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General Decoupled Method for Statistical Interconnect Simulation via Polynomial Chaos

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Abstract—This paper proposes a technique that allows to decouple the polynomial chaos equations for statistical interconnect analysis. The methodology is based on a transformation that renders the voltage and current polynomial chaos coefficients decoupled. Hence, these new decoupled coefficients are computed via repeated non-intrusive simulations. The advocated method maintains comparable accuracy with respect to the state-of-the-art approaches, nevertheless considerably easing the simulation procedure. Comparisons against literature results are provided to validate the proposed methodology.

Index Terms—Circuit modeling, circuit simulation, polynomial chaos, SPICE, statistical analysis, tolerance analysis, transmission lines, uncertainty.

I. INTRODUCTION

Modern interconnect designs often require statistical assessments to account for the inherent manufacturing variability. Circuit simulators (e.g., SPICE) usually provide features for statistical analysis based on the Monte Carlo (MC) method [1]. However, the computational cost is large or even prohibitive as the number of simulations required is typically on the order of (several) thousands.

To overcome this issue, alternative approaches have been recently investigated in this domain [2]–[9]. They are based on the polynomial chaos (PC) framework [10], i.e. on the representation of stochastic voltages and currents in terms of expansion of orthogonal polynomials. The expansion coefficients directly provide relevant statistical information on the interconnect behavior.

The PC-based methodologies can be divided into two classes depending on the strategy for the calculation of the coefficients: 1) pseudo-spectral [2] or collocation [3] methods use high-dimensional integration or interpolation techniques, respectively. They are non-intrusive and require to sample the stochastic responses at given points. As such, they can be considered as clever sampling-based (i.e., MC-like) approaches.

Nevertheless, compared to standard MC, the effectiveness rapidly decreases when a relatively large number of random variables (RVs) is considered, even when sparse grids are used [4]. Alternatively, 2) stochastic Galerkin method (SGM)-based techniques [5]–[9] require the single simulation of an augmented and coupled system of equations, which can be possibly given a circuit interpretation [7]. Compared to the sampling-based strategies, the overall problem dimension increases less rapidly with the number of RVs, but it does not have any advantageous sparsity pattern and yet requires the generation of new equations or of the corresponding equivalent circuit models, which might limit the applicability.

To mitigate the aforementioned problems, decoupling techniques have been recently proposed [11], [12]. Nonetheless, they rely on matrix approximations and only apply to Hermite-chaos (i.e., Gaussian variability). In this paper, an alternative, simple but yet effective decoupling technique is proposed. It is no longer based on a SGM, but rather on the point matching of PC equations via stochastic testing (ST) [13]. The advocated methodology preserves the reduced problem size characterizing the SGM, but with the considerable advantage that the equations are decoupled and the simulations can be performed iteratively in a non-intrusive manner. Validations against the state-of-the-art SGM-based approach [7] are provided.

II. PROPOSED DECOUPLING TECHNIQUE

This section summarizes the key features of the PC-based interconnect simulation and outlines the proposed decoupling technique.

A. The Polynomial Chaos Expansion

For illustration purposes, the discussion is based on the equations governing the behavior of a single lossless transmission line affected by one Gaussian random parameter ξ, i.e.

$$\frac{\partial}{\partial z}v(z,t,\xi) = -L(\xi)\frac{\partial}{\partial t}i(z,t,\xi) \tag{1a}$$

$$\frac{\partial}{\partial z}i(z,t,\xi) = -C(\xi)\frac{\partial}{\partial t}v(z,t,\xi), \tag{1b}$$

where z is the longitudinal coordinate and L and C are the per-unit-length (p.u.l.) inductance and capacitance of the line, respectively. The p.u.l. parameters, the voltage v and the current i inherently depend on the random parameter ξ, thus becoming stochastic themselves. The RV ξ is normalized so that it is has zero mean and unit variance.

The rationale of PC is to approximate stochastic responses (i.e., voltages and currents in this case) as expansions of orthogonal polynomials [10]. For example, assuming a second-order expansion and considering the first transmission-line

equation (1a) produces

$$\frac{\partial}{\partial z}v_0\varphi_0(\xi) + \frac{\partial}{\partial z}v_1\varphi_1(\xi) + \frac{\partial}{\partial z}v_2\varphi_2(\xi)$$
$$\approx -L(\xi)\left[\frac{\partial}{\partial t}i_0\varphi_0(\xi) + \frac{\partial}{\partial t}i_1\varphi_1(\xi) + \frac{\partial}{\partial t}i_2\varphi_2(\xi)\right], \tag{2}$$

where the dependence on z and t has been omitted for notational convenience. The basis functions φ_0, φ_1 and φ_2 are the first three normalized Hermite polynomials, i.e.

$$\varphi_0(\xi) = 1, \quad \varphi_1(\xi) = \xi, \quad \varphi_2(\xi) = \frac{1}{\sqrt{2}}(\xi^2 - 1).$$

The above polynomials are orthonormal with respect to the inner product

$$\langle f, g \rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(\xi)g(\xi)e^{-\xi^2/2}d\xi, \tag{3}$$

i.e. $\langle \varphi_i, \varphi_j \rangle = \delta_{ij}$ (Kronecker's delta). The PC-expansion coefficients (unknown and to be determined) directly provide statistical information. For example, the average voltage response is $\approx v_0(t)$, whereas the variance is $\approx v_1^2(t) + v_2^2(t)$, and similarly for the current.

To solve for the unknown PC coefficients, a deterministic set of equations relating such coefficients is constructed. Traditionally, by means of a SGM, a coupled augmented set of equations is obtained [5]–[9]. These equations can be given the interpretation of an equivalent augmented (i.e., multiconductor) transmission line. A SPICE-compatible implementation [7] allows to compute the PC expansion coefficients via a single simulation of an equivalent augmented network by means of standard circuit-analysis software. Here, a different approach is presented, which yields decoupled equations.

B. Decoupled Equations for the Expansion Coefficients

Assuming that a set of three distinct points $\{\xi_0, \xi_1, \xi_2\}$ in the random space is available, and forcing (2) to hold strictly for each of these points, leads to

$$\frac{\partial}{\partial z}a_{00}v_0 + \frac{\partial}{\partial z}a_{01}v_1 + \frac{\partial}{\partial z}a_{02}v_2$$
$$= -L(\xi_0)\left[\frac{\partial}{\partial t}a_{00}i_0 + \frac{\partial}{\partial t}a_{01}i_1 + \frac{\partial}{\partial t}a_{02}i_2\right]$$
$$\frac{\partial}{\partial z}a_{10}v_0 + \frac{\partial}{\partial z}a_{11}v_1 + \frac{\partial}{\partial z}a_{12}v_2$$
$$= -L(\xi_1)\left[\frac{\partial}{\partial t}a_{10}i_0 + \frac{\partial}{\partial t}a_{11}i_1 + \frac{\partial}{\partial t}a_{12}i_2\right] \tag{4}$$
$$\frac{\partial}{\partial z}a_{20}v_0 + \frac{\partial}{\partial z}a_{21}v_1 + \frac{\partial}{\partial z}a_{22}v_2$$
$$= -L(\xi_2)\left[\frac{\partial}{\partial t}a_{20}i_0 + \frac{\partial}{\partial t}a_{21}i_1 + \frac{\partial}{\partial t}a_{22}i_2\right],$$

where $a_{mk} = \varphi_k(\xi_m)$ $(m, k = 0, 1, 2)$. The above system of equations can be written in matrix form as

$$\left[\mathbf{A}\right]\begin{bmatrix}v_0\\v_1\\v_2\end{bmatrix} = -\begin{bmatrix}L(\xi_0)&&\\&L(\xi_1)&\\&&L(\xi_2)\end{bmatrix}\left[\mathbf{A}\right]\frac{\partial}{\partial t}\begin{bmatrix}i_0\\i_1\\i_2\end{bmatrix}, \tag{5}$$

where $\mathbf{A}$ is a matrix with the previously-defined entries a_{mk}.

The above equation is decoupled with respect to the "modified" voltage and current variables defined as

$$\begin{bmatrix}u_0(z,t)\\u_1(z,t)\\u_2(z,t)\end{bmatrix} = \left[\mathbf{A}\right]\begin{bmatrix}v_0(z,t)\\v_1(z,t)\\v_2(z,t)\end{bmatrix}, \tag{6a}$$

and

$$\begin{bmatrix}j_0(z,t)\\j_1(z,t)\\j_2(z,t)\end{bmatrix} = \left[\mathbf{A}\right]\begin{bmatrix}i_0(z,t)\\i_1(z,t)\\i_2(z,t)\end{bmatrix} \tag{6b}$$

respectively. Therefore, $\mathbf{A}$ can be interpreted as a matrix that transforms the PC coefficients $v_{0,1,2}$ and $i_{0,1,2}$ into the corresponding uncoupled quantities $u_{0,1,2}$ and $j_{0,1,2}$, respectively. Replacing (6) into (5) and into the analogous development of (1b), yields the following relation for the uncoupled PC coefficients of the voltage and current along the line:

$$\frac{\partial}{\partial z}u_m(z,t) = -L(\xi_m)\frac{\partial}{\partial t}j_m(z,t) \tag{7a}$$

$$\frac{\partial}{\partial z}j_m(z,t) = -C(\xi_m)\frac{\partial}{\partial t}u_m(z,t), \tag{7b}$$

$m = 0, 1, 2$. This implies that the uncoupled coefficients u_m and j_m are readily computed by solving the transmission-line equations for the pertinent samples $L(\xi_m)$ and $C(\xi_m)$ of the p.u.l. parameters.

The above procedure is readily extended to lossy multiconductor transmission lines and the transformation (6) is applicable to all the voltages and currents within a given network. Therefore, to compute the uncoupled PC coefficients, it suffices to sample network responses at the pertinent match points. Once all these coefficients are available, the classical PC coefficients v_m and i_m are retrieved via the inversion of (6).

C. Choice of the Match Points

A suitable and convenient choice for the match points ξ_m is represented by the nodes of a Gauss-Hermite quadrature rule, which in turn correspond to the roots of the Hermite polynomials. For the second-order expansion considered, these are the roots of the third-order Hermite polynomial $\xi^3 - 3\xi$, i.e. $\xi_0 = 0$, $\xi_1 = -\sqrt{3}$ and $\xi_2 = +\sqrt{3}$. The corresponding matrix $\mathbf{A}$ writes

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -\frac{1}{\sqrt{2}} \\ 1 & -\sqrt{3} & \sqrt{2} \\ 1 & \sqrt{3} & \sqrt{2} \end{bmatrix}$$

For problems with multiple random variables, the match points are chosen as a subset of the nodes of the pertinent multidimensional quadrature rule [13], so that the number of points equals the number of unknown PC expansion coefficients.

It is important to stress that the methodology is general and applies to any distribution type. It suffices to use the proper Gaussian quadrature rule (e.g., Gauss-Legendre or Gauss-Jacobi for uniform and beta distributions, respectively) for the generation of the match points and the related transformation matrix. Moreover, it should be noted that these do not depend on the specific problem, thanks to the normalization of the RVs, but only on the number of RVs and their distribution type. Therefore, match points and transformation matrices for a wide range of problems are pre-computed and made available offline.

III. Validation and Numerical Results

This section validates the advocated decoupling procedure by means of comparisons with literature results available in [7]. Comparisons against MC analysis are available therein and therefore not shown here. All the simulations are carried out in HSPICE [14] on an ASUS U30S laptop with an Intel(R) Core(TM) i3-2330M, CPU running at 2.20 GHz and 4 GB of RAM.

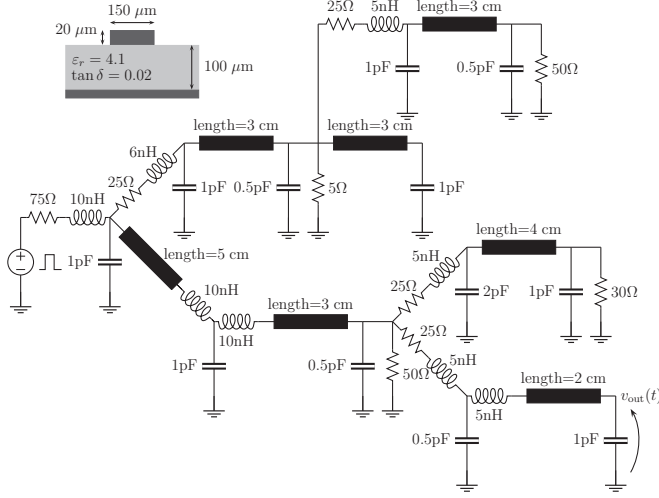### A. Single Transmission-Line Network

Fig. 1. Transmission-line network considered for the first application example.

The first application considers the transmission-line network in Fig. 1, where the variability is provided by three microstrip substrate parameters: thickness, permittivity and loss tangent, each exhibiting an independent Gaussian variation with a relative standard deviation of 10%. The voltage source is a trapezoidal pulse with an amplitude of 1 V, rise/fall times of 200 ps, and a width of 2.6 ns.

Fig. 2 shows in the top panel the average of the voltage v_{out} transmitted to one of the far-end terminations (see Fig. 1). The solid line is the result computed in [7] via the simulation of the equivalent augmented network, whereas the markers have been obtained by means of the proposed decoupled technique. Since $K = 10$ PC expansion coefficients are used for each voltage and current within the circuit, the former case requires the simulation of a network that is 10 times larger, whereas the latter case requires 10 simulations of the original network for the calculation of the uncoupled coefficients. The bottom panel provides a comparison on the estimation of the standard deviation instead. Excellent agreement between the state-of-the-art SGM-based technique and the novel approach is established.

B. Coupled Transmission-Line Network

The second example deals with the coupled transmission-line structure in Fig. 3, where the variability is in the geometry
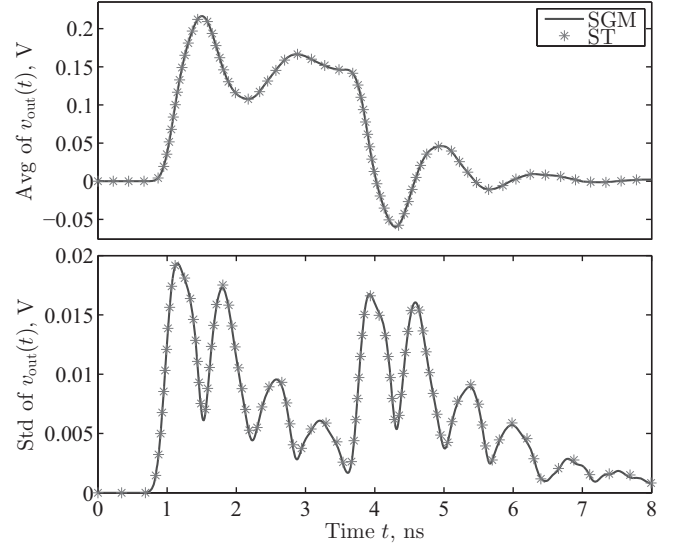

Fig. 2. Average (top panel) and standard deviation (bottom panel) of the voltage v_{out} transmitted to the far-end termination of the network of Fig. 1. Solid lines: results from the SGM-based simulation; markers: results obtained with the proposed decoupled technique.
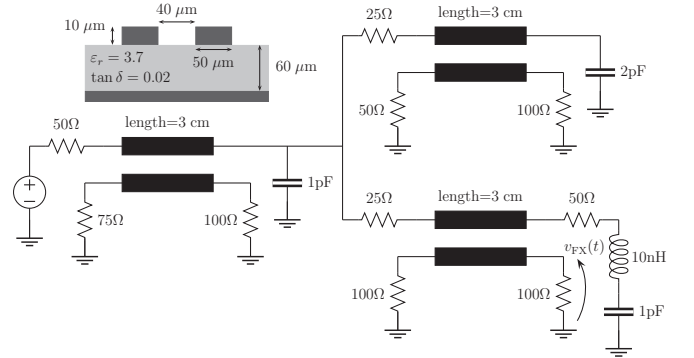

Fig. 3. Coupled transmission-line network for the second application example.

of the microstrip lines: the trace width, thickness and separation, together with the substrate thickness, are considered as four independent Gaussian RVs with a 10% relative standard deviation. The voltage source produces a Gaussian pulse with a peak of 1 V and a width of 0.177 ns at half amplitude.

As in the previous example, Fig. 4 compares the average (top panel) and standard deviation (bottom panel) computed with both the coupled state-of-the-art implementation (solid lines) and the decoupled methodology (markers). Here, $K = 15$ PC expansion terms are considered. Very good accuracy between the two methods is again revealed.

C. Performance Assessment

Tab. I collects the main figures concerning the performance of the PC-based simulations for the considered application examples. It is important to recall that, denoting as K the number of PC expansion terms, the state-of-the-art SGM
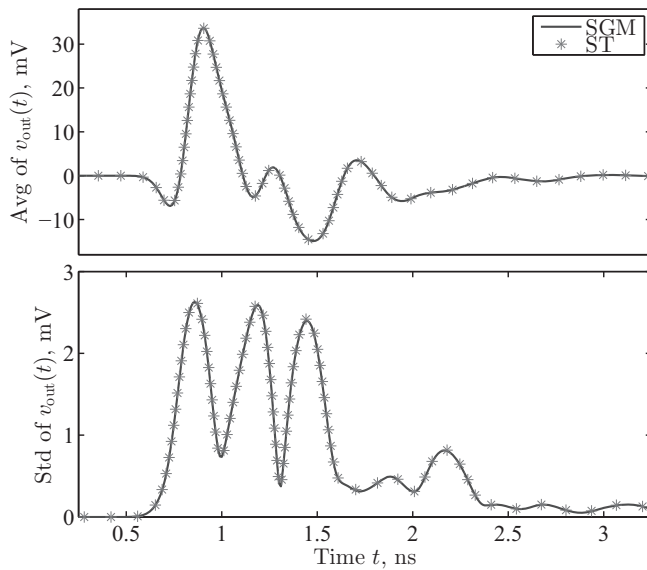
Fig. 4. Average (top panel) and standard deviation (bottom panel) of the far-end crosstalk voltage v_{FX} in the network of Fig. 3. Curve identification is as in the inset of Fig 2.

implementation requires a single simulation of a network which is K times larger. The ST-based approach requires K separate simulations of the original network at the match points of the RVs. In Tab. I, the time t_{sim} taken by a single simulation run of the original network is also provided. The ST figure includes the time to iteratively call the circuit simulator and to apply the inverse transformation. This explains why the overall simulation time is slightly larger than $K \cdot t_{\text{sim}}$.

In addition, it is worth noting how the simulation time of the SGM-augmented network is lower, despite the network being larger. This is due to the efficient handling of the coupled equations within SPICE and renders the SGM-based approach more efficient from a pure computational point of view. Nevertheless, for a fair comparison, it is relevant to point out that the ST-based approach is much simpler to implement and does not require the generation of the equivalent circuit models and of the overall augmented network.

TABLE I
COMPARISON BETWEEN ST- AND SGM-BASED PC SIMULATIONS.

test case	K	t_{sim} (single run)	ST-based	SGM-based
Fig. 1	10	1.7 s	20.9 s	7.5 s
Fig. 3	15	1.0 s	21.0 s	15.8 s

IV. CONCLUSIONS

This paper presents a simple yet effective transformation that decouples the PC coefficients of voltages and currents in the circuit-level simulation of high-speed interconnects

with random properties. The approach is based on the point matching of the governing equations of stochastic transmission lines. A transformation matrix for the voltage and current PC coefficients is constructed by evaluating the polynomial basis function at the match points. This considerably eases the simulation procedure as it now merely amounts to performing repeated simulations of the original network at the match points of the RVs. Compared to other sampling-based approaches, the advocated technique limits the amount of simulations to the number of PC-expansion terms. However, it is also shown that the state-of-the-art implementation based on the SGM is still more efficient from a pure computational viewpoint.

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