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# Use of the Karhunen–Loève Transform for interference detection and mitigation in GNSS<sup>☆</sup>

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## Abstract

Improving the Global Navigation Satellite System (GNSS) receiver robustness in a radio interfered environment has been always one of the main concerns for the GNSS community. Due to the weakness of the signal impinging the GNSS receiver antenna, GNSS receiver performance can be seriously threatened by the presence of stronger interfering signals. In these scenarios, classical interference countermeasures may fail due to the fact that interference detection and removal process causes also a non-negligible degradation of the received GNSS signal. This paper introduces an innovative interference detection and mitigation technique against the well-known jamming threat. This technique is based on the use of the Karhunen–Loève Transform (KLT) which allows for the representation of the received interfered signals in a transformed domain where interference components can be better identified, isolated and removed, avoiding significant degradation of the useful GNSS signal.
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*Keywords:* Interference; Jamming; Karhunen–Loève Transform; Chirp signal; Adaptive notch filter

## 1. Introduction

The effect of an interfering signal on the GNSS receiver performance can vary from the increase of the noise on the pseudo-ranges measurements, leading to large errors in the positioning domain, up to the complete disruption of the GNSS receiver operation thus causing the complete denial of the positioning service. Intentional interference generated by the jammers, known also as Personal Privacy Devices (PPDs), to the GNSS based services has become recently the main concern for the GNSS community. Such jammers can be easily purchased on-line even for few dollars despite their use being illegal in the United States and in several European Countries [1]. These devices are capable of transmitting strong Radio Frequency (RF) power overlapping a large part of the targeted GNSS frequency band thus preventing the receivers from operating correctly within

an area and causing hazardous outages of the GNSS based systems. Many documented incidents caused by PPDs have already occurred as for example, the infamous case at Newark Airport where one of the Local Area Augmentation System (LAAS) ground facility receiver was occasionally jammed by a Personal Privacy Device (PPD) installed in a vehicle passing along a nearby motorway [2].

Very detailed classification of existent civil GNSS jammers can be found in [3,4] and in [5]. The RF signal transmitted by most of the available in car jammers are chirp signals with unidirectional or bidirectional, linear and positive sweep functions.

Nowadays, professional GNSS receivers are equipped with interference detection and mitigation algorithms capable of dealing with a wide range of interfering signals. The adaptive notch filtering is the most known jamming mitigation algorithm [6]. This low-complexity technique is based on the use of a notch filter, characterized by a pass-band frequency response which rejects a very narrow portion of spectrum in correspondence of the interference frequency components, and an adaptive block tracking the instantaneous jamming frequency [7]. However, such a traditional countermeasure performs interference detection and excision in the frequency domain only, leading to a not negligible distortion on the useful received GNSS

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signals. This paper introduces an innovative interference detection and mitigation algorithm based on the use of an advanced signal processing technique: the Karhunen–Loève Transform (KLT). The KLT makes a projection of the digitized signal on the eigenfunctions domain where interference components can be better identified and isolated from the rest of the received signal.

The paper is organized as follows: after a brief description of the jamming signal characteristics and its signal model in Section 2, the KLT based detection and mitigation algorithm will be addressed in Section 3. A set of experimental test will be described in Section 4 showing the benefits in improving GNSS receiver robustness against jamming interference.

## 2. Jamming in GNSS: signal model

The composite digitized Intermediate Frequency (IF) signal at the input of the baseband processing block of a GNSS receiver under jamming interference can be modeled as

$$s[n] = \sum_{m=0}^{M-1} y_m[nT_s] + i[nT_s] + \eta[nT_s] \qquad (1)$$

where $y_m[nT_s]$ identify the useful GNSS signal coming from the $m$th Line-of-Sight (LoS) satellite, $i[nT_s]$ is the digitized jamming signal component and $\eta[nT_s]$ is the Additive White Gaussian Noise (AWGN) term. Neglecting the satellite index subscript for sake of simplicity of the notation, each useful digital GNSS signal at IF can be expressed as

$$y[n] = \sqrt{2C} \cdot d[nT_s - n_0] \cdot c[nT_s - n_0]$$
$$\cdot \cos(2\pi(f_{L1} + f_d)nT_s + \theta_0) \qquad (2)$$

where $C$ is the power at the antenna port, $d[nT_s]$ is the navigation data component, $c[nT_s]$ is the pseudo random sequence for spreading the signal spectrum, while $n_0$, $f_d$ and $\theta_0$ are the received code delay, the Doppler frequency and the phase introduced by the channel respectively. As mentioned in the Introduction, the RF signal generated by the majority of the available in-car PPDs is a chirp signal, which can be expressed, according to the model in [4], as

$$i(t) = a \cdot \sin\left[2\pi\left(f_0 + \frac{k}{2}t\right)t\right] \quad \forall t : 0 \leq t \leq T_{sw} \qquad (3)$$

where $f_0$ is the starting frequency, $k$ is the sweeping frequency rate, $T_{sw}$ is the sweeping frequency period and $a$ is the constant chirp signal amplitude. Fig. 1 shows the spectrogram of a chirp signal typically transmitted by an in-car jammer, characterized by a linear frequency sweep of 14 MHz and by a sweep period of 9 μs.

## 3. Advanced signal processing algorithms: the transformed domain techniques

The KLT based mitigation algorithm belongs to the family of the transformed domain techniques, which are based on the use of advanced signal processing techniques on the digitized GNSS signal. Such techniques offer the possibility to
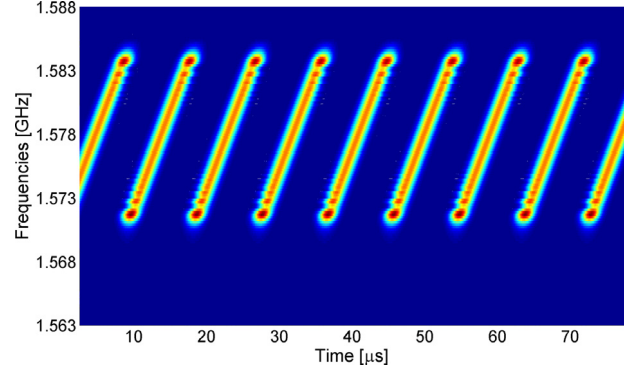


Fig. 1. Spectrogram of the RF jamming signal transmitted by a typical in-car cigarette lighter powered jammer.

perform interference detection and excision in a domain where interference components can be better identified and removed without causing large distortion of the received useful GNSS signal.

Several examples of transformed domain techniques for interference detection and mitigation can be found in literature, such as those based on the use of the Short Time Fourier Transform (STFT) [8], or those exploiting the properties of the Wavelet Packet Decomposition (WPD) as in [9] and [10].

### 3.1. The Karhunen–Loève transform

The KLT provides a decomposition of the digitized signal in a vectorial space using orthonormal functions which can have in principle any shape. The KLT decomposition of a general time dependent function is given by

$$x(t) = \sum_{j=1}^{\infty} Z_j \Phi_j(t) \qquad (4)$$

where $Z_j$ are scalar random variables that are statistically independent and $\Phi_j(t)$ are the basis functions, derived from the covariance matrix of a digitized version of the stochastic process $x(t)$. The KLT offers the better separation between the deterministic components within the received signal and the stochastic ones. The random variables $Z_j$ are obtained projecting the given stochastic process $x(t)$ over the corresponding eigenvector $\Phi_j(t)$, as

$$Z_j = \int_{-\infty}^{+\infty} x(t) \Phi_j(t) dt. \qquad (5)$$

In [11] it is stated that the KLT is the only possible statistical expansion in which all the expansion terms are uncorrelated from each other.

### 3.2. KLT for interference detection and mitigation

First use of KLT for Continuous Wave Interference (CWI) detection is described in [12], while application of the KLT decomposition against pulsed interference has been proposed first in [13]. The KLT decomposition has been implemented according to the following steps:
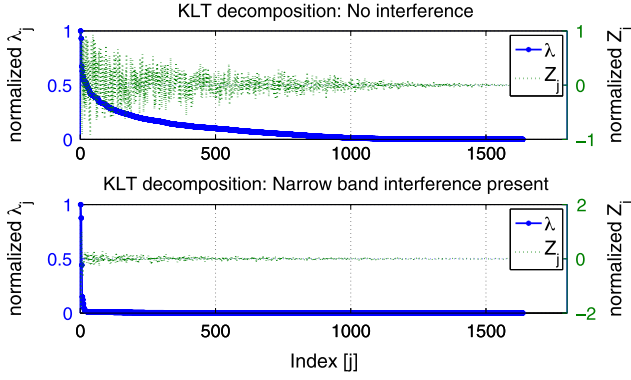
Fig. 2. Karhunen–Loève decomposition of the GNSS received signal in absence and in presence of narrow-band interference.

- computation of the Toeplitz matrix from $N$ samples of the autocorrelation $R_{ss}[n]$ of the received signal $s[n]$ in (1);

$$\mathbf{R_{toe}} = \begin{bmatrix} R_{ss}(0) & R_{ss}(1) & \dots & R_{ss}(N) \\ R_{ss}(1) & R_{ss}(0) & \dots & R_{ss}(N-1) \\ R_{ss}(2) & R_{ss}(1) & \dots & R_{ss}(N-2) \\ \vdots & \vdots & \vdots & \vdots \\ R_{ss}(N) & \dots & R_{ss}(1) & R_{ss}(0) \end{bmatrix} \quad (6)$$

- determination of the eigenvalues $\lambda_j$ of the Toeplitz matrix $\mathbf{R_{toe}}$ and of the related eigenfunctions $\Phi_j[\mathbf{n}]$ satisfying

$$\mathbf{R_{toe}} \, \Phi_\mathbf{j}[\mathbf{n}] = \lambda_\mathbf{j} \, \Phi_\mathbf{j}[\mathbf{n}] \quad (7)$$

- determination of the $Z_j$ coefficients according to (5).

Fig. 2 shows the capability of the KLT in separating the deterministic and stochastic components within the decomposed signal.

In this case, the KLT decomposition is achieved by solving the eigenvalues problem of the covariance matrix obtained from 100 µs of simulated Global Positioning System (GPS) C/A code signal in two cases:

- interference-free environment;
- interfered with a Narrow-band Interference (NBI) signal (10 kHz) centered on the intermediate frequency with a power equal to $-120$ dBW;

Fig. 2 reports the trend of the normalized eigenvalues and the coefficients $Z_j$ obtained from the KLT decomposition. It is possible to notice that, the distribution of the eigenvalues suggests a method for detecting interference. In fact, when the interference is present there is a small number of eigenvalues which have a great magnitude with respect to the others (bottom plot), differently from the case of interference-free environment (top plot). A detection method based on the eigenvalues magnitude observation is proposed in [14]. Basically, the highest magnitude eigenvalues, which represent the interference components, are detected and an inverse KLT is applied considering only the eigenfunctions representative of the noise in which the GNSS component is embedded.
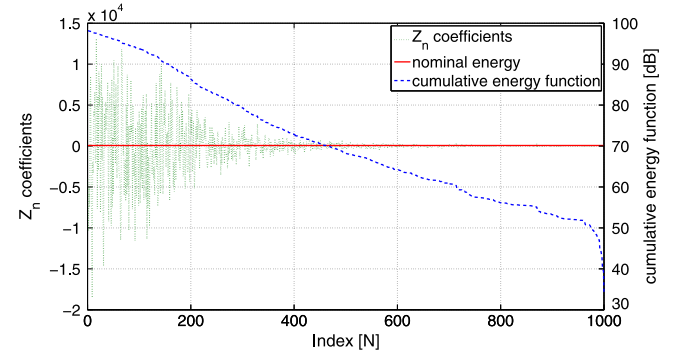


Fig. 3. KLT decomposition of the jammed interfered data in the GPS L1/Galileo E1 frequency band.

## 4. Experimental results

In this section, a set of experimental results compares the benefits of the KLT based mitigation algorithm with respect to the traditional adaptive notch filter. Several data collection of GNSS signals combined with realistic jamming signal at RF have been fed to the software implemented mitigation blocks in order to perform the desired signal conditioning. A fully software GNSS receiver is adopted in order to statistically assess the acquisition performance by means of the Complementary Cumulative Distribution Function (CCDF), computed with respect to the number of acquired Pseudo Random Noises (PRNs). For this purpose, a large number of signal acquisition tests have been performed on different and uncorrelated portions of the signal at the output of the software implemented KLT based block.

### 4.1. Jamming interference

Realistic GNSS data in the GPS L1/Galileo E1 frequency bands interfered with the typical in-car jamming signal reported in Fig. 1 have been considered. This dataset has been down-converted to an IF of 28.42 MHz and collected at a rate of 112 MHz exploiting a discrete component front-end with 30 MHz IF filter bandwidth. Jamming power is set constant thus simulating a GNSS receiver within the PPD effective range defined in [10]. The KLT decomposition of such interfered dataset is reported in Fig. 3 where the $Z_j$ coefficients, the cumulative energy function and the nominal energy threshold are reported.

The cumulative energy function is the energy of the reconstructed signal excluding the first $L$ highest magnitude $Z_j$ coefficients. From the distribution of the $Z_j$ coefficients (green dashed line) it is possible to observe that the strong jamming signal is spread over several eigenfunctions, and according to the energy based criterion, attenuation of the jamming interference can be achieved discarding the first 470 highest magnitude $Z_j$ over 1000 computed coefficients. According to this choice, the energy of the reconstructed signals equates the nominal energy threshold (intersection point shown in Fig. 3). A comparison between the Power Spectral Density (PSD) of the received signal before (blue curve) and after (red curve) the
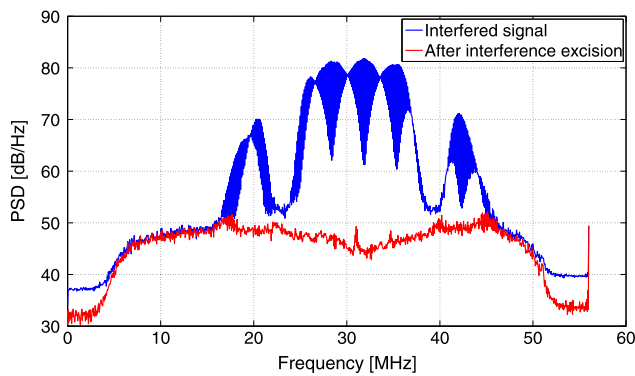
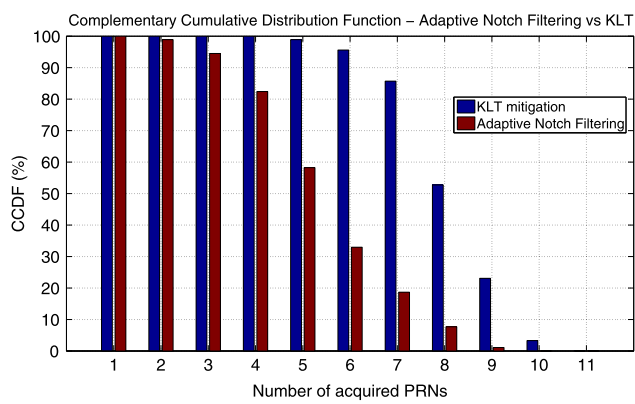Fig. 4. PSD comparison between the jammed dataset before and after the KLT based mitigation algorithm.



Fig. 5. Complementary Cumulative Distribution function of the number of acquired PRNs in the GPS L1 frequency band: Adaptive notch filtering vs KLT based method.

jamming removal by means of the KLT is reported in Fig. 4, while the achieved CCDF is reported in Fig. 5.

The use of the KLT improves considerably the acquisition performance, as demonstrated by the CCDF over the number of acquired PRN. In fact, acquisition of more than 4 PRNs happens with 100% probability (blue bar chart) in case jamming mitigation is performed by means of the KLT based algorithm, while when adaptive notch filtering is employed, more than 4 PRNs are detected with 82% of probability.

## 5. Conclusions

This paper demonstrated the capability of the use of a transformed domain technique based on the use of the KLT. By means of the KLT the signal is projected on the subspace spanned by the eigenfunction where interference components can be better identified and extracted from the received signal, avoiding a large degradation on the useful GNSS signal. Although the KLT-based method offers good performance in mitigating the interference, the computational burden of its implementation is  quite heavy especially with respect to the

traditional interference countermeasures. The complexity of the KLT is mainly caused by the eigenvalues problem that has to be solved. As mentioned in [11], if $N$ is the length of the autocorrelation, $N^2$ is the number of calculation requested to find the KLT. Although the use of the KLT is unsuitable for real-time processing, it can be considered a powerful tool for post processing operation for those applications where careful analysis of the interference environment on jamming critical area is required; in fact, according to the energy criterion for the eigenvalues–eigenfunctions selection, it is possible to reconstruct a synthetic version of the interfering signal from an inverse KLT starting from the eigenfunctions which contain the jamming information only.

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