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# Light as a probe of the structure of space-time

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**Abstract.** Light is an intrinsically relativistic probe and when used in an adequately sized array of ring lasers it is sensible to the curvature and to the chirality of space-time. On this basis the GINGER experiment is being implemented at the underground National Laboratories at Gran Sasso. The experiment, whose objective is the measurement of the terrestrial frame dragging effect or deviations from it, will be presented and discussed in its foundation. Furthermore, at a bigger scale, the possibilities given by the under way GAIA mission and the proposed AGP, will be analyzed with a special attention paid to the possibility of extracting information concerning the angular momenta of the sun and the main bodies of the solar system.

## 1. Introduction

The propagation of light in curved space-time happens along null geodesics and the latter are the same for all freely falling observers. In practice the structure of light cones covers and reproduces the 'shape' of space-time. For these reasons light is an ideal probe for the geometry of space-time, i.e. for the gravitational field.

## 2. Chiral symmetry with respect to time

The space-time surrounding a steadily rotating mass has a chiral symmetry about the time axis of the matter distribution of the source. The corresponding line element is written:

$$ds^2 = g_{00}c^2 dt^2 + 2g_{0i}cdtdx^i + g_{ij}dx^i dx^j \quad (1)$$

The elements of the metric tensor,  $g_{\mu\nu}$ , do not depend on time and on the rotation angle  $\phi$ . In the case of light it is of course  $ds = 0$  so that it is possible to solve Eq. (1) for the coordinated time of flight element  $dt$ :

$$dt = \frac{-g_{0i}dx^i \pm \sqrt{(g_{0i}dx^i)^2 - g_{00}g_{ij}dx^i dx^j}}{cg_{00}} \quad (2)$$

In order to have time flowing towards the future in any case, we choose the + sign in (2). Suppose now to have two light beams travelling along the same space trajectory in opposite directions. The coordinated time of flight element along the same stretch of the path depends on the direction of motion, as we can induce from the fact that  $dx^i$  reverses its sign for the two opposite directions. In practice the difference between the two time elements is obtained subtracting the left travelling (negative) result of (2) from the corresponding right traveling (positive) value. In practice the square root disappears and we are left with an elementary time of flight asymmetry:



$$d(\Delta t) = -2 \frac{g_{0i}}{cg_{00}} dx^i \quad (3)$$

The cumulated time of flight difference along a closed path in space may be obtained integrating (3) along the contour. For that purpose it is convenient to resort to the parametric equation of the space trajectory  $x^i = x^i(l)$ , being  $l$  the parameter. In this way Eq. (3) is converted into

$$d(\Delta t) = -2 \frac{g_{0i}}{cg_{00}} \frac{dx^i}{dl} dl \quad (4)$$

Finally, we should express the result of the integration in the proper time,  $\tau$ , of the observer fixed to the laboratory in which the round trip of the two light beams happens. It is:

$$\Delta\tau = \tau_+ - \tau_- = -\frac{2}{c} \sqrt{g_{00}} \oint \frac{g_{0i}}{g_{00}} \frac{dx^i}{dl} dl \quad (5)$$

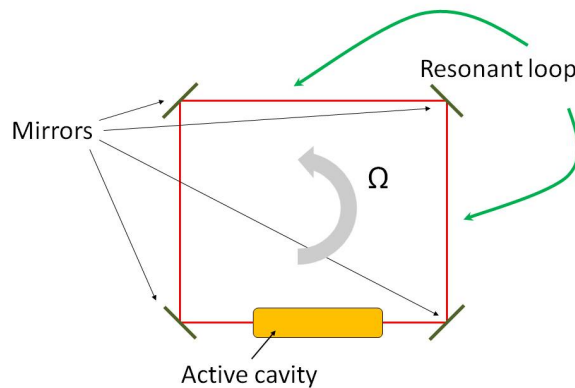
### 3. Ring lasers

The time difference (5) may be measured by means of an interferometer, as in the original Sagnac experiment [1]. When explicit expressions for the  $g_{\mu\nu}$ 's are inserted into Eq. (5) and under weak field conditions, one obtains that the  $\Delta\tau$  is proportional to the absolute angular velocity of the apparatus, or, to say better, to its component along the rotation axis [2]. At the lowest order in the small quantity  $v/c$ , where  $v$  is the absolute value of the peripheral rotation speed of the apparatus, it is:

$$\Delta\tau = 4 \frac{\mathbf{S} \cdot \boldsymbol{\Omega}}{c^2} \quad (6)$$

$\mathbf{S}$  is the area of the closed path of light expressed as a vector perpendicular to the rotation plane;  $\boldsymbol{\Omega}$  is the angular rotation vector of the instrument.

There is, however, another possibility to measure the asymmetric propagation: it is represented by a ring laser. The device is schematically sketched in Fig. 1.



**Figure 1.** The scheme of a square ring laser is shown. The laser is the active cavity, then four mirrors, connected with pipes with a low pressure appropriate atmosphere, form a resonating loop. The whole device is assumed to be rotating at the angular speed  $\Omega$  in its own plane.

The number of mirrors is not necessarily four and the shape of the ring is not necessarily a square. In the most accurate instruments light propagates in vacuo or, most often, in an

appropriate low pressure atmosphere, so the closure of the trajectory is insured by mirrors. In commercial devices, used, for instance, on airplanes to replace mechanical gyroscopes (the device is then called a gyrolaser), light propagates in an optical fiber and the loop may be circular or even consist of more than one winding.

The presence of the resonating ring insures that the two counter-propagating beams attain a steady state. The resonance converts the time of flight difference into a difference between the resonant frequencies of the right-, respectively, left-handed rays and the frequency difference turns out to be proportional to the the time of flight difference  $\Delta\tau$ . Since the two beams are superposed, the difference in frequency gives rise to a beat. The beat frequency, in turn, is half the frequency difference between the counter-propagating waves. Considering a laboratory rotating together with the earth and taking into account the gravitational field of the planet, approximated up to the lowest order term in the terrestrial angular momentum  $J$ , the beat frequency  $f_b$  is [3]:

$$f_b = 2 \frac{S}{\lambda P} \left[ \boldsymbol{\Omega} - 2 \frac{GM}{c^2 R} \Omega \sin \theta \hat{\mathbf{u}}_\theta + \frac{GJ}{c^2 R^3} (2 \cos \theta \hat{\mathbf{u}}_r + \sin \theta \hat{\mathbf{u}}_\theta) \right] \cdot \hat{\mathbf{u}}_n \quad (7)$$

Now  $P$  is the total length of the closed path;  $\lambda$  is the fiducial wavelength of the radiation;  $\boldsymbol{\Omega}$  is the angular velocity of the earth and  $R$  is its radius;  $M$  and  $J$  are the mass and, respectively, the norm of the angular momentum of our planet;  $G$  is Newton's constant; the angle  $\theta$  is the colatitude of the laboratory;  $\hat{\mathbf{u}}_\theta$ ,  $\hat{\mathbf{u}}_r$ ,  $\hat{\mathbf{u}}_n$  are unit vectors aligned respectively with the local meridian, the radius of the earth and the perpendicular to the ring.

The first term in the square brackets corresponds to the classical Sagnac effect due to the kinematical rotation of the ring (together with the earth). The second term, which we may call  $\Omega_G$ , expresses the so called geodetic precession or de Sitter effect, due to the interaction of the radial component of the gravitational field (the *gravito-electric* field) with the rotation of the laboratory. Finally the third term (let us dub it  $\Omega_{LT}$ ), comes from the interaction with the *gravito-magnetic* component of the field, depending on the off diagonal, non-reducible, terms of the metric tensor,  $g_{0i}$ ; the effect is also called frame dragging or Lense-Thirring effect (whence the *LT* indices).

On the surface of the earth, the second and third terms above, i.e. the general relativistic (GR) terms, have approximately the same order of magnitude and are expected to be  $10^{-9}$  times the kinematical contribution, corresponding to effective rotation rates of  $10^{-13}$  rad/s or less.

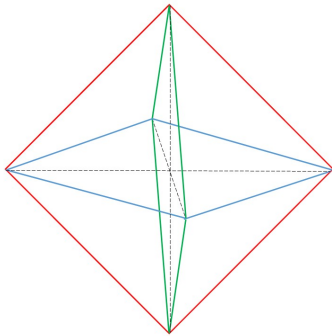
### 3.1. GINGER

The current laser technology has reached sensitivities which are not far from what is needed to detect the general relativistic contributions to the signal revealed by a ring laser. The best instrument at the moment is the G Ring, located in Wettzell (Bavaria), and already has a sensitivity close to 1 prad/s [4]. This promising technological condition is the starting point for the GINGER (Gyroscopes IN GEneral Relativity) project.

GINGER will be an array of three (or more) square rings mutually perpendicular to each other. The three-dimensional configuration, which could be in an octahedron shape like in Fig. (2), will allow for the detection of all three components of the effective rotation vector of the laboratory, including the GR terms.

The size of the rings will be over 6 m in side. The location of the laboratory will be in the Laboratory Nazionali del Gran Sasso, under a 1400 m cover of rock. This is because on the surface of the planet there is an enormous number of rotational noise sources, at the sensitivity level planned for the apparatus. The details of GINGER are presented in a poster for the present conference.

In principle it could be possible to fix new upper limits for at least two ppN parameters, which appear in the expected measured quantities according to the formulae:



**Figure 2.** Octahedral configuration for GINGER. Three square light rings, mutually orthogonal. The side of a ring is not less than 6 m long. The main diagonals (dashed in the figure) contain Fabry-Pérot cavities for the dynamical control of the geometry.

$$\begin{aligned}\Omega_G &= -[2 + (\gamma - 1)] \frac{GM}{c^2 R} \Omega \sin \theta \hat{\mathbf{u}}_\theta \cdot \hat{\mathbf{u}}_n \\ \Omega_{LT} &= \frac{2 + (\gamma - 1) + \alpha_1/4}{2} \frac{GJ}{c^2 R^3} (2 \cos \theta \hat{\mathbf{u}}_r + \sin \theta \hat{\mathbf{u}}_\theta) \cdot \hat{\mathbf{u}}_n\end{aligned}\quad (8)$$

Unfortunately it looks like a formidable task since the existing upper bounds to  $\gamma$  and  $\alpha_1$  would require accuracies better than  $10^{-22}$  rad/s, in order to be ameliorated.

#### 4. Other experimental opportunities

The interaction of light with gravitational fields offers a number of other interesting possibilities for experiment and observation.

An example is given by a linear cavity, like the one of a Fabry-Pérot interferometer. Recalling the fact that a light ray bouncing back and forth in the cavity actually covers a *surface* in space-time, we see that the electromagnetic wave (the Faraday tensor  $F^{\mu\nu}$ ) couples to the full Riemann tensor,  $R^\alpha_{\mu\nu\rho}$ , according to the formula:

$$\delta F^{\mu\nu} = (R^\mu_{\epsilon 0 i} F^{\epsilon\nu} + R^\nu_{\epsilon 0 i} F^{\mu\epsilon}) \delta S^{0i} \quad (9)$$

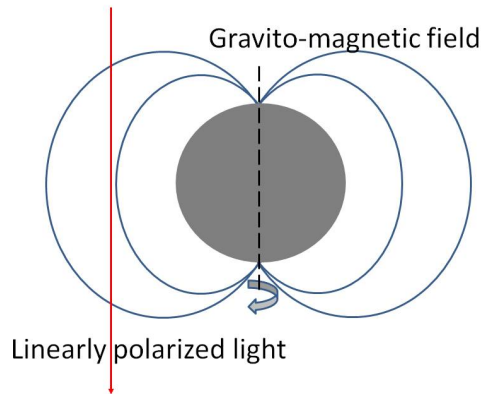
$\delta F^{\mu\nu}$  is the change in the components of the Faraday tensor (i.e. in the components of the electric and magnetic fields of the wave) for each single journey.  $\delta S^{0i} = (l^i)^2/c$  is the space-time area spanned during the journey and it is assumed that  $l^i$  is much smaller than the space-time curvature (here in practice  $l^i \ll R$ . The standard conventions hold (Greek indices run from 0 to 3, Latin indices go from 1 to 3) so that  $l^i$  is the  $i$  component of the length of the cavity vector  $\mathbf{l}$ . Expectedly, the effect of the coupling should show up as a rotation of the electric and magnetic fields of the wave, including, in principle, the appearance of a longitudinal component of the fields.

When considering the effect of gravity on the polarization of the electromagnetic waves, we have also a gravito-magnetic Faraday effect (analogous to the classical one induced by the propagation of light in a magnetic field). For linearly polarized light propagating along the field lines of a gravito-magnetic field (for example in a direction parallel to the axis of a rotating massive body) the expected rotation angle,  $\alpha$ , of the polarization plane is (at the lowest approximation) [5]:

$$\alpha = \frac{1}{2} \int_{\text{emission}}^{\text{reception}} \sqrt{g_{00}} (\nabla \wedge \mathbf{h}) \cdot \hat{\mathbf{u}}_k dl \cong \frac{GJ}{4c^3 b^2} \quad (10)$$

The components of the three-vector  $\mathbf{h}$  are  $h_i = g_{0i}/g_{00}$ . Doing the calculation, the geometric effect corresponding to the lensing has been considered negligible;  $b$  is the distance of the light

ray (assumed to be straight) from the center of the rotating mass;  $\hat{\mathbf{u}}_k$  is the unit vector along the propagation direction. A sketch of the situation is shown on Fig. (3).



**Figure 3.** Physical configuration of a system in which a rotation of the polarization plane of light propagating vertically (in the figure) in the gravito-magnetic field of a rotating mass is produced.

It is extremely unlikely that such a gravitational Faraday effect can be detected in a terrestrial experiment, but the story is different at an astronomical scale.

Another effect that could be investigated at the astronomical scale is the asymmetric Shapiro time delay, related to the rotation of the source of gravity, which is expected to happen between electromagnetic signals travelling in the equatorial plane and passing on the right rather than on the left of the central mass.

The sensitivity of the GAIA mission, even not flying for those purposes, let us hope for equally sensible missions destined to uncover all possibilities given by light when propagating in a curved space-time. It would be the case, for instance, of the AGN proposal, at the moment in stand by.

## 5. Conclusion

We have seen that light, being a fully relativistic phenomenon, lends a number of possibilities to explore the properties of gravity in the framework of General Relativity and beyond. The GINGER experiment, to test the terrestrial Lense-Thirring effect and deviations from it, is being developed. High sensitivity space missions can also extract interesting information observing the sky in order to spot and measure details of the propagation of light rays, so far unexplored. We may be confident of having new results in a few years in the future.

## References

- [1] Di Virgilio A 2014 *et al. Comptes Rendus Physique* **15**866874
- [2] Ruggiero M and Tartaglia A 2014 *Eur. Phys. J. Plus* **129** 126
- [3] Bosi F *et al* 2011 *Phys. Rev. D* **84** 122002
- [4] Schreiber K and Wells J 2013 *Rev. Sci. Instrum.* **84** 041101
- [5] Ruggiero M and Tartaglia A 2007 *MNRAS* **374** 847