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ENHANCING STRUCTURAL ROBUSTNESS BY COMPLEXITY MAXIMISATION

Structural robustness is considered a fundamental prerequisite in the design of structures. In particular, attention has to be paid to events that are unforecastable and with no known magnitude. Referring to an idea by Donald Rumsfeld, these are unknown unknowns. Among all the possible strategies for ensuring robustness, alternating the load paths on the structures may represent a feasible design solution. Structural complexity is a novel metrics for measuring the amount of interaction between hypothetical load paths on a structure. Maximum complexity corresponds to maximum interaction. In the paper, the links between structural complexity and robustness are investigated.

Keywords: robustness; structural engineering; extreme event; structural complexity.

Introduction

A simple observation can invalidate a general statement derived from millennia of confirmatory sightings. The use of locution "black swan" for indicating quasi-impossible events dates back to the Romans: Giovenale wrote in his sixth book of "Satire", Rara avis in terris nigroque simillima cycno as indicating something that is far from being usual in the everyday life. This is one of the main characteristics of Taleb's Black Swan. In 2007, after the subprime mortgage crisis, in the bookshops of the US a new book forecasting the future effects of the finance appeared. Taleb [1] states that the economy (and in more general, the World intended as a whole) is dominated by extreme events, which are unknown and not forecastable. The key point, which led to the crisis, is the fact that economists base their decisions on what they observe and know, while the world works completely different. Ergo, the predictions are wrong.

The book by Taleb does not concern economy, first. It is more a critical text on the use of statistics as an engine for solving and interpreting whichever natural phenomenon. Statistics is an extremely useful tool for all the situations in which the sensitivity to errors in the probability distribution can be neglected. A practical example is represented by such disciplines like measurement estimations, gambling theory, thermodynamics, and quantum mechanics.

In many other situations, the output of a mathematical model of the real world is not a linear combination of random parameters. Where nonlinearity is present, the sensitivity to estimation errors of the higher moments of probability distributions increases dramatically. Taleb criticizes the rigorous use of statistics and reliance on probability in domains where the current methods can lead us to make consequential mistakes (the "high impact") where, on logical grounds, we need to force ourselves to be suspicious of inference about low probabilities. An example of that is represented the estimation of volume discharges with very large return period on the base of a reduced range of measurements. That is the use of commoditized metrics such as "standard deviation", "shape ratio", "mean-variance" has no sense in fat-tailed domains where these terms have little practical meaning, and where reliance by the untrained has been significant, unchecked and, alas, consequential. In concise terms, the central idea behind Taleb's work is the confusion that most people make between absence of evidence and evidence of absence.

Disastrous combinations of events are extremely rare but entail large, say, enormous costs (damage + social). In this framework, the attention has to be put to low risk-high consequences events, i.e., black swans events, for which reduced data are available or, at worst, no statistic can be drawn.

Is there the possibility to deal with structural Black Swans? Luckily, the answer is positive. The basic approach is to shift the attention from the spectrum of actions to the gamut of damages on the structural scheme. That is, the consequences on the construction are the main interests of the designer. The philosophy is to prevent the propagation of damage to other structural components, which is in the field of interests of robustness concepts. That is, having a robust structure is a fundamental requisite for dealing with Black Swans situations.

As recalled previously, the design approach has to be based on the consequences. The inadequacy of the current design practices for particular situation has been already highlighted by Starossek and Wolff [2] when considering progressive collapse. Two deficiencies are identified: first, the global effects are lost in the design. In detail, all actions and resistances are statistically determined on the basis of empirical data. After the evaluation of an allowable probability of failure, the design values for actions and resistances can be calculated using probabilistic methods, but the resistance is usually considered only on a local level (cross section, structural element) while the global resistance remains disregarded. Then, the authors criticize the assumption that low probability events and unforeseeable incidents (accidental circumstances) need not be taken into consideration in the design, while they are the most dangerous for the construction.

The idea of implementing a design based on the consequences rather than on reliability takes its origins at the beginning of the new millennium [3]. Nafday [4] turned the concept to structural engineering. In a probabilistic framework, this complete lack of events, likelihood and data makes the design of structures for specific abnormal loads impossible and, therefore, there is little systematic code-based or regulatory guidance for limiting adverse system consequences due to unforeseen events. The fundamental aspect lies in the fact that the approach does not need a triggering event (or its likelihood), making it apt for Black Swan events. Capacity-based design strategies optimize robustness and general structural integrity by controlling adverse system consequences resulting from unexpected loads. In this sense, the idea of uniform reliability for all the structural members has to be rethought as an explicit variable reliability member design, to account for the differing system consequences of individual member failures. The design for Black Swan events is a secondary design. In the primary stage, structure is designed as usual using the current probabilistic member-based code provisions for normal loads, providing appropriate minimum joint resistance, continuity and inter-member ties. Thereafter, members are selectively redesigned for ensuring adequate structural system integrity, based on their role and importance in contributing to adverse consequences. These consequences can either be structural system collapse or any other pre-defined structural performance criterion. Unlike specific resistance method, where “key” members are empirically chosen for hardening based on threat-specific knowledge, Nafday’s proposed design method applies a logical quantitative approach to upgrade structural members based on their individual role and importance in contributing to pre-defined adverse structural consequences.

Strategies for structural robustness

In the early morning hours of May 16, 1968, the occupant of apartment 90 on the 18th floor of the 22-stories Ronan Point apartment tower, in London, lit a match to brew her morning cup of tea [5]. A small gas explosion occurred and the resulting pressure increase blew out the walls of her apartment, and initiated a partial collapse (both upwards and downwards) of the structure that killed four people and injured 17. This was the spark for robustness discussions in the domain of structural engineering.

A unique definition of robustness has not formulated yet as highlighted in [6]. Specifically, in the field of structural engineering, the design codes implemented the concept and formulated a proper definition. Many of these codes specify that structures should be robust in the sense that the consequences of structural failure should not be disproportional to the effect causing the failure [7].

Various strategies can be adopted to increase the robustness of a structure [8, 9]. First, event control strategies point at avoiding or protecting the construction against an incident that might lead to its disproportionate failure. This approach does not increase the inherent resistance of a structure, but limits the possibility of occurrence of the event. If sufficient strength is provided to structural elements, they would be able to resist overloads. This is the principle at the base of the Specific Load Resistance strategy for robustness. Key elements have to be preserved and their capacity increased in order to limit the propagation of the damage. This strategy can be applied in those cases in which the key elements are few in number and easy to identify [7].

Providing alternatives for a load to be transferred from the point of application to a point of resistance, namely the foundations, may result in an appropriate strategy for increasing the robustness of a structure. Provided that the alternative paths are sufficiently strong, this enables redistribution of forces originally carried by failed components to prevent a failure from spreading. In order to achieve this requirement, the remaining structural elements must be strong enough, collectively, to resist the loads corresponding to the situation after the event. The resistance of the elements must be associated with a proper capacity in deformation without loss of resistance. In any case, it is necessary that, after the failure, the overall stability is guaranteed. An important issue has to be addressed in these cases in which the redundancy of the structural scheme is increased in order to achieve robustness requirements: multiple load-paths may sometimes involve brittle situations or limit deformations with negative consequences. These situations must be avoided [8]. An opposite strategy for reducing the consequences of events consists in isolating parts of the structure in order to prevent the spreading of the damages [10]. Structural segmentation, namely compartmentalization, has been demonstrated to be effective in various cases [11, 12].

Alternate Load Path strategy, as well as structural compartmentalization, is an effective measures in case of both hazard-specific and non-hazard specific situations because the notional damage to be considered in the application of the alternative-paths method is non-threat-specific [13]. Because of that, these approaches are useful in dealing with Black Swan situations.

The role of structural complexity

De Biagi [14], noting that Nature, intended an evolutionary entity able to adapt the surrounding environment, developed various strategies for dealing with extreme situations (such drought, starve, climate change), tried to export his observations on structural engineering domain. In this sense, it seems that some natural structures (such DNA-RNA transcription, blood circulation, food-chain,..) are robust because they are complex. That is why the author developed the idea of structural complexity in order to assess the possible links between the novel metric and the robustness [14].

Consider the following example. A load set composed by vertical, horizontal forces and torques has to be carried by a structure: the two possible solutions are sketched in Fig. 1a and 1b. Note first that the two schemes have the same loads and the same number and position of beam-column connections. A deeper top beam and a large column characterize the frame on left-hand side. On the contrary, the frame on right-hand side has elements with similar cross-section. A structural engineer can easily imagine the load path in the left-hand side scheme, i.e. the forces are carried by the elements with higher stiffness; on the contrary it is difficult to assess the real operating method for the structure on right-hand side because no patterns are easily found for this load set. In the limit case in which the load case is represented by only vertical forces acting at nodes, the columns themselves would identify the load path from the elevation to the foundation in both schemes. We consider the first structure as being simple, while the second one complex. In general the presence of a defined load path makes the structure simple. We say that a structure is complex if it composed by a large number of parts that interact in a non-simple way under an arbitrary loading scheme [15]. This definition, which can be interpreted as an extension to structural mechanics of the work by Simon [16], accounts for the shape of the structure, its stiffness and the acting loads. The metrics for determining the structural complexity is based on the so-called Information Content introduced by Shannon [17]. Here, the information content is represented by the effectiveness of the load paths across the structure. A simple structure is the one that has a reduced number of effective load paths. On the contrary, when all the possible load paths are equally effective, the structure reaches its maximum complexity [15].

As a matter of evidence, in statically determinate schemes, like a cantilever, the load path is unique. That is why, in the proposed theory, the load paths are derived from statically determinate structures, called "fundamental structures" extracted from the original statically indeterminate scheme. In this framework, the elastic work of deformation is the parameter that better describes the behaviour of a structure subjected to loads. First, it accounts both for stiffness and loads, and, in case of nonlinear analysis, it considers the ductility of the elements composing the scheme. In linear elastic structures, the deformation work can be computed by means of the well-known Clapeyron's Theorem.

The effectiveness of a load path is measured as the ratio between the deformation work in the original structure and the deformation work performed on the statically determinate structure, i.e. the fundamental structure. This ratio is called performance ratio and ranges from 0 to 1 since the denominator is always larger than the numerator. These bounds represent, respectively, the limit cases in which the load path is not effective, or viceversa. The number of fundamental structures and, consequently, of performance ratios, n, depends on the original scheme. The measure of the "amount" of information required to describe the structural behaviour, is based on the definition of information entropy stated by Shannon [17]. In particular, the Structural Complexity Index SCI, is represented by

$$\mathrm{SCI} = -\sum_{i=1}^{n}\left(\frac{\psi_i}{\sum_{j=1}^{n}\psi_j}\log_b\frac{\psi_i}{\sum_{j=1}^{n}\psi_j}\right). \quad (1)$$

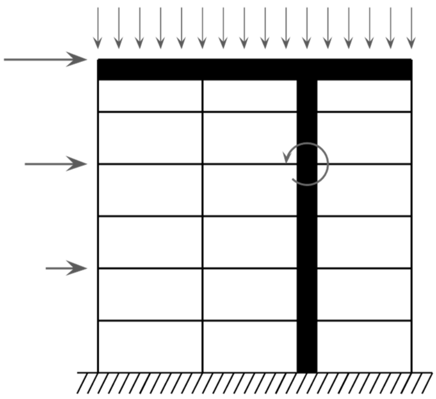

Fig. 1a. Sketch of a scheme that can be considered simple

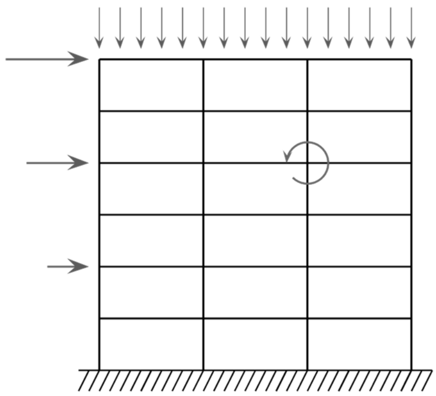

Fig. 1b. Sketch of a scheme that can be considered complex

The base of the logarithm, b, is not relevant (if 2, the measure is in bit). The entropy measure possesses many interesting properties [18]. The identification of the load paths can be easily performed if the structural scheme is studied under the framework of Graph Theory [15].

In order to compare the complexities of various structures with different size and element number, a normalized parameter is introduced. The SCI is divided by its maximum possible value, which represents the situation in which each possible load path has the same effectiveness (i.e. the same performance factor). This situation, representing the maximum corresponds to a SCI equal to log n, where n is the number of load paths. Thus, the Normalized Structural Complexity Index, NSCI, is expressed as

$$NSCI = \frac{SCI}{\log_b n}. \tag{2}$$

The NSCI ranges between 0 and 1. As much as the parameter approaches to 0+, the scheme is simple. On the opposite side, values of NSCI tending to 1− refer to complex structures [15].

Physically speaking, maximum complexity, i.e., maximum disorder, can be intended as a white noise, that is a constant power spectral density signal, in the range of the spectrum between 0 and 1. Minimum complexity is gained by a simple harmonic oscillation, which representation in a spectral plot is a Dirac delta function at $1/n$ [19]. The metrics has been proved to have interesting properties. The complexity of the structure is invariant to scaling as much as specific requirements are satisfied [20].

Structural complexity and robustness

The effects of the complexity of structures on the robustness of the frame are analysed through an example. The metrics has been formulated on frame structures. Because of that, the attention would be focused on that. The reference structural scheme is constituted by 15 beams (6 horizontal beams and 9 columns) joined together in 9 elevation nodes and by a unique foundation node; see the sketch in Figure 2. All the elements of the arbitrary scheme are made of linear elastic material with Young's Modulus equal to 25 GPa, squared cross-section (40×40 cm) [21, 22]. The frame is exclusively loaded with nodal forces applied on all elevation nodes, i.e. *A–I*. That is V_i = 100 kN and H_i = 100 kN, with i = *A–I*. The number n of fundamental structures is about 1183. The structural complexity parameters are computed with the previous expressions and are SCI = 9,5849, NSCI = 0,9389.

For studying the effects of the values of the normalized complexity index on the robustness of the structure, the following iterative procedure has been implemented on the proposed loaded structural scheme:

1) a structure, namely j, is generated. For doing that, a random set of cross-section sizes, referring to all the elements of the frame except AB (which is set kept constant to a reference length), is generated. External loads are applied to the scheme;

2) the work of deformation, $W_{in,j}$, and complexity of the structure, the $NSCI_j$, is computed;

3) alternatively, each element (named i) of the frame is removed, and the work of deformation, $W_{i,j}$, is computed on the resulting structure.

A first analysis on a sample of 10k structures has been done. Various observations can be done. First, the Normalized Structural Complexity Indices derived from the generation of random structures range from 0,2691 to 0,9808. Obviously, structures, which NSCI is outside the previous bounds, do exist, but they were not generated. Table 1 reports the cardinality of a subset of structures having the complexity index ranging in a specific interval [14]. It seems that the associated probability distribution can be classified as an extreme values one. In this situation, considering the random generation, the median value of NSCI is equal to 0,7881. Since the number of different undamaged schemes is equal to 10k and the number of possible damage situations, i.e. element removal, is 15, 150k values of the ratio $W_{i,j}/W_{in,j}$ are computed, ranging between 1 and 1013. That is, there are damage situations that produce extremely high impacts on the scheme: despite their very reduced number, they affect the distribution of values since their magnitude is extremely elevated. Analysing the structural schemes belonging to the range NSCI = (0,80; 0,90), which are 3719 in number, one discovers that the

$$\min_i \frac{W_{i,j}}{W_{in,j}} = 1{,}00000008347837$$

while

$$\max_i \frac{W_{i,j}}{W_{in,j}} = 1658849976505{,}83$$

i.e. a range of twelve order of magnitude. Observing the dataset, one notes that as much as the ratio $W_{i,j}/W_{in,j}$ increases, the number of occurrences diminishes. Outliers, which are extreme or atypical data values that are notably different from the rest of the data, are thus present.

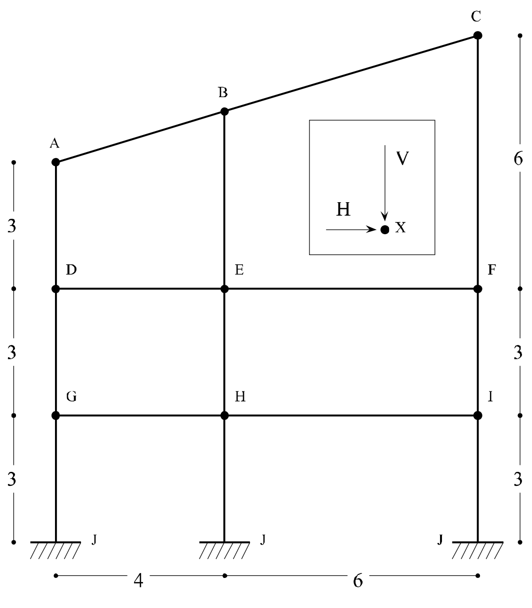

Fig. 2. Sketch of the proposed frame for robustness considerations

The data have been analysed statistically. Four statistical parameters are evaluated in the sample of 15: the minimum, the maximum and 50 and

90 percentiles. Obviously, the presence of outliers would affect, locally, the previous parameters. Now, for each set of structures, grouped by the corresponding value of NSCI, the median of the parameters is computed. In other words, referring to the range (0,80; 0,90), the median is over a sample of 3719.

Table 1

Number of structures with a NSCI ranging in a specific interval (set of 10k simulations)

NSCI range	No. of structures
0,00–0,10	0
0,10–0,20	0
0,20–0,30	3
0,30–0,40	21
0,40–0,50	109
0,50–0,60	516
0,60–0,70	1529
0,70–0,80	3303
0,80–0,90	3719
0,90–1,00	800

The results are plot in Figure 3. A clear trend emerges: as much as the complexity increases, the statistical parameter, which relates to the behaviour of the ratio $W_{i,j}/W_{in}$, decrease. This leads to a preliminary conclusion: as much as the complexity increases, the impact of element removal in the loaded structural scheme reduces.

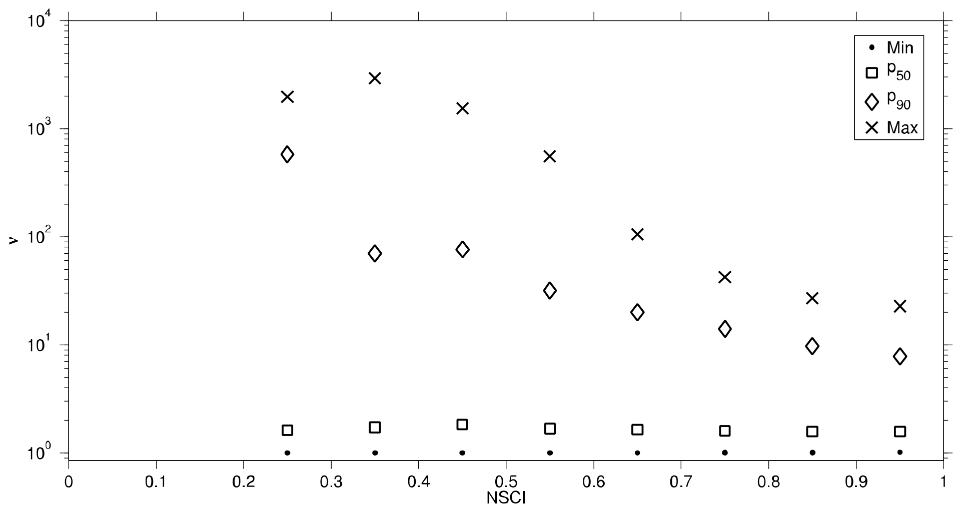

Fig. 3. Plot of the median of minimum, 50 and 90 percentiles and maximum of $W_{i,j}/W_{in,j}$ for each NSCI range (set of 10k simulations), from [14]

In another simulation, the steps reported at the beginning of the paragraph have been repeated 25k, i.e. 25k structures have been generated. The impact of element removal has been evaluated through the mean of the increment of deformation work due to the damage. In other words, the parameter M_j as been computed as

$$M_j = -\frac{1}{15}\sum_{i=1}^{15}\frac{W_{i,j}-W_{in,j}}{W_{in,j}}. \quad (3)$$

At the end of the numerical experiment, 25k couples (NSCI_j, M_j) are obtained. Each one of these has been plotted on a graph, as shown in Figure 4. One can easily note that, as much as the complexity increases, the average impact of element removal decreases. De Biagi and Chiaia [19], in an on-going research paper, have determined the bounds of the cloud of points in case of systems made of parallel rods.

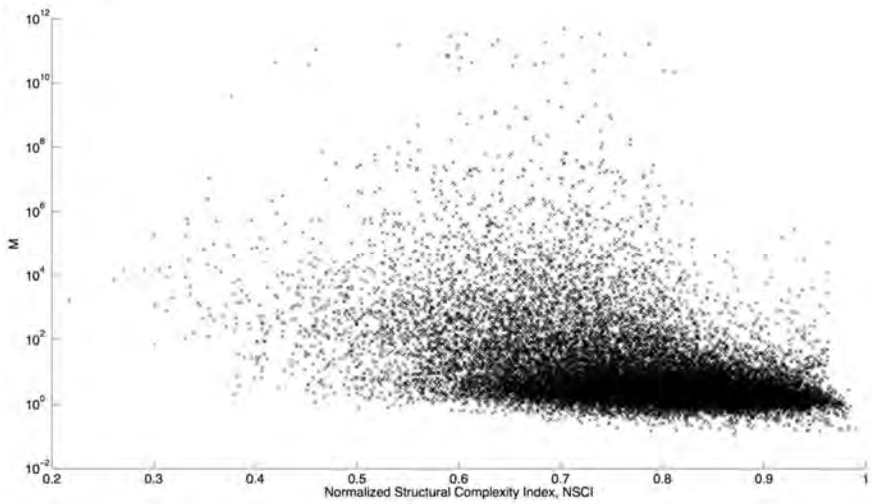

Fig. 4. Normalized Structural Complexity Index *vs* M-value. Each point corresponds to a simulation (set of 25k simulations)

Conclusion

Structural robustness is considered a fundamental prerequisite in the design of structures. As shown, for preventing collapse from Black Swan events (or, better, Black Swan damage scenarios), alternate load paths strategy represents a feasible design solution. Natural systems shows spontaneous tendency to be robust to adverse situations. Nature implemented and implements various strategies in order to survive to ordinary and extreme situations that may occur. Complexity is one of these: evidences of complexity are visible in biological, social and spontaneous systems. The connectedness between the components of the system has been shown to be a powerful way for ensuring the transfer between networks even if nodes are removed.

Structural complexity, which theoretical bases have been illustrated in this text, is a novel metrics for measuring the amount of interaction between hypothetical load paths on a structure, that are represented by the so called fundamental structures. Maximum complexity corresponds to maximum interaction.

The second part of the paper is devoted to the investigation of possible links between the value of the normalized structural complexity index and the robustness of the structure. The last parameter has been computed through the elastic energy stored in the system before and after a random damage. It has been shown that as much as the complexity increases, the impact of element removal on the system reduces. Theoretically, increasing the complexity of a structure would imply an increase of its robustness to random damage.

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