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***Abstract*—In this paper, we propose an algorithm for the generation of guaranteed passive state-space models of one-port immittances from finite frequency response samples. Differently from conventional approaches, which are based on a two-step process that first fits a rational function to the samples, and only in a second stage checks and enforces passivity via perturbation, our approach provides directly a guaranteed passive model. This is achieved by computing a stable rational approximation of a spectral factor associated to the immittance function under modeling. Several examples demonstrate the feasibility of the proposed technique.**

***Index Terms*—State-space modeling, Model Order Reduction, Loewner interpolation, Passivity, Immittance.**

I. Introduction

The construction of passive macromodels of linear time-invariant systems starting from sampled frequency data is now a common step in state-of-the-art electronic design automation flows. The frequency data can come from direct measurements or from numerical electromagnetic simulations. In both cases, the sampled data are processed by an approximation, identification, or *data-driven Model Order Reduction (MOR)* algorithm, in order to derive a compact, reduced-order simulation model. Such model should be passive, in order to guarantee stable transient analyses.

Several algorithms are available for data-driven MOR, the most prominent being Vector Fitting [1] and Löwner interpolation [4], [6]. Both these techniques are however unable to provide a guaranteed passive model, and usually a post-processing model perturbation is required [3]. Most available perturbation schemes [7] are based on approximate passivity constraints, leading to iterative schemes that, although very effective in most cases, may not converge or may lead to non-optimal solutions. Alternatively, one can enforce algebraic passivity constraints such as the KYP Lemma [2] during model construction. The latter method suffers from high computational cost and is not applicable for medium to large-scale systems.

In this work, we show that, for the particular case of scalar one-port immittance systems, it is possible to combine a standard rational approximation method with a spectral factorization process, in order to obtain an algorithm that provides directly a passive state-space model. The proposed technique is based on an alternative formulation and does not require a post-processing passivity enforcement. The method is demonstrated on a simple academic test case and on a real package interconnect, for which the model is extracted from full-wave solver data.

The proposed technique can be further used to improve the accuracy of an already passive model, e.g. obtained by a standard perturbation approach. The passive model is first subjected to a spectral factorization, and the residues of the resulting spectral factor are optimized. The formulation guarantees that the resulting optimized model is passive at each stage of the optimization process. An example is provided to demonstrate this approach.

II. Formulation

Let us consider a one-port LTI system described by a finite set of its immittance frequency samples $\{(\omega_k, \breve{Y}_k),\ \ k = 1,\dots,K\}$. The main objective is to compute a state-space (descriptor) model

$$Y(s) = \mathbf{C}(s\mathbf{E} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \tag{1}$$

where all state-space matrices are real-valued, such that $Y(\mathrm{j}\omega_k) \approx \breve{Y}_k$. In addition, we require the state-space model (1) to be passive, so that $Y(s)$ is a positive real function, i.e., subject to the following constraints [2], [3]

- $Y(s)$ analytic for $\Re(s) > 0$;
- $Y(s^*) = Y^*(s)$;
- $Y(s) + Y(s)^* \geq 0$ for $\Re(s) > 0$;

where * denotes complex conjugate. Since the inverse of a positive real immittance is also positive real, the above conditions imply that the real part of both poles and zeros of $Y(s)$ cannot be positive.

As a first step in this formulation, we determine a state-space (descriptor) system, whose transfer function is

$$R(s) = Y(s) + Y(-s)\,. \tag{2}$$

For $s = \mathrm{j}\omega$, we see that $R(\mathrm{j}\omega) = 2\Re(Y(\mathrm{j}\omega))$. This system can be obtained from various different identification algorithms; in this work we use the so-called Löwner method [4], [5], [6], which provides directly real-valued descriptor matrices $\{\mathbf{E}_R, \mathbf{A}_R, \mathbf{B}_R, \mathbf{C}_R\}$ such that

$$R(s) = \mathbf{C}_R(s\mathbf{E}_R - \mathbf{A}_R)^{-1}\mathbf{B}_R\,. \tag{3}$$

This descriptor form is obtained directly from a truncated Singular Value Decomposition (SVD) applied to a matrix collecting data samples $\{(\omega_k, \breve{R}_k),\ k = 1,\dots,K\}$, where $\breve{R}_k = 2\Re(\breve{Y}_k)$, see [4] for details. Note that we assume $\breve{R}_k \geq 0$ for all k, since the starting dataset is assumed to

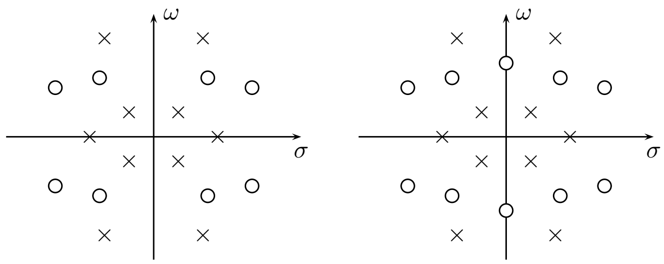

Fig. 1. Zeros (circles) and poles (crosses) of $R(s)$ in the two situations without (left) and with (right) purely imaginary spectral zeros.

comply with passivity conditions at discrete frequencies; if $\breve{R}_k < 0$ for some k, it is redefined in the following as $\breve{R}_k = 0$.

A closer look at $R(s)$ reveals that:

- due to (2), the set of zeros and poles of $R(s)$ is symmetric with respect to $s = 0$;
- due to realness of the descriptor matrices in (3), the set of zeros and poles of $R(s)$ is symmetric with respect to the real axis (all singularities are either real or they appear in complex conjugate pairs).

We conclude that the sets of zeros $\{z_i\}$ and poles $\{p_i\}$ of $R(s)$ are symmetric with respect to both real and imaginary axes. The poles p_i are found by computing the finite generalized eigenvalues of the matrix pencil $(\mathbf{A}_R, \mathbf{E}_R)$. Conversely, the zeros z_i are determined by first constructing a descriptor realization associated to $R(s)^{-1}$

$$R(s)^{-1} = \widetilde{\mathbf{C}}_R(s\widetilde{\mathbf{E}}_R - \widetilde{\mathbf{A}}_R)^{-1}\widetilde{\mathbf{B}}_R\,, \tag{4}$$

where

$$\widetilde{\mathbf{E}}_R = \begin{bmatrix} \mathbf{E}_R & \mathbf{0} \\ \mathbf{0}^\mathsf{T} & 0 \end{bmatrix}, \quad \widetilde{\mathbf{A}}_R = \begin{bmatrix} \mathbf{A}_R & \mathbf{B}_R \\ -\mathbf{C}_R & 0 \end{bmatrix}, \tag{5}$$

$\widetilde{\mathbf{C}}_R = \widetilde{\mathbf{B}}_R^\mathsf{T} = (\mathbf{0}^\mathsf{T}, 1)$, and then by finding the finite generalized eigenvalues of the matrix pencil $(\widetilde{\mathbf{A}}_R, \widetilde{\mathbf{E}}_R)$. Two situations may arise, as depicted in Fig. 1 and discussed below.

A. Case of no purely imaginary spectral zeros

If there are no purely imaginary zeros, then it is possible to split $R(s)$ into a stable and an antistable part, which can be assigned to $Y(s)$ and $Y(-s)$, according to (2). This is achieved by computing a (generalized) eigendecomposition (or an ordered QZ factorization) of pencil $(\mathbf{A}_R, \mathbf{E}_R)$, and by performing an additive decomposition into two descriptor systems associated to the stable and antistable subspaces. In this operation, special care must be taken in handling infinite eigenvalues, whose "fast" subsystem is used to determine the high-frequency direct coupling constant Y_∞. We remark that, due to passivity requirements, the real part of $Y(s)$ must be bounded at all frequencies, so that system (3) is at most index-1. We omit the detailed derivations due to lack of space. As a result, we obtain a descriptor realization

$$Y(s) = \mathbf{C}_p(s\mathbf{E}_p - \mathbf{A}_p)^{-1}\mathbf{B}_p + Y_\infty + Y_I(s)\,, \tag{6}$$

where the generalized eigenvalues of $(\mathbf{A}_p, \mathbf{E}_p)$ coincide with $\{p_i^-\}$. In (6), $Y_I(s)$ is a (still unknown) lossless immittance function characterized by purely imaginary poles (including possibily $s = 0$ and $s = \infty$), for which the following Foster representation holds

$$Y_I(s) = \frac{K_0}{s} + \sum_{\ell=1}^{n_\mathrm{im}} \frac{K_\ell s}{s^2 + \omega_\ell^2} + K_\infty s\,, \tag{7}$$

where all coefficients K_0, K_ℓ and K_∞ are real and nonnegative. Note that the contribution from $Y_I(s)$ disappears in $R(s)$, which is thus expected to have no purely imaginary poles.

In most applications $Y_I(s) = 0$. However, this part can be identified by applying another Löwner interpolation to the data samples $\breve{I}_k = 2\mathrm{j}\Im(\breve{Y}_k)$. The resulting descriptor system with matrices $\{\mathbf{E}_I, \mathbf{A}_I, \mathbf{B}_I, \mathbf{C}_I\}$ is then partitioned into three subsystems corresponding to stable, antistable, and purely imaginary eigenvalues by means of another generalized eigendecomposition (or ordered QZ factorization), and the subsystem with purely imaginary eigenvalues (including $s = \infty$) is extracted, leading to a descriptor representation of $Y_I(s)$. Combining this system with (6) leads to a descriptor model that is guaranteed passive.

B. Handling purely imaginary spectral zeros

It may be the case (see Fig. 1, right panel) that $R(s)$ includes some zeros z_i^0 that are purely imaginary. If this happens, and if these zeros have odd multiplicity, there is no way to split them into two symmetric and disjoint subsets, to be assigned to the stable and antistable factors of $R(s)$. Such zeros in fact correspond to the crossover frequencies $z_i^0 = \mathrm{j}\omega_i^0$ at which the real part of $Y(\mathrm{j}\omega)$ crosses the zero baseline, leading to passivity violation bands [8]. Such zeros must be eliminated in order to obtain a passive model $Y(s)$.

To this end, we consider the following spectral factorization

$$R(s) = W(-s)W(s)\,, \tag{8}$$

where the spectral factor $W(s)$ is defined to be stable. We seek for a rational model of the spectral factor $W(s)$, which is parameterized as

$$W(s) = \sum_i \frac{r_i}{s - p_i^-} + W_\infty, \tag{9}$$

with unknown residues r_i and direct coupling constant W_∞, subject to the fitting condition

$$|W(\mathrm{j}\omega_k)|^2 \approx \breve{R}_k, \quad k = 1, \ldots, K. \tag{10}$$

The evaluation of (9) for $s = \mathrm{j}\omega_k$ leads to the compact representation

$$W(\mathrm{j}\omega_k) = \boldsymbol{\phi}_k^\mathsf{T}\boldsymbol{x}\,, \tag{11}$$

where vector $\boldsymbol{x}$ collects the residues r_i of real poles, real/imaginary parts of r_i for complex pole pairs, and the constant W_∞. The vector $\boldsymbol{\phi}_k$ is complex-valued, constant and known. Using now (8)–(10), we obtain the following fitting condition

$$|\boldsymbol{\xi}_k^\mathsf{T}\boldsymbol{x}|^2 \approx b_k^2\,, \quad ||\boldsymbol{\xi}_k|| = 1, \tag{12}$$

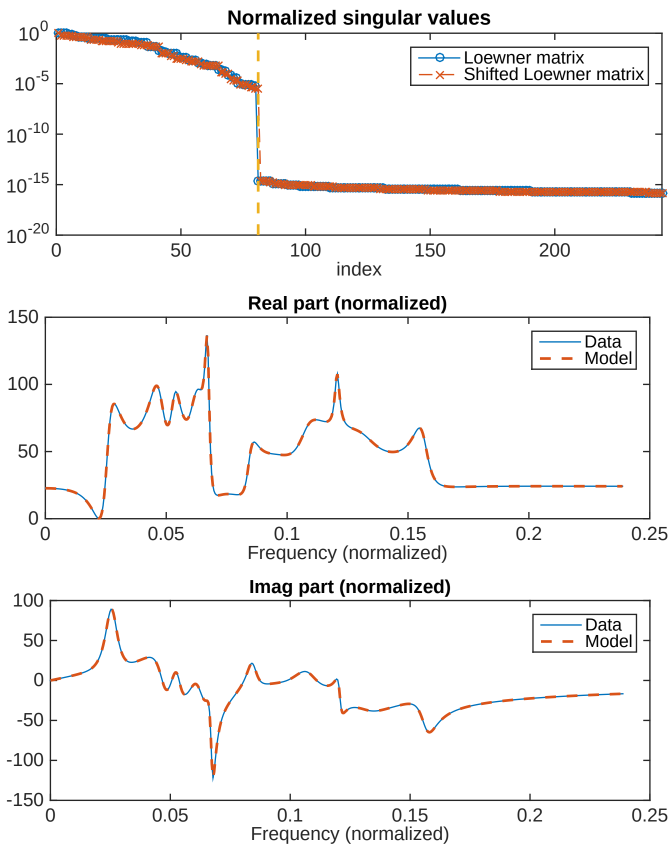

Fig. 2. Synthetic test case: singular values of Löwner and shifted Löwner matrices (top panel), the vertical dashed line indicates the detected model order (of $R(s)$, which is twice the order of the final model). A comparison between model and data samples is depicted in the middle and bottom panels.

to be enforced for $k = 1, \ldots, K$, where $\boldsymbol{\xi}_k = \boldsymbol{\phi}_k / ||\boldsymbol{\phi}_k||$ and $b_k^2 = \breve{R}_k / ||\boldsymbol{\phi}_k||^2$. Problem (12) simply amounts to finding $\boldsymbol{x}$ by controlling the magnitude of its components along a (possibly large) set of directions $\boldsymbol{\xi}_k$.

Once all constants in (9) have been computed, a regular state-space realization $\{\mathbf{A}_W, \mathbf{B}_W, \mathbf{C}_W, \mathbf{D}_W\}$ for $W(s)$ is constructed, following standard techniques. Then, the same procedure of of Sec. II-A can be used to extract the stable subsystem from the product $R(s) = W(-s)W(s)$, possibly complemented by the lossless submodel $Y_I(s)$.

III. Results

We start with a first academic test case, consisting of a synthetic passive immittance system with randomly generated poles and residues (order 40). The proposed approach was applied to identify a descriptor model, starting from $K = 500$ linearly spaced frequency samples. The top panel in Fig. 2 shows how the model order is determined, by truncation of the singular values of Löwner and shifted Löwner matrices. Middle and bottom panels of Fig. 2 illustrate that the proposed scheme is able to identify perfectly the original system. The middle panel confirms that the model is passive, since the real part is uniformly nonnegative throughout the frequency axis. This was further confirmed by the Hamiltonian passivity test [8].

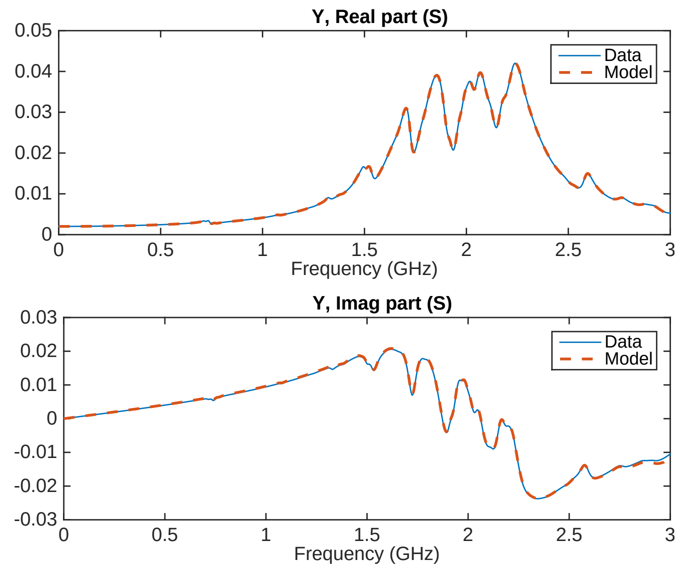

Fig. 3. Model vs. data comparison for a package interconnect.

The second example we consider is a package interconnect, for which we determine a model of the driving point admittance of a single wire terminated into a $500\,\Omega$ impedance. The starting frequency samples are obtained from a full-wave electromagnetic simulation of a CAD model of the structure. Using a truncation threshold 10^{-4} for the singular values of the Löwner matrices leads to a very accurate model (the resulting RMS relative error is 0.013), whose frequency response is compared to the raw data in Fig. 3. We remark that the model includes also a capacitive contribution $C_\infty = 0.22157$ pF, as resulting from the extraction of the high-frequency leading linear term of the model at $s = \infty$.

IV. Passive Model Refinement

In this section, we show how the proposed technique can be used to improve the accuracy of an original, already passive model. If the model is already passive, the spectral factorization problem (8) is guaranteed to be solvable (note that the resulting spectral factor $W(s)$ is not unique, since multiplication by an arbitrary all-pass factor leads to the same result for $R(s)$).

Let us consider a passive model in form (1), that for simplicity we assume in a regular state-space form with $\mathbf{E} = \mathbf{I}$. A state-space realization of the spectral factor $W(s)$ is first derived by solving an associated Algebraic Riccati Equation, as discussed in [9]. A straightforward postprocessing is then applied to extract a pole-residue representation of the spectral factor as in (9). The poles p_i^- are then retained, and the optimization framework discussed in Section (II-B) is applied to obtain a new set of residues $r_i \to \hat{r}_i$ and direct coupling $W_\infty \to \hat{W}_\infty$ by solving (10) or its normalized form (12). Once the optimized spectral factor is available, reconstruction

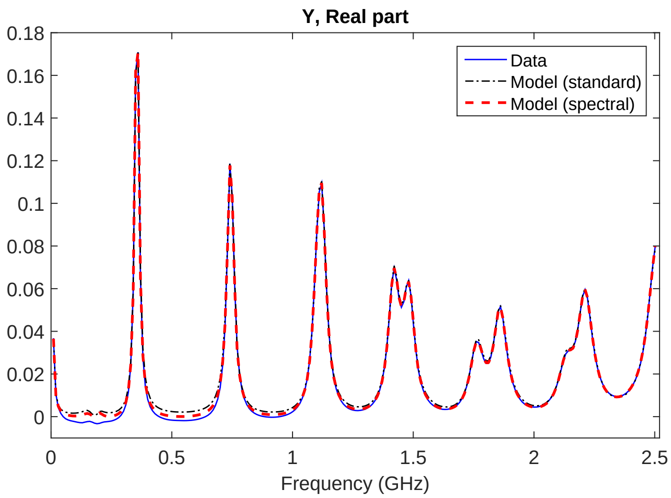

Fig. 4. Model vs. data (real part) comparison for a PCB interconnect.

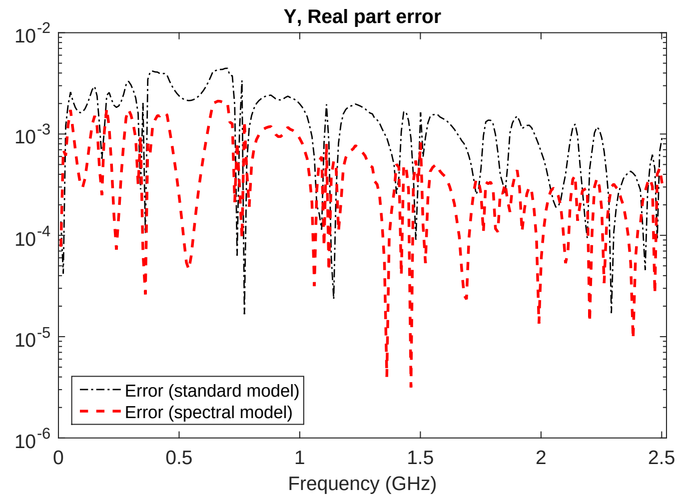

Fig. 5. Model vs. data error for a PCB interconnect, before and after optimization.

of the full immittance model $Y(s)$ is performed, as discussed in Section II.

We illustrate the performance of the proposed passive model refinement on a PCB interconnect example, known via measured frequency responses (1500 samples up to 15 GHz). We extract a passive model for the input admittance $Y_{11}(s)$, whose extracted frequency samples from the measurements are affected by localized passivity violations in the low frequency band up to 500 MHz. Figure 4 compares the real part of the input immittance model before and after optimization to the corresponding original (non-passive) data. The accuracy improvement of the model after optimization is demonstrated in Fig. 5.

V. Conclusions

We proposed a new approach to compute a guaranteed passive state-space (descriptor) model from a finite set of frequency samples of a one-port immittance system. This is achieved by a special formulation of the rational function approximation problem, applied to a spectral factor of the considered immittance. As formulated, the approach is applicable only for one-port systems. The generalization to the multiport case is under way.

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