5TH INTERNATIONAL CONFERENCE
OF THE EUROPEAN SOCIETY FOR THE HISTORY OF SCIENCE

SCIENTIFIC COSMOPOLITANISM AND LOCAL CULTURES: RELIGIONS, IDEOLOGIES, SOCIETIES

Proceedings

ATHENS, 1-3 NOVEMBER 2012

European Society for the History of Science
ESHS

ΕΘΝΙΚΟ ΙΔΡΥΜΑ ΕΡΕΥΝΩΝ
1958
National Hellenic Research Foundation

HELLENIC REPUBLIC
National and Kapodistrian University of Athens

Edited by
Gianna Katsiampoura
Logo designed by
Nefeli Papaioannou
Published by
National Hellenic Research Foundation/Institute of Historical Research/ Section of Neohellenic Research/ Programme of History, Philosophy and Didactics of Science and Technology

Section of Neohellenic Research/IHR/NHRF, Series Conference-Symposia 2.8

ISBN 978-960-98199-3-0
Athens, 2014

Leonardo on hydrostatic force: a research engineering approach towards the idea of hydrostatic pressure?

Paolo Cavagnero
Department of Environment, Land and Infrastructure Engineering, Politecnico di Torino, Italy

Introduction

Leonardo studied problems of fluid mechanics and hydraulic engineering for most of his life (Roberts 1982, 13-22; Di Teodoro 2002, 258-277; Marani 2010, 329-346). His activities and results in these fields, that are often aimed at the solution of actual engineering problems, often present a very original methodology. Leonardo's work on fluid mechanics and hydraulic engineering is based on a research engineering approach, a mix of confrontation with other scholars and technicians, observations of natural phenomena and laboratory experiments, ideal or not (Bellone, 1982; Macagno 1985, 71-96). A chronological examination of notes and illustrations drawn by Leonardo on hydraulics subjects may lead to a better understanding of his attitudes to them, and may give account of how these attitudes change through time. Adopting this point of view, my contribution will focus on a particular theme studied by Leonardo, the hydrostatic force exerted by water on vertical plane surfaces, closely related to the concept of hydrostatic pressure.

Potenzia vs Resistenzia

We will start examining a part of a quite late page of Leonardo's production, the folio 6 recto of Codex Leicester (1506-1508 ca.) which can be considered as a good summary of his mature ideas on the subject. In this page Leonardo defines the problem: *Che potenzia fia quella, che fa l'acqua contro all'argine che la sostiene*, and gives its solution: *L'acqua sostenuta dall'argine sospinge nell'altezza d'essa argine, dalla superfizie al fondo, con varia potenzia; e questa tal varietà è causata dalla disformità, over dalla inequalità della altezza d'ess'acqua, con ciò sia che, quanto più s'appressa al fondo, con maggior forza spinge in essa argine*. After that, he immediately suggest an application for determining the design *resistenzia* of banks: *L'acqua sostenuta dall'argine sospinge nell'altezza d'essa argine, dalla superfizie al fondo, con varia potenzia; e questa tal varietà è causata dalla disformità, over dalla inequalità della altezza d'ess'acqua, con ciò sia che, quanto più s'appressa al fondo, con maggior forza spinge in essa argine*. This page is important for one key point: it demonstrates that at the end of his life Leonardo understands the action exerted by water from both a qualitative and quantitative point of view. From a qualitative point of view, his understanding comes from the observation of jet and ranges: *Come mostra le cannelle, che versano in varie altezze del bottino*; from a quantitative point of view, it comes from considerations and experiments on weights in equilibrium: *Possi misurare la inequalità di tal potenzie [...] col bottino*.

Jets and ranges

Leonardo indeed often uses the range and discharge of a jet that flows out from a container as an indicator of the intensity of the push exerted by the fluid against a face of the container itself. Let's examine a chronological series of notes and drawings on jets that could have influenced Leonardo's ideas on the characteristics of hydrostatic force. One of the first testimonies of this kind in Leonardo's manuscripts belongs to the Codex Atlanticus, folio 303 verso, dated by Pedretti around 1490 (Pedretti, 1978). It is a small drawing (fig. 1), accompanied by a very short note, in which Leonardo represents two containers (*a* e *b*) filled with water, with very different volumes but with the same height. From their bottoms two jets of identical shape and size flow out; Leonardo writes: *Tanto gitta a quanto b*. Although the explicit topic of this drawing is the comparison between the discharges, the note is important because Leonardo recognizes that the ranges of water jets, caused by the "push" of water, don't depend on the volume of the containers, but only on the height of water inside them.

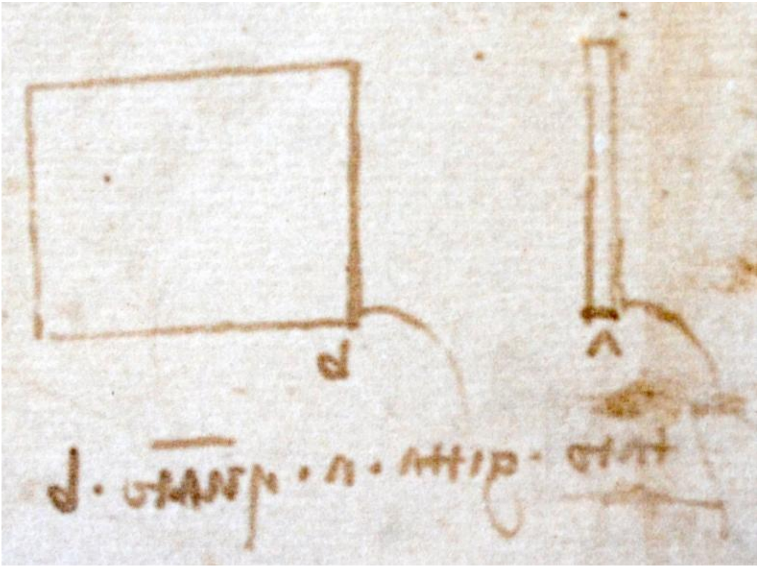

Figure 1: *Codex Atlanticus, folio 303 verso*

On another note, belonging to the folio 5 verso of Manuscript C (ca 1490-91) Leonardo suggests in a more explicit way a direct proportionality between the ranges of the jets and the heights of the fluid in a container (fig. 2).

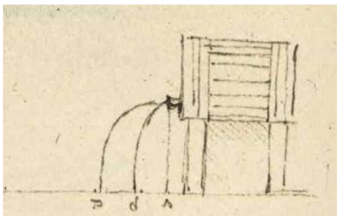

Figure 2: *Manuscript C, folio 5 verso*

In this case the fluid is wine, and it is contained within a small barrel, that lies down on two vertical supports. Leonardo writes: *Quella proporzione che arà b c con a c,*

tale proporzione troverai nelle 2 quantità del vino che si trova in nel vasello, ch'è cagione d'esse mutazione di versare più presso o lontano. In other words, Leonardo says that if double is an observed range (with respect to a shorter one) then double is the corresponding fluid level inside the container and, implicitly, the push exerted by the liquid on the orifice. From the point of view of contemporary hydraulics, Leonardo provides a wrong quantitative interpretation of the phenomenon; two levels which ratio is 2, infact, produce two ranges which ratio is the square root of 2, not 2. Probably the interpretation provided by Leonardo (correct from a qualitative point of view) is the result of a deduction suggested by the observation of the phenomenon itself, not supported by a deeper analysis.
A drawing (fig. 3) on folio 151 recto of Codex Madrid I (1492-97 ca.) depicts what seems to be an apparatus for the 'experimental' verification of the note written by Leonardo on the folio 303 verso of the Codex Atlanticus.

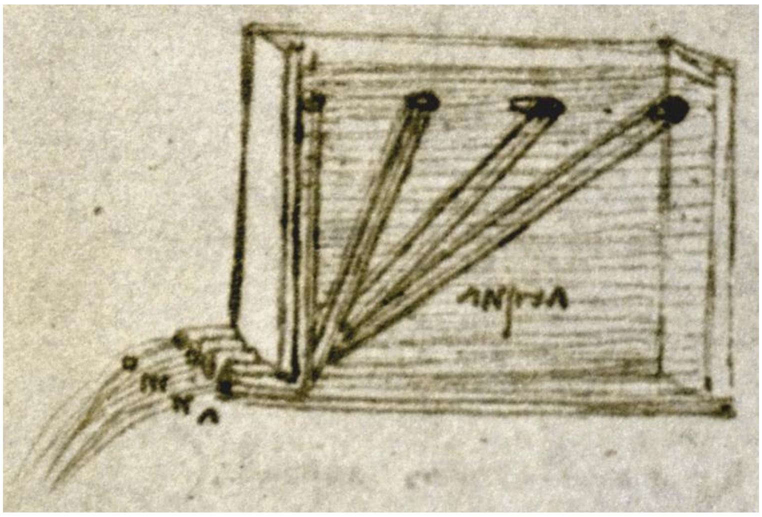

Figure 3: *Codex Madrid I, folio 151 recto*

The drawing represents four pipes protruding from the lowest part of the container; the upper portions of the pipes convey water from different points near the surface of the liquid to the four orifices. The distances of the points from the wall where the outlets are opened increase progressively. Leonardo asks: *Dimando quale di queste 4 canne spignierà più lontan da ssé le sue acque*, so we can infer that this kind of experiment aims precisely to verify that the push in a point near the outlets (in present words the hydrostatic force that generates the jets) depends only on the 'vertical' amount of fluid imposed on the outlet itself, and not on the 'horizontal' one, determined by the other dimensions of the container. Leonardo represents all the four jets flowing out from the pipes with identical ranges, just as if they were produced by pushes of the same magnitude. We don't know if the experiment have been carried out or not, but this detail of the drawing sounds like a confirmation and an experimental justification of Leonardo's belief expressed in Codex Atlanticus, folio 303 verso.
The comparison between the jets produced by a large container and a narrower one characterizes also the folio 117 verso of the Codex Forster IIb (1495 ca.). A drawing (fig. 4) that is very similar to that of the folio 303 verso of the Codex Atlanticus is

here accompanied by a note in form of a question: *Qual gitterà più distante da sé o n o m*?

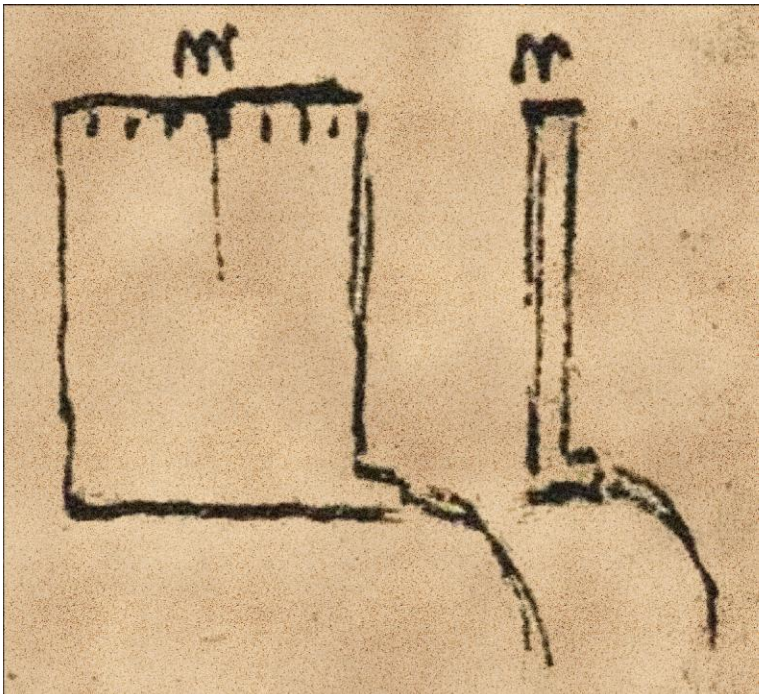

Figure 4: *Codex Forster IIb, folio 117 verso*

The interrogative form can be interpreted here as one of the many questions used by Leonardo to propose different kinds of problems to a fictional interlocutor, with whom he often imagines dialogues aimed at refuting the thesis that he considers false, as well as illustrating the results of his research. The small marks made along the top of the wider container maybe support the hypothesis, because their spacing is equal to the width of the smaller container, which has the shape of a cane.
On the same theme, the folio 114 recto of the Manuscript I (1497-99 ca.) as well contains the drawing of a nozzle supplied by a container that is reduced to the proportions of a "cane" (fig. 5).
This is an element whose transverse dimension is negligible when compared to the vertical one, that univocally defines the height of the water in it. The importance of the vertical dimension is further underlined by Leonardo through the subdivision of the cane in four *gradi di altezza*, which, passing *colla immaginazione* to the description of an ideal experiment, may even become infinite in number: in this case, also the range of the jet flowing out from the nozzle would be infinite. The passage to the infinity, along with the opening sentence of this folio of Manuscript I, reveals a qualitative leap in Leonardo's way of thinking on this subject. He, in fact, asserts that the dependence of the range on water head "is in nature", thus summarizing the data and the intuitions coming from experience and observation in a more general statement, that sounds like a physical law which applies to every similar situation.

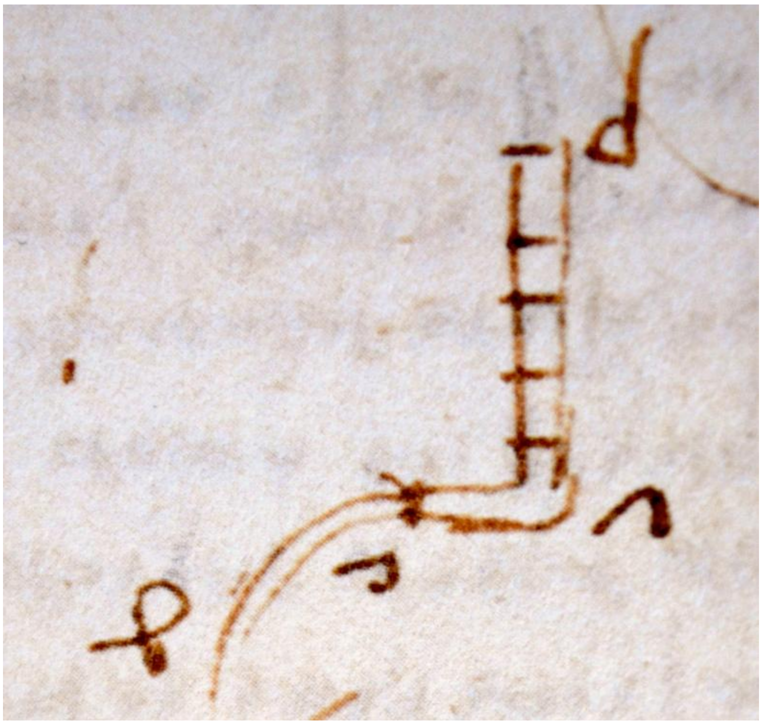

Figure 5: *Manuscript I, folio 14 recto*

This is an element whose transverse dimension is negligible when compared to the vertical one, that univocally defines the height of the water in it. The importance of the vertical dimension is further underlined by Leonardo through the subdivision of the cane in four *gradi di altezza*, which, passing *colla immaginazione* to the description of an ideal experiment, may even become infinite in number: in this case, also the range of the jet flowing out from the nozzle would be infinite. The passage to the infinity, along with the opening sentence of this folio of Manuscript I, reveals a qualitative leap in Leonardo's way of thinking on this subject. He, in fact, asserts that the dependence of the range on water head "is in nature", thus summarizing the data and the intuitions coming from experience and observation in a more general statement, that sounds like a physical law which applies to every similar situation.

Weights in equilibrium

Leonardo tries to describe, characterize and measure the magnitude of the push exerted by water on a vertical surface also in other ways, in particular by studying the equilibrium conditions of systems of hanging weights. A drawing (fig. 6) on the folio 149 verso of the Codex Madrid I (1492-97 ca.) Leonardo represents a quite ingenious instrument.

A container with a square has a mobile vertical face, that is connected to the other faces and to the bottom with a strip of leather. An object (probably an interchangeable body of known weight) is connected to the mobile face with a rope guided by a pulley, in order to counteract the push exerted by the fluid on

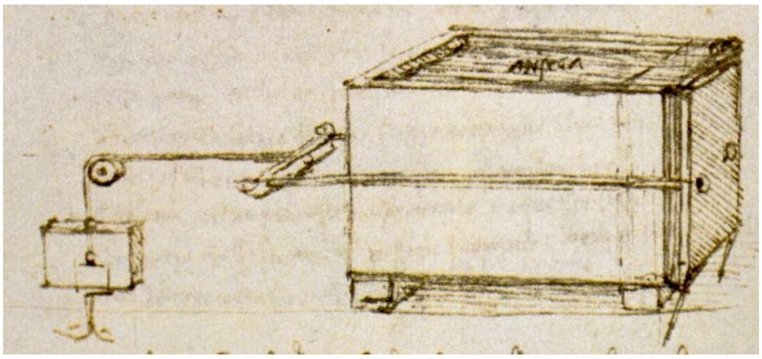

Figure 6: *Codex Madrid I, folio 149 verso*

the face itself. The weight of the object that maintains the mobile wall in contact with the edges of the others gives a direct estimation of the global push acting on the face. The drawing demonstrates Leonardo's intention of studying nature (in this case some properties of the fluids at rest) *ben isperimentando*. The design of some details in this device and the way in which Leonardo intends to use it, however, suggest us that he never conducts any experience with it, not before the draft of the note at least. This experimental apparatus actually contains an error; Leonardo applies the load in the center of the mobile face, while in this case an effective contrasting force must be applied at one third of the height of the face with respect to its bottom. In Leonardo's arrangement, the mobile face is subjected to a counterclockwise torque, and therefore it is not in a condition of static equilibrium. If Leonardo had really built the device as it is shown and had performed experiments with it, very likely the rotation of the mobile face might have suggested him a shift of the point of application of the counterbalancing weight. Moreover, Leonardo, asserts that the global push acting on one of the faces is identical to those acting on the other three (because all the faces are identical) but he goes wrong when he states that the magnitudes of these four pushes may be subtracted from the global weight of the fluid to compute the magnitude of the push exerted by water on the bottom of the container. If the weight of water and the magnitudes of the four identical pushes had been really determined from a performed experiment, such a computation would have seriously led Leonardo to the paradoxical result of a negative value for the push exerted by water on the bottom, at least in containers with a base side shorter than double the height of the vertical walls.
The research on the action exerted by water on a vertical surface, as well as the development of methods to calculate its magnitude, occupy Leonardo on several occasions, and for quite a long time of his life. The initially mentioned folio 6 recto of the Codex Leicester perhaps illustrates the highest degree of elaboration of Leonardo's ideas in this field (Fassò, 1987). This is suggested as well by a method for measuring water *potenzia* illustrated in the folio 6 recto itself, whose very original conception directly reminds the drawing on the folio 149 verso of Codex Madrid I. Leonardo proposes an instrument (fig. 7) that consists

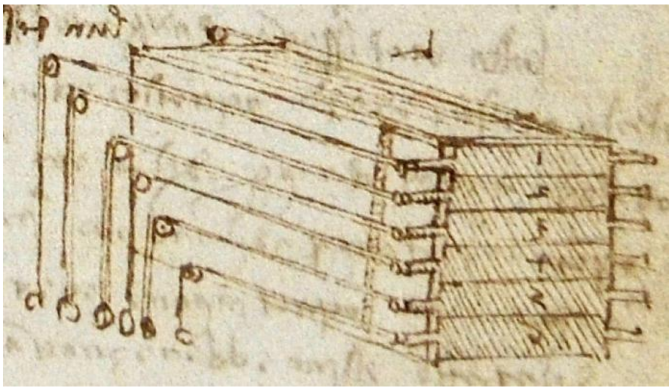

Figure 7: *Codex Leicester, folio 6 recto*

of a parallelepiped container, with a side walls replaced by a flexible parchment. Externally, the parchment is supported by a series of rigid and parallel horizontal bands. Each band is connected to a pair of hanging weights; through a system of pulleys, the loads exert *tanto peso per opposito, che con precisione sostenghino esse righe al contatto della fronte del predetto bottino*. Leonardo does not say anything else about this instrument. Its conception, however, demonstrates that he is aware that the only system of *resistentie* able to keep flat the segmented wall consists of a series of loads whose weights increase with the closeness of the respective bands to the bottom of the container. The values of these *resistentie* indeed are identical to those of the *potentie* that the water exerts on each band. The total *potenzia* acting on the wall is therefore determined by the sum of all those relating to the individual bands.

Even if Leonardo does not develop a calculation technique independent of instrumental measurements to determine the push exerted by water, it is undeniable that with the described instrument Leonardo performs a real mechanical, instead of mathematical, differentiation of the hydrostatic force on the wall of the container, a process that undoubtedly is very advanced at the time. Leonardo actually intends to evaluate the whole action that he calls *potenzia* by summing a finite number of its parts. All these parts are related to an identical portion (the band) of the original wall, but their magnitudes are always different and vary with depth. The design of this instrument, whose conception is even more significant than its incidental construction, moves therefore Leonardo very close to the present representation of the hydrostatic pressure distribution.

It is unclear whether the term *potenzia* uniquely refers to an idea similar to the contemporary concepts of hydrostatic force or pressure. Recalling how easily Leonardo goes wrong when he states that the action exerted on the bottom of a container can be computed by subtracting from the weight of the fluid the sum of the actions that it exerts on the walls of the container itself, it could be argued that in the field of hydrostatics he has some difficulties in understanding the concept of force. Under this point of view, the term *potenzia* probably refers to something analogous to the contemporary idea of pressure. Leonardo, however, often uses the word *potenzia* with different meanings. In the folio 6 recto of Codex Leicester, for example, the measurement of the *potenzia* performed through the weights applied to the segmented face is actually the measurement of a force. At the same time, the *potenzia* of water that increases *in ogni grado della sua profondità* seems again to be more similar to the modern concept of pressure. It is possible, after all, that Leonardo uses this term as common as evocative, with multiple meanings, because he is looking for the right way to call ideas and concepts that maybe are still slightly unclear in his mind, but that he likely feels new (Macagno 1985, 71-96).

Conclusions

This paper presents an overview of the notes and drawings that Leonardo drew up in connection with his studies on the pushes exerted by a fluid at rest on flat vertical surfaces, actions that we now describe through the concepts of hydrostatic pressure and force. According to what can be deduced from the surviving manuscripts and their more likely dating, it seems that Leonardo's ideas on this subject have not undergone radical changes over time, but rather subsequent enrichment brought by different experiences. The observation of jets outflowing from multiple orifices made in the wall of a container may have suggested to Leonardo, since the early Nineties of the XVth century, the idea of a push exerted by the fluid on the wall. The variation of the ranges of these jets, as well as the processes of efflux from containers of different sizes, seem to be the experiences that have led Leonardo to believe, correctly, that the push in one point depends only on the depth of the fluid and not on its volume. Significantly, some of his notes treat the problem by using ideal and 'infinite' dimensional devices, actually a visual translation of the law of variation with depth grasped in its essence. In spite of the often qualitative character of Leonardo's research, he also attempts to quantify water push, probably in order to solve some engineering problems that had stimulated his curiosity. There is no evidence about a resolution of the doubts on the nature of this push that must have accompanied him for a long time, and that drew him more than once in error with regard to the relationship between the weight of the fluid and the actions that it exerts on vertical and horizontal surfaces. It is significant, however, that he considers this action similar to the contemporary concept of hydrostatic force, a quantity whose distribution along the vertical direction may be measured through the balance with a series of increasing weights applied to elements of equal areas. This detail brings Leonardo extremely close to the idea of force per unit area that is nowadays associated with the concept of pressure, whose effects on fluid properties and behaviour seems to be the core of many other notes and drawings by him.

References

Roberts, J. (1982), "Il contesto del Codice Hammer", in Roberts, J. (ed.), *Il Codice Hammer di Leonardo da Vinci: le acque, la terra, l'universo*, Firenze: Giunti Barbera, 13–22

Di Teodoro, F. P. (2002), "L'architettura idraulica negli studi di Leonardo da Vinci: fonti, tecniche costruttive e macchine da cantiere", in Hopkins, A., and Conforti, C. (eds.), *Architettura e tecnologia. Acque, tecniche e cantieri nell'architettura rinascimentale e barocca*, Roma: Nuova Argos, 258–277

Marani P. (2010), "Leonardo e le acque in Lombardia: dal 'Primo libro delle acque' ai Diluvi", in Calzona, A., and Lamberini D. (eds.), *La civiltà delle acque tra Medioevo e Quattrocento: atti del Convegno Internazionale*, Firenze: Olschki, 329–346
Bellone E., Rossi P. (1982), Leonardo e l'età della ragione, Milano: Scientia
Macagno E. (1985), "Hidrostatica vinciana en el Codice Hammer", in *Estudios in honor de Francisco Javier Domínguez*, Santiago de Chile: Universidad de Chile, 71–96
Fassò C. (1987), "Birth of hydraulics during the Renaissance period", in Garbrecht G. (ed.), *Hydraulics and hydraulic research: a historical review - IAHR 1935-1985*, Rotterdam: Balkema
Pedretti C. (1978), *The Codex Atlanticus of Leonardo da Vinci: a catalogue of its newly restored sheets*, New York: Johnson Reprint Corporation

ISBN 978-960-98199-3-0