POLITECNICO DI TORINO
Repository ISTITUZIONALE

Teaching Reitlinger Cycles To Improve Students' Knowledge And Comprehension Of Thermodynamics

Original
Teaching Reitlinger Cycles To Improve Students' Knowledge And Comprehension Of Thermodynamics / Sparavigna, A.C.. - In: MECHANICS, MATERIALS SCIENCE & ENGINEERING JOURNAL. - ISSN 2412-5954. - ELETTRONICO. - 2016:2(2016), pp. 78-83.

Availability:
This version is available at: 11583/2623524 since: 2016-02-15T10:33:18Z

Publisher:
Magnolithe

Published
DOI:

Terms of use:

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

Publisher copyright

(Article begins on next page)

12 October 2026

Teaching Reitlinger Cycles To Improve Students' Knowledge And Comprehension Of Thermodynamics

Amelia Carolina Sparavigna[1]

1 – Department of Applied Science and Technology, Politecnico di Torino, Torino, Italy

Keywords: Thermodynamics, Thermodynamic cycles, Regenerative cycles, Thermal efficiency.

Abstract. The second law of thermodynamics puts a limit on the thermal efficiency of heat engines. This limit value is the efficiency of the ideal reversible engine represented by the Carnot cycle. During the lectures on physics, the emphasis on this cycle is generally so strong that students could be induced to consider the Carnot cycle as the only cycle having the best thermal efficiency. In fact, an entire class of cycles exists possessing the same maximum efficiency: this class is that of the regenerative Reitlinger cycles. Here we propose to teach also these cycles to the engineering students of physics classes, to improve their knowledge and comprehension of thermodynamics.

Introduction: Generally, the Carnot cycle is the only thermodynamic cycle that, during the lectures on physics, is discussed as having the maximum possible thermal efficiency. This happens because Carnot cycle is directly connected to the second law of thermodynamics, which puts a limit on the thermal efficiency of heat engines. This limit value is the efficiency of the ideal reversible engine cycle represented by the Carnot cycle. Sometimes, an approach considering only Carnot engines with emphasis on their efficiency, could yield the following result: it is unknown that an entire class of cycles exists, having a thermal efficiency which is the same of that of Carnot cycle. This is the class of the regenerative Reitlinger cycles. Of course, since a large part of engineering students will be required as engineers to deal with relatively simple thermodynamic problems, a discussion of Reitlinger cycles could appear as unnecessary. However, it is unquestionable that a proper knowledge of the fundamentals of thermodynamics is necessary for engineers as well as for scientists in general. For this reason, in the following discussion, we will propose some notes suitable for teaching these cycles to students of physics classes, to improve their knowledge and comprehension of thermodynamics.

Reitlinger cycles. The Reitlinger cycles consist of two isothermal and two polytropic processes of the same kind [1,2], so that the heat which is absorbed during a polytropic, is exactly the same that it is rejected on the other polytropic process. Therefore, if we have a perfect regeneration of heat, by means of which the heat rejected during the polytropic is transferred to a thermal storage (the regenerator) and then transferred back to the working fluid, the thermal efficiency of the Reitlinger cycle equals that of the Carnot cycle (in fact, it is a Reitlinger cycle too).

Of all the Reitlinger cycles, the Carnot cycle is unique in requiring the least regeneration, namely, none at all because its polytropics are adiabatics [1]. Let us note that the mechanical work of the Carnot cycle is not the best we can obtain between extremal states. We can easily evidence this fact from the diagram in Figure 1, which is comparing Carnot and Stirling cycles, having the same temperature and volume extremes [1]. In the Figure 2, we can see how, in general, a Reitlinger cycle can be different from a Carnot cycle, in a p-V diagram. Working between the same isothermals, with the same thermal efficiency, a regenerative Reitlinger cycle can give more work or less work, depending on the polytropic process the cycle is performing between the same extremal states.

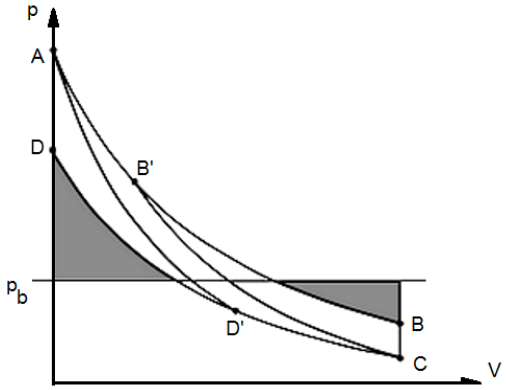

Fig. 1. The figure (adapted from Ref.1) shows a Carnot cycle inscribed in a Stirling cycle in a p-V diagram. The optimum constant buffer pressure is also shown. The work of the Stirling cycle ABCD is greater than the work of Carnot cycle AB'CD'

Let us note that the ideal Stirling cycle is also a Reitlinger cycle, having as polytropics two isochoric segments. It is the most popular example of a cycle having the same thermodynamic efficiency of the Carnot cycle; however, to attain this result, the Stirling cycle makes quite heavy demands on the process of regeneration [3].

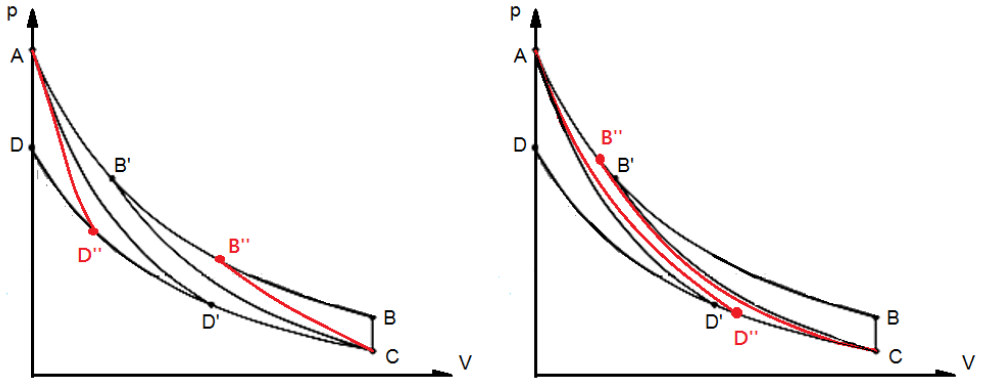

Fig. 2. The figure shows how a Reitlinger cycle can be different from a Carnot cycle, in a p-V diagram. Working between the same isothermals, with the same thermal efficiency, a regenerative Reitlinger cycle AB''CD'' can give more work or less work, depending on the polytropic process the cycle is performing between the same extremal states

As observed in [6], there are ten elementary power cycles which follow from the combinations of five typical thermodynamic changes of state. In the Figure 3, we can see them and the names of their inventors (for other cycles, see [7]). In [6], Carnot, Ericsson and Stirling cycles are distinguished from the Reitlinger cycles, which have the most general form in idealized cycles [4,5], because they have a specific importance in thermodynamics. In these cycles we have, besides the two isothermal processes, the two polytropic regenerative processes realized by adiabatics, isochoric and isobaric processes, respectively.

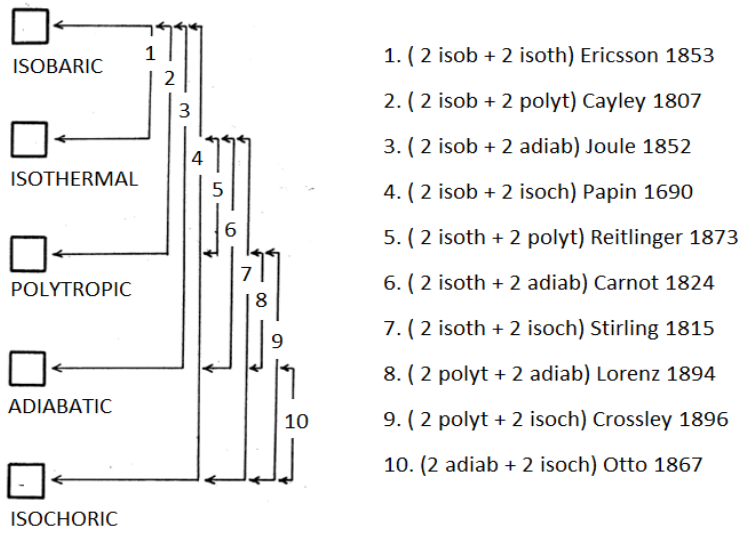

Fig. 3. The elementary thermodynamic cycles (figure adapted from [6])

Thermal Efficiency: For any thermodynamic cycle, reversible or irreversible, after one cycle, the working fluid is again in its initial state and thus the change of its internal energy is zero. In this manner, the first principle of thermodynamics tells us that the mechanical work produced by the cycle is the difference of input heat energy Q_{in} minus the energy dissipated in waste heat Q_{out}. Heat engines transform thermal energy into mechanical energy or work, W, so that $W = Q_{in} - Q_{out}$. We can calculate the thermal efficiency of the cycle as the dimensionless performance measure of the use of thermal energy. The thermal efficiency of a heat engine is the percentage of heat energy which is transformed into work, so that:

$$\eta = \frac{W}{Q_{in}} \tag{1}$$

For a Carnot engine, it is $\eta = 1 - T_C/T_H$, where T_H, T_C are the temperatures of the furnace and of the cold sink, respectively.

Let us discuss the thermal efficiency of the Stirling cycle. Using a p-V diagram, the cycle appears as in the Figure 4. In the same figure, the Ericsson cycle and Reitlinger cycle are also shown.

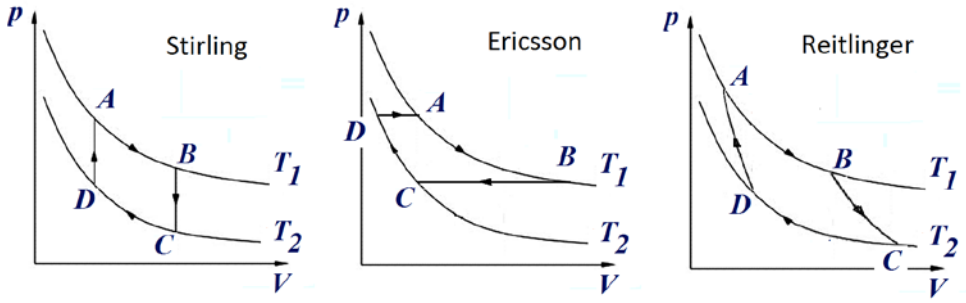

Fig. 4. Stirling, Ericsson and Reitlinger cycles in p-V diagrams.

The work can be easily calculated as:

$$W = nR(T_1 - T_2)\ln\frac{V_B}{V_A} \tag{2}$$

In (2), *n* is the number of moles and *R* the universal gas constant. Heat is gained by the thermodynamic system from the reversible isochoric transformation from D to A and during the isothermal path AB. During isochoric process, heat gained is: $Q_{isoc} = nC_V(T_1 - T_2)$. C_V is the molar specific heat for an isochoric process. During isothermal process, the heat gained is $Q_{isot} = nRT_1 \ln(V_B / V_A)$.

Let us note that, during the isochoric process, the fluid is obtaining heat from an infinite number of thermal reservoirs [8]. This same amount of heat is lost during the isochoric cooling process, with a thermal exchange with the same reservoirs. Then, for each of the infinite thermal reservoirs that we meet during the isochoric reversible process, it happens what we see in the Figure 5. In this figure, we have two thermal machines that must have the same efficiency, to satisfy the second principle of thermodynamics. Let us suppose the efficiency of the right machine larger than that of the left one. Let us consider the same work W produced by the two machines, and operate the machine on the left in reversed manner. It is easy to see that, if we consider the net result of these two machines operating together, we have that some heat is transferred from the low temperature reservoir to the high temperature reservoir, violating the Clausius statement of the second principle of thermodynamics. We have the same result if we consider the efficiency of the left machine larger than that of the right one, and operate this last machine in reversed manner.

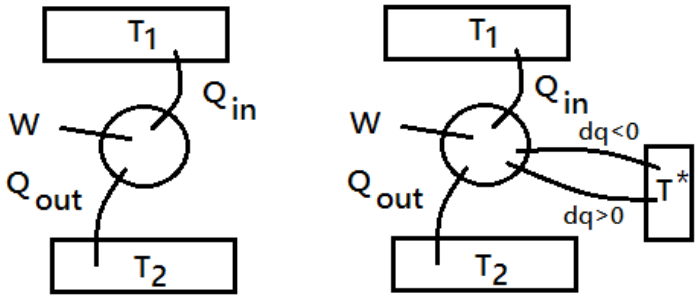

Fig. 5. The two reversible cycles in the figure have the same efficiency. If it were not so, we should violate the second principle of thermodynamics. Let us suppose the efficiency of the right machine larger than that of the left one. Let us consider the same work W produced by the two machines, and operate the machine on the left in reversed manner. It is easy to see that the net result of these two machines operating together is that of transferring some heat from the low temperature reservoir to the high temperature reservoir, violating the Clausius statement of the second principle of thermodynamics. We have the same result, supposing the efficiency of the left machine larger than that of the right machine

Calculating efficiency of working fluid with regeneration: If we consider a regenerative Stirling cycle from an engineering perspective, we have in it the regenerator which is storing the heat. Therefore the abovementioned thermal reservoirs are not involved. Consequently, considering the system made of working fluid and regenerator, the thermal efficiency is:

$$\eta = \frac{W}{Q} = \frac{nR(T_1 - T_2)\ln\frac{V_B}{V_A}}{nRT_1 \ln\frac{V_B}{V_A}} = 1 - \frac{T_2}{T_1} \tag{3}$$

In (3), *Q* is the heat the system receives during the high temperature isothermal process, because the heat received from the regenerator is that lost by the fluid during cooling isochoric process. This efficiency is equal to that of a Carnot cycle which is working between the same two isothermal processes. We can repeat the calculation for the Ericsson cycle. The work is:

$$\begin{aligned} W &= nC_P(T_1 - T_2) + nRT_1 \ln\frac{V_B}{V_A} - nC_P(T_1 - T_2) + nRT_2 \ln\frac{V_D}{V_C} \\ &= nRT_1 \ln\frac{V_B}{V_A} - nRT_2 \ln\frac{nRT_2\, p_D}{p_C nRT_2} = nRT_1 \ln\frac{V_B}{V_A} - nRT_2 \ln\frac{p_A}{p_B} \\ &= nRT_1 \ln\frac{V_B}{V_A} - nRT_2 \ln\frac{nRT_1 V_B}{V_A nRT_1} = nRT_1 \ln\frac{V_B}{V_A} - nRT_2 \ln\frac{V_B}{V_A} \end{aligned} \tag{4}$$

In (4), *Cp* is the molar specific heat at constant pressure. It is clear that the heat lost and gained during the two isobaric processes is the same. Therefore, the thermal efficiency, in the case of a perfect regeneration, is given by:

$$\eta = \frac{W}{Q} = \frac{nR(T_1 - T_2)\ln\frac{V_B}{V_A}}{nRT_1 \ln\frac{V_B}{V_A}} = 1 - \frac{T_2}{T_1} \tag{5}$$

In (5), *Q* is the heat the system receives during the high temperature isothermal process. Let us conclude with a Reitling cycle, where polytropics are given by equations $pV^{\alpha} = const$ and $TV^{\alpha-1} = const$. The molar specific heat of such polytropic process is C_α. Let us note that from polytropic equation we have (see Figure 4):

$$\begin{aligned} T_1 V_A^{\alpha-1} &= T_2 V_D^{\alpha-1} \\ T_1 V_B^{\alpha-1} &= T_2 V_C^{\alpha-1} \end{aligned} \tag{6}$$

Therefore, we have:

$$\frac{V_A^{\alpha-1}}{V_B^{\alpha-1}} = \frac{V_D^{\alpha-1}}{V_C^{\alpha-1}} \rightarrow \frac{V_A}{V_B} = \frac{V_D}{V_C} \tag{7}$$

Then:

$$\begin{aligned} W &= nC_\alpha(T_1 - T_2) + nRT_1 \ln\frac{V_B}{V_A} - nC_\alpha(T_1 - T_2) + nRT_2 \ln\frac{V_D}{V_C} \\ &= nRT_1 \ln\frac{V_B}{V_A} - nRT_2 \ln\frac{V_B}{V_A} \end{aligned} \tag{8}$$

Again, we find a thermal efficiency of the system (fluid and regenerator), which is equal to that of the Carnot engine. Therefore, since the polytropic index α can have any value, we have an infinite number of thermodynamic cycles that have the same value of thermal efficiency, equal to that of the Carnot cycle when operating between the same two isothermal processes. Let us stress that these cycles incorporate a regenerative heat transfer process, in place of adiabatic compression and expansion of the Carnot cycle [5], or, if preferred, an infinite number of mono-thermal processes, not influencing the efficiency of the cycle. Moreover, during lectures, it is better to remark that the

fact of possessing the same thermal efficiency does not mean that the same work is obtained from different reversible cycles, when they are operating between the same extremal states.

References

[1] J.R. Senft, Mechanical Efficiency of Heat Engines, Cambridge University Press, 2007.

[2] I. Kolin, The Evolution of the Heat Engine, Longman, 1972.

[3] J.R. Senft, An Introduction to Stirling Engines, Moriya Press, 1993.

[4] J. Reitlinger, Uber Kreisprozesse zwischen zwei isothermen. Z. Ost. Ing. Arch. Ver. 1876.

[5] G. Walker, Cryocoolers, Part 1: Fundamentals, Plenum Press 1983.

[6] I. Kolin, S. Koscak-Kolin, M. Golub, Geothermal Electricity Production by means of the Low Temperature Difference Stirling Engine, Proceedings World Geothermal Congress 2000, Kyushu - Tohoku, Japan, May 28 - June 10, 2000 , 3199-3203.

[7] J.Selwin Rajadurai, Thermodynamics and Thermal Engineering, New Age International, 2003.

[8] P. Mazzoldi, M. Nigro, C. Voci, Fisica, S.E.S. 1991.