# Investigating the properties of interfacial layers in planar Schottky contacts on hydrogen-terminated diamond through DC/small-signal characterization and radial line small-signal modelling 

Supplemental Material<br>Mathematical derivation of the distributed radial line input admittance

We refer in the following to the equivalent circuit in Fig. 4 of the manuscript.
The per-unit-length parameters (p.u.l) of the radial line can be derived by considering a circular section of length $\mathrm{d} r$ with radius $r$, whose parallel capacitance and conductance model the gate/IL/channel cross section, whereas the series resistance describes hole transport across the channel:

$$
\begin{align*}
\mathrm{d} C & =C_{\mathrm{GC}} 2 \pi r \mathrm{~d} r \\
\mathrm{~d} G & =G_{\mathrm{d}} 2 \pi r \mathrm{~d} r  \tag{1}\\
\mathrm{~d} R & =R_{\mathrm{ch}} \frac{\mathrm{~d} r}{2 \pi r}
\end{align*}
$$

where, with reference to Fig. 4, the gate-channel capacitance $C_{\mathrm{GC}}(V)$ is defined as

$$
\begin{equation*}
C_{\mathrm{GC}}(V)=\frac{C_{\mathrm{IL}} C_{\mathrm{ch}}}{C_{\mathrm{IL}}+C_{\mathrm{ch}}} . \tag{2}
\end{equation*}
$$

The parameters $C_{\mathrm{IL}}$ and $C_{\mathrm{ch}}(V)$ are the per-unit-area IL and channel capacitance, respectively. In Eq.(1), $G_{\mathrm{d}}$ is the per-unit-area conductance accounting for the gate-channel leakage current. Finally, $R_{\mathrm{ch}}=1 /\left(q \mu_{h} p_{s}\right)$ is the equivalent channel sheet resistance, where $\mu_{h}$ and $p_{s}$ are the 2 DHG mobility and sheet density, respectively. The 2 DHG concentration is evaluated self-consistently with the gate-channel capacitance of the circular section as

$$
\begin{equation*}
p_{\mathrm{s}}\left(V_{\mathrm{G}}\right)=-\frac{1}{q} \int_{V_{\mathrm{G} 0}}^{V_{\mathrm{G}}} C_{\mathrm{GC}}(V) \mathrm{d} V \tag{3}
\end{equation*}
$$

where $V_{\mathrm{G} 0}$ is a bias point in the off state, such that $p_{\mathrm{s}}\left(V_{\mathrm{G} 0}\right)=0$.
Applying the Kirchhoff's voltage and current laws to a line section of radial length $\mathrm{d} r$ located in $r$, the line equations result as

$$
\begin{align*}
\frac{d V}{d r} & =-\frac{R_{\mathrm{ch}}}{2 \pi r} I \\
\frac{d I}{d r} & =-Y_{\mathrm{p}} 2 \pi r V \tag{4}
\end{align*}
$$

with $Y_{\mathrm{p}}=G_{\mathrm{d}}+j \omega C_{\mathrm{GC}}$. Differentiating, substituting and exploiting the variable change $x=\sqrt{Y_{\mathrm{p}} R_{\mathrm{ch}}} r$, we have for the line voltage

$$
\begin{equation*}
x^{2} \frac{\mathrm{~d}^{2} V}{\mathrm{~d} x^{2}}+x \frac{\mathrm{~d} V}{\mathrm{~d} x}-x^{2} V=0 \tag{5}
\end{equation*}
$$

i.e. a zero-order Bessel differential equation, admitting a closed-form solution in terms of modified Bessel functions of the first $(I)$ and second $(K)$ kind. The resulting values for the line current and voltage are, respectively:

$$
\begin{align*}
I(r) & =A 2 \pi r \sqrt{\frac{Y_{\mathrm{p}}}{R_{\mathrm{ch}}}} I_{1}(k r)+B 2 \pi r \sqrt{\frac{Y_{\mathrm{p}}}{R_{\mathrm{ch}}}} K_{1}(k r) \\
V(r) & =A I_{0}(k r)+B K_{0}(k r) \tag{6}
\end{align*}
$$

where $A$ and $B$ are integration constants, and $k=\sqrt{Y_{\mathrm{p}} R_{\mathrm{ch}}}$. The line current is derived from the voltage by differentiation. Finally, considering a line with radius $R$ and applying the boundary conditions $I(r=0)=0$ and $V(r=R)=V_{0}$, the line input admittance is calculated as

$$
\begin{equation*}
Y_{\mathrm{i}}=\frac{I(R)}{V(R)}=2 \pi R \sqrt{\frac{Y_{\mathrm{p}}}{R_{\mathrm{ch}}}} \frac{I_{1}(k R)}{I_{0}(k R)} . \tag{7}
\end{equation*}
$$

The total equivalent impedance of the device is then computed by including the effect of the parasitic access and contact resistances $\left(R_{s}\right)$ as $1 / Y_{\mathrm{i}}+R_{\mathrm{s}}$.

The analytic model has been validated against 3D physics-based simulations ${ }^{1}$, under AC condition, of a planar device with 5 nm thick IL ( $\epsilon_{r}=9$ ), carrier mobility of $50 \mathrm{~cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$, and radius of $25 \mu \mathrm{~m}$. Fig. S1 shows the parallel capacitance and conductance associated with the small-signal admittance $Y_{\mathrm{i}}=G_{\mathrm{i}}+j \omega C_{\mathrm{i}}$ as derived from the physics-based simulation and as predicted by Eq. (7), demonstrating the high accuracy of the analytic distributed model.

As discussed in the manuscript, reducing the gate radius to a few $\mu \mathrm{m}$ will make the dispersive effect of the channel resistance ( $R_{\mathrm{ch}}$ in Fig. 4 of the manuscript) negligible, thus allowing the accurate extraction of the physical paramaters by a lumped model that can be derived from the series expansion of Eq. (7) when $k R \rightarrow 0$. Truncating the series at the 4th order, one obtains

$$
\begin{equation*}
Y_{\mathrm{i}, \text { lumped }}=\pi R^{2} Y_{\mathrm{p}}-\frac{\pi R^{4}}{8} Y_{\mathrm{p}}^{2} R_{\mathrm{ch}} \tag{8}
\end{equation*}
$$

The approximation admits for an interpetation in terms of a lumped equivalent circuit made by the series of the admittance $\pi R^{2} Y_{\mathrm{p}}$ and the resistance $R_{\mathrm{ch}} / 8 / \pi$, as can be demonstrated from the series expansion of the input impedance $1 / Y_{\mathrm{i}}$. Thus, in the lumped limit,


FIG. S1. $C_{\mathrm{i}} / V$ (a) and $G_{\mathrm{i}} / V(\mathrm{~b})$ curves at various frequencies of a large area planar device as predicted by physics-based simulations (symbols) and by the analytical distributed model in Eq. (7) (solid lines). Frequency points are logarithmically equally spaced between 10 kHz and 10 MHz .
the equivalent resistance of the line is the sheet resistance $\left(R_{\mathrm{ch}}\right)$ corrected by a suitable scale factor $(1 / 8 / \pi)$.

## REFERENCES

1 "Sentaurus Device, Synopsys Inc." (2014).

