On tuning passive black-box macromodels of LTI systems via adaptive weighting

In this work, we concentrate on electronics applications, for which reliable models of the Power Distribution Network (PDN) at chip, package, board and system level are required [1–3]. The PDN can be regarded as a large-scale Linear and Time-Invariant (LTI) dynamic system [4, 5]. A first-principle formulation would lead to a state-space or descriptor formulation with billions of states and hundreds of inputs/outputs. However, such detailed first-principle descriptions are usually not available to the power ingegrity engineer, who is responsible for compliance verification at the system level. Moreover, even if such descriptions were available, the resulting complexity of system-level verification would be overwhelming. Hence, there is a strong need for accurate and broadband reduced-order models.

We concentrate here on the construction of state-space PDN macromodels in a black-box setting, via identification from a finite set of frequency response samples. The main tool that we employ is frequency-domain rational approximation, for which several good algorithms exist, such as Vector Fitting [6–10], followed by a postprocessing step aimed at enforcing passivity [11–15]. Passivity is in fact a fundamental requirement for ensuring model robustness and global stability of successive system-level transient simulations.

The main problem that we address is the sensitivity of the state-space macromodel to the termination networks to which the model will be connected during normal operation. This sensitivity may be the root cause for major accuracy degradation, so that a model that is very accurate in the input-output representation that is adopted for its construction may result quite inaccurate during normal operation. This degradation results from the feedback mechanisms that the terminations induce on the model dynamics [16, 17].

We propose a simple algorithm to alleviate this accuracy degradation, based on the definition of suitably and adaptively defined frequency-dependent weights, which are used to construct an optimized cost function embedding information on the nominal termination scheme for the model. Minimization of this cost function during model identification and passivity enforcement leads to an effective compensation of the model sensitivity, with resulting improved accuracy. Various examples from real applications demonstrate the effectiveness of this approach.

2 Problem statement

Let us consider a P-port PDN system, known through a set of K frequency samples of its $P \times P$ scattering matrix

$$\hat{\mathsf{S}}_k \approx \hat{\mathsf{S}}(\mathrm{j}\omega_k), \quad k = 1, \ldots, K. \tag{1}$$

The scattering representation is such that $\mathsf{b}(\mathrm{j}\omega) = \hat{\mathsf{S}}(\mathrm{j}\omega)\mathsf{a}(\mathrm{j}\omega)$ where a, b are the power waves that are incident into and reflected from the structure, respectively. This representation is preferred here since it is guaranteed to exist for any LTI system. We want to construct a regular state-space model

$$\begin{aligned} \dot{\mathsf{x}}(t) &= \mathsf{A}\mathsf{x}(t) + \mathsf{B}\mathsf{a}(t) \\ \mathsf{b}(t) &= \mathsf{C}\mathsf{x}(t) + \mathsf{D}\mathsf{a}(t) \end{aligned} \tag{2}$$

with transfer (scattering) matrix

$$\mathsf{S}(s) = \mathsf{C}(s\mathsf{I} - \mathsf{A})^{-1}\mathsf{B} + \mathsf{D}\,, \tag{3}$$

so that

- a cumulative least squares fitting error with respect to the original data (1)

$$E_w^2 = \sum_{k=1}^{K} E_{w,k}^2 = \sum_{k=1}^{K} w_k^2 \|\mathsf{S}(\mathrm{j}\omega_k) - \hat{\mathsf{S}}_k\|_F^2 \tag{4}$$

  is minimized, where w_k are appropriate frequency-dependent weights and $_F$ denotes the Frobenius norm;
- the model is passive, so that

$$\sigma_{\max}(\mathsf{S}(\mathrm{j}\omega)) = \|\mathsf{S}(\mathrm{j}\omega)\|_2 \leq 1\,, \quad \forall \omega \in \mathbb{R}\,, \tag{5}$$

  where $\sigma_{\max}$ denotes the maximum singular value of its matrix argument.

In standard applications, the weights in (4) are uniformly set to $w_k = 1$, or at best to $w_k = 1/\varsigma_k^2$ when the variance ς_k^2 of noise affecting raw data is known. Here, we want to construct these weights such that a second objective is met. We assume that the nominal termination scheme is fully known and characterized in the frequency-domain as

$$\begin{aligned} \mathsf{a}(s) &= \mathsf{M}(s)\mathsf{b}(s) + \mathsf{N}(s)\mathsf{u}(s), \\ \mathsf{y}(s) &= \mathsf{P}(s)\mathsf{b}(s) + \mathsf{Q}(s)\mathsf{u}(s), \end{aligned} \tag{6}$$

where u is a vector collecting independent sources embedded in the termination network, y collects the output variables of interest, and $\mathsf{M}, \mathsf{N}, \mathsf{P}, \mathsf{Q}$ are suitable transfer matrices. Note that the port inputs b of the termination network (6) are the outputs of the macromodel (2), and viceversa. Our objective is minimization of the error

$$\Delta^2 = \sum_{k=1}^{K} \Delta_k^2 = \sum_{k=1}^{K} \|\mathsf{H}(\mathrm{j}\omega_k) - \hat{\mathsf{H}}_k\|_F^2\,, \tag{7}$$

where $\mathsf{H}(\mathrm{j}\omega_k)$ and $\hat{\mathsf{H}}_k$ are the transfer functions between input u and output y, based on the model $\mathsf{S}(\mathrm{j}\omega_k)$ of (3) and on the raw data $\hat{\mathsf{S}}_k$ of (1), respectively.

3 Iterative rational approximation via adaptive weighting

A simple first-order approximation of the relationship between the frequency-dependent model error E_k and transfer function error Δ_k leads to

$$\Delta_k \approx \mathscr{S}_k E_k, \tag{8}$$

where $\mathscr{S}_k$ can be interpreted as a sensitivity of $\mathsf{H}(j\omega_k)$ with respect to perturbations in the model responses $\mathsf{S}(j\omega_k)$ under nominal termination conditions (6). Therefore, if we set $w_k = \mathscr{S}_k$ and we minimize (4) during model construction, we expect that the resulting model will achieve an equivalent minimization of (7). As documented in [16], this approach still leaves margins for improvement, in addition to requiring the explicit computation of the sensitivity. We resort to a simpler and more effective iterative approach, based on the following steps.

1. At the first iteration $\mu = 0$, we initialize the weights as $w_k^{(0)} = 1$ for all k.
2. For each iteration $\mu = 0, 1, \ldots,$ we compute a state-space macromodel (3) by minimizing (4). This is obtained by a standard application of the Vector Fitting (VF) algorithm [6–10].
3. Once the model is available, the corresponding frequency-dependent transfer function error $\Delta_k^{(\mu)}$ is computed. If $\Delta_k^{(\mu)} < \delta$ at all frequencies, where δ is the desired target accuracy, the iteration is stopped.
4. Otherwise, a new frequency-dependent weight for next iteration is defined as

$$w_k^{(\mu+1)} = w_k^{(\mu)} \cdot \mathscr{F}(\Delta_k^{(\mu)}), \tag{9}$$

   where $\mathscr{F} : \mathbb{R}^+ \mapsto \mathbb{R}^+$ denotes a non-decreasing function such that $\mathscr{F}(\xi) = 1$ for $\xi \le \delta$. Then, the iteration index is increased $\mu \leftarrow \mu + 1$, and the scheme is restarted from step 2.

The redefinition of the weights in (9) further emphasizes those frequencies for which the transfer function error is significant, without affecting the other frequencies. The result of this process is both the termination-tuned model $\mathsf{S}(s)$ and the corresponding set of optimal weights w_k. The convergence properties of this iteration are related to the specific choice of $\mathscr{F}$. A detailed convergence analysis is in progress and will be documented in a future report.

4 Passivity enforcement

Once a state-space macromodel is available, its passivity should be verified. We perform this check by computing the set $\mathscr{I}$ including all purely imaginary eigenvalues $\lambda_i = j\omega_i$ of the associated Hamiltonian matrix [12] (we assume $\|\mathsf{D}\|_2 \le 1$)

$$\mathscr{M} = \begin{pmatrix} \mathsf{A} + \mathsf{B}(\mathsf{I} - \mathsf{D}^\mathsf{T}\mathsf{D})^{-1}\mathsf{D}^\mathsf{T}\mathsf{C} & \mathsf{B}(\mathsf{I} - \mathsf{D}^\mathsf{T}\mathsf{D})^{-1}\mathsf{B}^\mathsf{T} \\ -\mathsf{C}^\mathsf{T}(\mathsf{I} - \mathsf{D}\mathsf{D}^\mathsf{T})^{-1}\mathsf{C} & -\mathsf{A}^\mathsf{T} - \mathsf{C}^\mathsf{T}\mathsf{D}(\mathsf{I} - \mathsf{D}^\mathsf{T}\mathsf{D})^{-1}\mathsf{B}^\mathsf{T} \end{pmatrix}. \tag{10}$$

If $\mathscr{I}$ is empty, the model is already passive and no other action is required. Otherwise, the model needs to be corrected to eliminate local passivity violations, intended as violations of condition (5) within localized frequency bands

$\Omega_i = (\omega_i, \omega_{i+1})$. The boundary points of each violation band Ω_i correspond to the imaginary part of some Hamiltonian eigenvalue in set $\mathscr{I}$.

The passive model to be determined is parameterized by perturbing the state-output map $\widetilde{\mathsf{C}} = \mathsf{C} + \Delta\mathsf{C}$, corresponding to a model perturbation

$$\widetilde{\mathsf{S}}(s) = \mathsf{S}(s) + \Delta\mathsf{S}(s), \quad \Delta\mathsf{S}(s) = \Delta\mathsf{C}(s\mathsf{I} - \mathsf{A})^{-1}\mathsf{B}. \tag{11}$$

A set of local passivity constraints is obtained by considering each individual singular value trajectory $\sigma_r(\mathrm{j}\omega)$ that exceeds one within a given violation band Ω_i, finding its local maximum $\bar{\sigma}_{i,r} = \sigma_r(\mathrm{j}\bar{\omega}_{i,r})$ with $\bar{\omega}_{i,r} \in \Omega_i$, and linearizing the relationship between this singular value and the decision variables $\Delta\mathsf{C}$. Imposing that this linearized singular value falls below one gives the linear inequality constraints

$$\mathsf{z}_{i,r}^{\mathsf{T}} \mathrm{vec}(\Delta\mathsf{C}) \leq 1 - \bar{\sigma}_{i,r}, \quad \forall i, r, \tag{12}$$

to be enforced concurrently while minimizing the model perturbation (11).

Most existing passivity enforcement schemes [11–15] aim at minimizing the $\mathscr{L}_2$ norm of the model perturbation, which can be characterized as

$$\|\Delta\mathsf{S}\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \|\Delta\mathsf{S}(\mathrm{j}\omega)\|_2^2 \,\mathrm{d}\omega = \mathrm{tr}\left(\Delta\mathsf{C}\,\mathsf{G}_c \Delta\mathsf{C}^{\mathsf{T}}\right), \tag{13}$$

where G_c is the controllability Gramian of the original model. Minimization of (13) subject to (12) optimizes the model accuracy, but may degrade the accuracy of the target transfer function $\mathsf{H}(s)$, since no weighting is considered. We propose two different approaches to overcome this limitation.

The first approach is to consider a frequency-weighted controllability Gramian G_w instead of G_c in (13). This Gramian is constructed based on an augmented state-space system providing a realization of

$$\Delta\mathsf{S}_w(s) = \Delta\mathsf{S}(s)F(s), \tag{14}$$

where $F(s)$ is a minimum-phase transfer function such that $|F(\mathrm{j}\omega_k)|^2 \approx w_k^2$, where w_k are the optimal weights from the fitting. More details can be found in [17].

A second and more straightforward approach is to construct a data-based cost function. We consider the model deviation at frequency $\mathrm{j}\omega_k$, which we write as

$$\mathscr{E}_k^2 = \left\|\widetilde{\mathsf{S}}(\mathrm{j}\omega_k) - \hat{\mathsf{S}}_k\right\|_F^2 = \left\|\Delta\mathsf{C}\,\mathsf{K}_k + \mathsf{S}(\mathrm{j}\omega_k) - \hat{\mathsf{S}}_k\right\|_F^2, \tag{15}$$

where $\mathsf{K}_k = (\mathrm{j}\omega_k\mathsf{I} - \mathsf{A})^{-1}\mathsf{B}$, and where $\hat{\mathsf{S}}_k$ are the original frequency samples. Based on this expression, we define a weighted cost function as

$$\mathscr{E}^2 = \sum_{k=1}^{K} w_k^2 \mathscr{E}_k^2, \tag{16}$$

to be minimized subject to the passivity constraints (12).

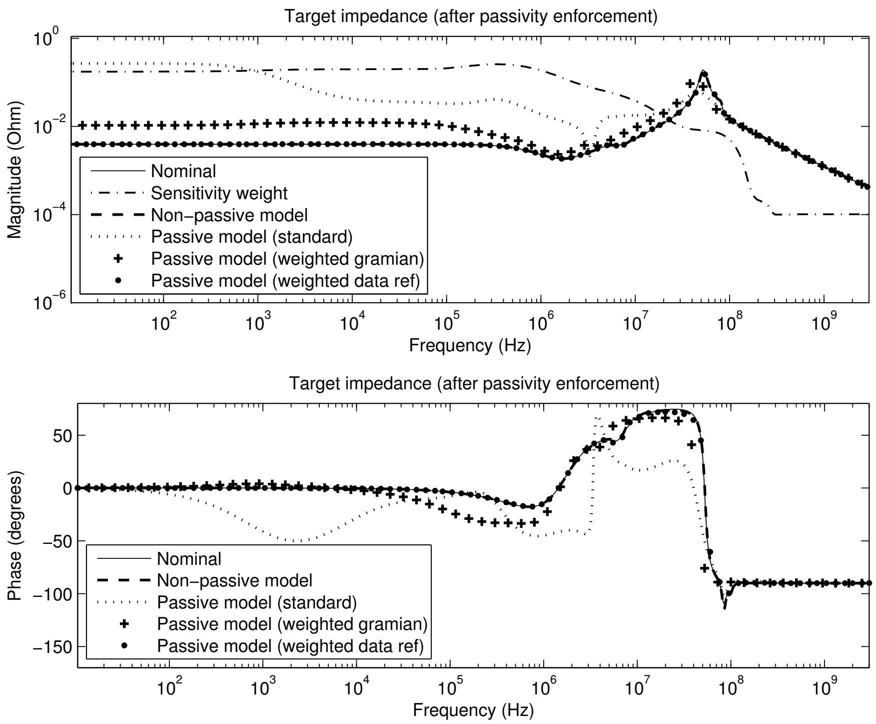

Fig. 1 Magnitude (top) and phase (bottom) of the input impedance for different models of PDN example 1, compared to the nominal impedance. See text for a detailed description.

5 Numerical examples

We apply the proposed passive model identification process to two different PDN structures, whose scattering responses are available through a broadband electromagnetic simulation. In both cases, the nominal termination conditions are also available in terms of current sources with an RC internal impedance to represent on-chip loading, various decoupling capacitors of different sizes to be placed at the package and board ports, and one Voltage Regulator Module (VRM). The transfer function of interest is the input impedance observed from one of the on-chip ports, subject to the above loading conditions at all other ports.

Figure 1 reports magnitude and phase of the reference (exact) PDN impedance for the first structure (thin solid line), based on nominal terminations, and computed using the raw scattering data. This response is compared to the non-passive model obtained from the proposed iteratively reweighted rational approximation (dashed line). We see that the accuracy of this initial model is excellent. The passive model obtained by perturbation based on a standard cost function (13) is seriously degraded (dotted line), as can be justified by the (rescaled) sensitivity function, also depicted in the top panel (dash-dotted line). The model obtained using a frequency-

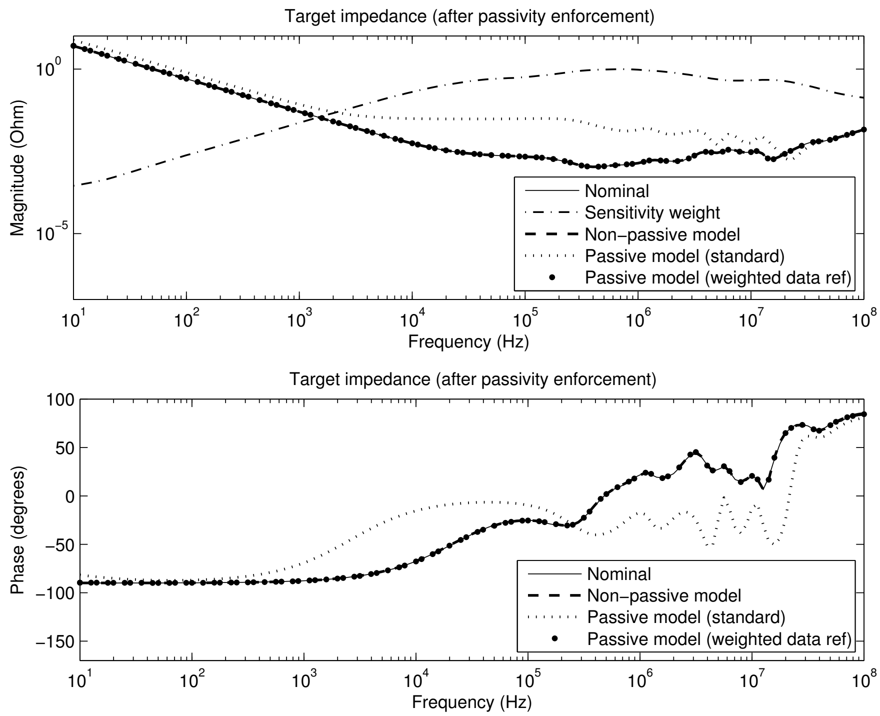

Fig. 2 As in Figure 1, but for PDN example 2.

weighted Gramian (plus markers) shows some improvement, but only using the proposed data-based cost function we are able to match almost perfectly the reference (black dot markers).

Similar conclusions can be drawn from a second application example, which refers to a different PDN structure, with similar overall characteristics and nominal termination scheme. The corresponding curves are depicted in Figure 2.

6 Conclusions

We have presented a simple approach for the identification of broadband black-box macromodels of LTI systems subject to passivity constraints, and with an input-output accuracy tuned to particular loading conditions. The proposed algorithm is based on a set of adaptively defined frequency-dependent weigths, which are used in both rational approximation and passivity enforcement stages of model identification. Numerical results obtained for two chip-package power distribution networks demonstrate the excellent performance of proposed technique.

References

1. M. Swaminathan, A.E. Engin, *Power Integrity Modeling and Design for Semiconductors and Systems*, Prentice Hall, 2007.
2. Swaminathan, M.; Joungho Kim; Novak, I.; Libous, J.P., "Power distribution networks for system-on-package: status and challenges," *IEEE Trans. on Advanced Packaging*, vol. 27, no. 2, pp. 286–300, May 2004.
3. Haihua Su; Sapatnekar, S.S.; Nassif, S.R., "Optimal decoupling capacitor sizing and placement for standard-cell layout designs," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, vol. 22, no. 4, pp. 428–436, Apr 2003.
4. Swaminathan, M.; Daehyun Chung; Grivet-Talocia, S.; Bharath, K.; Laddha, V.; Jianyong Xie, "Designing and Modeling for Power Integrity," *IEEE Trans. on Electromagnetic Compatibility*, vol. 52, no. 2, pp. 288–310, May 2010.
5. Kose, S.; Friedman, E.G., "Distributed On-Chip Power Delivery," *IEEE Journal on Emerging and Selected Topics in Circuits and Systems*, vol. 2, no. 4, pp. 704–713, Dec. 2012
6. B. Gustavsen, A. Semlyen, "Rational approximation of frequency responses by vector fitting", *IEEE Trans. Power Delivery*, Vol. 14, N. 3, pp. 1052–1061, July 1999.
7. D. Deschrijver, B. Haegeman, T. Dhaene, "Orthonormal Vector Fitting: A Robust Macromodeling Tool for Rational Approximation of Frequency Domain Responses", *IEEE Transactions on Advanced Packaging*, Vol. 30, No. 2, pp. 216–225, May 2007.
8. D. Deschrijver, M. Mrozowski, T. Dhaene, D. De Zutter, "Macromodeling of Multiport Systems Using a Fast Implementation of the Vector Fitting Method," *IEEE Microwave and Wireless Components Letters*, Vol. 18, N. 6, June 2008, pp.383–385.
9. A. Chinea, S. Grivet-Talocia, "On the Parallelization of Vector Fitting Algorithms," *IEEE Trans. on Components, Packaging, and Manufacturing Technology*, Vol. 1, n. 11, Nov. 2011, pp. 1761–1773.
10. S. Grivet-Talocia, S.B. Olivadese, P. Triverio, "A compression strategy for rational macromodeling of large interconnect structures," EPEPS 2011, San Jose, CA (USA), October 23-26, 2011, pp. 53–56.
11. C. P. Coelho, J. Phillips, L. M. Silveira, "A Convex Programming Approach for Generating Guaranteed Passive Approximations to Tabulated Frequency-Data", *IEEE Trans. CAD*, Vol. 23, No. 2, pp. 293–301, Feb. 2004.
12. S. Grivet-Talocia, "Passivity enforcement via perturbation of Hamiltonian matrices", *IEEE Trans. CAS-I*, Vol. 51, No. 9, pp. 1755-1769, Sept. 2004.
13. D. Saraswat, R. Achar and M. Nakhla, "Global Passivity Enforcement Algorithm for Macromodels of Interconnect Subnetworks Characterized by Tabulated Data", *IEEE Trans. VLSI Systems*, Vol. 13, No. 7, pp. 819–832, July 2005.
14. S. Grivet-Talocia, A. Ubolli "On the Generation of Large Passive Macromodels for Complex Interconnect Structures", *IEEE Trans. Adv. Packaging*, vol. 29, No. 1, pp. 39–54, Feb. 2006.
15. B. Gustavsen, A. Semlyen, "Enforcing passivity for admittance matrices approximated by rational functions", *IEEE Trans. Power Systems*, Vol. 16, N. 1, pp. 97–104, Feb. 2001.
16. S.Grivet-Talocia, A.Ubolli, M.Bandinu and A. Chinea. An iterative reweighting process for macromodel extraction of power distribution networks. *IEEE 22nd Conference on Electrical Performance of Electronic Packaging and Systems (EPEPS 2013), San Jose, CA, USA*, pp. 125–128, October 27–30, 2013.
17. A.Ubolli, S.Grivet-Talocia, M.Bandinu and A. Chinea. Sensitivity-based weighting for passivity enforcement of linear macromodels in power integrity applications. in *DATE 2014 - Design, Automation and Test in Europe, Dresden, Germany*, pp. 1–6, March 24–28, 2014.