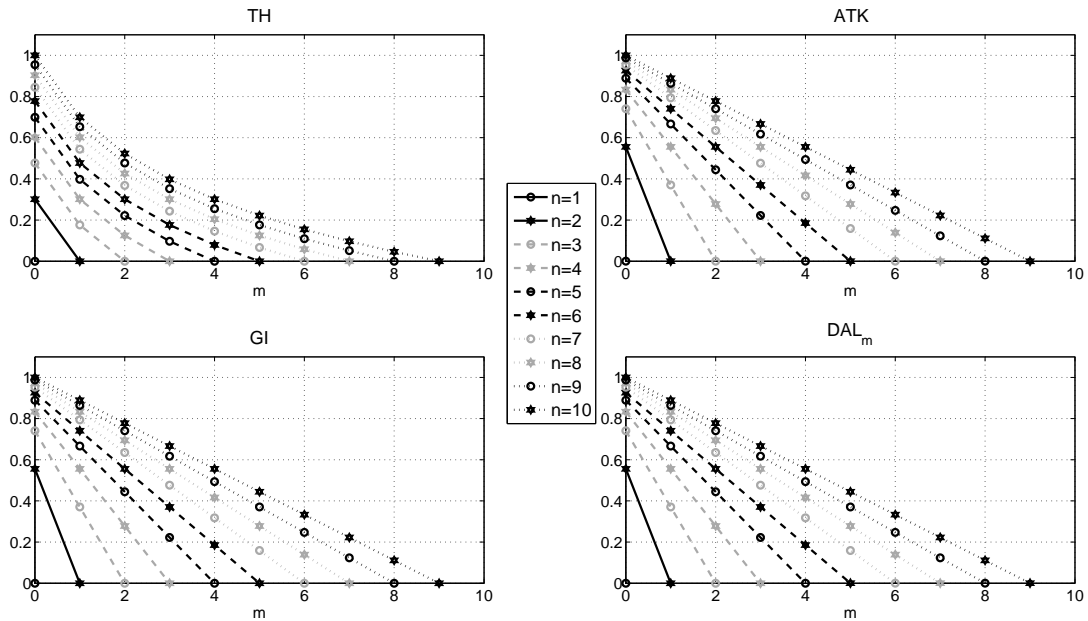

Figure 4. Variation of the indices TH, ATK, GI and DAL_m after min-max normalisation. All points of a given index with the same value of n are joined by lines.

problems (Figure 3).

The other considered indices qualitatively present desirable behaviours, showing greater multimodality values (indices decrease) when m is increasing with n constant and also when both n and m increase, and lesser multimodality (indices increase) when n is increasing with m constant. Let us note in passing that GI, ATK and DAL_m provide the same results and, for n constant, these three along with OM_MI follow a linear rela-
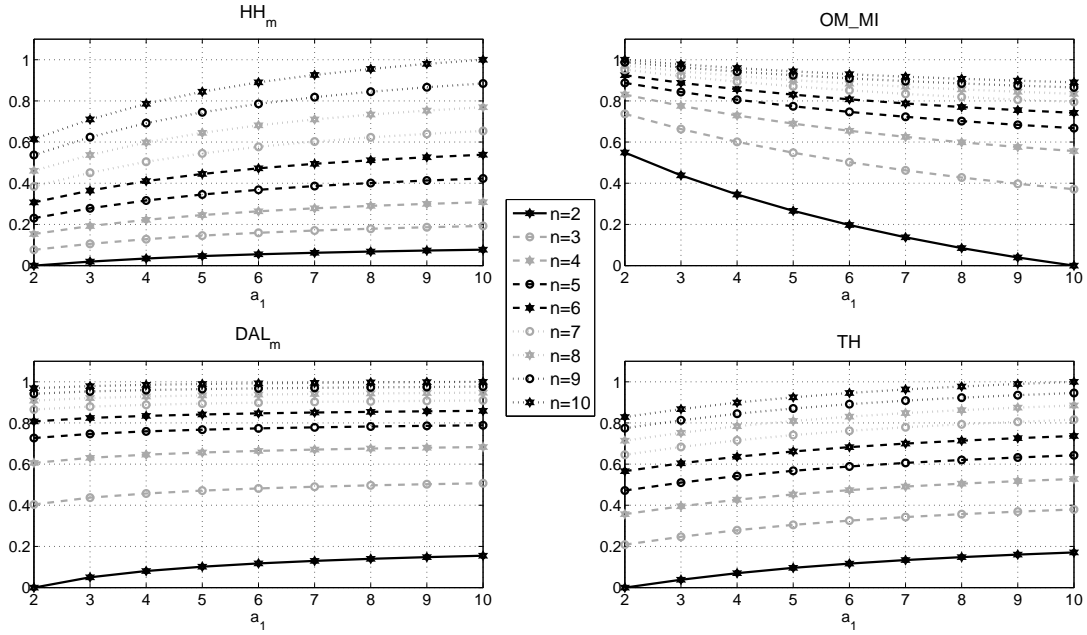
Figure 5. Variation of the indices HH_m, OM_MI, DAL_m and TH after min-max normalisation. All points of a given index with the same value of n are joined by lines.

tionship (Figure 3 and 4). Authors wishing to have a clearer insight of the relationships displayed in these two figures are referred to an Excel spreadsheet that is available as supplemental material linked to this article.

6.3. *Bimodal Analysis*

We now assume that only two means among the available ones have an intensity of use different from zero. It follows that $B = 1$, $m = 2$ and the values, normalised, of some of the indices as a function of n and a_1 are shown in Figure 5. The above mentioned spreadsheet also encompasses this latter figure.

In this case, the considered indices qualitatively behave in a similar way, monotonically increasing with both independent variables. In line with the preceding discussions, this is a desirable feature, since we expect that multimodality is increasing with both the number of means and the intensity of use of the less used ones. The two exceptions are OM_MI and DAL (the plots of this latter are not reported in Figure 5). The first, in fact, is too sensitive to the overall intensity of use of different means and not enough sensitive to the inter-modal variability, thus contrasting with Herfindahl under this point of view but being equally not optimal for the case under consideration. On the other hand, a functional study has shown that DAL is not monotonically increasing with a_1.

This analysis has therefore shown some problems of OM_MI, DAL and HH when using them to measure multimodality. Furthermore, OM_PI shows an increasing sensitivity to a_1 as n becomes larger, unlike the indices HH_m, DAL_m and TH represented in Figure 5. This is again not desirable, since when considering a larger set of travel means the influence on the multimodality index of the ratio between the second least and the least used means should decrease.

6.4. *Application of the Principle of Transfers*

Finally, let us now consider a set of cases where four modes are used: the first two have an average intensity of use that is half that of the remaining two. We shift a unit of intensity within each couple, thus obtaining for example the following sequence: [5 5 10 10] → [5 5 9 11] → [4 6 10 10]. According to the above discussion, we would expect that the index measures a greater multimodality in the second than in the third case.

Yet both GI and HH_m give the same values for these two cases, although for different reasons. If we order the modes by increasing intensity, the magnitude of change of the Gini index depends only on the relative difference in the ranking between the two modes that change, rather than on their actual intensities of use. On the other hand, Herfindahl follows the above introduced strong principle of transfers with $\beta = 1$. All the other parameters, instead, show the desired behaviour. These results were confirmed by running a sensitivity analysis where we define Δ_1 and Δ_2 as the difference of intensities of respectively the second minus the first and the fourth minus the third mode within the set of four and we systematically vary such differences from 0 to 10. ATK and DAL_m generate the same rankings of cases by increasing multimodality, whereas the other indices share a slightly different ranking. However, both rankings were rather meaningful and there is not a compelling reason to prefer one over the other.

6.5. *Summary of the findings*

The computational study of the candidate multimodality indices that was carried out in the Sections 5 and 6 has evidenced desirable and less desirable behaviours of each measure under a variety of viewpoints. We present a summary of the related findings in Table 1.

The first substantial result is that the use of some of the indices seems not recommendable in any of the considered cases to measure multimodality, since they are suboptimal in the multicriteria assessment exercise synthetised in the table. This is the case of Gini, Atkinson and Theil, beyond the 'original' formulations of both Herfindahl and Dalton indices. It is particularly interesting to note that Gini and Herfindahl are two of the most commonly used indices to study market concentration or social equity issues also in the transport sector, according to our above discussed review. However, they seem not be the best choice when multimodality has to be measured.

Apart from these five measures, Table 1 shows that there is not an index among the remaining four that is clearly outperforming all the others. However, some measures give the best results in specific cases: for example, HH_m, OM_PI and OM_MI are not replication invariant (desirable propriety in most cases), while DAL_m is scale invariant (again, a desirable propriety). It is therefore of interest to develop some guidelines on the use of these latter measures, by identifying their best ambits of use on the basis of the combinations of positive and negative properties that were evidenced. In the concluding section we elaborate some examples of possible applications, where each on these indices should be preferred over the others.

7. Conclusions and Guidelines for the Use of the Indices

The present paper has offered a comparative study of several potential measures of multimodality. The peculiarity of the problem of measuring multimodality, compared to other variability measures that are commonly employed in fields such as economics or biology, has been clarified by analysing which properties are desirable and which are not in our framework. Subsequent analyses, namely a factorisation of the considered indices

Table 1. Assessment of the behaviour of the indices in the considered case studies ("+" = desirable behaviour, "−" = undesirable behaviour).

	GI	HH	HH_m	OM_PI	OM_MI	TH	DAL	DAL_m	ATK
Replication invariance	−	+	+	+	+	−	−	−	−
Scale invariance	−	−	−	−	−	−	+	+	−
Bilevel analysis	+	−	+	+	+	+	+	+	+
Bimodal analysis	+	−	+	−	−	+	−	+	+
Principle of transfers	−	−	−	+	+	+	+	+	+

and sensitivity analyses through simulations, have then been designed to understand which properties are held by each index formulation.

The above research has shown that some indices, namely Gini, Atkinson, Theil, Herfindahl and Dalton, are not recommended to measure multimodality because they perform relatively worse than others. On the other hand, HH_m, OM_PI, OM_MI and DAL_m present different attractive combinations of desirable and not desirable characteristics. This points to the fact that each of these latter indices could be used in different problem instances: in the following we discuss some examples where considering any of these indices could be the best option.

The set of available travel means is often varying across subjects in the study group, for example when comparing the multimodality of those that have or do not have a driving licence. Incidentally, this problem is typically encountered also in the choice set definition of mode choice models. In such cases, the researcher could be interested in measuring the 'real' multimodality behaviour by simply considering the number of travel means that the individual is using from a pre-defined set, irrespective of the fact that some of these are not accessible to some individuals. As already mentioned, an index that is not replication invariant is more appropriate in this case: HH_m, OM_PI or OM_MI are therefore recommended. When the focus is in properly comparing multimodal behaviours of different social groups (e.g. teenagers versus adults, or rural versus urban dwellers), for example to build socioeconomic profiles of travellers as in Diana and Mokhtarian (2009), this is probably the best option. On the other hand, the potential drawback in such approach is that some individuals could appear less multimodal simply because they have access to a smaller number of travel modes. If the focus of the analysis is on measuring multimodality perceptions and attitudes rather than behaviours (such as in the case of studies on modality styles, e.g. Lavery, Páez, and Kanaroglou 2013; Vij, Carrel, and Walker 2013), it could therefore be preferable to take the complementary approach and consider a replication invariant index, only considering the travel means that are available to each individual.

Another common situation is when the mean intensities of use of the different modes are widely different across respondents, yet some modes in the set are never used. Most of the indices are scale invariant, which could lead to undesirable effects since the multimodality measure is not affected by the increasing gap between modes that are used and those that are not used. Suppose for example to include a measure of multimodality as an exogenous variable to improve travel behavioural models, as prospected by some of the studies reviewed in the introduction. The calibration sample might well contain both highly mobile and much less mobile individuals, while niche travel modes (e.g. taxi or waterborne) are used only by a small minority. In this case, the use of the DAL_m index in the model seems recommendable, whereas OM_MI would probably give the worst results according to our factorisation analysis.

Beyond these two examples that were already prospected in existing studies, we believe that the consideration of a multimodality measure can contribute in advancing the state of the art on a number of other issues in travel research. A multimodality index can be seen as a generalised measure of modal habit that is jointly considering the use of several

different modes rather than only one. Such measure could also help in identifying if there are modes that clearly dominate others in their use. From a modelling perspective, the study of the stability of travel behaviours, including mode choices, is a central aspect in activity-based models that are striving to reduce the number of activity-travel combinations to consider. A potential symplification in the model structure by merging seemingly unrelated tour typologies that show similar degrees of multimodality could therefore be explored. On a policy viewpoint, it would be interesting to study the relationship between the multimodality degree of individual behaviours and environmental impacts, in order to help decision makers to understand if the goal of promoting multimodality, that is recalled in many strategic documents, is for example effective in lowering emissions. Assessing a policy that is typically compounding several different individual measures to boost modal changes, such as park and ride, integrated ticketing or shared vehicles schemes, is in fact not an easy task if the analysis tries to separately quantify the effects of each measure.

The present study was intended to contribute to a better understanding of the best method to measure multimodality behaviour, that has proven to be an intricate issue basically because existing variability measures have properties that are not completely desirable in this framework. Future research endeavours will be aimed at clarifying how different measures of travel intensity (number of trips, travel time or travel distance) affect multimodality values, and assessing through empirical analysis the added value of including these indices in behavioural travel demand models according to the above recommendations.

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