# Prioritization of Engineering Characteristics in QFD in the case of Customer Requirements orderings

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**Abstract**

Quality Function Deployment (QFD) is an effective tool to orient the design of a product and related production processes towards the real exigencies of the end-user. Its first phase – the House of Quality – is aimed at translating Customer Requirements (*CR*s) into Engineering Characteristics (*EC*s) of the product of interest, also determining an *EC*s' prioritization. All of the techniques proposed for tackling this problem are based on the assumption that the importance of each *CR* is expressed on interval or ratio scales (i.e. cardinal scales). To this end, customer evaluations – naturally expressed on ordinal scales – are artfully turned into numbers.

This paper introduces a novel technique – denominated as Ordinal Prioritization Method – that can be applied to prioritize *EC*s. The method addresses the problem of the prioritization of ECs when the importance of *CR*s is given on an ordinal scale. The description of the method is supported by a some application examples.

**Keywords:** *Quality Function Deployment; House of Quality; data fusion; aggregation; prioritization; engineering characteristics; customer requirements; Independent Scoring Method, ordinal scale*.

## 1. Introduction

Quality Function Deployment (QFD) is a tool that is used to guide the design of a product toward the needs of the end-user (Kogure and Akao 1983). Akao et al. (1996) defined QFD as a "method to transform user demands into design quality, to deploy the functions forming quality, and to deploy methods for achieving the design quality into subsystems and component parts, and ultimately to specific elements of the manufacturing process".

Over the years, an extensive literature on the subject has been produced and today QFD is widely recognized as an effective approach to pursue customer satisfaction (Matzler and Hinterhuber 1998; Cristiano, Liker et al. 2001; Franceschini 2001; Chen and Chuang 2008; Mehrjerdi 2010; Nahm, Ishikawa et al. 2013). The implementation of QFD proved to bring significant improvements in the process of product development, including earlier and fewer design modifications, fewer start-up issues, improved cross-functional communications, improved product quality, reduced product development time and cost, etc. (Hauser and Clausings 1996; Cristiano, Liker et al. 2001; Mehrjerdi 2010).

Operatively, a complete QFD is composed of four phases which deploy Customer Requirements (*CR*s) throughout the planning process. In QFD, each phase's output becomes the input of the next phase. The construction of House of Quality (HoQ) is the first phase of QFD. The goal of this phase is to transform *CR*s into Engineering Characteristics (*EC*s) of the product/service of interest, also determining an *EC*s' prioritization. However, this conversion requires two controversial assumptions (Franceschini and Rupil 1999; Andronikidis, Georgiou et al. 2009):

- The importance of each *CR* is assumed to be expressed on a cardinal scale (interval or ratio scales), i.e. in the form of a number. This number is generally obtained by translating customer feelings – normally expressed on ordinal scales – into a numerical scale. This artificial encoding can lead to errors or inconsistencies in the evaluation.
- The prioritization of *EC*s is traditionally carried out through methods – such as the Independent Scoring Method – that generally require the numerical conversion of the relationship matrix symbols into numbers. This is again an artificial promotion of an information given on an ordinal scale into a cardinal one.

This paper proposes an alternative approach to prioritize *EC*s.

The remainder of this paper is organized into six sections. Section 2 briefly presents QFD particularly focusing on HoQ. Some criticalities about the deployment of HoQ are also presented. Section 3 discusses a possible analogy between decision-making processes and *EC*s prioritization. Section 4 introduces a novel approach to prioritize *EC*s and Section 5 formalizes its application to HoQ. The two concluding section highlights the main implications, limitations and original contributions of this work. Finally, the appendix contains a more articulated example for a deeper comprehension of the proposed method.

## 2 Quality Function Deployment

### *2.1 The four phases of QFD*

Typically, a complete QFD system is composed of four phases which deploy the customer requirements throughout the planning process (see Fig. 1). Operatively, the editing of each phase is demanded to a cross-functional team. In the first phase, (*House of Quality* or *Product Planning Matrix*), Customer Requirements (*CR*s) are related to Engineering Characteristics (*EC*s) of the product. Then, *EC*s are associated to critical part characteristics in the second phase (*Part Deployment Matrix*). The *Process Planning Matrix* relates the characteristics of the single subsystem with its respective production process. Finally the *Process and Quality Control Matrix* defines inspection and quality control parameters and methods to be used in the production process.

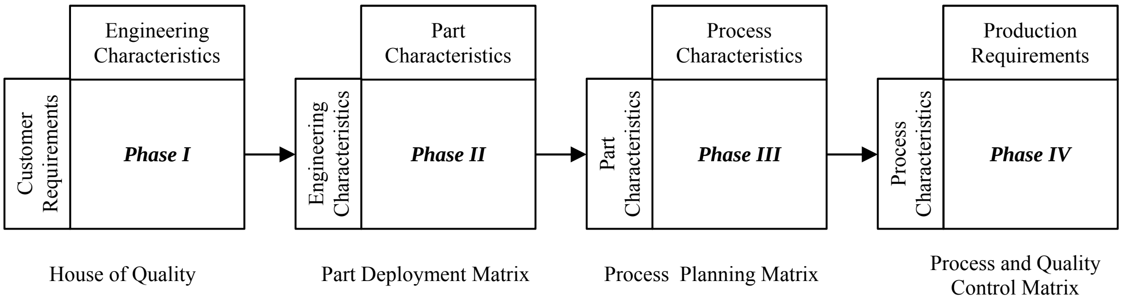


**Fig. 1 Scheme of the four phases of QFD. Adapted from (Hauser and Clausings 1996).**

Being the first phase, the House of Quality is widely recognized as fundamental and strategic (Franceschini 2001; Tontini 2007; Li, Tang et al. 2009; Li, Tang et al. 2010). Errors made at this stage can propagate throughout all the subsequent phases of QFD.

## *2.2 Eight steps of HoQ*

With reference to its structure (Fig. 2), the construction of HoQ can be broadly structured into ten steps:

**Step 1**: The first step is to define *CR*s for the product/service concerned. Possible approaches for collecting the so called "Voice of the Customer" (VoC) include surveys, focus groups, individual interviews, etc.. Then, the VoC is generally reorganized into basic *CR*s by means of several techniques (customer requirements tree, affinity diagrams, hierarchical cluster analysis, etc.).

**Step 2**: *CR*s are prioritized basing on several alternative approaches. The simplest and most widely used methods require the involvement of customers who are asked to translate their preferences on cardinal scales (for example by providing judgments on a scoring scales from 1 to 5 or 1 to 10) (Griffin and Hauser 1993; Hauser and Clausings 1996). Notice that judgments are naturally expressed by customer on ordinal scales, their conversion into number represents an artificial promotion into a cardinal scale. Depending on the (arbitrary) choice of the scale, the results of the prioritization may change significantly (Franceschini and Rupil 1999). An important and widely used alternative to this approach is Analytic Hierarchy Process (AHP) (Saaty 1988). This method – along with its many variants – has been widely used to measure the relative degree of importance of each customer need. A representative sample of customers is required to compare each pair of *CR*s. However, since customers are asked for a large number of comparisons – even for relatively simple cases – these approaches take the risk of becoming tedious, leading the customer to inconsistent judgments (Nahm, Ishikawa et al. 2013).

Many other methods, which partially improve the aforementioned approaches, have been proposed (Saaty 2003; Partovi 2007; Tontini 2007; Lee, Wu et al. 2008; Nahm, Ishikawa et al.

2013). The common feature of these approaches is the translation of customer evaluations into cardinal information, i.e. the promotion – without any metrological foundation – of subjective orderings into an artificial rating of *CR*s – generally named as *relative importance*.

**Step 3 and 4**: Subsequently (and optionally) the obtained rating is corrected considering the perception of competitors positioning and according to some strategic considerations so as to obtain the so called *relative weight* (Franceschini 2001). Again, the ultimate result is an artificial rating of *CR*s.

**Step 5**: *EC*s related to *CR*s are identified by the cross-functional team. Proper *EC*s can be generated from current (or competitors') product/service standards or selected by cause-effect analyses (Franceschini 2001).

**Step 6**: The construction of HoQ requires the definition of a relationship matrix. In this step, the cross-functional team has to indicate how the technical decisions affect the satisfaction of each *CR*. In other words, the team is asked to qualitatively express the relationships between customer requirements and *EC*s. The relationships are expressed on an ordinal scale, typically codified into specific conventional symbols.

**Step 7**: The analysis of correlation among *EC*s allows to determine which *EC*s are redundant or supporting each other and which ones are in conflict.

**Step 8**: This step is to prioritize *EC*s. To this purpose, several approaches are possible. The traditional and most used method is the *Independent Scoring Method* (Akao 1988). Basing on the ratings of *CR*s and the relationship matrix, it provides a rating of *ECs*. It requires two operative steps. The first and more controversial step consists in converting the relationship matrix into a cardinal matrix according to an arbitrary convention: the most typical approach is that of expressing the relationships between *CR*s and *EC*s on four levels – i.e. strong, medium, weak and absent relationships – and then encode them into four numeric values, respectively, 9, 3, 1 and 0. Then the so called *relative importance* (or the *relative weight*) of each *EC*, i.e. its rating, is evaluated as a function of the relative importance of *CR*s and the transformed relationship matrix (Akao 1988). The typical model used is:

$$w_j = \sum_{i=1}^{n} d_i r_{ij} \tag{1}$$

where $w_j$ is the relative importance of the *j*-th *EC*, $d_i$ is the relative importance of the *i*-th *CR* and $r_{ij}$ is the numerical value corresponding to the relationship between the j-th *EC* and the *i*-th *CR*.

Alternatively, other less diffused approaches can be used for the prioritization of *EC*s: (i) Multi

Criteria Decision Aid (MCDA) techniques (Franceschini and Rossetto 1995; Han, Kim et al. 2004); (ii) Borda's method and equivalent techniques based on pairwise comparison (Dym, Wood et al. 2002); (iii) techniques based on fuzzy logic (Büyüközkan and Çifçi 2012; Yan, Ma et al. 2013); (iv) hybrid methods using typical approaches deriving from decision-making contexts (Chan and Wu 1998; Li, Jin et al. 2014; Zaim, Sevkli et al. 2014); etc.

**Step 9 and 10**: The technical benchmarking compares the company and its competitors in terms of quality performance regarding each *EC*. Then, for each *EC* a target value is established according to the results of the benchmarking and the importances of *EC*s. These target values are used as input data for the design of the final product/service.

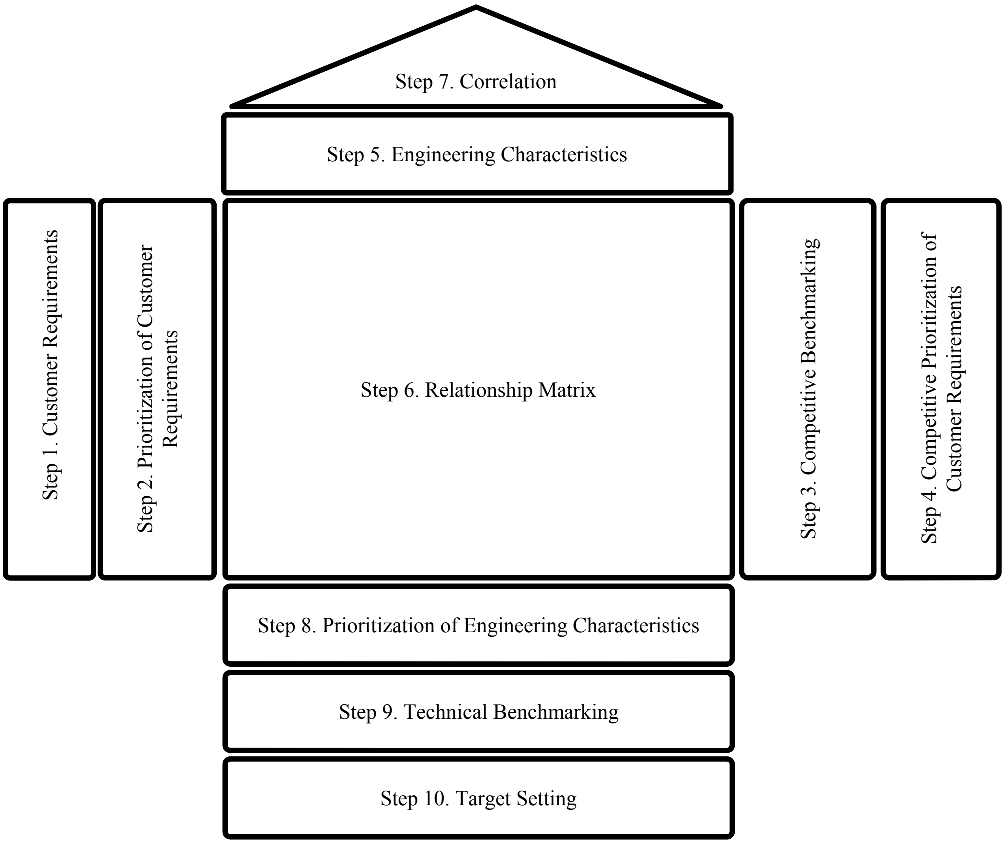


**Fig. 2 Main steps of House of Quality (Nahm, Ishikawa et al. 2013).**

### *2.3 Aim of the paper*

Andronikidis et al. (2009) propose an analysis of QFD highlighting the main criticalities of the traditional approach. In particular the prioritization of *CR*s and *EC*s (i.e. Steps 2 and 8) are among the most controversial and discussed steps of HoQ:

i. With regard to step 2, a critical point is the assignment of an importance rating to each *CR* so as to define an ordering among *CR*s. To this end, subjective information such as customer feelings are – more or less artfully – converted into numerical weights (relative importance).

ii. The prioritization of *EC*s, instead, is traditionally carried out through the Independent

Scoring Method. Such approach requires the numerical conversion of the relationship matrix symbols into numbers. This is again a promotion of an ordinal scale into a cardinal one. Depending on the arbitrary choice of the adopted numerical scale, the result may change significantly (Franceschini and Rupil 1999).

This paper aims at defining an alternative approach to overcome the above-mentioned issues.

With reference to the prioritization of *CR*s, the approaches briefly listed in the previous section substantially differ in the way they collect customers' opinion about the importance of CRs. This information can be collected through different type of judgments:

i. pairwise comparisons between different *CR*s. In this case a sample of customers is asked to compare each pair of *CR*s, stating which one is more important;
ii. assessments of *CR*s on cardinal scales. In this framework, the sample of customer has to rate all the *CR*s on an arbitrary numerical scale;
iii. preference ordering. Each customer has to sort *CR*s in order of importance (eventually including ties or incomparabilities); etc..

Pairwise comparison is an effective approach when dealing with a small number of *CR*s, since the number of *CR*s increases the complexity of data collection.

On the other hand the assessments of *CR*s on cardinal scales is by its nature more complex: in this case, the customer is asked to translate his feelings in a numerical scale (Franceschini and Rupil 1999). The effort required by this encoding can lead to errors or inconsistencies in the evaluation. Moreover, the choice of the resolution of the rating scale is arbitrary and can lead to different results.

For these reasons, authors believe that the option to ask for an importance order of *CR*s is easier and more “natural” for customers. Therefore a legitimate issue is that concerning the problem of *EC*s prioritization when, instead of a set of ratings, an importance ordering between the *CR*s is available.

In other terms, the purpose of this paper is to answer the question: "how to define a hierarchy of *EC*s when the importance of *CR*s is given on an ordinal scale?"

In order to answer this question, an analogy between the HoQ problem and a decision-making process is proposed in the following section.

## 3 Prioritization of *EC*s as a decision-making process

In general, decision making can be regarded as the cognitive process driving to the selection of a course of action among several alternatives. In this sense, the prioritization of *EC*s can be interpreted as a decision-making process in which multiple decision-makers need to define an importance order among a set of possible actions. *CR*s can be considered as the decision-makers and $EC$s as the possible alternatives. For each $CR$, the relationship matrix specifies an importance ordering among $ECs$. In detail, the following general assumptions are considered:

- $CR$s and $EC$s uniquely defined;
- The relationship matrix is also defined by the cross-functional team on an ordinal scale with $Q$ levels ($L_1, L_{2,\ldots,} L_Q$).
- A weak ordering between the $CR$s exists, i.e. an importance order with possible ties is admitted.

In general, the $ECs$ prioritization problem can be stated as the problem of identifying the best fused ordering of $ECs$, considering (i) the importance order among $CR$s and (ii) the set of orderings among $EC$s related to each $CR$.

As an example consider the schematic HoQ proposed in Tab. 1. This HoQ has four $CR$s and five $ECs$. In this specific framework, $CR$s are assumed to be ranked as: $CR_1 > CR_2 > CR_3 \sim CR_4$.

$CR$s are related to the $ECs$ through the relationship matrix which is encoded on an ordinal scale with $Q$ = 4 levels. In detail, strong, medium and weak relationship are the first three levels of the ordinal scale, while the fourth one corresponds to the absence of a relationship.

**Tab. 1. Schematic example of HoQ, Relationship Matrix.**

| | $EC_1$ | $EC_2$ | $EC_3$ | $EC_4$ | $EC_5$ |
|---|---|---|---|---|---|
| $CR_1$ | ○ | ● | △ | | |
| $CR_2$ | | ● | | ● | |
| $CR_3$ | | △ | ● | △ | |
| $CR_4$ | | | ○ | ○ | ○ |

| Relationship | |
|---|---|
| ● | strong |
| ○ | medium |
| △ | weak |

With reference to $CR_1$, Tab. 1 shows that $EC_2 > EC_1 > EC_3 > EC_4 \sim EC_5$. Also, it is known that the relationship between $CR_1$ and $EC_2$ is "strong"; between $CR_1$ and $EC_1$ is "medium" and between $CR_1$ and $EC_3$ is "weak". Furthermore $EC_4$ and $EC_5$ are not related to $CR_1$. Referring to $CR_2$, Tab. 1 indicates that $EC_2 \sim EC_4 > EC_1 \sim EC_3 \sim EC_5$. It is also known that the relationship between $CR_2$ and both $EC_2$ and $EC_4$ is "strong", while $CR_2$ is not related to $EC_1$, $EC_3$ and $EC_5$. Similar considerations hold for $CR_3$ and $CR_4$. All this is summarized in Tab. 2.

**Tab. 2. Importance ordering related to *CR*s**

| *Relationship* | $CR_1$ | $CR_2$ | $CR_3$ | $CR_4$ |
|---|---|---|---|---|
| *Strong* | $\{EC_2\}$ | $\{EC_2, EC_4\}$ | $\{EC_3\}$ | Null |
| *Medium* | $\{EC_1\}$ | Null | Null | $\{EC_4, EC_3, EC_5\}$ |
| *Weak* | $\{EC_3\}$ | Null | $\{EC_2, EC_4\}$ | Null |
| *Absent* | $\{EC_4, EC_5\}$ | $\{EC_1, EC_3, EC_5\}$ | $\{EC_1, EC_5\}$ | $\{EC_1, EC_2\}$ |

At this stage, the issues of HoQ is how to obtain the best fused ordering out of the ones presented in Tab. 2. To this purpose, next section introduces the Ordinal Prioritization Method.

## 4 Ordinal Prioritization Method

The *Ordinal Prioritization Method* (OPM) is a variant of Yager's algorithm (2001) specifically defined for being more "suitable" for the application to HoQ. Since the OPM is derived from a decision-making context, the concept of "importance ordering" is herein deliberately confused with that of "preference ordering".

Assume that there are $M$ decision-makers ($D_1$, $D_2$, …, $D_M$), each of which defines an evaluation of $n$ alternatives ($a$, $b$, $c$, …) on an ordinal scale with $Q$ levels ($L_1$, $L_{2,\ldots,}$ $L_Q$). This evaluation scale is assumed to be shared among decision-makers. Notice that the present discussion is made in general terms, leaving $Q$ as a parameter. However, in HoQ, $Q$ is generally set to a relatively low value, typically $Q = 4$ as for the examples in this paper. This choice comes from the necessity to take into account two aspects: (i) $Q$ must be large enough to ensure a sufficient discrimination between different judgments and (ii) small enough as to guarantee the simplicity of evaluation. In fact, the greater the number of levels $Q$, the greater is the probability that the evaluator may confuse neighbouring levels.

Since each alternative is evaluated by any decision-maker, a preference vector corresponding to each decision-maker can be defined. The preference vector directly stems from the opinion expressed by decision-makers. To this end, the following convention is adopted. For each ($j$-th) decision-maker, alternatives are ordered in a preference vector of size $Q$. For simplicity, this vector will be denominated as the decision-maker itself, i.e., $D_j$. Alternatives are positioned in the component of the preference vector according to the relevant level of the ordinal scale.

As an example, consider the situation in Tab. 3 in which six alternatives ($a$, $b$, $c$, $d$, $e$ and $f$) are evaluated on a 5 level ordinal scale ($L_1$, $L_{2,\ldots,}$ $L_5$).

**Tab. 3. Example of alternatives evaluation.**

| Alternatives | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| Evaluation | $L_1$ | $L_2$ | $L_5$ | $L_2$ | $L_3$ | $L_1$ |

Six alternatives are considered: $a$, $b$, $c$, $d$, $e$, $f$.
Each alternative is evaluated on a 5 level ordinal scale ($L_1$, $L_2$,…, $L_5$)

The resulting preference vector will conventionally be $[\{a, f\}, \{b, d\}, \{e\}, \text{Null}, \{c\}]^T$. By adopting this convention, the number of elements of a vector will coincide with the number of levels of the ordinal scale in use.

Also, it is assumed an ordering over the decision-makers, based on their individual importance. In these ordering, for any two decision-makers $D_i$ and $D_j$, exactly one of the statements $D_i > D_j$, $D_i \sim D_j$, $D_j > D_i$ is true, where symbols ">" and "~" respectively mean "preferred to" and "indifferent to".

The problem of interest is to combine decision-makers' individual preference orderings into a "fused" preference ordering according to a specific synthesis logic.

### *4.1 Algorithm Description*

The description of the OPM can be organized into three phases, as illustrated in Tab. 4. Each phase is presented in the following sub-sections.

**Tab. 4 Fundamental phases of the Ordinal Prioritization Method (OPM).**

| | |
|---|---|
| Phase 1 | Construction and reorganization of decision-makers' preference vectors. |
| Phase 2 | Definition of the reading sequence. |
| Phase 3 | Generation of the fused ordering. |

#### 4.1.1 Phase 1: Construction and reorganization of decision-makers' preference vectors

For each decision-maker, a preference vector of the alternatives is constructed according to the convention introduced in the previous section.

Preference orderings are then reorganized considering the ordering among the decision-makers: decision-maker(s) with the same importance are aggregated into a single reorganized vector ($D_j^*$) in which each component contains the alternatives with corresponding level of importance.

As an example, consider the case exemplified in Tab. 5 where four fictitious decision-makers ($D_1$ to $D_4$) provide four corresponding orderings of six alternatives ($a$, $b$, $c$, $d$, $e$, $f$) given on an ordinal scale with 7 levels ($L_1 > L_2 > L_3 > L_4 > L_5 > L_6 > L_7$).

**Tab. 5. Preference vectors related to the orderings by four fictitious decision-makers ($D_1$ to $D_4$).**

| | | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
|---|---|---|---|---|---|
| Preference vector | $L_1$ | $\{c\}$ | Null | Null | $\{a\}$ |
| | $L_2$ | $\{b\}$ | $\{a\}$ | $\{a, f\}$ | $\{b\}$ |
| | $L_3$ | $\{a\}$ | $\{d, e\}$ | $\{b\}$ | $\{c\}$ |
| | $L_4$ | Null | $\{b\}$ | $\{c, d, e\}$ | $\{d, e\}$ |
| | $L_5$ | Null | $\{f\}$ | Null | Null |
| | $L_6$ | Null | $\{c\}$ | Null | $\{f\}$ |
| | $L_7$ | $\{f, d, e\}$ | Null | Null | Null |

Six total alternatives are considered: $a$, $b$, $c$, $d$, $e$, $f$.
The decision-makers' importance ordering is $D_4 > (D_2 \sim D_3) > D_1$.
$i$ is the position of an element, starting from the top.

Assuming that there is an importance ordering between decision-makers $D_4 > (D_2 \sim D_3) > D_1$, the resulting reorganized vectors can be constructed as exemplified in Tab. 6.

**Tab. 6. "Reorganized" vectors related to the orderings by the four decision-makers in Tab. 5. Vectors ($D_j^*$) are**

**strictly decreasing in terms of importance. The alternatives in the second vector ( $D_2^*$ ) are duplicated, since this vector originates from the level-by-level union of the alternatives from two preference vectors ($D_2$ and $D_3$).**

| $D_1^*$ ($D_4$) | $D_2^*$ ($D_2 \sim D_3$) | $D_3^*$ ($D_1$) |
|---|---|---|
| {*a*} | Null | {*c*} |
| {*b*} | {*a*, *a*, *f*} | {*b*} |
| {*c*} | {*b*, *d*, *e*} | {*a*} |
| {*d*, *e*} | {*b*, *c*, *d*, *e*} | Null |
| Null | {*f*} | Null |
| {*f*} | {*c*} | Null |
| Null | Null | {*f*, *d*, *e*} |

Vector $D_2^*$ – given by the union of two decision-makers with equal importance (i.e., $D_2 \sim D_3$) – contains two occurrences of each alternative. Since aggregation is performed through a level-by-level union of the alternatives and both $D_2$ and $D_3$ have no alternatives with maximum importance, the first level of vector $D_2^*$ does not contain any alternative.

The total number ($m$) of "reorganized" vectors will be smaller than or equal to that ($M$) of the initial vectors (3 against 4 in the example presented).

#### 4.1.2 Phase 2: Definition of the reading sequence

The next step is to define the reading sequence of the elements in the reorganized vectors. The reading procedure can be summarized as follows:

**1.** Initialise the sequence number to $S = 0$

**2.** Consider the most important vector $D_j^*$, by setting $j = 1$

**3.** Consider the element with highest scale level and set $i = 1$

**4.** Set $S = S + 1$

**5.** Associate the element of interest with the sequence number $S$

**6.** If $j = m$, Go To Step 9

**7.** Set $j = j + 1$

**8.** Consider the element with position $i$, related to the $j$-th vector $D_j^*$ and Go To Step 4

**9.** If $i = n$, Go To Step 12

**10.** Set $i = i + 1$

**11.** Consider $j = 1$ and Go To Step 4

**12.** End

In practical terms, the procedure establishes a reading sequence for the reorganized vectors, based on a level-by-level reading of vector elements, moving from vectors of greater importance to those of lesser importance. The logic of the sequence is to read the most preferred alternatives first.

Considering the reorganized vectors in Tab. 6, the resulting sequence is shown in Tab. 7. A sequence number ($S$) is associated to each vector element.

**Tab. 7 Resulting sequence number ($S$), related to the reorganized vectors in Tab. 5.**

| $D_1^*$ ($D_4$) | $D_2^*$ ($D_2 \sim D_3$) | $D_3^*$ ($D_1$) |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| 10 | 11 | 12 |
| 13 | 14 | 15 |
| 16 | 17 | 18 |
| 19 | 20 | 21 |

4.1.3 Phase 3: Generation of the fused ordering

The procedure for determining the total fused ordering is as follows:

1. Initialise the gradual ordering
2. Initialise the counter of the occurrences, for each ($k$-th) alternative, to $O_k = 0\ \forall k$
3. Initialise $S = 1$
4. Initialize the set of residual elements ($R$) as the set of all the alternative(s)
5. Consider the element ($I$) with the sequence number $S$
6. If $I$ is "Null", Go To Step 14
7. Initialise the set of alternatives to be excluded $E = \emptyset$
8. For each ($k$-th) alternative in $I$, set $O_k = O_k + 1$
9. If the $k$-th alternative is in $R$ and $O_k \geq T_k$, include the alternative of interest in the set of those to be excluded ($E$)
10. If $E = \emptyset$, Go To Step 14
11. Include the alternative(s) in $E$, at the bottom of the gradual ordering. In case of multiple alternatives, consider them as indifferent.
12. Remove the alternative(s) in $E$ from $R$, i.e. $R = R \backslash (R \cap E)$
13. If $R = \emptyset$, Go To Step 15
14. Increment $S = S + 1$ and Go To Step 5
15. End

It may be noticed that the selection of a $k$-th alternative and its addition to the fused ordering is more "gradual" than in Yager's original approach, as it is performed when the number of occurrences ($O_k$ ) overcomes a predefined threshold ($T_k$). The choice of the threshold value ($T_k$) is deliberately left to the user so as to allow a tuning of the method as well as a testing of the robustness of the obtained results.

Tab. 8 shows the results of the step-by-step application of the algorithm when $T_k$ = 2 for all the alternatives. The last columns contains the total ordering. Applying the OPM, the total fused preference ordering related to the example in Tab. 5 is $a > b > c > d \sim e > f$.

**Tab. 8 Step-by-step application of the OPM. The first three columns are related to the reading sequence: *S* is the sequence number, while *j* denotes the decision-maker selected. The subsequent columns refer to the construction of the total ordering. We remark that an alternative is added to the total ordering only when the number of occurrences is greater than or equal to $T_k$. Data are related to the example of Tab. 5 and Tab. 6.**

| $S$ | $j$ | Element ($I$) | $E$ | Occurrences ($O_k$) | | | | | | Residual elements ($R$) | Gradual ordering |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | | |
| 0 | - | - | - | - | - | - | - | - | - | $\{a, b, c, d, e, f\}$ | - |
| 1 | 1 | $\{a\}$ | - | 1 | 0 | 0 | 0 | 0 | 0 | $\{a, b, c, d, e, f\}$ | - |
| 2 | 2 | Null | - | 1 | 0 | 0 | 0 | 0 | 0 | $\{a, b, c, d, e, f\}$ | - |
| 3 | 3 | $\{c\}$ | - | 1 | 0 | 1 | 0 | 0 | 0 | $\{a, b, c, d, e, f\}$ | - |
| 4 | 1 | $\{b\}$ | - | 1 | 1 | 1 | 0 | 0 | 0 | $\{a, b, c, d, e, f\}$ | - |
| 5 | 2 | $\{a, a, f\}$ | $\{a\}$ | 3 | 1 | 1 | 0 | 0 | 1 | $\{b, c, d, e, f\}$ | $a$ |
| 6 | 3 | $\{b\}$ | $\{b\}$ | 3 | 2 | 1 | 0 | 0 | 1 | $\{c, d, e, f\}$ | $a > b$ |
| 7 | 1 | $\{c\}$ | $\{c\}$ | 3 | 2 | 2 | 0 | 0 | 1 | $\{d, e, f\}$ | $a > b > c$ |
| 8 | 2 | $\{b, d, e\}$ | - | 3 | 3 | 2 | 1 | 1 | 1 | $\{d, e, f\}$ | $a > b > c$ |
| 9 | 3 | $\{a\}$ | - | 4 | 3 | 2 | 1 | 1 | 1 | $\{d, e, f\}$ | $a > b > c$ |
| 10 | 1 | $\{d, e\}$ | $\{d, e\}$ | 4 | 3 | 2 | 2 | 2 | 1 | $\{f\}$ | $a > b > c > d$~e |
| 11 | 2 | $\{b, c, d, e\}$ | - | 4 | 4 | 4 | 3 | 2 | 1 | $\{f\}$ | $a > b > c > d$~e |
| 12 | 3 | Null | - | 4 | 4 | 4 | 3 | 2 | 1 | $\{f\}$ | $a > b > c > d$~e |
| 13 | 1 | Null | - | 4 | 4 | 4 | 3 | 3 | 1 | $\{f\}$ | $a > b > c > d$~e |
| 14 | 2 | $\{f\}$ | $\{f\}$ | 4 | 4 | 4 | 3 | 3 | 2 | $\emptyset$ | $a > b > c > d$~e $> f$ |
| End | - | - | - | - | - | - | - | - | - | - | - |

Deriving from Yager's algorithm, the OPM has different strengths, discussed by Yager (2001) in detail. Apart from its simplicity and ability to be automated, the fusion technique of preference vectors satisfies some interesting properties, such as:

- *Idempotency*. If all of the preference vectors are the same, the resulting fused preference vector is this one.
- *Monotonicity*. If alternative *a* is preferred to alternative *b* in all preference vectors, then this relationship will be respected in the fused ordering.
- *Positive association*. Assume $D_1^*$, ..., $D_m^*$ are a collection of preference vectors, resulting in a fused preference where alternative *a* is preferred to alternative *b*. Let $\hat{D}_1^*$, ..., $\hat{D}_m^*$ be another collection of preference vectors where, if $\hat{D}_i^*$ differs from $D_i^*$, it only differs in that either *a* has moved up or *b* has moved down or both; then in the fused ordering of these new vectors, alternative *a* will be preferred to alternative *b*.

## 5 Examples

### *5.1 OPM Application Example*

With reference to the example introduced in Section 3, the goal of *EC*s prioritization is to produce the best aggregate ordering out of the multiple orderings related to each *CR*. Thus, the OPM can be suitably applied in this context. The first phase requires the definition of the reorganized vectors (see Tab. 9).

**Tab. 9. Reorganized vectors deriving from Tab. 2**

| *Relation* | $CR_1$ | $CR_2$ | $CR_3$~ $CR_4$ |
|---|---|---|---|
| *Strong* | $\{EC_2\}$ | $\{EC_2, EC_4\}$ | $\{EC_3\}$ |
| *Medium* | $\{EC_1\}$ | Null | $\{EC_4, EC_3, EC_5\}$ |
| *Weak* | $\{EC_3\}$ | Null | $\{EC_2, EC_4\}$ |
| *Absent* | $\{EC_4, EC_5\}$ | $\{EC_1, EC_3, EC_5\}$ | $\{EC_1, EC_1, EC_2, EC_5\}$ |

Then, the step by step application of the second and third phase of the OPM produces the results in Tab. 10. Depending on the choice of the threshold value, different aggregate orderings can be obtained. For instance, if the threshold is equal to 1 ($T_k = 1 \forall k$), the fused preference ordering is $EC_2 > EC_4 > EC_3 > EC_1$. Otherwise, if the threshold is equal to 2 ($T_k = 2 \forall k$), the fused preference ordering is $EC_2 > EC_3 \sim EC_4 > EC_1$.

**Tab. 10. Results of the step by step application of the proposed method to the example of Tab. 2.**

| Pass | Element ($I$) | Cumulative Occurrences ($O_k$) | | | | Gradual Ordering | Gradual Ordering |
|---|---|---|---|---|---|---|---|
| | | $EC_1$ | $EC_2$ | $EC_3$ | $EC_4$ | ($T_k$ = 1) | ($T_k$ = 2) |
| 0 | - | 0 | 0 | 0 | 0 | - | - |
| 1 | $\{EC_2\}$ | 0 | 1 | 0 | 0 | $EC_2$ | - |
| 2 | $\{EC_2, EC_4\}$ | 0 | 2 | 0 | 1 | $EC_2 > EC_4$ | $EC_2$ |
| 3 | $\{EC_3\}$ | 0 | 2 | 1 | 1 | $EC_2 > EC_4 > EC_3$ | $EC_2$ |
| 4 | $\{EC_1\}$ | 1 | 2 | 1 | 1 | $EC_2 > EC_4 > EC_3 > EC_1$ | $EC_2$ |
| 5 | Null | 1 | 2 | 1 | 1 | $EC_2 > EC_4 > EC_3 > EC_1$ | $EC_2$ |
| 6 | $\{EC_4,EC_3\}$ | 1 | 2 | 2 | 2 | $EC_2 > EC_4 > EC_3 > EC_1$ | $EC_3 \sim EC_4 > EC_2$ |
| 7 | $\{EC_3\}$ | 1 | 2 | 3 | 2 | $EC_2 > EC_4 > EC_3 > EC_1$ | $EC_3 \sim EC_4 > EC_2$ |
| 8 | Null | 1 | 2 | 3 | 2 | $EC_2 > EC_4 > EC_3 > EC_1$ | $EC_3 \sim EC_4 > EC_2$ |
| 9 | $\{EC_2, EC_4\}$ | 1 | 3 | 3 | 3 | $EC_2 > EC_4 > EC_3 > EC_1$ | $EC_3 \sim EC_4 > EC_2$ |
| 10 | $\{EC_4\}$ | 1 | 3 | 3 | 4 | $EC_2 > EC_4 > EC_3 > EC_1$ | $EC_3 \sim EC_4 > EC_2$ |
| 11 | $\{EC_1, EC_3\}$ | 2 | 3 | 4 | 4 | $EC_2 > EC_4 > EC_3 > EC_1$ | $EC_1 > EC_3 \sim EC_4 > EC_2$ |
| 12 | $\{EC_1, EC_1, EC_2\}$ | 4 | 4 | 4 | 4 | $EC_2 > EC_4 > EC_3 > EC_1$ | $EC_1 > EC_3 \sim EC_4 > EC_2$ |

Varying the threshold allows a sensitivity analysis of the resulting fused ordering. In the original version of the method, Yager proposes to use a drastic value of the threshold ($T_k = 1 \forall k$), while the OPM generalizes this constraint delegating the choice to the user. It is worth noting that, greater values of $T_k$ assign less significance to the ordering of decision makers (*CR*s).

### *5.2 A QFD application example from the literature*

This example refers to the design of a pencil, for a more complex example please refer to the Appendix. HoQ – adapted from (Wasserman 1993) – relates N = 4 *CR*s to M = 5 *EC*s. The relationships are expressed on a 4 level ordinal scale, as shown in the legend of Tab. 11.

**Tab. 11. House of Quality of a pencil.**

| Customer Requirements | Engineering characteristics | Length of the pencil | Time between sharpening | Lead dust generated | Hexagonality | Minimal erasure residual |
|---|---|---|---|---|---|---|
| | | $EC_1$ | $EC_2$ | $EC_3$ | $EC_4$ | $EC_5$ |
| Easy to hold | $CR_1$ | ○ | | | ● | |
| Does not smear | $CR_2$ | | ○ | ● | | ● |
| Point lasts | $CR_3$ | △ | ○ | ● | | ● |
| Does not roll | $CR_4$ | △ | | | ● | |

Legend

| Relation | |
|---|---|
| ● | strong |
| ○ | medium |
| △ | weak |

Consistently with the original problem, it is assumed that CRs are ordered according to the following preference ordering: $CR_3 > CR_2 > CR_1 \sim CR_4$. Thus, the reorganized vectors are shown in Tab. 12.

Tab. 13 shows the results of the step-by-step application of the method.

**Tab. 12. Reorganized vectors for the pencil example (Wasserman 1993)**

| $CR_3$ | $CR_2$ | $CR_1 \sim CR_4$ |
|---|---|---|
| $\{EC_3,EC_5\}$ | $\{EC_3,EC_5\}$ | $\{EC_4,EC_4\}$ |
| $\{EC_2\}$ | $\{EC_2\}$ | $\{EC_1\}$ |
| $\{EC_1\}$ | Null | $\{EC_1\}$ |
| $\{EC_4\}$ | $\{EC_1,EC_4\}$ | $\{EC_2,EC_2,EC_3,EC_3,EC_5,EC_5\}$ |

As a result, the total fused preference function– respectively corresponding to $T_k = 1$ and $T_k = 2$ for all the alternatives – are both coincident to EC3~EC5 > EC4 > EC2 > EC1. Notice that the result obtained is consistent (in this case exactly coincident) with the one obtained in the original example by the use of the Independent Scoring Method (Wasserman 1993).

**Tab. 13. Steps of the OPM.**

| Pass | Element ($I$) | Cumulative Occurrences ($O_k$) | | | | | Residual elements ($R$) | Gradual Ordering |
|---|---|---|---|---|---|---|---|---|
| | | $EC_1$ | $EC_2$ | $EC_3$ | $EC_4$ | $EC_5$ | ($T_k$ = 1) | ($T_k$ = 1) |
| 0 | - | 0 | 0 | 0 | 0 | 0 | $\{EC_1, EC_2, EC_3, EC_4, EC_5\}$ | - |
| 1 | $\{EC_3,EC_5\}$ | 0 | 0 | 1 | 0 | 1 | $\{EC_1, EC_2, EC_4\}$ | $EC_3 \sim EC_5$ |
| 2 | $\{EC_3,EC_5\}$ | 0 | 0 | 2 | 0 | 2 | $\{EC_1, EC_2, EC_4\}$ | $EC_3 \sim EC_5$ |
| 3 | $\{EC_4,EC_4\}$ | 0 | 0 | 2 | 2 | 2 | $\{EC_1, EC_2\}$ | $EC_3 \sim EC_5 > EC_4$ |
| 4 | $\{EC_2\}$ | 0 | 1 | 2 | 2 | 2 | $\{EC_1\}$ | $EC_3 \sim EC_5 > EC_4 > EC_2$ |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | $\{EC_2\}$ | 0 | 2 | 2 | 2 | 2 | $\{EC_1\}$ | $EC_3 \sim EC_5 > EC_4 > EC_2$ |
| 6 | $\{EC_1\}$ | 1 | 2 | 2 | 2 | 2 | - | $EC_3 \sim EC_5 > EC_4 > EC_2 > EC_1$ |
| 7 | $\{EC_1\}$ | 2 | 2 | 2 | 2 | 2 | - | $EC_3 \sim EC_5 > EC_4 > EC_2 > EC_1$ |
| 8 | Null | 2 | 2 | 2 | 2 | 2 | - | $EC_3 \sim EC_5 > EC_4 > EC_2 > EC_1$ |
| 9 | $\{EC_1\}$ | 3 | 2 | 2 | 2 | 2 | - | $EC_3 \sim EC_5 > EC_4 > EC_2 > EC_1$ |
| 10 | $\{EC_4\}$ | 3 | 2 | 2 | 3 | 2 | - | $EC_3 \sim EC_5 > EC_4 > EC_2 > EC_1$ |
| 11 | $\{EC_1,EC_4\}$ | 4 | 2 | 2 | 4 | 2 | - | $EC_3 \sim EC_5 > EC_4 > EC_2 > EC_1$ |
| 12 | $\{EC_2,EC_2,EC_3,EC_3,EC_5,EC_5\}$ | 4 | 4 | 4 | 4 | 4 | - | $EC_3 \sim EC_5 > EC_4 > EC_2 > EC_1$ |

## 6 Conclusions and future developments

The most common approach to prioritize *EC*s in QFD is the Independent Scoring Method (Akao 1988). Although widely adopted, this method requires two questionable assumptions: (i) the definition of ratings in the prioritization of *CR*s (ii) the conversion of the symbols in the relationship matrix into a corresponding cardinal matrix.

This paper proposes the application of an alternative approach deriving from an algorithm initially proposed by Yager (2001). The method addresses the problem of aggregating preference/importance orderings of multiple, ordered decision-makers with respect to a set of possible alternatives. Interpreting the *EC*s prioritization as a decision-making problem, this paper suggests a novel approach to face up this problem in the case of *CR*s evaluated on ordinal scales.

The main aspects of the method are here summed up:

- It can be used to rank *ECs* when the importance of *CR*s is expressed on an ordinal scale;
- Also, it can be used as an alternative to traditional approaches (such as the Independent Scoring Method) when the relative importances or weights are available by simply ordering *CR*s according to their weights.
- It does not require artificial numerical coding of the symbols contained into the relationship matrix;
- It can adapt to relationship matrices expressed on ordinal scales with a number of levels at will (not necessarily the 4 levels of the classic HoQs);
- The method is simple to implement, its simplicity in terms of processing is comparable with that of the Independent Scoring Method.
- It allows to carry out a sensitivity analysis of the total fused ranking modifying the threshold value ($T_k$).

Summarizing, there are three advantages in the application of this new method: (i) it can be applied when *CR*s are simply ranked according to a preference/importance ordering; (ii) it does not require special data manipulations of the information gathered in the QFD process and (iii) it does not require an artificial promotion of the scale properties of the symbols contained in the relationship matrix of HoQ and of the *CR*s importances.

Future development of this research will include the proposed method into a more general framework, with the aim of designing a QFD completely based on information given on ordinal scales.

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## Appendix - HoQ for a climbing harness

This application example refers to the design of a climbing harness. HoQ – adapted from (Hunt 2013) – relates N = 8 *CR*s to M = 8 *ECs*. The relations are expressed on a 4 level ordinal scale, as shown in the legend of Tab. 14.

**Tab. 14 House of Quality for a climbing harness.**

| | | Engineering characteristics | Meets Safety Standards | Harness weight | Webbing strength | No. of colours | No. Of sizes | Padding Thickness | No. Of buckles | No. Of gear loops |
|---|---|---|---|---|---|---|---|---|---|---|
| Customer Requirements | | | $EC_1$ | $EC_2$ | $EC_3$ | $EC_4$ | $EC_5$ | $EC_6$ | $EC_7$ | $EC_8$ |
| Usability | Easy to put on | $CR_1$ | | | | | ○ | | ● | |
| | Comfortable when hanging | $CR_2$ | | | | | ○ | ● | ○ | |
| | Fits over different clothes | $CR_3$ | | | | | ○ | ○ | ● | |
| | Accessible gear loops | $CR_4$ | | | | | | | | ● |
| Performance | Does not restrict movement | $CR_5$ | | ○ | | | ○ | ● | ○ | |
| | Lightweight | $CR_6$ | | ● | ○ | | | ○ | △ | △ |
| | Safe | $CR_7$ | ● | ○ | ● | | | | | |
| | Attractive | $CR_8$ | | △ | | ● | | △ | △ | |

Legend

| Relation | |
|---|---|
| ● | strong |
| ○ | medium |
| △ | weak |

Consistently with the original problem, it is assumed that *CR*s are ordered according to the following preference ordering: $CR_2$~$CR_5$~$CR_7$ > $CR_4$~$CR_6$> $CR_1$~$CR_8$ > $CR_3$. Given this ordering, it is possible to define the reorganized vectors as described in Sect. 2. The aggregation result is shown in Tab. 15.

**Tab. 15. Reorganized vectors**

| $CR_2$~$CR_5$~$CR_7$ | $CR_4$~$CR_6$ | $CR_1$~$CR_8$ | $CR_3$ |
|---|---|---|---|
| $\{EC_6,EC_6,EC_1,EC_3\}$ | $\{EC_8,EC_2\}$ | $\{EC_7,EC_4\}$ | $\{EC_7\}$ |
| $\{EC_5,EC_7,EC_2,EC_5,EC_7,EC_2\}$ | $\{EC_3,EC_6\}$ | $\{EC_5\}$ | $\{EC_5,EC_6$ |
| Null | $\{EC_7,EC_8\}$ | $\{EC_2,EC_6,EC_7$ | Null |
| $\{EC_1,EC_2,EC_3,EC_4,EC_8,EC_1,EC_3, EC_4,EC_8,EC_4,EC_5,EC_6,EC_7,EC_8\}$ | $\{EC_1,EC_2,EC_3,EC_4,EC_5, EC_6,EC_7,EC_1,EC_4,EC_5\}$ | $\{EC_1,EC_2,EC_3,EC_4,EC_6, EC_8,EC_1,EC_3,EC_5,EC_8\}$ | $\{EC_1,EC_2,EC_3,EC_4, EC_8\}$ |

So far, the proposed method can be applied. Tab. 16 shows the results of the step-by-step application of the method.

**Tab. 16. Steps of the method**

| Pass | Element | Cumulative Occurrences | | | | | | | | Gradual Ordering |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $EC_1$ | $EC_2$ | $EC_3$ | $EC_4$ | $EC_5$ | $EC_6$ | $EC_7$ | $EC_8$ | ($T_k$ = 1) |
| 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| 1 | $\{EC_6,EC_6,EC_1,EC_3\}$ | 1 | 0 | 1 | 0 | 0 | 2 | 0 | 0 | $EC_6$ |
| 2 | $\{EC_8,EC_2\}$ | 1 | 1 | 1 | 0 | 0 | 2 | 0 | 1 | $EC_6$ |
| 3 | $\{EC_7,EC_4\}$ | 1 | 1 | 1 | 1 | 0 | 2 | 1 | 1 | $EC_6$ |
| 4 | $\{EC_7\}$ | 1 | 1 | 1 | 1 | 0 | 2 | 2 | 1 | $EC_6 > EC_7$ |
| 5 | $\{EC_5,EC_7,EC_2,EC_5,EC_7,EC_2\}$ | 1 | 3 | 1 | 1 | 2 | 2 | 4 | 1 | $EC_6>EC_7>EC_2$~$EC_5$ |
| 6 | $\{EC_3,EC_6\}$ | 1 | 3 | 2 | 1 | 2 | 3 | 4 | 1 | $EC_6>EC_7>EC_2$~$EC_5>EC_3$ |
| 7 | $\{EC_5\}$ | 1 | 3 | 2 | 1 | 3 | 3 | 4 | 1 | $EC_6>EC_7>EC_2$~$EC_5>EC_3$ |
| 8 | $\{EC_5,EC_6\}$ | 1 | 3 | 2 | 1 | 4 | 4 | 4 | 1 | $EC_6>EC_7>EC_2$~$EC_5>EC_3$ |
| 9 | Null | 1 | 3 | 2 | 1 | 4 | 4 | 4 | 1 | $EC_6>EC_7>EC_2$~$EC_5>EC_3$ |
| 10 | $\{EC_7,EC_8\}$ | 1 | 3 | 2 | 1 | 4 | 4 | 5 | 2 | $EC_6>EC_7>EC_2$~$EC_5>EC_3>EC_8$ |
| 11 | $\{EC_2,EC_6,EC_7\}$ | 1 | 4 | 2 | 1 | 4 | 5 | 6 | 2 | $EC_6>EC_7>EC_2$~$EC_5>EC_3>EC_8$ |
| 12 | Null | 1 | 4 | 2 | 1 | 4 | 5 | 6 | 2 | $EC_6>EC_7>EC_2$~$EC_5>EC_3>EC_8$ |
| 13 | $\{EC_1,EC_2,EC_3,EC_4,EC_8,EC_1,EC_3,EC_4,EC_8,EC_4,EC_5,EC_6,EC_7,EC_8\}$ | 3 | 5 | 4 | 4 | 5 | 6 | 7 | 5 | $EC_6>EC_7>EC_2$~$EC_5>EC_3>EC_8>EC_1$~$EC_4$ |
| 14 | $\{EC_1,EC_2,EC_3,EC_4,EC_5,EC_6,EC_7,EC_1,EC_4,EC_5\}$ | 5 | 6 | 5 | 6 | 7 | 7 | 8 | 5 | $EC_6>EC_7>EC_2$~$EC_5>EC_3>EC_8>EC_1$~$EC_4$ |
| 15 | $\{EC_1,EC_2,EC_3,EC_4,EC_6,EC_8,EC_1,EC_3,EC_5,EC_8\}$ | 7 | 7 | 7 | 7 | 8 | 8 | 8 | 7 | $EC_6>EC_7>EC_2$~$EC_5>EC_3>EC_8>EC_1$~$EC_4$ |
| 16 | $\{EC_1,EC_2,EC_3,EC_4,EC_8\}$ | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | $EC_6>EC_7>EC_2$~$EC_5>EC_3>EC_8>EC_1$~$EC_4$ |

The fused preference function – corresponding to $T_k = 2$ for all the alternatives – is $EC_6 > EC_7 > EC_2 \sim EC_5 > EC_3 > EC_8 > EC_1 \sim EC_4$. Notice that this result is consistent with that obtained in the original example by the use of the independent scoring method (Kogure and Akao 1983), i.e. $EC_6 > EC_7 > EC_2 > EC_5 > EC_3 > EC_1 > EC_8 > EC_4$. Of course, different threshold values could lead to different results.