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Doctoral Thesis

Power Systems Vulnerability and Performance: Application from Complexity Science and Complex Network

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Abstract

Power system has been acknowledged as a complex system owing to its complexity resulting from interactions of different layers which include physical layer like generators, transformers, substations and cyber layer like communication units and human decision layer. Complex network theory has been widely used to analyze the power grids from basic topological properties to statistic robustness analysis and dynamic resilience property. However, there are still many problems need to be addressed. This thesis will pay more attention on the application and extension of complexity science and complex network theory in power system analysis from different aspects:

In the first place, one of our aims is using an extended topological method to effectively explore the structural property and analyze vulnerability of power systems by introducing some electrical engineering features into traditional complex network approach. Based on this consideration, some features such as line impedance, line flow limit are introduced into characteristic path length and clustering coefficient as well as degree metrics in order to examine if power grids share structural features of exist complex network models. And aiming at analyzing static robustness of power grids, a new metric named electrical betweenness is proposed and used by introducing power transmission capability and line flow limit into betweenness centrality which is a metric to measure the importance of a vertex or an edge in network. In the meantime, a metric named net-ability is used to replace the original network efficiency to quantify the performance of power grid from a global perspective. Through our extended complex network methodology, the bulk interconnecting power transmission network UCTE is analyzed to see the efficiency and accuracy on the spotting component importance and the robustness of network with respect to different attacks.

Secondly, although power grids have been thoroughly studied as complex network and many topological measures have been used in order to classify their structure, evaluate their behavior in terms of robustness or model their dynamic

response to malfunctions. Their results have been mainly theoretical and no correlation between power grids' realistic behavior (i.e., malfunctions and major events) and any structural measure has been found. Therefore, a first attempt to correlate these new measures with real malfunctions data for some major European power transmission grids is given in this thesis. Based on our proposed new metrics, similar behavior is found in four major power transmission networks (Germany + Italy, France + Spain), in terms of robustness to selected attacks to buses, between different networks. This is measured by means of extended topological indexes electrically better defined. These behaviors can be (weakly) correlated with similar probability distributions of major events, identifying similar dynamical response among topologically similar grids. It would raise hopes in finding a more meaningful and significant linkage between structural measures and the real dynamical output (i.e., major events) of a grid.

Thirdly, as complex systems are usually characterized by some level of hierarchy, which spans in time and space at different scales. This hierarchical structure commonly allows reducing costs in terms of reliably transmitted information but at the same time involves different dynamical responses to malfunctions. In the case of critical infrastructures like transmission power grids, different hierarchical structures may lead to different behaviors in terms of accumulated major events. We compare and evaluate the evolution of hierarchy for some real different power transmission networks when buses are attacked selectively in decreasing order of some topologically and electrically defined metrics. The simulation results show that: hierarchy increases when the network is being attacked and a low variability of hierarchy implies an increased probability of accumulated major events.

Finally, in the smart grid scenario, new energy generation facilities (mainly based on renewable sources) are becoming widely accessible and becoming more and more numerous. In this situation, distribution grids gain more and more importance, while requiring a major update. Most of the researches focus on modeling the power grid as a simple graph, and the differences are undirected or directed, unweighted or weighted. However, the power grids have significant spatial characteristics: the coordination of the generator, transformer and substation, the wiring direction/shape and length of power cables, etc. Therefore, in our research we apply complex network theory to power distribution network analysis, and model the power grid as a spatial network. Some real distribution networks: SDN1 and SDN2 (Spanish distribution network 1&2) and NL1-NL12 (the Netherlands distribution network 1-12) are analyzed using complex network methodology. The cumulative distribution functions of degree, betweenness and real length of cable exhibit some significant differences. In order to explain these differences better, we study the role of branch wiring in spatial model of power grid. Two methods: edge exchange shuffling and vertex swapping shuffling are used to reveal the relation between branch wiring and performance optimality. The simulation shows that SDN2 network and one Dutch network don't achieve their optimal branch wiring compared with other networks. Again, the real malfunctions data will be used to verify our simulation results aforementioned.

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Chapter 1.

Introduction

1.1. TASKS ENCOUNTERED

Although complexity science or complex network in short is widely used in the power systems analysis especially the vulnerability studies, there are still some problems are neglected and needed to be addressed further.

Firstly, to our best knowledge, the initial researches mainly focus on traditional pure topological method which overlooks electrical engineering specificity so that the analysis and understanding of power systems could be far from the reality. However, power grid has its own features like line impedance, flow-based network and transmission capacity, etc. It seems that there needs an update of the pure topological method evolving specific electrical characteristics together.

Secondly, although batch of metrics have been proposed to evaluate the importance or robustness both for the components and the whole network, there is a lake of linkage between measures (such as structure or capability) and malfunctions (such as the failures of real power systems' operation). This linkage would be beneficial to verify the feasibility and efficiency of the application of complex network methodology to the vulnerability analysis of power systems.

Thirdly, the hierarchy seems to pervade complexity in both living and artificial systems. As an important complex system, how power grids' hierarchy characteristics will influence its own performance and robustness is a pretty good point to be studied. On the other hand, whether this hierarchy property will affect the cascading failure evolution could help us to guide the construction of the power grid at the beginning to improve its robustness.

Last but not least, most of the researches focus on high voltage networks or power transmission networks. While with the emerging development of smart grid, the power distribution networks gain more and more importance. Therefore, a comprehensive study is needed to power distribution networks involving complexity science and complex network methodology.

1.2. NOVELTY AND FOUNDATIONAL CONTRIBUTION

In this thesis, complexity science or complex network theory is applying and extending to power systems analysis. Multiple aspects of novelty can be found from the encountered task and possible solutions:

First of all, to our best knowledge, exist studies about the application of complex network theory in power systems neglect the specific electrical features. However, power grids have obvious different characteristics compared with other kinds of complex networks. Based on this consideration, this thesis tries to introduce the electrical features of power systems such as line impedance, power flow and transmission capacity into traditional pure topological methodology. A novel extended method including three new metrics: entropy degree, electrical betweenness and net-ability are proposed and used to assess the vulnerability of power systems.

When using complex network theory to analyze the vulnerability of power systems, a natural consideration is restoring to some metrics both traditional and extended. Batch of papers have addressed this problem and some metrics have been proposed. However, there are still no papers mentioned how to verify the feasibility and efficiency of these metrics. Therefore, a first attempt to build a linkage between our proposed extended topological metrics and malfunctions dataset of power grids is given in the thesis. Four major power grids: France, Germany, Italy and Spain are chosen to be analyzed using our proposed electrical metrics. And the cumulative probability distribution functions of the malfunctions for these grids are checked respectively. A linkage is built between topological measures and malfunctions in the similar topologically characterized networks.

Hierarchy is another important feature of complex network. The novelty of this thesis is that we investigate the hierarchy evolution of power transmission networks under a morphosapce coordinates that evaluates and quantifies the hierarchy properties of complex networks. The interesting point is that the evolution arbitraries of different power grids have a strong correlation with their real operation malfunctions. Based on this observation, the influence of hierarchy property to the cascading failure is further studied.

In order to investigate the power distribution networks not only in terms of vulnerability but also in the performance optimality, a spatial model considering the geographic coordinates of node and branch is built. In this spatial model, the topological and spatial properties are checked. Additionally, the role of branch wiring in spatial model of power grid is studied as well. Two methods: edge exchange shuffling and vertex swapping shuffling are used to reveal the relationship between branch wiring and optimality of performance. Furthermore, the real malfunctions data of each grid will be used to verify our simulation results aforementioned.

1.3. STRUCTURE OF THE THESIS

In addressing aforementioned methods and resolutions, the main chapters of the thesis are organized as following:

- Chapter 2 gives a short but comprehensive description about the complexity science and complex network theory. In the meantime, the complexity of power systems is addressed and modeling power network as complex network is also introduced. After that, a comprehensive review about the application of complex network theory in power systems is given.
- Chapter 3 starts from the analysis of the neglecting of specific electrical features in power systems of traditional pure topological method in complex network theory. Based on this consideration, an extended topological methodology is proposed which involving complex network theory and electrical features of power systems together and three new metrics are proposed to analyze the vulnerability of power systems.

- Chapter 4 illustrates the lack of efficient method to verify the feasibility and efficiency of topological metrics derived from complex network theory to assess the vulnerability of power systems. A first attempt is given in this chapter to link the analysis results of proposed topological metrics with the real malfunctions of power systems. Results show statistically meaningful (although weak) correlations among similar topologically characterized networks, which could finally help in defining a linkage between topological measures and malfunctions in power grids.
- Chapter 5 analyzes the hierarchy evolution of power transmission networks under a morphorspace coordinates. The different evolution arbitraries have a strong correlation with their real malfunctions data. This phenomenon illustrates that the hierarchy property of complex network has a strong influence to cascading failure. Therefore, the relationship between hierarchy and cascading failure is further analyzed in this chapter.
- Chapter 6 extends complex network theory from power transmission networks to power distribution networks. On the one hand, the traditional properties are checked for distribution networks, on the other hand, the spatial network model is built for distribution networks and their optimality property under spatial constraints is analyzed. Different performance is obtained and analyzed. Furthermore, the malfunctions datasets are used to verify the results as well.

Chapter 2.

Complexity in Power Systems and Complex Network Approach

Since the traditional methods of science and analytical philosophy were not sufficiently efficient and feasible to capture the dynamics exhibited in complex systems. Late last century there was a trend for geographers, biological, environmental, human, societal scientists and engineers applying a new theory, called complexity theory to topics ranging from economic growth, herds behavior, cultural dissimilation to the braiding of rivers, the urban geography [1-3], etc. in sharp contrast to the Newtonian Science based on reductionism, determinism and objective knowledge, the complexity science provides a new way of thinking that values the non-linear, interaction, asymmetrical relationships of elements in a system much more than analytically accurate models themselves which are used represent the behavior of a specific component. The significance of the elements was obtained through the complex interactions and interplays in a network.

The same story happens in power systems as well, especially after the deregulation since last century and the emergence of smart grids in the new millennium and low voltage networks which involved human distributed decision making scenarios. In the smart grids environment, the interactions among a considerable number of participants in various levels have been greatly enhanced. At present, many researches begin to exploit the complexity science and techniques to the study of power systems and to develop a new synthetic perspective to view the power systems.

In this chapter, the definitions of complexity and relevant issues regarding complexity science and techniques will be given. Then the complexity in power systems and a scene of power system as a complex system are discussed. Furthermore, the new application about complex network theory to power systems is presented and a corresponding and comprehensive review is also given.

2.1. COMPLEXITY AND COMPLEX SYSTEMS

There is no general accepted formal definition of complexity science like Heyligen gave an answer to this question: “Conceptually, the most difficult aspect of complexity is still its definition and the deeper understanding that goes with it” [4]. There have been many different forms of endeavors in complexity over almost all disciplines which cover various definitions and measures of complexity. Generally, they could be categorized into three groups: Algorithmic complexity [5][6], Deterministic complexity [6] and Aggravate complexity [6]. The first group covers the complexity of describing system characteristics, such as mathematical complexity theory and information theory. The second group includes the interaction of very few key variables that create largely stable systems prone to sudden discontinuities, such as chaos theory, catastrophe theory, etc. the last one which mostly interests us concerns how individual elements work in a synergy that generates complexity in a system.

To explicitly and clearly define the notion of complexity used in this dissertation, while avoiding too generalizing the concept to be workable or losing any useful positive meaning, in this thesis we adopt the definition of complexity as follows: “Complexity is a property that makes it difficult to analytically formulate its overall behavior even when knowing the complete information about its elements and their relationships”. Here “difficult” could involve several aspects such as size, depth, computational indication, efforts in a search for the most apt representation, etc.

Accordingly, a general and logical definition of complex system is a system that exhibits complexity: “A system, that can be decomposed in a set of different types of elementary parts with autonomous behaviors, goals and attitudes and an environment, is complex if its modeling and related simulation tools cannot be done resorting to a set of whichever type of equations expressing the overall performance of the system, in terms of quantitative metrics, or a function on the basis of state variables and other quantitative inputs". It's noticed that this definition is more like an articulated and practical way which targeting at engineering systems.

Over the last several years, complexity science has changed the way scientists approach all fields of life, form biology to medicine, from economics to engineering [7-11]. The concepts or techniques such as self-organization, genetic algorithm, cellular automata, criticality, artificial life or chaos theory are now widely accepted and used as new means of understanding the always changing reality. The history complex systems research including these concepts begins in the 1950's, emerging with the advent of von Bertalanffy's systems theory, the appearance of nonlinear phenomena in scientific fields away from physics, like chemistry and biology, and the study of feedback concepts in communication and control in living organisms, machines and organizations. From these early stages, the idea of threshold turned up to be the cornerstone of much of the complexity science developments of the 1980's, especially in the cellular automata and artificial life fields, where complex behavior seemed to appear suddenly [12-14]. From then on many books, journals, conferences, and even whole institutes devoted to the field have flourished everywhere, and even computer modeling of complex systems has become widely accepted as a valid scientific activity. A conceptual map which cited from the dissertation of Marti Rosas-Casals ("Topological Complexity of the Electricity Transmission Network: Implications in the Sustainability Paradigm") is shown in *Fig. 2-1* highlights various aspects involved in the characterization of complexity. In this conceptual framework complexity pervades both the (a) structure (i.e., formal arrangement of the constituent parts), (b) dynamics (i.e., functional behavior) and (c) evolution (i.e., the way it has reached its actual formal and functional state) of any system. It covers comprehensive majors of complexity science and different technology and method.

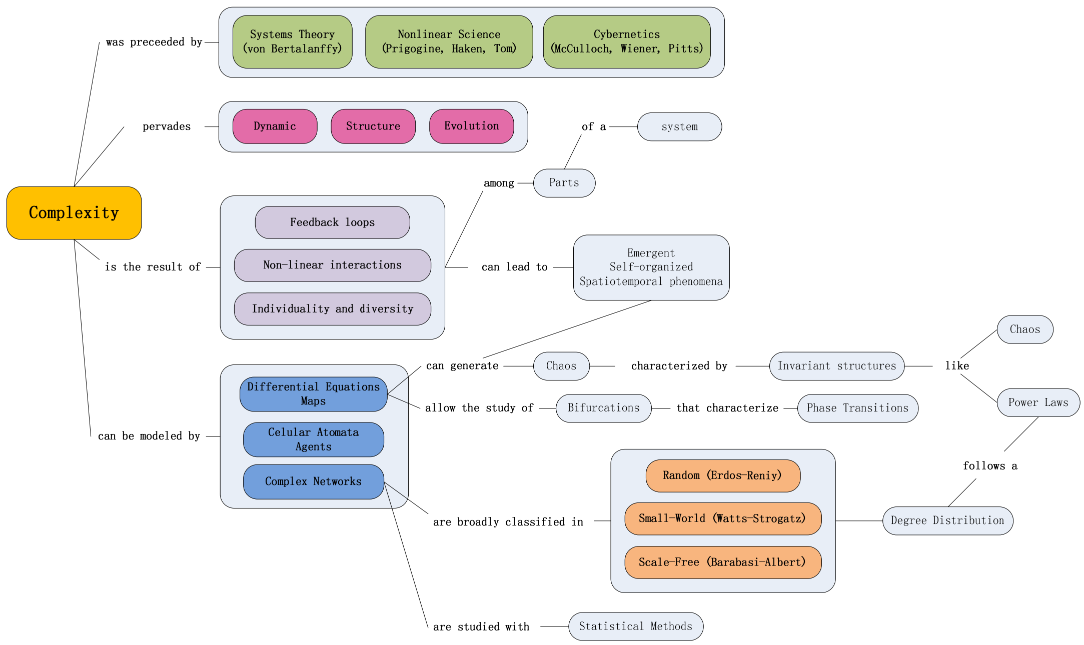

Fig. 2-1 The conceptual map to highlight the various aspects involved in complexity's characterization. (Source: Marti Rosas-Casals, "Topological Complexity of the Electricity Transmission Network. Implications in the Sustainability Paradigm", Ph.D Thesis)

Reference [15] gave a try to review the methods and techniques of complex systems, and grouped them into three categories: (1) those for analyzing data, (2) those for building and understanding models, and (3) those for measuring complexity as such. The techniques for the purpose (1) and (3) are out of the scope of this thesis, hence, what we focus on are those commonly used in power systems like multi-agent modeling and complex network theory which will be described specifically in the next section.

2.2. COMPLEX NETWORK

As introduced in the last section we have noticed that complex system, or complexity in short, is a new approach to science that studies how relationships between parts give rise to the collective behaviors of a system and how the system interacts and forms relationships with its environment. Complex system is a very hot research arena which is studied by many areas of natural science, mathematics, and social science. Many complexity models have been proposed include human economies and social structures, climate, nervous systems, as well as modern energy like power systems or telecommunication infrastructures. One of important branch of complexity techniques is called complex network theory which is conceptualized as the intersection of graph theory and statistical methodology [15]. Complex network theory pays attention on top level properties, i.e., a global level, to analyze the emergent pattern of the system mapping on a graphic representation. It examines the interconnections in diverse physical, engineering, social, etc., networks, seeking for principles, algorithms governing the network patterns and leading to predictive models.

Over the last decade, mainly due to advances in computational capacity and database accessibility of computer science, modeling and computational methods have stimulated the interest of the scientists to analyze complex systems as networks. In its broadest sense, a network is a formal and functional representation of a complex system, where vertices are the elements of the system and an edge represents the interactions between any two of vertices. For example, living cells are supported by large molecular genetic networks, whose vertices are proteins and edges represent the chemical interactions among them. Similarly, complex networks occur in social sciences, where vertices are individuals, organizations or countries and the edges characterize the social, economic or cultural interactions among them [16]. Examples from nowadays biological science, which is shown in *Fig. 2-2* left part, a network showing 3200 protein interactions between 1700 proteins; or the information science, which is shown in *Fig. 2-2* right part, the world wide web whose vertices are HTML documents connected by links pointing from one page to another [17][18]. When power network as a complex network, the generators, transformers and substations could be abstracted as the vertices and the power cables could be modelled as edges.

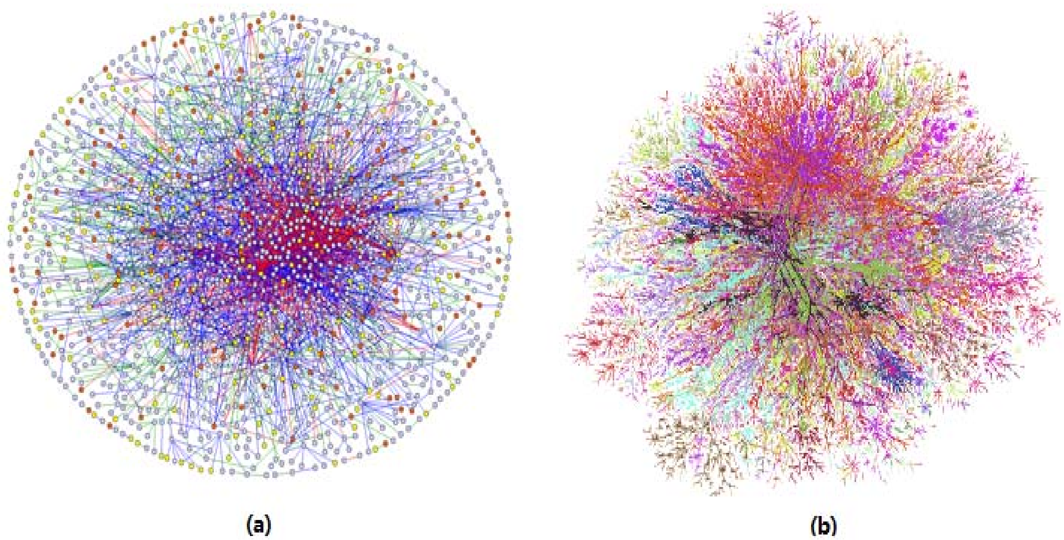

Fig. 2-2 A complex system viewed as a network. Left: protein interactions. (Source: [17]). Right: part of the actual internet, retrieved from the Internet Mapping Project.(Source: http://www.visualcomplexity.com).

In order to study the complex systems from the topological point of view, complex network approach has become popular. Following will give a review of the complex network approach application in complex system including structural properties and structural robustness.

Structural properties of a network and its evolution process could be analyzed by defining and calculating a set of metrics. The metrics can in turn be used to categorize real networks into several classes with different properties, as different classes of networks have different features that can be characterized by the class itself. In fact, network models based on real systems show some special features, such as community structure (the presence of groups of vertices more densely interconnected), power law degree distributions (the probability distribution of the number of edges connected to a vertex follows a mathematical power law) [19] and hubs (vertices linked to a large part of the edges of the network) [20]. Three specific works have made particular contributions to this field: the model of random network by Bollobás [21], the investigation of small-world networks by Watts and Strogatz [22], and the characterization of scale-free models by Barabási and Albert [19][20].

Besides structural classification of networked complex systems, another research topic of complex systems from topological point of view is its structural robustness, which can be defined as the ability of a network to avoid malfunctioning when a fraction of its components is damaged. This was one of the first issues having been explored in the literature on complex networks [20][23] and it can be encountered in two different groups: static robustness and dynamic robustness. Static robustness is the act of deleting nodes without the need of redistributing any quantity that is transmitted in the network; while dynamic robustness refers to the situation that dynamics of the redistribution of flows has been taken into consideration. Deletion is the most common method for detecting the vulnerability of the networks which usually refers to the components in the networks, such as deleting vertices or edges. At the same time, both groups can be implemented in two ways: Errors (or random failures) and Attacks (or selective failures). Errors are the ability of the system to maintain its connectivity properties after the random deletion of a fraction of its vertices or edges. Attacks are the ability of the system to maintain its connectivity properties when a deletion process is targeted to a particular class of vertices like the highly connected ones.

The static robustness of a network to maintain its connectivity obviously depends on its original topology and the way to modify its structure (i.e., errors and attacks by means of successive deletion). For example, scale-free networks, i.e. World Wide Web links, are extremely sensible to attacks but very resilient to error failures; while random networks, i.e. Erdos-Renyi model, react similarly to any kind of deletion of the components of the networks [20]. Also, it is significant to find the critical components in networked systems. In this respect, efficiency [24] seems to be a promising metric for analyzing the overall structural vulnerability in a networked infrastructure such as routing network [25], internet [26], subway network [27], power systems [28], and so on.

Dynamic robustness is another important problem considered in complex networks research since it refers to modeling the dynamics of flows of the physical quantities of interest over a network. When it comes to modeling the dynamics, the situation is far more complicated since the components of a network may have different dynamical behaviors and the flows are often highly variable, both in space and time. In the traditional topological method, the betweenness centrality is used to evaluate the flow of the physical quantities over a network. Betweenness centrality is a measure of a node's centrality in a network, equal to the number of shortest paths from all vertices to all others that pass through that node [29]. Since in traditional topological method, it consumed that the physical quantities is always passing through the shortest path, therefore betweenness can be seen as a useful measure of the load over a network. By reviewing each element characterized by a finite capacity (defined as the maximum load that the element can handle), the dynamic robustness of the network is then evaluated in the following way: 1) a deletion of node, which obviously changes the shortest paths between vertices. Consequently, the redistribution of betweenness, possibly creating overloads on some other vertices. 2) All the overloaded vertices are removed simultaneously from the network. This leads to a new redistribution of loads and subsequent overloads may occur again. 3) The new overloaded vertices are removed and the redistribution process continues until at a certain time all the value of betweenness of the remaining vertices under or equal to its capacity [30][31].

2.3. POWER SYSTEM AS A COMPLEX SYSTEM

2.3.1. Complexity in power systems

Before the arising of complexity science, people used complexity to refer to a very complex situation in power systems, such as ref. [32] used it to describe the increase of heavily computational burden on solving power flows as the increase of system size. Ref. [33] defined complexity over the failures and employed it to state problems that could become potentially quite difficult to solve. Ref. [34] described a system involving DC lines or asynchronous operation, loss of synchronism, etc., as complex system. However, these situations were conceptually incorrectly described as complex while they were actually complicated.

In recent years, the difference between complexity and complication has been noticed by the researchers of power system. A few researches regarding complexity and the applications of its theories to power systems were reported. Instead of looking at the details of particular blackouts, ref. [35] studied the statistics and dynamics of series of blackouts with approximate global models. Ref. [36] employed topology analysis to figure out the vulnerability of a given transmission system and concluded that when a network is attacked following a delicately sequence corresponding to their criticality, the network would illustrate more vulnerability. A very important research using complex network features to analyze the topological structure and static tolerance to errors and attacks of the Union for the Coordination of Transport of Electricity (UCTE) power grids [37] was published. The authors found that the nodal removal behavior can be logarithmically related to the power grid size, which suggests that though size favors fragility, growth can reduce it.

Energy infrastructures, such as power systems, are characterized by a large number of components and many different types of interactions among them. Size itself does not infer complexity. Continental-scaled power grid, for example, is the biggest dynamic system in the world but from a physical point of view it can be modeled by a huge set of differential and algebraic equations. It may conjure complexity in the computational efficiency; however, it is somehow solvable by using computationally powerful facilities and advanced algorithms. In contrast, complexity arises when the physical substrate interacts with the rest of hierarchical levels governing and using the infrastructure. The overall expected performance and dynamic evolution are related to those interactions at the "individual" scale. These phenomena cannot be handled nor studied with a set of equations in any form. Studies and applications related to the deregulation towards market environment have thrived in both academia and industries. This change brought a great challenge to power systems in the production and transmission. With the prevalence of the distributed generation and smart grids, the distribution and utilization are confronting with new scenarios in which a large number of users transformed from passive receipts to active participants. The emerging situation and newly introduced players with clear self-interest display an important role for the future power system which will increase the complexity of power system further.

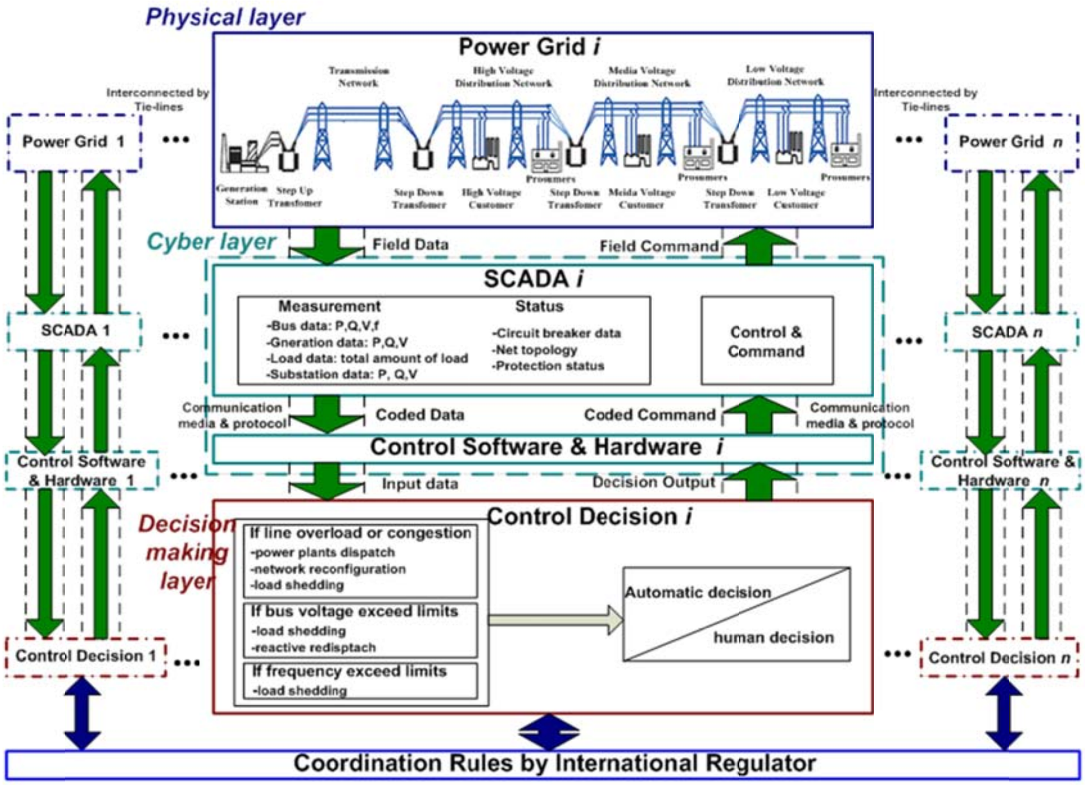

Fig. 2-3 Complex interactions in the power systems

The power system is a typical system of Multi-layer Interacting Reality (MLIR) as shown in *Fig. 2-3*. In power systems, various layers involving different physical, technological, human decision-making systems interact among themselves to determine the overall performance of the system that can be measured by a set of meaningful metrics, such as energy savings, environmental pollution, market efficiency, etc., which had not been seriously considered before. The overall "system control" can be exerted only in terms of policy action, implemented by laws and regulations to influence the behavior of various players. For example, a regulator may issue a set of mandatory codes to compel generators providing necessary auxiliary services to keep system feasible in terms of providing quality power supply without endangering the system itself. Market designers may set prohibition for participants to game collusively in any phase of any market. Incentive or disincentive could be of great use in modifying ratios of various energy sources. In contemporary power industry, green energy is incredibly encouraged for replacement of high-pollutant fossil resources, which can be achieved by discounting access tariff for renewable energy and subsidizing green-energy price.

The complexity of power systems also increases with the change of its administrative mechanism. Initially, in power systems, each utility and/or pool of utilities has control centers which support today's hierarchical monitoring and control of the grid. Moreover, electrical market is gradually introduced into power systems in order to transmit the least expensive power in power grids. The important consequence of this situation is that utilities require systematic integration of monitoring, computing and controlling for improved performance. Therefore, the interaction between power grids and decision information via cyber layer is more complex than before.

Besides, renewable energy such as wind power, solar energy, fuel cell and so on is drastically emerging and developing in the distribution level of traditional power industrial. This trend apparently increases the complexity in power systems as a whole.

2.3.2. Power grid as a complex network

Power grids have been widely acknowledged as a typical complex network because of both their huge sizes of components and the complex interactions among them. For example, the UCTE transmission network has about 5910 nodes and 7970 transmission lines. The North American power grid has about 14,099 nodes and 19,657 transmission lines. A typical paradigm about abstracting power grid as a complex network is shown in *Fig. 2-4*. In part (a) we can see that the power systems is composed of multiple and diverse elements, such as generator, transformers, switching stations, etc., connected physically by electric cable lines; part (b) is an Italian (220-400 KV) transmission power grid, where each node is a substation or transformer; part (c) is an abstract network of the Italian transmission power grid in part (b) from pure topological point of view.

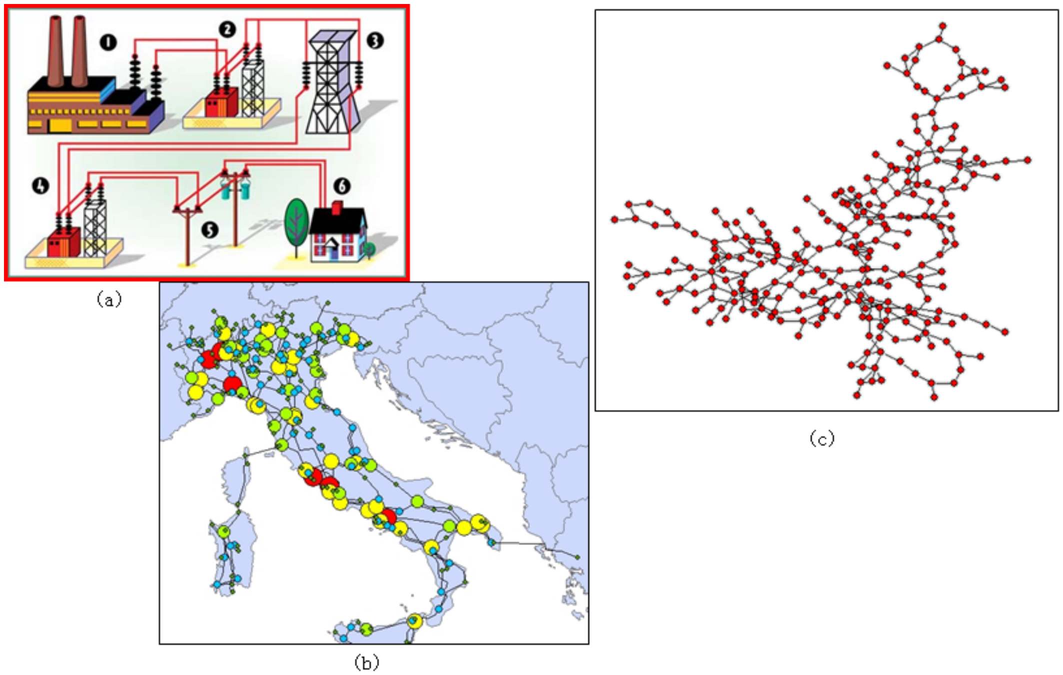

Fig. 2-4 Power grid as a complex network

Mathematically, from the graph theory point of view, when applying complex network methodology to power systems, the electrical power grid as a weighted and directed network identified by a set Y = {B, L, W} where B (dim {B} = N_B) is the set of vertices (or nodes), L (dim {L} = N_L) is the set of edges (or links) and W is set of line weights. Vertices are identified by index *i*. Edges are identified by l_{ij}, which represents a connection between vertex *i* and vertex *j*. And the weight element w_{ij} in the set W is associated with each line l_{ij}.

With the development of complex system theory, power grids arise as natural objects of study under the conceptual frame of complex systems, particularly as complex networks. Therefore, complex network methodology as one of approaches to study complex systems has been used to analyze and understand power systems from topological point of view. How complex network theory and methodology applied in power systems study especially its vulnerability analysis will be addressed in the following sections.

2.4. REVIEW OF COMPLEX NETWORK METHODOLOGY APPLIED IN POWER SYSTEMS

Complex network theory has received considerable attention recently which has been used in many different fields. A lot of researches including basic characteristics, statistical global graph properties, small-world property, scale-free property, degree distribution, betweenness distribution and vulnerability analysis, have been performed to power grids since they are infrastructures in our society. It is noticed that there is a strong link between the topological structure and operation performance in power systems because the structural change could alter operational condition of a power system and thus change its operation performance. As a result, there is an increasing interest in analyzing structural vulnerability of power grids by means of complex network methodology.

In this section, to our best knowledge, a brief but comprehensive review about the application of complex network theory in power systems from basic structure property analysis to vulnerability assessment.

2.4.1. Structure property analysis

As mentioned in section 2.2, there are three main models of complex networks: small-world, scale-free and random networks. Different network will exhibit different structure and vulnerability property. Therefore, the first question to analyze power grid is what type of power grid is. The first reference comes from Watts and Strogatz [22] who analyzed the graph of the United States western power grid. It was deduced that the western power grid seemed to be a small-world network. After that, Barabasi and Albert in 1999 [19] firstly published that degree distribution of a power grid was supposed to be scale-free following a power law distribution function, but few of the subsequent later references would support this finding. Exponential cumulative degree function was detected in Californian power grid [38] and the whole United States grid [39]. The topological features of the UCTE (Union for the Co-ordination of Transport of Electricity) power grid and its individual nation grids are analyzed and results showed these national transmission power grids' topologies are similar in terms of mean degree and degree distribution, which could suggest similar topological constraints, mostly associated with technological considerations and spatial limitations [37]. Besides, the topologies of the North American eastern and western electric grids were analyzed to estimate their reliability based on the Barabasi–Albert network model. The results were compared to the values of power system reliability indices previously obtained from some standard power engineering methods, which suggested that scale-free network models are applicable to estimate aggregate electric grid reliability [40].

2.4.2. Vulnerability analysis

✓ ***Traditional approach to assess power systems vulnerability***

Power systems are one of critical infrastructures since they are widely distributed and indispensable to modern society. Both accidental failures and intentional attacks can cause disastrously social and economic consequences. For example, in August 2003, the historic blackout of United States and Canada in which 61,800 MW of power were disconnected to an area spanning most of the north-eastern states of United States and two provinces of Canada, and more than 50 million people remained without electricity for 15 hours [41]. Therefore, electrical utility operators need to analyze the vulnerability of power systems and identify the critical components whose protection or back-up will result in a more robust system against natural or malicious threats.

The concept of a vulnerable system is defined in [42] as a system that operates with a "reduced level of security that renders it vulnerable to the cumulative effects of a series of moderate disturbances". Vulnerability is a measure of the system's weakness with respect to a sequence of cascading events that may include line or generator outages, malfunctions or undesirable operations of protection relays, information or communication system failures and human errors.

According to the functions and structures of power systems, there are four basic parts in our security interests of a power network, listed as following:

- Transmission network (e.g. 380kV and 220kV in Italian transmission system)

- High voltage (HV) distribution network (e.g. 150kV and 132kV)
- Substations
- Power plants

Compared with substations and power plants, the networks are much more widely distributed in geography, and this makes them more easily and possibly to be targeted by intentional threats than substations and power plants where strict protections may be implemented, such as on-site police guard, access control, anti-burglary system or perimeter detection system in substations. However, on the contrary, if the substations or power plants are really successfully attacked, the consequences and impacts may be more serious. For example, the failure of a key substation can be considered as the failure of all transmission lines connected to it.

A vulnerability assessment is the process of identifying, quantifying, and prioritizing (or ranking) the vulnerabilities in a system [43]. Energy utilities should routinely perform vulnerability assessments to better understand threats and vulnerabilities, determine acceptable levels of risks, and stimulate action to mitigate identified vulnerabilities. The direct benefits of performing a vulnerability assessment include:

- Build and broaden awareness.
- Establish or evaluate against a baseline.
- Identify vulnerabilities and develop responses.
- Categorize key assets and drive the risk management process.
- Develop and build internal skills and expertise.

In the traditional study of vulnerability of power systems, the vulnerability is analyzed using methods completely based on operational data and physical models such as static security assessment [44][45] and dynamic security assessment [46].

However, these traditional methods evaluate the security and reliability relying on a given contingency and operating condition. On the one hand, it is computationally infeasible to check all possible combinations of contingencies that could cause serious blackouts in practical power grids; on the other hand, operating conditions of power systems change in time due to load variations, switching actions, etc. So it is difficult to prevent the collapse of electrical power grids owing to unforeseen operating conditions. Besides, due to the size of large-scaled power systems, physical behaviors and the interaction among many operators over power grid add difficulty to perform a comprehensive analytic analysis and simulation of the electromagnetic processes over the whole grid. Hence, in practical, reduced systems or some simplifying hypothesis are applied to these conventional methods to simulate the network's response to various external disturbances, but the simulation results cannot reflect the exact response of power systems.

As a result, frequent blackouts occurred all over the world although advanced technologies and huge investments have been exploited in maintaining the reliability and security of power systems. To deepen the insight into power systems, it is necessary to develop and complement the conventional analysis technology with new point of view.

✓ ***Structural robustness analysis***

The vulnerability analysis of network is the main motivation for the studies involving CN analysis into power grids. The first power grid whose robustness was analyzed was the North American power grid [39]. The authors removed vertices randomly and in decreasing order of their degrees for both generation vertices and transmission vertices, and monitored the connectivity loss which measured the decrease of the ability of distribution substations to receive power from the generators. The loss of generating substations does not significantly alter the overall connectivity of the grid owing to a high level of redundancy at the generating substations. However, the grid is sensitive to the loss of transmission nodes. Even the removal of a single transmission node can cause a slight connectivity loss. Especially, the connectivity loss is substantially higher when intentionally attacking higher degree or high load transmission hubs. They concluded that the transmission highly connected hubs guarantee the connectivity of the power grid but meanwhile they are also its largest liability in case of power breakdowns. The first reference to European power grids was made by Crucitti *et al.* The authors studied and compared the topological properties of the Spanish, Italian and French power grids, finding those components whose removals seriously affected the structure of these graphs [47]. Since the proposed improvements also treat power grid as a simple graph and no physical features are taken into consideration, we think that the power grid vulnerability results obtained with this approach could be different from the real situation. Rosato *et al.* studied the topological properties of high-voltage power grid in Italy (380 kV), France (400 kV) and the Spain (400 kV) [48]. An assessment of the vulnerability of the networks has been implemented by analyzing the level of damage caused by a controlled removal of links. Such topological studies could be useful to make vulnerability assessment and to design specific action to reduce topological weaknesses. Since the grids are the same as used in the former case, most of the results are consistent. Robustness of the whole European power grid is studied in [48][49], where also includes the resilience against to the failures and attacks of every national power grid. The authors found that European power grid composed of the thirty three EU power grids could broadly be classified into two separate groups, fragile and robust.

It is noticed that cascading failures have frequently occurred throughout electrical power grids of various countries. The cascading failures firstly were analyzed in electrical power grid of the western United States [30]. The degree distribution in this network appeared exponential and was thus relatively homogeneous. The distribution of loads, however, was more skewed than what displayed by semi-random networks with the same distribution of links. This implied, to a certain extent, that the power grid may have structures not being captured by existing complex network models. As a result, global cascades are supposed to be triggered more probably by load-based intentional attacks than by random or degree-based removal of vertices. The attack on a single vertex with large load may make the largest connected component decrease to less than a half of its initial size, though the network is highly tolerant. In North American Power Grid, the cascade phenomenon was also modeled [50]. It was observed that the loss of a single substation can lead to a 25% loss of transmission efficiency caused by an overload cascade in the network. A systematically study of the damage caused by the loss of vertices suggested that 40% of the disrupted transmission substations may lead to cascading failures. While the loss of a single vertex can exacerbate primary substantial damage, the subsequent removals only make the situation worse. Crucitti *et.al* applied cascading failure model into the Italian power grid where they neglected the details of the electromagnetic processes and only focused on the topological properties of the grid [51]. The objective of this study was to demonstrate that the structure of an electric power grid may provide important information about the vulnerability of the system under cascading failures. The power grid has 341 vertices (substations) and 517 edges (transmission lines). Different kinds of vertices have been distinguished. Although the degree distribution is not very different through the network, it still exhibits a high heterogeneity in the vertex load. Most of the vertices are only responsible for a small load, but a few other vertices have to carry an extremely high load. Large scale blackouts can be triggered by the failure of vertices with high loads. Perhaps it is due to the fact that some highly connected vertices may be not necessarily involved in a high number of paths. However, the used model is quite simplified for a real electric power grid, so that this result may be not very credible since the definition of degree and load here are not very meaningful for power grids. Jiang-wei, *et al.* [52] proposed a cascading failure model based on degree centrality to analyze the Western United States power grid. A counterintuitive result is found that the attack on the vertices with the lowest loads is more harmful than the attack on the ones with the highest loads. Simonsen *et al.* [53] studied cascading failures in power grids using a dynamical flow model based on simple conservation and distribution laws. Within the framework, it is studied that the role of the transient dynamics of the redistribution of loads towards the steady state after the failure of network edges. It is found that considering only flow of loads in the steady state gives a best case estimate of the robustness; the worst case of robustness can be determined by the instantaneous dynamic overload failure model. Bakke *et al.* [54] analyzed the power blackout of Norwegian high-voltage power grid using a model with Kirchhoff equations and the same line conductance. The results showed that the size distribution of power blackouts in Norwegian power grid seems to follow a power law probability distribution.

2.4.3. From static to dynamic

The works reviewed so far are mainly about the static properties such as the categorization of power networks and vulnerability assessment of the components (buses and lines) in power systems. Recently published papers extend these static analyses to dynamic ones. For example, a Kuramoto oscillator model is introduced as a phase model to analog the synchronous generator in order to analyze the synchronization stability property of the coupled generators in the whole power networks. The Kuramoto oscillator is motivated by the behavior of systems of chemical and biological oscillators, and it is also adopted as the synchronization model in the complex network.

In Bullo and Dorfler's papers [55][56], Kron reduction of graphs was introduced to eliminate the load buses of the power network and Kuramoto oscillator like model is used to model the synchronous machine, then the whole power grid is a coupled Kuramoto oscillator like network. The explicit necessary and sufficient condition on the critical coupling strength to achieve synchronization is studied. Similar results are also addressed by M. Rohden *et al.* [57] and S. Lozano *et al.* [58]. In H. Sakaguchi's paper [59], this Kuramoto like model has been used to analyze the cascading failure in power grid. In the meantime, some other dynamic features are also taken into consideration: a dynamical flow model is used by Helibing *et al.* to study the cascading failure in a power grid [53]. Restrepo *et al.* [60] proposed instead a general theoretical approach to study the effects of network topology on dynamic range. All these works extend the complex network theory from steady-state analysis to dynamic, which significantly improves the studies about CN theory application in power systems.

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Chapter 3.

Extended Topological Methodology

Complex network theory proposed by physicists is an increasingly popular method to analyze complex systems such as social interacting species, internet, computer network and so forth. In the framework of complex network theory, complex systems are abstracted as networks consisting of a set of edges connecting a set of vertices and then inherent structure features of these networks are analyzed statistically by some metrics. Initially, the complex network approach is applied in some abstracted networks such as random network [1], small world network [2], scale-free network [3][4]. It is found that there is relatively short characteristic path length between any pair of nodes and high clustering coefficient in small world networks [2]; scale-free networks are robust against random failures of nodes but fragile to intentional attacks [5]; intentional attacks also more easily trigger a cascading failure in scale-free networks than random networks [6][7].

Power systems have been considered as complex systems where the complexity of power systems not just is resulted from the instant power balance of generators and consumers in large-scale transmission network across multitude of countries, but also from the intricate decision-making of system operators in order to keep the system secure and reliable. Furthermore, it is noticed that there is a strong link between topological structure and operation performance in power systems. For instance, a large scale blackout is more possible to be triggered by removing some critical buses or lines which are essential elements of topology structure of power systems. As a consequence, power systems are naturally analyzed under the framework of complex networks [8-13]. Structural vulnerability is analyzed in the North American power grid [8] and European power grids [9-11]. Cascading failure is modeled in North American power grid [12]. Critical transmission lines are located in Italian electrical power grid [13].

However, when applied to power systems, the complex networks method neglects the specific engineering features; therefore, the analytical results may be far from the reality in power systems, and so it seems more appropriate to analyze the structure vulnerability of electrical power grid combining the electrical engineering features with complex networks theory. Coming from this thought, in this section, the specific physical features of power systems such as electrical distance, line flow limit and power transmission distribution are introduced into the traditional complex networks metrics: degree [14], betweenness [15] and efficiency [16]. Three extended topological metrics: entropy degree, electrical betweenness and net-ability are proposed to assess the vulnerability of components and the whole power network. In addition, a simplified UCTE power grid is used to test our extended topological methodology for its structural vulnerability analysis.

3.1. PURE TOPOLOGICAL METHOD

In complex network approach, there exists a fundamental and important set of centrality indices measuring importance of a vertex or an edge in a network according to one or another criterion. Basically, these centrality indices can be divided into three classes: one is based on the idea that the centrality of a vertex in a network is related to how it is near to other vertices such as degree centrality; the other is grounded on the thought that central vertices stand between others, playing the role of intermediary, such as betweenness centrality. Besides, a class of delta centrality is recently proposed which measures the contribution of a vertex or an edge to a network performance when removal of it leads to the variation of such performance. These metrics compose the base of the pure topological method to measure the criticality of the components in networks.

There are four main types of complex networks, which include weighted digraphs (directed graphs), unweighted digraphs, weighted graphs and unweighted graphs [17]. In current literature most of the researches on infrastructure systems is focused on the unweighted graphs.

Initially, networked complex systems such as internet network can be abstracted as a unweighted graph **Y** = {**B**, **L**} to analyze their inherent structure features, where **B**(dim{**B**}=N_B) is the set of vertices (or nodes) and **L** (dim{**L**}=N_L) is the set of edges (or links). Each vertex can be identified by i; the edge is identified by l_{ij} that represents a connection going from vertex i to vertex j.

3.1.1. Degree

The connectivity of a node is measured by its degree, k_i, which is defined as the number of edges connected to a given vertex i [10].

$$k_i = \sum_j a_{ij} \tag{3.1}$$

On the one hand, the elementarily topological features of a graph can be obtained in terms of degree cumulative distribution P(k≥K) which is the probability that the degree of a node randomly selected is not smaller than K. Generally, if the degree cumulative distribution of a network follows a Poisson distribution, then it is a homogeneous network where each node has the same degree; on the other hand, if the distribution is a power law or exponential, then it is a heterogeneous network where there are some vertices which have higher degree than others.

On the other hand, since the degree indicate the connectivity of a node, if a node have higher connectivity, it means that this node has more connections between other nodes, in other word, it has more importance. Therefore, degree could be treated as a metric to measure the criticality of the nodes in networks.

3.1.2. Distance and efficiency

The shortest path plays a fundamental and important role to analyze topological structure of a network since it is usually assumed that a shortest path is an optimal path along which physical quantity can be transmitted faster and more effectively.

A walk from vertex i to vertex j is a sequence of vertices and edges that begins with i and end with j while a path is a walk in which no vertex is visited more than once [1]. A shortest path between a pair of vertices is the path which has minimal number of edges between the two vertices. Shortest path length d_{ij} is the number of edges in the shortest path between vertex i to vertex j.

In a graph, the separation degree among vertices can be quantitatively measured by average shortest path length, also known as characteristic path length. Characteristic path length L can be defined as the average of shortest path lengths over all pairs of vertices in a graph [18].

$$L = \frac{1}{N_{\mathcal{B}}(N_{\mathcal{B}} - 1)} \sum_{i \neq j \in \mathcal{B}} d_{ij} \tag{3.2}$$

The concept of efficiency is closely related to that of distance. The distance, as we discussed above, is generally assumed as a measure of the difficulty, cost or effort needed to transfer physical quantities over a network. So an efficiency e_{ij} can be associated to a pair of vertices i and j and defined as:

$$e_{ij} = \frac{1}{d_{ij}}, \ (i, j \in \mathcal{B}, i \neq j) \tag{3.3}$$

By averaging the efficiencies, the performance of network Y is able to quantify as global efficiency E(Y)

$$E(\mathcal{Y}) = \frac{1}{N_{\mathcal{B}}(N_{\mathcal{B}} - 1)} \sum_{i \neq j \in \mathcal{B}} \frac{1}{d_{ij}} \tag{3.4}$$

where d_{ij} is shortest path length between vertices i and j; N_B is the total number of vertices in a network.

Assume a unit of information or energy is transmitted along a shortest path between a pair of vertices. The smaller shortest path length between the pair of vertices is, more efficiently the information or energy transmits. Therefore, E(Y) quantifies the average performance of a network as how efficiently the information exchanges along the shortest path between any pair of nodes, and the global efficiency of a network is proportional to the reciprocal of shortest path length between any pair of nodes. Also, because global efficiency is associated with the performance of network, it is possible to spot critical components of a network by the ranking of the relative drop of global efficiency after nodes or lines removed [19].

3.1.3. Betweenness

In networks, if a vertex or edge participates in more number of paths, we generally consider this component more important for the transmission in the whole network. Therefore, if we assume that the interactions or transmission always through the shortest paths between two vertices, we can quantify the importance of a vertex or an edge in terms of its betweenness.

In form of formula, the betweenness of a vertex or an edge can be represented as:

$$B(v) = \sum_{m}^{N_{\mathcal{B}}} \sum_{n}^{N_{\mathcal{B}}} \frac{\sigma_{mn}(v)}{\sigma_{mn}},\ m \neq n \neq v \in \mathcal{B} \tag{3.5}$$

$$B(l_{ij}) = \sum_{m}^{N_{\mathcal{B}}} \sum_{n}^{N_{\mathcal{B}}} \frac{\sigma_{mn}(l_{ij})}{\sigma_{mn}},\ l_{ij} \in \mathcal{L},\ i \neq j \in \mathcal{B}, m \neq n \in \mathcal{B} \tag{3.6}$$

where $\sigma_{mn}(v)$ and $\sigma_{mn}(l_{ij})$ are respectively the number of the shortest paths between vertices m and n that pass through vertex v and edge l_{ij}. σ_{mn} denotes the total number of the shortest paths connecting vertices m and n.

A component with higher betweenness value means a greater number of shortest paths passing through the component and so implies a higher criticality of the component. Thus, the critical components of a network can be identified by ranking the betweenness value of the components in the network.

3.2. EXTENDED TOPOLOGICAL METHDOLOGY

Pure topological method is introduced in last section. However, the investigations using existing metrics could give rise to the deviation of really structural features of power systems because of ignorance of electrical engineering specificity. When these centrality indices are applied to study electrical power grids, they need to be redefined. Degree centrality has been redefined as entropic degree in which both weights of each line and their distributions can be taken into account simultaneously by incorporating entropy concept into original degree centrality. Efficiency used in delta centrality has also been redefined as net-ability where line flow limit on each line and electrical distance were introduced into efficiency index. And betweenness centrality is redefined as electrical betweenness by incorporating line flow limit on each line and Power Transfer Distribution Factors (PTDF) [20] which is a matrix that reflects the sensitivity of the power flow on the lines to the change in the injection power of buses and withdrawn at a reference bus.

3.2.1. Electrical consideration to complex network approach

The initial research works on complex networks developed many common concepts and measures which are supposed to be applicable to different types of networks. However, the functions and physical rules of different networks would be totally different and many specific characteristics cannot be dealt with by the general methodologies. When the complex networks methodology is directly applied to some fields with neglect of the specific features of these networks, analyzing results is unavoidably deviated from reality. Consequently, complex networks approach needs to be extended with the consideration of the electrical properties when applying the methodology to analyze power systems.

✓ ***Distance***

The distance between a pair of vertices and length of a path are crucial concepts in definitions of several important metrics in complex networks, such as average characteristic path length, betweenness and global efficiency, and so forth. In unweighted and undirected graphs, the number of edges in a path connecting vertices i and j is called the length of the path. A geodesic path (or shortest path) between vertices i and j is one of the paths connecting these vertices with minimum length; the length of the geodesic paths is the distance between the two vertices.

However, from the perspective of electrical engineering, distance should have more practical meaning which should be a measure of the “cost” when physical quantity is transmitted between the two vertices through the network. For electrical power grids, the cost of power transmission between two buses can be described from both economic and technological point of view, such as transmission loss or voltage drop. Therefore, for electrical engineering, the description of distance by pure topological approach cannot effectively reflect these related features and must be replaced by the description of “electrical distance”.

✓ ***Bus classification***

In general theory of complex networks, to avoid those difficulties involved in their differentiation and dynamical behavior characterization, all elements have been treated identically. Correspondingly, vertices are considered identically in definition of several metrics, such as betweenness and global efficiency, where the physical quantity was considered to be transmitted from any vertex to any other, even for power grids. However, the essential function of power grids is to transmit electrical power from any generator bus to any load bus with acceptable quality. Generally, we can classify the buses in power transmission networks as generation buses, transmission buses and load buses. Power transmission should be only considered from generation buses to load buses.

✓ ***Line flow limit***

In pure topological approach, edges are generally described in an unweighted way in definition of several related metrics, such as distance, degree and betweenness. However, in electrical engineering, transmission lines have line flow limits which restrict the ability of one line for power transmission due to many economic and technological factors. As this feature is critical for the networks to perform their essential function, it cannot be neglected in analysis related to security issues. Different lines may have distinct flow limits; therefore the distribution of this feature may also be important for vulnerability assessment.

✓ ***Flow-based network***

As defined by distance, the physical quantity transmission between two vertices is always supposed to be through the shortest path. This assumption is also in many works like power grids. This is the most unrealistic assumption from the point of view of electrical engineering. Power transmission from a generator bus to a load bus will involve most lines or a huge number of paths with different extent contribution. In a linear model of power flow, the different contributions of lines in power transmission can be described by the PTDF.

The network model in pure topological description of complex networks is unweighted and undirected. The identification of possible paths connecting two vertices is based on graph theory where transmission lines are assumed to be bidirectional, whereas, as we have discussed, the power transmission behavior between two vertices completely depends on physical rules which can be illustrated by PTDF. As each element in PTDF has sign, the lines connecting to one vertex should be classified as input or output lines. Therefore, some paths in undirected model may be not valid in the directed power transmission networks.

3.2.2. Basic conceptions extend to power grids

As mentioned in chapter 3.2.1, there are four main aspects of power grid consideration we want to extend to pure topological method. This chapter we will give detail description about these basic conceptions which will be used to define some metrics in the following section.

✓ ***Equivalent impedance as distance***

The distance is measured in topological model by the characteristic path length due to the assumption that physical quantity is transmitted along shortest path. However, in an electrical power grid, the flow is transmitted not just along the shortest path but along the remaining path as well. Hence, the electrical distance between a pair of buses should be defined as the equivalent impedance Z_i^j which considers the impedance of all transmission lines between buses i and j [21]. Suppose U_i^j is the voltage between bus i and bus j; I_i is the current injected at bus i and withdrawn at bus j (I_i=-I_j). According the electrical circuit theory, the equivalent impedance can be expressed as:

$$Z_i^j = \frac{U_i^j}{I_i} \tag{3.7}$$

Moreover, assume a unit current is injected at bus i and withdrawn at bus j (i.e., I_i=1 and I_j=-1) while no current is injected or withdrawn at other buses, then equivalent impedance can be calculated as:

$$Z_i^j = \frac{U_i^j}{I_i} = U_i^j = U_i - U_j = (z_{ii} - z_{ij}) - (z_{ij} - z_{jj}) = z_{ii} - 2z_{ij} + z_{ij} \tag{3.8}$$

where z_{ij} is the ith, jth element of the impedance matrix which is the inverse of bus admittance matrix.

✓ ***Bus classification***

Buses have different function in a power grid and so these buses can be classified as generator buses (**G** dim(**G**)=N_G), transmission buses (**T** dim(**T**)=N_T) and load buses (**D** dim(**D**)=N_D). **G** is a set of buses that injects power in power grid while **D** is a set of buses withdrawing power from power grid; **T** is a set of buses that transmit power rather than injects and withdraws power in power grid.

✓ ***Flow-based network***

In the linear model of power systems, the contribution of each transmission line to power transmission can be computed by PTDF [20]. PTDF reflects the sensitivity of the power flowing on each line for a power injection/withdrawal at a couple of buses. Therefore, PDTF matrix is used to denote the flow-based feature of power grids.

PTDF can be represented by a $N_L \times N_B$ matrix $\boldsymbol{F}$ in which each element $f_{lij}{}^m$ express the change of power on each line l_{ij} for a unit change of power injected at bus m and withdrawn at the reference bus; $f_{lij}{}^{gd}$ is the change of the power on line l_{ij} ($l_{ij} \in$ L) for injection at generation bus g and withdrawal at load bus d, and $f_{lij}{}^{gd}$ can be computed as follows:

$$f_{l_{ij}}^{gd} = f_{l_{ij}}^{g} - f_{l_{ij}}^{d}\ ,\ l_{ij} \in \mathcal{L} \tag{3.9}$$

where $f_{lij}{}^g$ and $f_{lij}{}^d$ are respectively the l_{ij}th row, g th column and l_{ij}th, row d-th column of $\boldsymbol{F}$.

A conceptual power grid (lines with pure reactance) is used as an example to illustrate the conception of PDTF calculation. *Table 3-1* reports the PTDF on each line, for injection at generation 1 and withdrawal at load bus 3.

It is worth noting that f_{lij} may be positive or negative, though all the values of PTDF in table 1 are positive. If f_{lij} is positive, then the power flows along the reference direction of the line l_{ij}, otherwise, the power flows along the direction opposite to the reference direction. For instance, in *Fig. 3-1*, assume the reference direction of line l_{42} is from bus 4 to bus 2, then if $f^{13}{}_{l42} > 0$, it means that the power flows from bus 4 to bus 2.

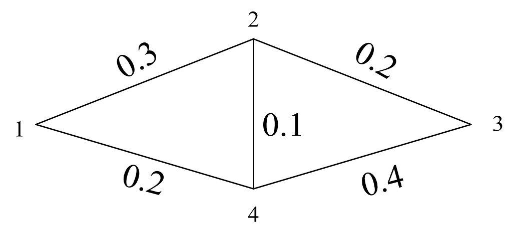

Fig. 3-1 Conceptual power grid

Table 3-1 PTDF for the conceptual power grid

	f^{13}_{l12}	f^{13}_{l23}	f^{13}_{l43}	f^{13}_{l14}	f^{13}_{l42}
Value	0.44	0.64	0.36	0.56	0.20

✓ ***Transmission capacity***

In order to maintain the stability and security operation of a power grid, each transmission line l_{ij} has its own transmission limit P_{lij}^{max}. The line flow limit plays a significant role in safe power transmission between generation buses and load buses. In fact, for power transmission, not all the lines will reach their line flow limit at the same. In other words, if one line reaches its transmission limit, the power transmitted between this pair of buses reaches its upper limit.

To evaluate the feature mentioned above quantitatively, we define the power transmission capacity C_g^d as the power injected at bus g when the first line in all lines connecting generation bus g and load bus d reaches its limit:

$$C_g^d = \min_{\substack{l_{ij} \in \mathcal{L} \\ i \neq j \in \mathcal{B}}} \left(\frac{P_{l_{ij}}^{\max}}{\left| f_{l_{ij}}^{gd} \right|} \right) \tag{3.10}$$

In the example of *Fig. 3-1* (with the line flow limits in *Table 3-2*) the power transmission capacity is C_1^3= 15.77 MW.

Table 3-2 Transmission line limits for the conceptual power grid (MW)

	P_{l12}^{max}	P_{l23}^{max}	P_{l43}^{max}	P_{l14}^{max}	P_{l42}^{max}
Value	15	10	10	15	5

3.2.3. Entropy degree

As a measurement of connectivity for a vertex, the definition of degree in aweighted network model should reflect the following factors:

- the strength of connections in terms of the weight of the edges;
- the number of edges connected with the vertex;
- the distribution of weights among edges.

Fig. 3-2 Different distributions of weights

In a weighted graph, the weighted connectivity of a vertex is measured by strength which is defined as the sum of weights on lines connected to a given bus. Higher strength of a bus means the bus more closely connects its neighbor buses. In *Fig. 3-2*, for node A and node B, they both have two connections and the sum of weights is both equal to 1.

The result of the example shown in *Fig. 3-2* has been noticed that the strength fails to distinguish the centrality of nodes especially when two nodes have the same strength with different distribution of weights among lines. Thus, entropic degree k_i^w was proposed to define weighted connectivity to solve the existing problem:

$$k_i^w = (1 - \sum_j p_{ij} \log p_{ij}) \sum_j w_{ij} \text{ , } p_{ij} = \frac{w_{ij}}{\sum_j w_{ij}} \tag{3.11}$$

where weight w_{ij} is defined as the line flow limit on line l_{ij} because the electrical parameter can reflect the strength that two buses connect.

Return to the example, the result calculated according to entropy degree is D(A) = 1.3 and D(B) = 1.14. We can see that, the connectivity of node A is higher than node B because the distribution of weights is more equal in node A.

As degree is a traditional concept in graph theory and widely applied for the analysis in complex networks, the proposed entropic degree may be a good replacement for research in weighted network models which include not only power grids but also other weighted networked systems. For power grids, it may directly give a quantitative measurement to indicate the importance of buses and their difference. The more important vertex may have higher connectivity in network. The most important buses may need more resource to be protected or be more likely to be selected as targets of intentional attacks. If measured with the pure topological concept of degree, the corresponding results may be far from reality. Therefore, this entropic degree can give more reasonable evaluation of the importance of buses by taking into account not only the total strength of the connection but also the distribution of strength that may be sensitive for malicious attacks.

3.2.4. Electrical betweenness

In traditional topological method, betweenness is defined as the sum of the probability for a vertex or an edge to belong to a randomly selected geodesic path linking any other pair of vertices. Betweenness is a more useful measure of the load placed on the given node/edge in the network as the node/edge's importance to the network than just connectivity. It is also a local metric as the degree centrality to measure the criticality of components (vertices and edges) in complex networks.

As it was mentioned that electrical engineering features need be considered in complex networks approach when electrical power grid is studied from topological point of view. According to above-mentioned specific features of power grids, the bus betweenness can be redefined as:

$$B_e(v) = \frac{1}{2}\sum_{g\in\mathcal{G}}\sum_{d\in\mathfrak{D}} C_g^d \sum_{\substack{l_{ij}\in\mathcal{L}^v \\ i\neq j\in\mathcal{B}}} \left| f_{l_{ij}}^{gd} \right|,\ v\neq g\neq d\in\mathcal{B} \tag{3.12}$$

$\sum_{l_{ij}\in\mathcal{L}^v, i\neq j\in\mathcal{B}} | f_{l_{ij}}^{gd} |$ is the sum of the PTDF of all lines connecting a bus *v* when a unit of power injected at bus *g* and withdrawn at bus *d*.

According to electrical circuit theory, the input power of a bus is equal to the output power of the bus, so $1/2\cdot C_g^d \sum_{l_{ij}\in\mathcal{L}^v, i\neq j\in\mathcal{B}} | f_{l_{ij}}^{gd} |$ represents the power taken by the bus *v* and the power is the half of the sum of power flowing on all lines connecting the bus *v* when power transmission capacity C_g^d is injected at generator bus *g* and withdrawn at load *d*.

$B_e(v)$ is the total power flowing through the bus *v* and the total power is equal to the half of the total sum of power flown on all lines connecting the bus *v* when various scenarios of power transmission capacity are transmitted from any generator bus to any load bus in whole power grid.

Similarly, line betweenness can be redefined as:

$$B_e(l_{ij}) = \max[B_e^p(l_{ij}), \left|B_e^n(l_{ij})\right|],\ l_{ij}\in\mathcal{L} \tag{3.13}$$

where $B_e^p(l_{ij})$ and $B_e^n(l_{ij})$ represent respectively the positive electrical betweenness and the negative electrical betweenness of line l_{ij}:

$$B_e^p(l_{ij}) = \sum_{g\in\mathcal{G}}\sum_{d\in\mathfrak{D}} C_g^d f_{l_{ij}}^{gd} \text{, if } f_{l_{ij}}^{gd} > 0 \tag{3.14}$$

$$B_e^n(l_{ij}) = \sum_{g\in\mathcal{G}}\sum_{d\in\mathfrak{D}} C_g^d f_{l_{ij}}^{gd} \text{, if } f_{l_{ij}}^{gd} < 0 \tag{3.15}$$

f_{lij}^{gd} is the PTDF on line l_{ij} when a unit of power injected at generation bus *g* and withdrawn at load bus *d*.

$C_g^d f_{lij}^{gd}$ represents the transmitting power on the line l_{ij} when the power C_g^d is transmitted from the generation bus *g* to the load bus *d*.

$B_e(l_{ij})$ is the total transmitting power on line l_{ij} when various scenarios of power transmission capacity C_g^d are transmitted from any generator bus to any load bus in whole power grid.

The concept of betweenness has been extended by introducing PTDF and power transmission capacity associated with line flow limit. The set of extended betweenness centrality quantifies the contribution of a component to power transmission in a power grid and in this respect the components (buses or lines) of the power grid can be ranked according to their criticality.

3.2.5. Net-ability

In traditional topological method, the performance of network Y is able to quantify as global efficiency *E*(Y).

Aiming to analyze the performance of a power grid in consideration of their above mentioned engineering features, the shortest path length distance should be replaced with electrical distance while the whole performance of a power grid should be averaged by all pairs of generators and loads rather than all pairs of nodes since the power is transferred only from generators to loads in a power grid. Besides, the power transmission capacity can also be considered. Therefore, the global efficiency was redefined as net-ability:

$$A(\mathcal{Y}) = \frac{1}{N_{\mathcal{G}} N_{\mathcal{D}}} \sum_{g \in \mathcal{G}} \sum_{d \in \mathcal{D}} \frac{C_g^d}{Z_g^d} \tag{3.16}$$

Where N_G and N_D respectively are the number of generation buses and load buses in a power grid; Z_g^d is the equivalent impedance for injection at generation bus g and withdraw at load bus d.

The general goal of a power grid is the feasible and economic transmission of power from generation buses to load buses. Feasibility refers to technical issues (losses, voltage drop, stability, etc.). Economy is related to other aspects (transmission costs, market efficiency, etc.). Net-ability measures the ability of a power grid to perform properly its function under normal operating conditions; the possibility to perform its function properly depends on the maximum line flow limits (transfer arbitrary amounts of power) and on the impedance of the lines (economic and technical convenience). The unit for net-ability is MW/ohm which indicates with one unit of cost (ohm) how many benefits (power transmission) can be achieved through the considered power grid from any generator to any load.

3.3. CASE STUDY

The Union for the Co-ordination of Transmission of Electricity (UCTE) coordinates the operation and development of the electricity transmission grid for large part of EU countries. Over more than fifty years, UCTE has been issuing all technical standards for a co-ordination of the international operation of high voltage grids, providing electricity supply for 430 million people in one of the biggest electrical synchronous interconnections worldwide. UCTE provide as well comprehensive statistics on electricity generation and transmission in the European mainland. In this section, our proposed extended topological methodology is applied to a simplified UCTE power network which contains 1254 buses and 1944 branches for its vulnerability analysis. The gird map of UCTE network is shown in *Fig. 3-3* and its member counties are reported in *Table 3-3*.

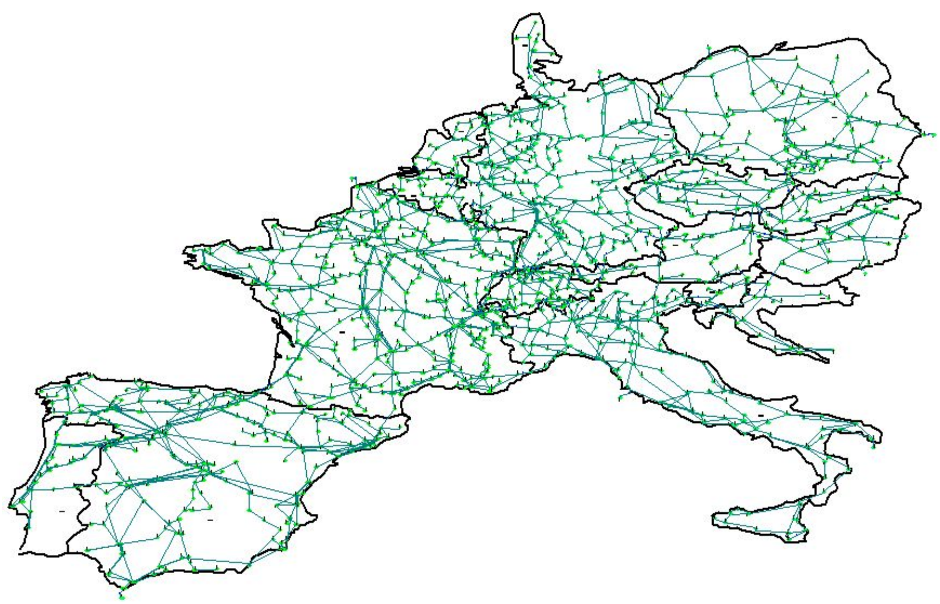

Fig. 3-3 Simplified UCTE power grid

Table 3-3 Members of simplified UCTE power grid

Member countries of UCTE [22]	Member countries of simplified UCTE power grid
Austria	√
Belgium	√

Bosnia and Herzegovina	
Bulgaria	
Croatia	√
Czech Republic	√
Denmark (west)	√
France	√
FYROM	
Germany	√
Greece	
Hungary	√
Italy	√
Luxemburg	√
Montenegro	
Netherlands	√
Poland	√
Portugal	√
Romania	
Serbia	
Slovakia	√
Slovenia	√
Spain	√
Switzerland	√

The entropy degree is calculated for the simplified UCTE network and reported in *Fig. 3-4*. As mentioned above, for power grids, entropy degree may directly give a quantitative measurement to indicate the importance of buses.

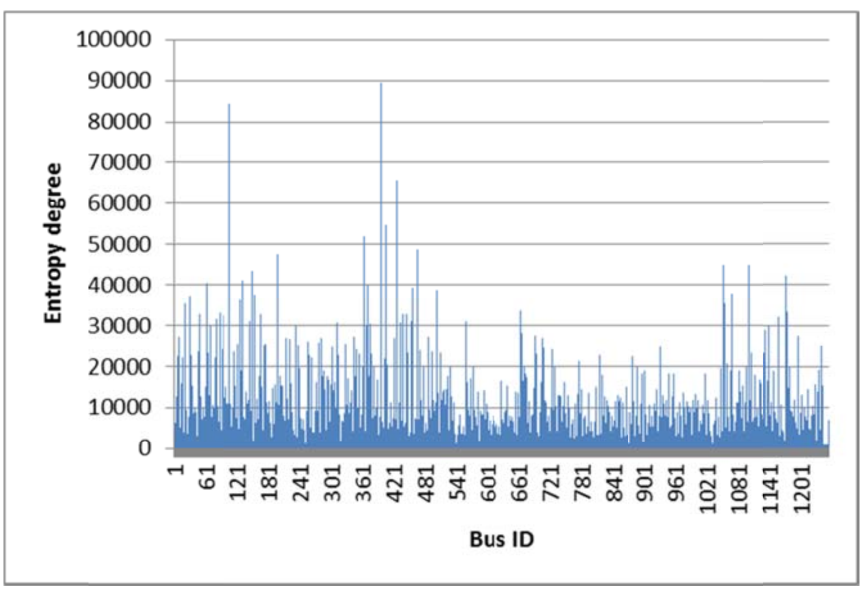

Fig. 3-4 Entropy degree of simplified UCTE network

The electrical betweenness of buses and branches are calculated and reported in *Fig. 3-5* and *Fig. 3-6.* Besides degree (entropy degree), electrical betweenness is another metric to measure the importance of a vertex or a line in a network. From *Fig. 3-5* and *Fig. 3-6* we can see the importance of each bus and branch in simplified UCTE network.

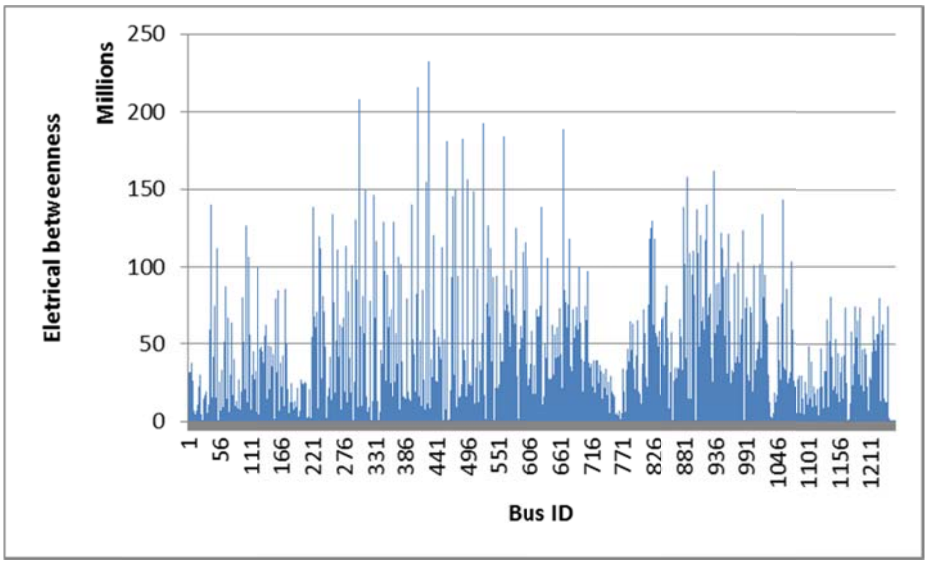

Fig. 3-5 Electrical betweenness of buses in simplified UCTE network

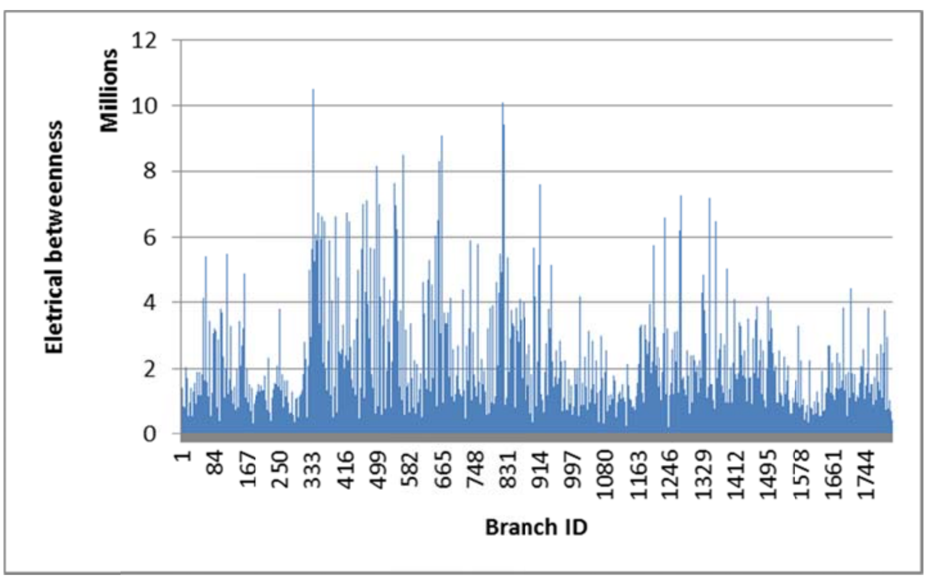

Fig. 3-6 Electrical betweenness of branches in simplified UCTE network

The relative drop of net-ability when a bus or a branch is cut from the network in simplified UCTE network is calculated. The results are shown in *Fig. 3-7* and *Fig. 3-8*.

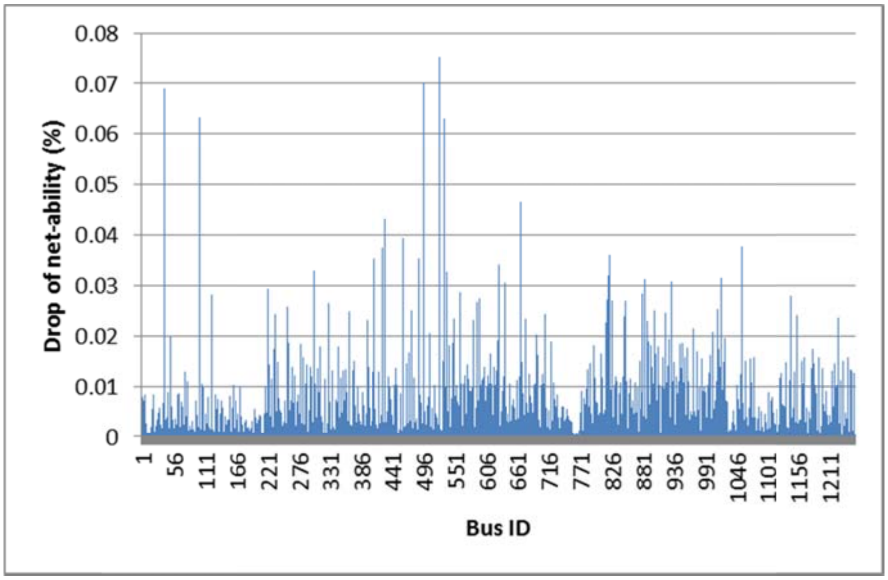

Fig. 3-7 Normalized drop of net-ability by removing bus in simplified UCTE network

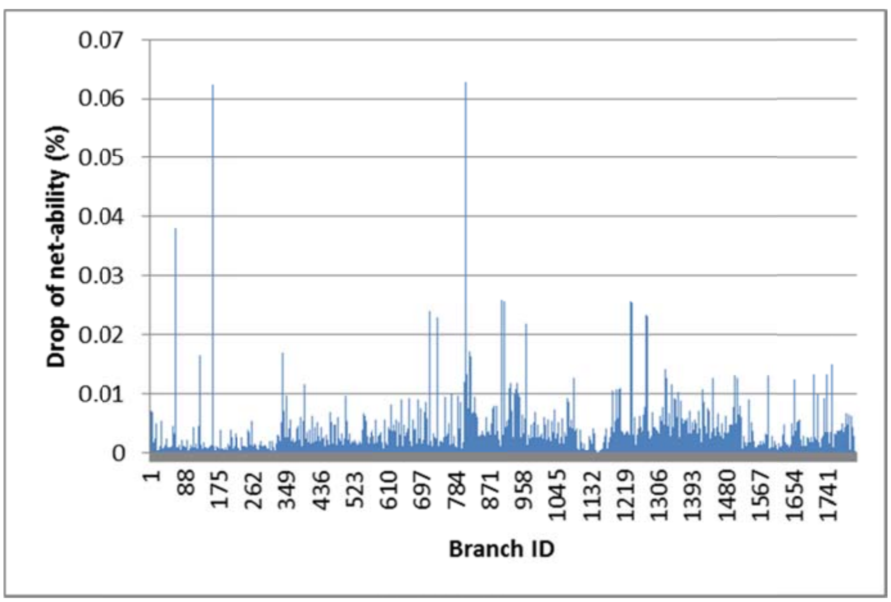

Fig. 3-8 Normalized drop of net-ability by removing branch in simplified UCTE network

The drop of net-ability is normalized by the original net-ability of simplified UCTE case without node/edge failure. Since each components (buses and branches) removed from the network will cause the drop of net-ability, the criticality of components in a power grid can be identified by ranking their drops in net-ability as well.

3.4. CONCLUSION

In our proposed extended topological method, electrical specificity is introduced into traditional pure topological method, and three metrics: entropy degree, electrical betweenness and net-ability are proposed to examine the criticality of the components (buses and lines) in power grids.

It is conformed that the metrics mentioned above might more suitably analyze the structural characteristics of power systems as complex systems and are superior to purely topological metrics in analyzing the criticality of components in power grids both from local and global point of view. It is more helpful for us to pay more attention on the improvement of power grid infrastructure protection.

Our proposed extended topological method can also be used in the analysis of cascading failure in power systems. An improved cascading failure model is proposed to model the cascading failure in which real line flow limit and maximum load capacity are introduced so that reality of cascading failure in electrical power grids is able to be more approximately modeled.

Moreover, since a complex system may be made up of multiple complex systems, power systems could be abstracted as not only power grids but also other complex networks (like cyber networks) interacted with power grids. As a consequence, the interaction and interdependency among various complex networks could reflect the inherent characteristics of intact power systems which cannot be uncovered by single power grid.

The extended topological method is applied to analyze the simplified UCTE network to find the importance of its components. The criticality of the components in the simplified UCTE network calculated by extended topological method. However, to verify the result's feasibility, it should be checked by the field data in real operation condition.

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Chapter 4.

Correlating Empirical Data with Extended Topological Measures

During the last years, in order to classify their structure, dynamics and evolving patterns, new topological measures, algorithms and models have been widely used in networks from different fields such as biology, chemistry, social sciences, computer networks, etc. A considerable amount of studies have been performed on a remarkable technological network such as the power grid, where buses and transmission lines are considered nodes and links respectively, in order to define a graph. As far as the structure is concerned, power grids, at least at the transmission level, have been thoroughly studied and different aspects, such as basic topological characteristics and statistical global graph properties have been performed on many grids around the world [1]. Among the latter, static robustness (or vulnerability) analysis based on evaluating the variation in global connectivity due to random failure (i.e., random bus deletion) or selective attack (i.e., in decreasing order of some bus topological feature) of nodes has been mostly used. For most grids, global connectivity decreases exponentially, with a higher variability when buses are "attacked" in decreasing order of degree [2].

On the other hand, power grids are complex multilayered networks where many decision processes, involving different objectives, are at play. The global behavior of the grid is thus mainly driven by the complex interaction between its structure, its dynamical processes and economic and environmental constraints. Since this complex interaction is difficult to unveil at a global level, research has been focused on detecting whether malfunctions, turned into emergent outcomes such as blackouts, can be related to topological constraints, the rationale behind this procedure being that structure affects dynamics and vice versa [3]. Until now, most of the literature has been concerned on relating purely topological measures, such as analytical results coming from the aforementioned static vulnerability analysis, with aggregated malfunctions outcome (i.e., total loss of power, energy not supplied or restoration time) [4]. But this approach has failed when it has been applied to power systems with different topological characteristics, mainly due to the poor definition of purely topological measures, away from the real physical and electrical definition of the system. In order to overcome this limitation, more specific topological measures have been defined in last chapter: entropy degree (ED) and electrical betweenness (EB) have been presented as useful means to characterize the topology of the nodes of a power network.

In this chapter, ED and EB are used in order to characterize the buses of the four biggest transport networks in Europe (i.e., France, Germany, Italy and Spain) and a static robustness analysis is performed. Similar statistical behavior is observed between Germany and Italy (GI networks), and Spain and France (SF networks), with respect to attacks performed in decreasing order of ED and EB. This behavior can be correlated with disaggregated cumulative probability distributions of major events. Results show statistically meaningful (although weak) correlations among similar topologically characterized networks, which could finally help in defining a linkage between topological measures and malfunctions on power grids.

4.1. VULNERABILITY ANALYSIS TO MAJOR NATIONAL POWER GRIDS

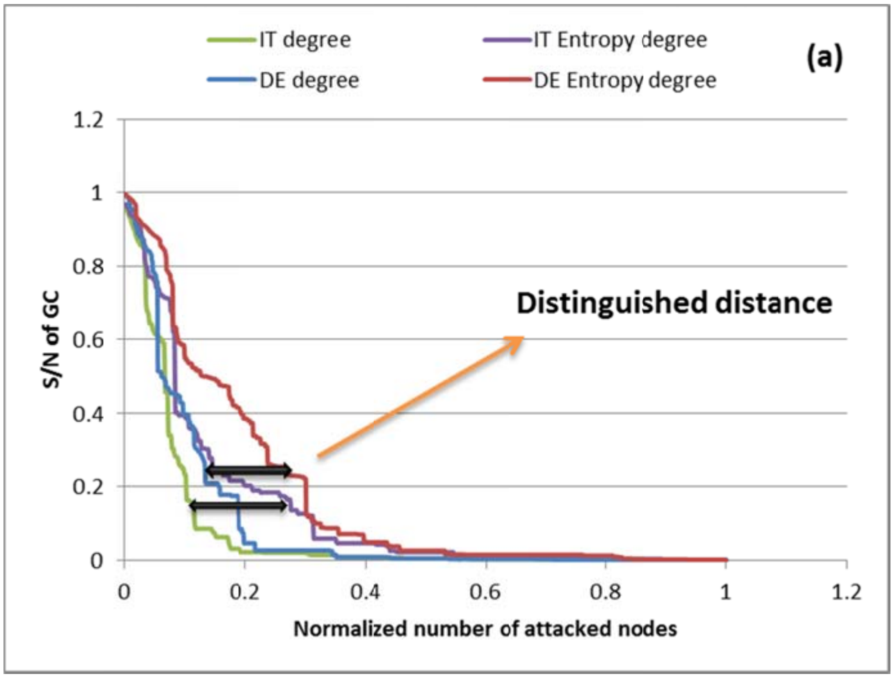
The robustness of the power grid is an example of a generalized feature of most complex networks, from the Internet to the genome [3][5-7]. Specifically, real networks are often characterized by a considerable resilience against random removal or failure of individual units but experience important short-comings when the highly connected elements are the target of the removal. Such directed attacks have dramatic structural effects, typically leading to network fragmentation [8-12]. In this subsection the evolution of this fragmentation is evaluated in the case of four European power grids: France, Germany, Italy and Spain. Essential features of these networks are reported in *Table 4-1*.

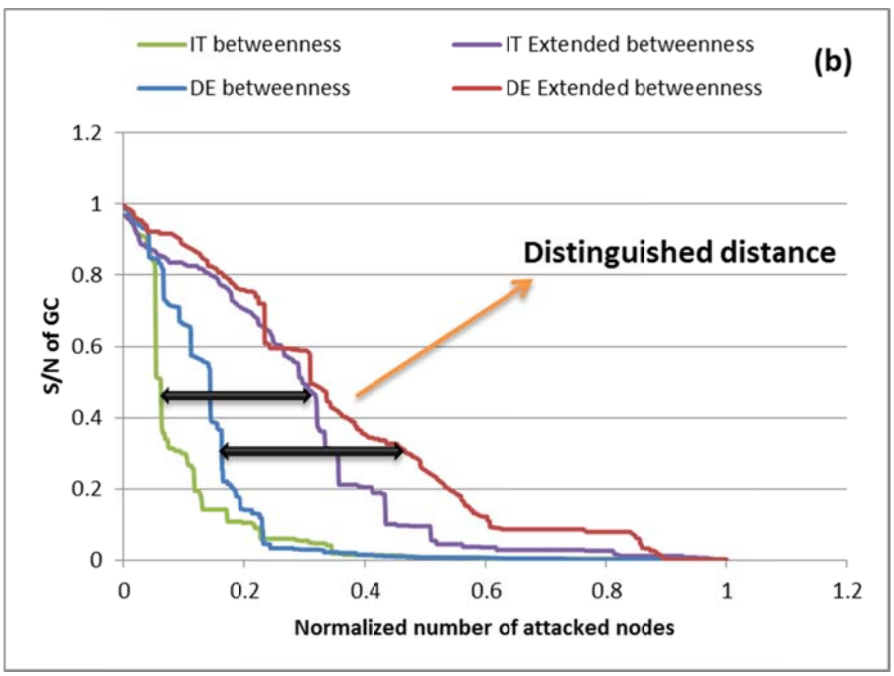
Table 4-1 Basic characteristics of the four major national power grids

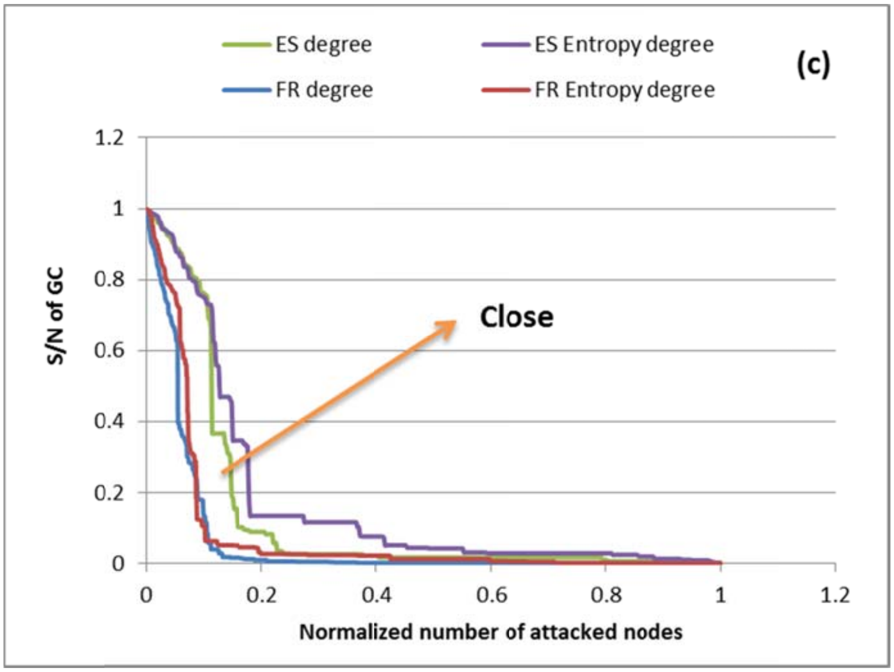
Number of	France	Germany	Italy	Spain
Buses	1401	1197	535	447
Lines	1819	1714	645	644
Generators	136	156	126	100
Loads	881	602	249	349

Entropy degree and electrical betweenness could be used as new metrics to evaluate how differently the power grids behave when random or selective nodes are eliminated and compared to traditional purely topological metrics. The calculation method is proposed in last chapter. However, since entropy degree and electrical betweenness imply already an ordered list of nodes, random deletion could be neglected in this case, and only selective attacks are considered instead. Therefore, in this chapter we will adopt the decreasing entropy degree and electrical betweenness as the elimination sequence.

Fig. 4-1 shows the static tolerance to selective removal of a fraction of nodes, by decreasing order of each metric and for the four major national grids studied. Robustness is measured by the relative size of the largest connected component which is normalized to the network size (*S/N)*. As it is shown, German and Italian power grids present a distinguished pattern between traditional and entropy degree static tolerance procedures, the situation is more significant in traditional and electrical betweenness scenario. However, in Spain and France power grids, the curves under different scenarios are much more similar and follow a similar trend.


IT degree
IT Entropy degree
DE degree
DE Entropy degree
(a)
S/N of GC
Distinguished distance
Normalized number of attacked nodes
0
0.2
0.4
0.6
0.8
1
1.2


IT betweenness
IT Extended betweenness
DE betweenness
DE Extended betweenness
(b)
S/N of GC
Distinguished distance
Normalized number of attacked nodes
0
0.2
0.4
0.6
0.8
1
1.2



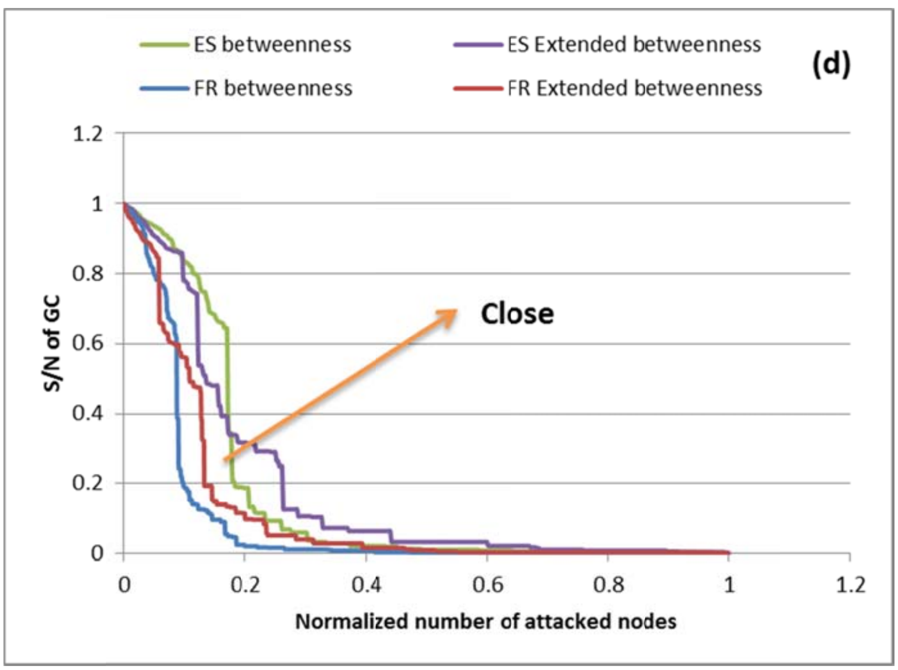


***Fig. 4-1**. Effects of attacks on the topology of France (FR), Germany (DE), Italy (IT) and Spain (ES) power grids.*

Except the obvious observation above, a quantified analysis is implemented to simulation result. We analyze the maximal information coefficient (MIC) between all data which is reported in *Table 4-2* [13]. As far as the electrical betweenness is concerned, there exists a higher correlation between France and Spain, and Germany and Italy. As far as the entropy degree is concerned, results are less conclusive although Germany and Italy are significantly correlated.

Table 4-2 Maximal information coefficient (MIC) for electrical betweenness and entropy degree among France, Germany, Italy and Spain power grids.

MIC strength		Electrical Betweenness	Entropy Degree
France	Germany	0.99624	0.97894
France	Italy	0.98761	0.97313
France	Spain	**0.99668**	0.951
Germany	Italy	**0.99976**	**0.99825**
Germany	Spain	0.99639	**0.99825**
Italy	Spain	0.98456	**0.99844**

The evolution of the largest connected component during the attack is obviously different between GI power grids and SF power grids when extended metrics (especially electrical betweenness) are used instead of traditional metrics. Furthermore, this dissimilar behavior coincides with the conclusion published by Solé and collaborators [14], where GI networks and SF networks were segregated in different groups, in this case termed as robust (γ < 1.5) and fragile (γ > 1.5) according to γ, the exponential degree distribution characteristic parameter respectively as shown in *Fig. 4-2*. In this same reference, the authors provide an evidence for the correlation between topological structure and vulnerability performance in terms of aggregated values of major events.

Country	γ	Errors f_c^{theor}	Errors f_c^{real}	Errors $\lvert\Delta f_c\rvert$	Attacks f_c^{theor}	Attacks f_c^{real}	Attacks $\lvert\Delta f_c\rvert$	N	L	$\langle k\rangle$
Belgium	1,005	0,011	0,395	0,384	0,010	0,131	0,121	53	58	2,18
Holland	1,086	0,147	0,387	0,240	0,034	0,126	0,092	36	38	2,11
Germany	1,237	0,322	0,565	0,243	0,097	0,229	0,132	445	560	2,51
Italy	1,238	0,322	0,583	0,261	0,097	0,241	0,144	272	368	2,70
Austria	1,409	0,450	0,506	0,056	0,159	0,191	0,032	70	77	2,20
Rumania	1,418	0,455	0,579	0,124	0,162	0,238	0,076	106	132	2,49
Greece	1,457	0,477	0,492	0,015	0,174	0,183	0,009	27	33	2,44
Croatia	1,594	0,543	0,525	0,018	0,214	0,202	0,012	34	38	2,23
Portugal	1,606	0,548	0,595	0,047	0,217	0,250	0,033	56	72	2,57
EU	1,630	0,557	0,629	0,072	0,223	0,275	0,052	2783	3762	2,70
Poland	1,641	0,562	0,594	0,033	0,226	0,249				
Slovakia	1,660	0,569	0,563	0,006	0,231	0,227				
Bulgaria	1,763	0,604	0,570	0,034	0,256	0,232				
Switzerland	1,850	0,629	0,610	0,020	0,275	0,260				
Czech Republic	1,883	0,638	0,634	0,004	0,281	0,279				
France	1,895	0,641	0,647	0,006	0,285	0,289				
Hungary	1,946	0,654	0,617	0,036	0,295	0,266				
Bosnia	1,952	0,655	0,588	0,067	0,295	0,244				
Spain	2,008	0,668	0,689	0,020	0,307	0,324				
Serbia	2,199	0,705	0,655	0,051	0,339	0,296				

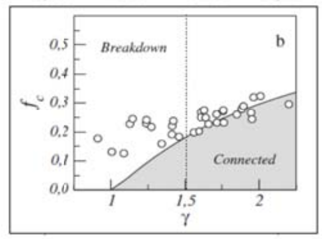


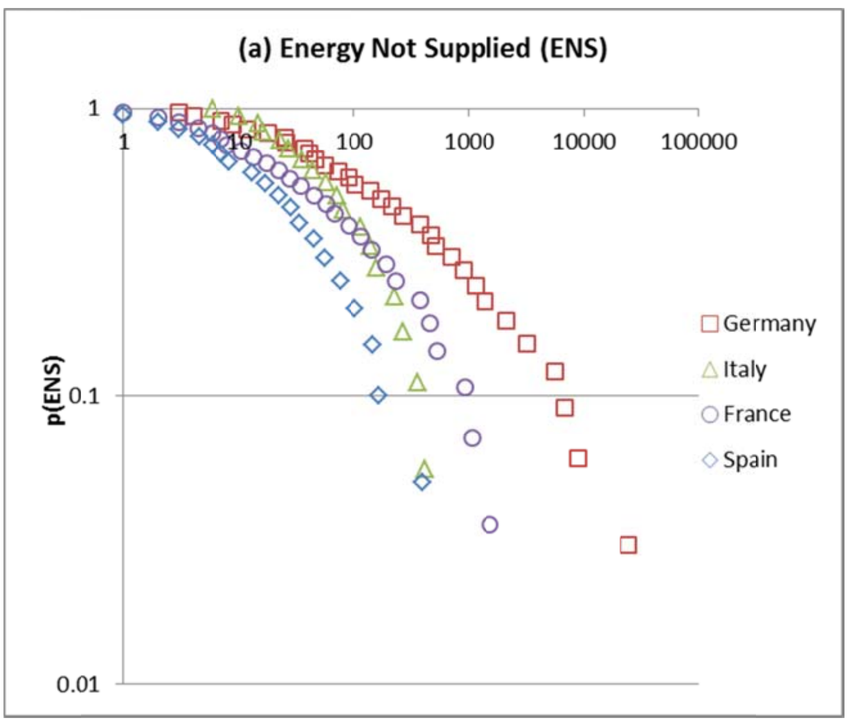
Fig. 4-2 A summary of the exponential degree distribution exponent of the European power grids. (Source: [14]).

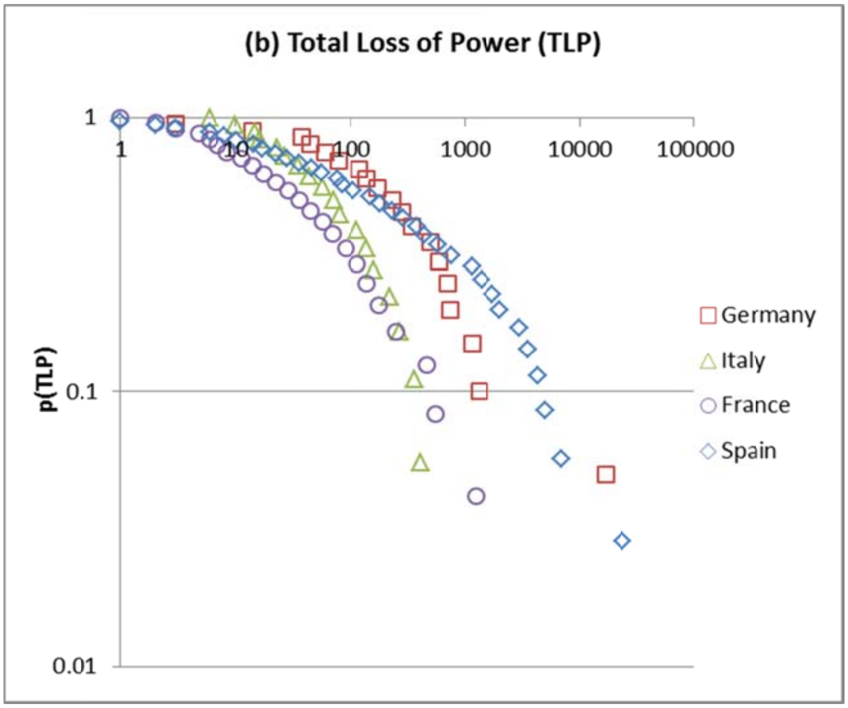
Through the analysis above we can see that our defined extended metrics can be used to discriminate the topological difference according to the static tolerance to selected attacks. Although our proposed new metrics can illustrate the difference between two particular types of network, it is difficult to directly assume that these extended metrics can be correlated with any real dynamic feature of the grid. Therefore, a linkage between structural measures and the real dynamical outputs (i.e., major events) of a grid is needed. Therefore, the natural consideration is to check the dynamical outputs of the four major networks which will be described in the following section.

4.2. PROBABILITY DISTRIBUTIONS OF MAJOR EVENTS

Probability distribution analysis is one of the methods to study the statistics and dynamics of empirical data with approximate global models. Heavy tailed probability distributions seem to be ubiquitous statistical features of self-organized natural and social complex systems [15], and the appearance of the power law distribution is often thought to be the signature of hierarchy, robustness, criticality and basically, non-random behavior [16]. In this sense, European power transmission grids major events data (UCTE/ENTSO-E) provide us with a set of real malfunctions data for the vulnerability analysis in power transmission grids. Probability distribution analysis is used in order to detect correlations between real dynamic output and topological measures.

European power transmission grids reliability data is given through three measures: energy not supplied (ENS), total loss of power (TLP) and restoration time (RT). These statistic data can be found in the UCTE/ENTSO-E webpage and they are publicly available from 2002 onwards [17]. The data are collected and investigated using the probability distribution analysis for four major power networks. *Fig. 4-3* shows the cumulative distribution functions for the aforementioned reliability measures and for the four major power grids. Logarithmic binning has been used in order to diminish the noise associated with statistical fluctuations [18].





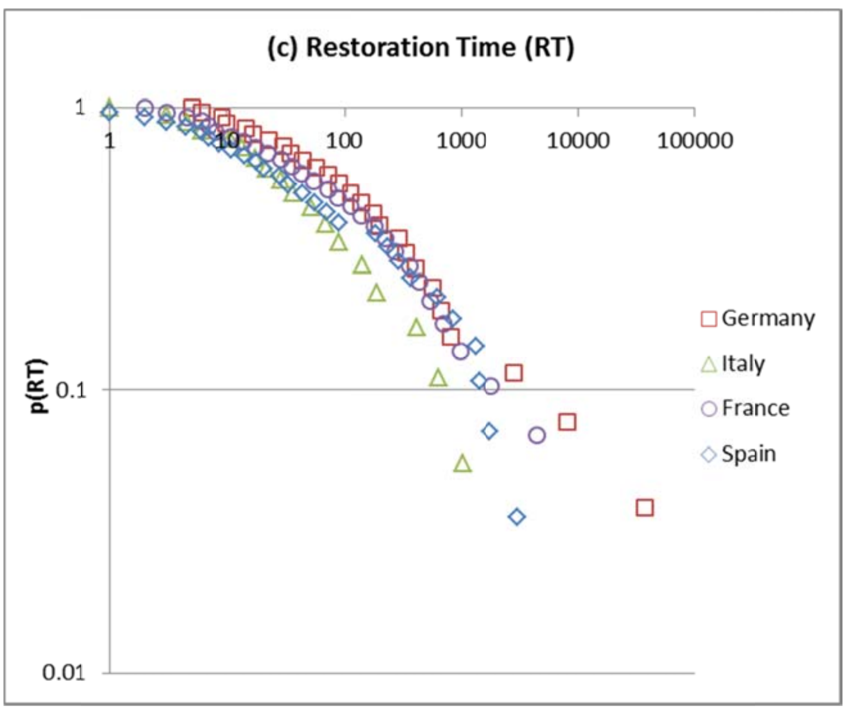


Fig. 4-3. Cumulative distribution functions for the four major power grids reliability measures: ENS, TLP and RT.

The fitting function for the cumulative probability distribution of the reliability indexes of each power network is needed to be investigated for the pattern recognition to see whether there is a difference between these curves. The methodology described by Clauset and collaborators offers the possibility to statistically fitting a function to the tail of the distribution. This methodology has been followed in this section, where a maximum likelihood approach is proposed to estimate the heavy tailed function from the data and a significance test is constructed to evaluate the plausibility of some specific distributions. *Table 4-3* shows likelihood ratios and p-values with respect to log-normal, exponential, stretched exponential and power law with cut off distributions, all of them with power law function taken as comparative means. Positive likelihood values favor the power law hypothesis and p-values higher than 0.1 imply no significance on the results. As we can see, although power law could be accepted only for the TLP (total loss of power) in Spain, the value of the likelihood ratio does not support this option. In general terms, results are not conclusive and no function can be adjusted with enough statistical significance.

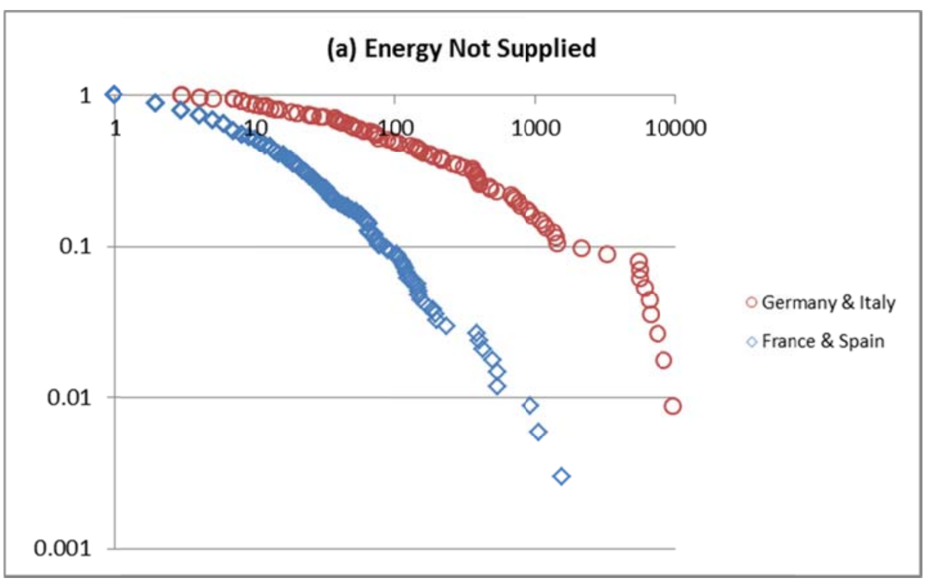
Table 4-3 Test of fat-tailed behavior taking the power law as comparative function for ENS, TLP and RT of each power grid.

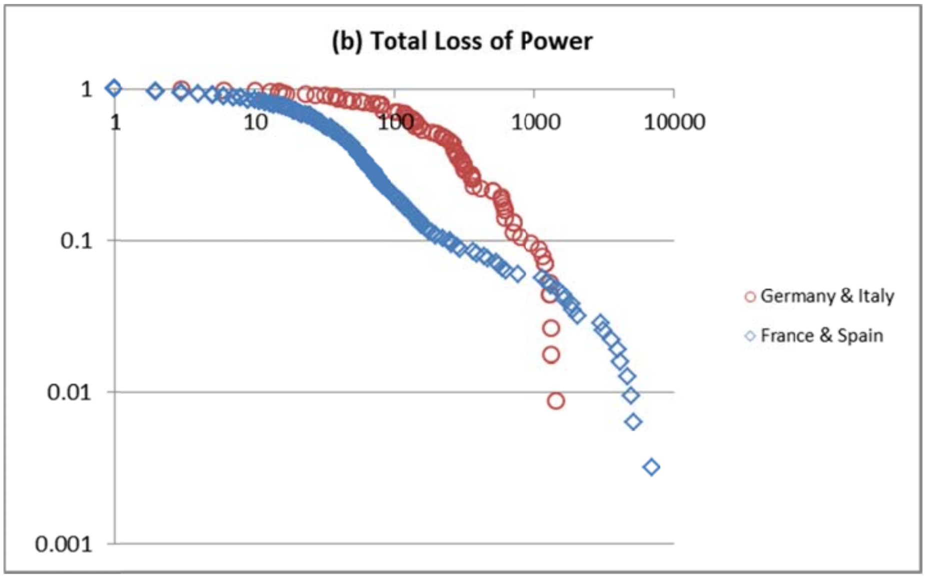
		power law	log-normal		exponential		stretched exp.		power law + cut-off	
		p	LR	p	LR	p	LR	p	LR	p
ENS	France	0.11	-1.26	0.21	0.91	0.36	-1.23	0.22	13.55	1.00
	Germany	0.80	-0.68	0.50	1.04	0.30	-0.63	0.53	122.08	1.00
	Italy	0.14	-0.87	0.39	-0.57	0.57	-0.76	0.45	9.41	1.00
	Spain	0.72	-0.42	0.68	0.30	0.76	-0.57	0.57	37.31	1.00
TLP	France	0.81	-0.34	0.73	0.79	0.43	-0.52	0.61	66.15	1.00
	Germany	0.65	1.03	0.31	-0.42	0.67	0.00	1.00	82.00	1.00
	Italy	0.13	-0.87	0.39	-0.57	0.57	-0.76	0.45	9.41	1.00
	Spain	**0.07**	-1.65	0.10	0.47	0.64	-1.79	**0.07**	68.33	1.00
RT	France	0.86	0.05	0.96	0.91	0.36	-0.18	0.86	114.54	1.00
	Germany	0.91	0.43	0.67	1.58	0.11	0.66	0.51	80.16	1.00
	Italy	0.80	-0.51	0.61	0.89	0.38	-0.47	0.64	26.38	1.00
	Spain	0.28	-1.19	0.23	1.56	0.12	-1.19	0.24	9.03	1.00

4.3. VULNERABILITY, EXTENDED TOPOLOGICAL MEASURES AND MAJOR EVENTS

4.3.1. Probability distribution for aggregated major events

Based on the previous research, it is worthy to point out again that the two groups networks (DE + IT and FR + ES) exhibit a difference in the static tolerance to failures according a specific order by entropy degree and electrical betweenness. However, the probability distribution of the major events of each network cannot tell us very significant conclusions since we didn’t see any information to separate these two kinds of power networks. One of the important reasons is due to the short of the major events number from the statistic science point of view. Therefore, on the one hand, the major events are separated and aggregated as two groups: DE + IT and FR + ES; on the other hand, new analysis method of statistic theory will be found and performed to test the malfunctions data. The corresponding probability distribution for the aggregated two groups of major events are analyzed again and reported in *Fig. 4-4*.





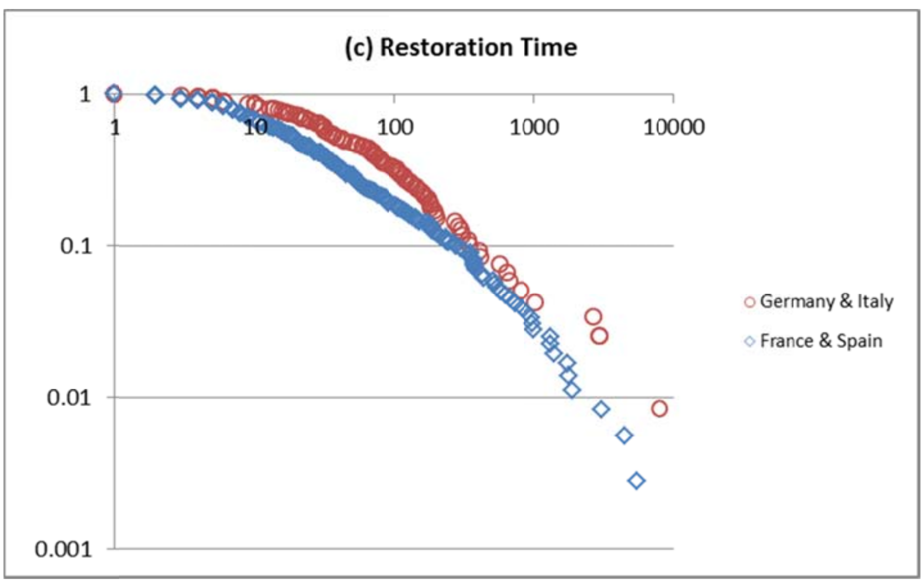


Fig. 4-4 Cumulative distribution functions for the aggregated two group power grids reliability measures: ENS, TLP and RT.

It still noticed that there exists a difference between these two group power grids even in a first glimpse. However, since the method used in last section couldn't give us a statistical significant conclusion. A new method is needed to address this problem.

4.3.2. Kolmogorov-Smirnov test for aggregated major events

One drawback observed in the previous section is the amount of major events data considered, which might be less than desired when fitting any fat tailed function. In this section aggregated data for all combinations of major events has been considered. On the other hand, although no conclusions can be drawn from the previous probability distribution analysis, cumulative distributions shown in *Fig. 4-3* present obvious differences which make them depart from or approach to fitting functions. This can be detected with other statistical tests like the Kolmogorov-Smirnov (KS) test, defined as the maximum distance *D* between the cumulative distribution functions of the data *S(x)* and the fitted model *P(x):*

$$D = \max \left| S(x) - P(x) \right| \tag{4.1}$$

KS test is used in order to detect how close a theoretical probability distribution function is from the real one. It is performed with the aim of detecting whole function approximation and not only fitting the tail of the function. *Table 4-4* shows KS test results for the meaningful combination of pairs of grids. The dark black number in each raw denotes that which distribution function is more sound for the data. From the table we can see that the exponential distribution can be ruled out completely.

Table 4-4 Values of the KS test for different fitting functions to ENS, TLP and RT probability distribution functions.

		power law + exp. cut-off	log-norm	stretched exp.	exp.
ENS	Germany + Italy	0.096	**0.064**	**0.064**	0.387
	France + Spain	**0.083**	**0.083**	**0.083**	0.250
TLP	Germany + Italy	0.107	**0.071**	**0.071**	0.357
	France + Spain	**0.071**	**0.071**	**0.071**	0.321
RT	Germany + Italy	0.090	0.121	**0.090**	0.424
	France + Spain	**0.062**	**0.062**	**0.062**	0.375

In the power law with exponential cut-off scenario, it's coincident with the previous selection: Germany and Italy on one side, and France and Spain on the other. We can see that although log-normal and stretched exponential distributions cannot be ruled out completely, power law with exponential cut-off can be ruled out for energy not supplied; total loss of power and restoration time for Germany and Italy but not for France and Spain combined major events data.

4.3.3. Correlating extended measures to major events

Even though statistically speaking the evidence is somehow weak, these results would favor the existence of a linkage between structure and dynamics. Some grids, in this case France and Spain, can be adjusted by power law with cut-off, lognormal and stretched exponential. Germany and Italy, on the other side, can be adjusted by lognormal and stretched exponential but not by power law with cut-off. Although firm conclusions cannot be drawn, the probability distributions of major events for these networks would suggest a different performance in terms of vulnerability, distinguished by frequency of major events and MW, MWh and minutes (i.e., restoration time) involved in these failures. From the physics point of view, an exponential cut-off could be understood in the following manner:

- For the Energy Not Supplied (ENS), which means the loss of energy from consumption side, it reveals the physical constraints on the maximum energy consumption from consumers (residential, commercial and industrial).
- For Total Loss of Power (TLP), which means the loss of production from the generation side, the fast decaying tail is consistent with the maximum power output of the generator at each vertex.
- For Restoration Time (RT), it is the signature of an obvious upper bound since the power facilities cannot be damaged forever.

The physical meaning described above can help us suggesting the meaning of this dissimilar behavior. Spain and France grids' dynamic behavior (i.e., major events) is closer to what would seem the limit of their reality, while Germany and Italy power grids are not, since there is no exponential decay in their probability functions. Back to their topological structure, the metrics (i.e., the extended metrics

EB and ED or the exponential degree distribution characteristic parameter γ cited by Solé and collaborators [14] also discriminate the four major power grids in two groups, this is Germany and Italy, and Spain and France. So a direct linkage can be suggested between structural measures and the real dynamical output: on the one hand, the topological structure of Spain and France power grids indicates that these networks nearly reach their maximum power transmission ability. In other words, the networks are more fragile and, correspondingly, their dynamic output (in terms of major events) shows the existence of maximum constraints. On the other hand, Germany and Italy power grids seem not yet at their maximum capacity, and there is still a margin to reach the upper bound of their dynamic output. Equivalently, they could be considered (for the time being) more robust.

4.4. CONCLUSION

Although a contradiction as it seems, complex networks science allows a simplified view of the reality. Algorithms, measures and models involved in studying complex systems as networks, have allowed an understanding of some common features which characterize their topology and, in a lesser extent, their dynamic processes. Power grids have been thoroughly studied as complex networks and many topological measures have been used in order to classify their structure, evaluate their behavior in terms of robustness or model their dynamic response to malfunctions. Results have been mainly theoretical and no correlation between real grid's dynamical behavior (i.e., malfunctions and major events) and any structural measure has yet been found. In this paper new extended topological measures have been used in order to quantify the ability of four European power grids (i.e., France, Germany, Italy and Spain) to sustain selective removal of buses. A maximal information coefficient has been used to find similar robustness behavior between Spanish and French networks on one side, and German and Italian networks on the other. In order to find a correlation with any dynamical output (i.e., blackouts), binned cumulative probability distributions of majors events in terms of energy not supplied, total loss of power and restoration time have been fitted to some characteristic fat-tailed functions, with no success. This could be probably due to the small amount of major events data actually available for the studied power grids (or simply because real cumulative probability distributions do not follow any of the fat-tailed function used for the fitting). To avoid the first drawback, aggregated data for every two networks has been used to significantly increase the amount of values included in the probability distributions. Although a favorable fitting is not found, the paper shows that a significant (although weak) statistical approximation appears when Germany and Italy on one side and France and Spain on the other are considered in aggregated manner, thus identifying similar dynamical response among topologically similar grids. Although much research must be done, such as extending this methodology to distribution networks or exploring the cascading failure in power grids, combining topological measures that include electrical engineering perspectives, this evidence would raise hopes in finding a more meaningful and significant linkage between structural measures and real dynamical output, in terms of major events, of a power grid.

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Chapter 5.

Evolution of Hierarchy in Power Transmission Networks

Complex systems are usually characterized by some level of hierarchy, which spans in time and space at different scales. This hierarchical structure commonly allows reducing costs in terms of reliably transmitted information but at the same time involves different dynamical responses to malfunctions. In the case of critical infrastructures like transmission power grids, different hierarchical structures may lead to different behaviors in terms of accumulated major events. In this chapter, we compare and evaluate the evolution of hierarchy for four real different power transmission networks when buses are attacked selectively in decreasing order of some topologically and electrically defined values. Two important simulation results occur: firstly, hierarchy increases as the network is being attacked and secondly a low variability of hierarchy implies an increased probability of accumulated major events.

5.1. THE COORDINATES OF HIERARCHY

A morphospace is phenotype space where a small set of quantitative traits can be defined as the axes [1]. This conception is widely used in the biology science and then extended to the complexity science and complex network studying [2-7]. Recently, the researcher in [1] has proposed three coordinates in the morphospace to study the hierarchy property in complex network.

The hierarchy coordinates are applied in a directed graph G(V,E), where $v_i \in V$ (i

= 1, …, n) is a node and < v_i , v_j > ∈ E is an arrow going form v_i to v_j. The proposed morphospace is a metric space defined from three coordinates [1]: Treeness (T), Feedforwardness (F), and Orderability (O), which properly quantify graph hierarchy. In the paper, the author also analyze the position of different real networks in this morphospace which shown in *Fig. 5-1*. The electronic circuits (TECH) placed at the O(G) ≈ 1 plane shows a narrow band of feedforward with -0.2 < T(G) < 0.2 and slightly biased to negative T(G) values.

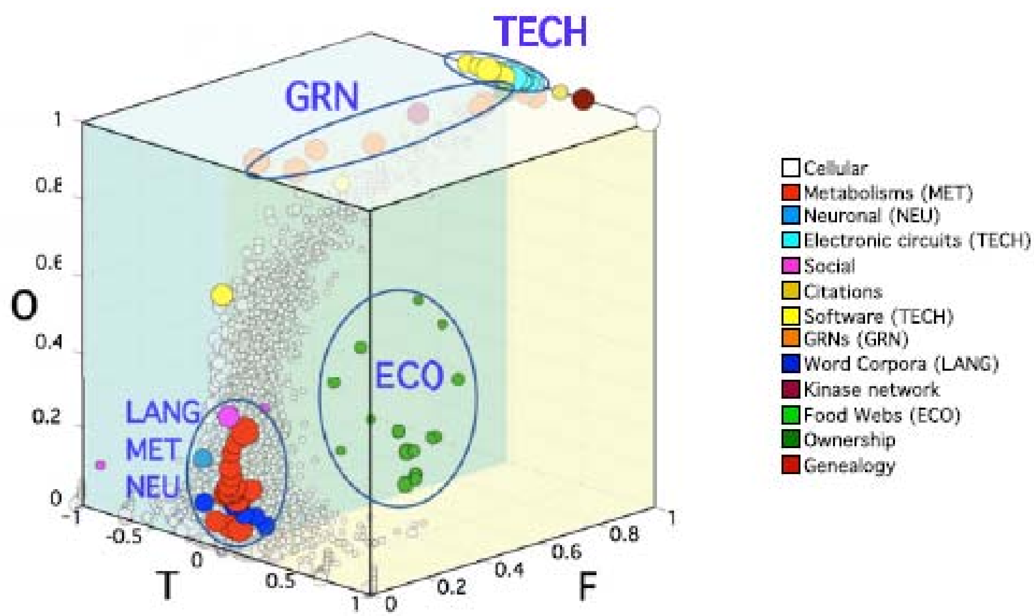

Fig. 5-1 The coordinates of the 125 real networks (Source: [1])

Based on this observation, a question has been wandered and made hypothesis: How about power transmission networks? To verify our guess, we testified with the IEEE 118-bus case as a typical power transmission network. Before the validation, we need to get the directed graph model for power transmission networks. The most popular method is using either AC or DC power flow to generate the directed graph model [8-11]. After the power flow calculation, the flow direction can be obtained. A simple example using IEEE 4-bus system is shown in *Fig 5-2*.

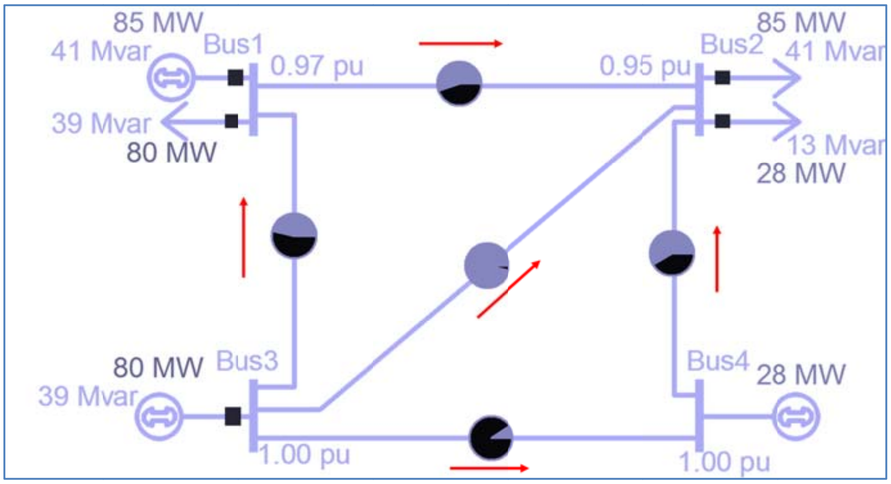

Fig. 5-2 A illustration of directed graph from AC power flow

Using similar procedure, we got the directed graph model of IEEE 118-bus case and also the hierarchy coordinates are calculated and reported as following:

- Treeness (T) = 0.2899
- Feedfordwardness (F) = 1
- Orderability (O) = 1

The position of IEEE 118-bus system is consistent with the position exhibited in the TECH group in *Fig. 5-1*. Therefore, according to the proposed method for analyzing vulnerability in chapter 4, the evolution of hierarchy coordinates when eliminating buses or branches from the network is worth of being analyzed. Similarly, we will also try to correlate the real malfunctions data of power grids with the evolution of hierarchy coordinates. More detailed information would be addressed in the next section.

5.2. ASSESSING HIERARCHY IN POWER NETWORKS

Based on the dataset we have, four European power grids: France, Germany, Italy and Spain are used in this section to analyze their hierarchy evolution. The basic features of these networks are shown in *Table 5-1*.

Table 5-1 Basic characteristics of the four major national power grids

Number of	France	Germany	Italy	Spain
Buses	1401	1197	535	447
Lines	1819	1714	645	644
Generators	136	156	126	100
Loads	881	602	249	349

The hierarchy evolution is corresponding to the eliminating of buses or branches according to a specific order. The order can be random or calculated by certain metrics. Here we have chosen to use metrics of decreasing electrical betweenness, net-ability and randomly generator as the elimination order. The simulation procedure as following: the Tressness values will be calculated when each bus (node) is removed from each power transmission network according to the orders mentioned above.

5.2.1. Hierarchy Evolution in decreasing electrical betweenness

The node electrical betweenness of the four major power networks are calculated by the formula 3.12. The hierarchy coordinates are calculated one by one when eliminating the buses according to the decreasing order of electrical betweeness for each network. The Feedfordwardness and Orderability values are always 1. Only Treeness values are changing with the elimination of buses. Therefore, here we just report the Treeness results which are shown in *Fig. 5-3*, and their mean values and deviations are shown in *Fig. 5-4*. It worth to denote that because the four grids have different bus number, for better illustration, the bus number is normalized to the maximum value and an interpolation is used to make they have same length in x axis.

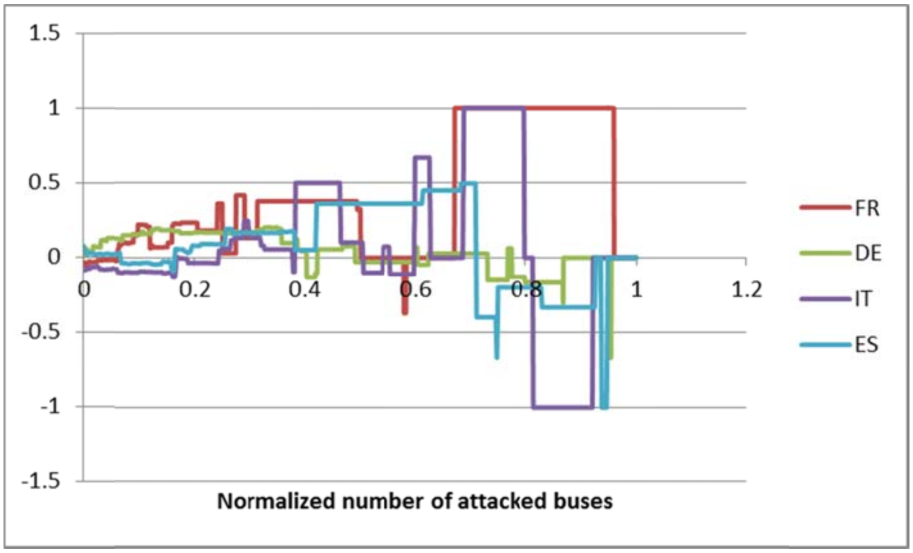

Fig. 5-3 Treeness evolution of four major power grids by electrical betweenness order

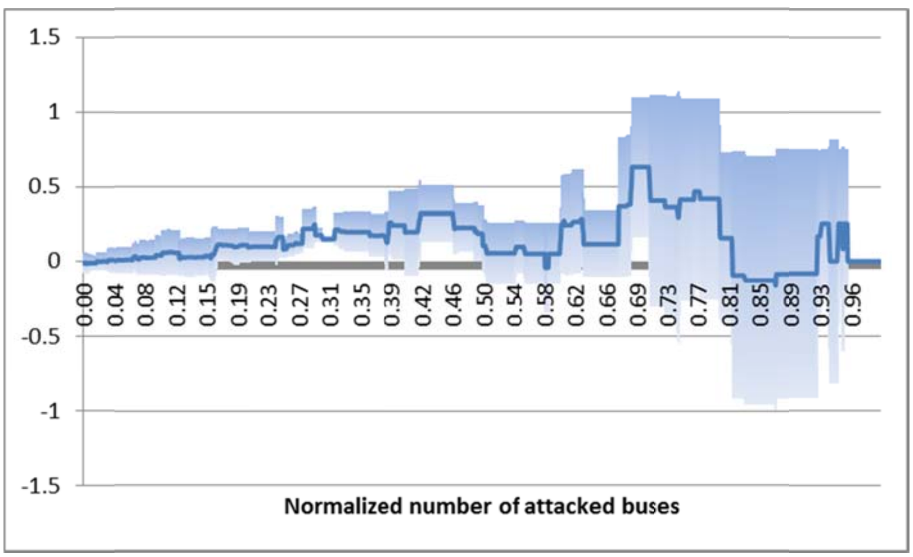

Fig. 5-4 Mean (in blue) and deviation (in shadow) of Treeness evolution by electrical betweenness order

Two important features can be observed: firstly, from *Fig. 5-3* we can see that the hierarchy coordinates evolution of Germany power network is smoother than other networks. Secondly, from *Fig. 5-4* it is noticed that the Treeness value evolves to positive values mainly.

5.2.2. Hierarchy Evolution in decreasing net-ability

The node net-ability of the four major power networks is calculated by the formula 3.16. Similar simulation is performed according to the decreasing order of net-ability for each network. The Feedfordwardness and Orderability values are also equal to 1. The Treeness results are shown in *Fig. 5-5*, and their mean values and deviations are shown in *Fig. 5-6*. The x axis is also normalized.

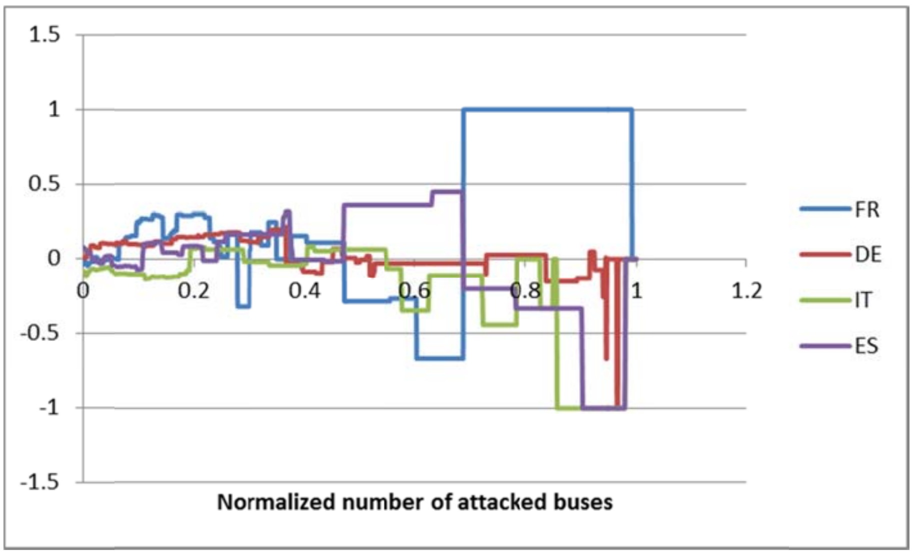

Fig. 5-5 Treeness evolution of four major power grids by net-ability order

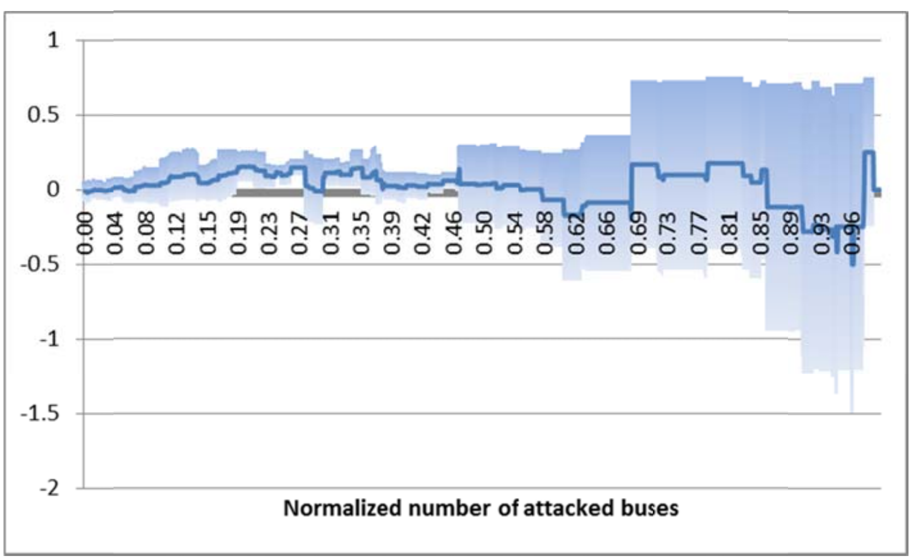

Fig. 5-6 Mean (in blue) and deviation (in shadow) of Treeness evolution by net-ability order

From *Fig. 5-5* and *5-6* we can get the same conclusion with the electrical betweenness scenario in *section 5.2.1*. The Germany power network can be identified with a uniqueness compared to other networks in the Treeness coordinate evolution, which would be introduced more specifically in the random generator deletion scenario in the following section.

5.2.3. Hierarchy Evolution in randomly generator elimination

Although in the electrical betweenness and net-ability scenarios, the Germany network exhibits difference. However, the difference is not significant enough. We need more evidence to highlight this feature. Except for topological metrics to generate the elimination order, the generator in power network can be used because it usually plays an important role as source. Because it's difficult to identify which generator is more important or not, therefore, only random attack to the generator can be used. Thus considerable times (i.e. 50 times) simulations are performed. The simulation results and their mean values and deviations are reported from *Fig. 5-7*, to *Fig. 5-10* for four power grids respectively.

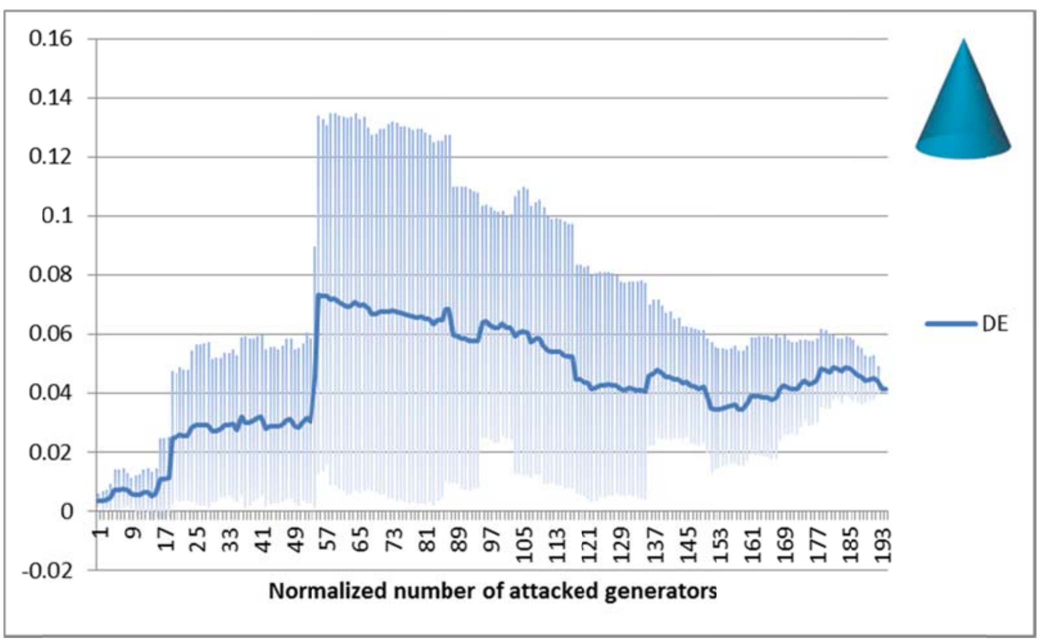

Fig. 5-7 Treeness evolution of DE power grid by random generator elimination. Mean (in blue) and deviation (in shadow)

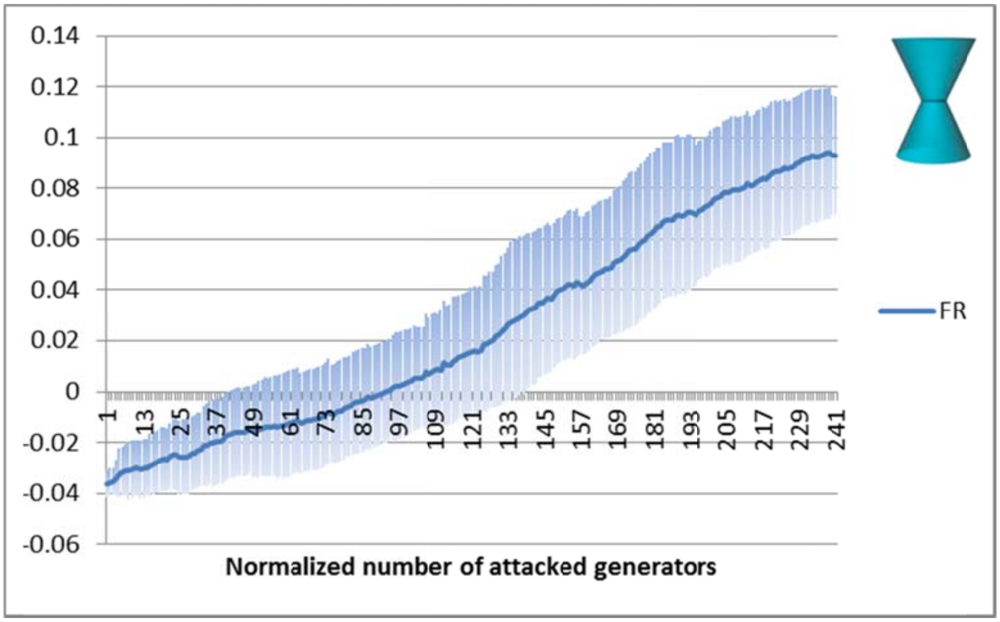

Fig. 5-8 Treeness evolution of FR power grid by random generator elimination. Mean (in blue) and deviation (in shadow)

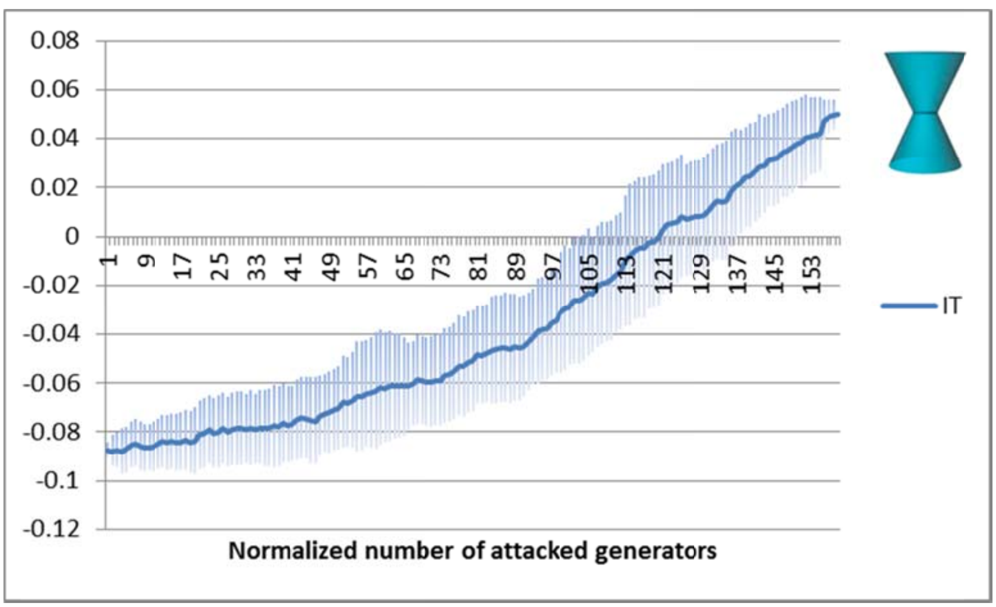

Fig. 5-9 Treeness evolution of IT power grid by random generator elimination. Mean (in blue) and deviation (in shadow)

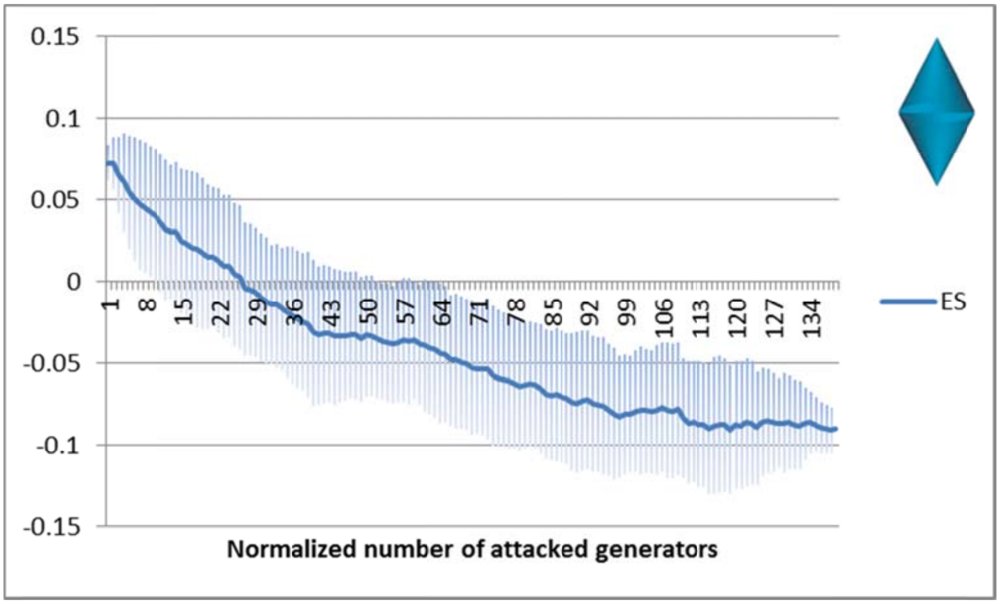

Fig. 5-10 Treeness evolution of ES power grid by random generator elimination. Mean (in blue) and deviation (in shadow)

The upper right parts of *Fig. 5-7* to *Fig. 5-10* denote the network hierarchy direction when generators are removing from the power network. For Germany network, the Treeness evolution values remain positive which means the hierarchy direction is from top to bottom. While for other networks, because their Treeness values cross x axis, it means their hierarchy direction change from top to bottom or vice versa. In short, in generator deletion scenario, the Germany network again exhibits difference with other networks which consistent to the conclusion in electrical betweenness and net-ability scenarios.

From the analysis to the hierarchy coordinates evolution of four major power transmission networks above, there are enough evidences to say the Germany power network has different footprint with other networks:

1) Treeness value always remain positive in each scenario;
2) lower variability Treeness value than other networks

In other words, the proposed morphospace hierarchy coordinates [1] can be used to analyze the power networks from complex network point of view. However, more important question is to find out what the rationality behind the different behaviors in the hierarchy coordinates, and what these results can be translated as meaningful features. In the next section, an attempt is given to link this evolution property with specific characteristic of power systems.

5.3. HIERARCHY AND RELIABILITY IN POWER NETWORKS

Hierarchy property is an important characteristic of complex network [12-21]. Complex systems are usually characterized by some level of hierarchy, which spans in time and space at different scales. Power system as a typical complex system would contain this feature as well. Base on this consideration, we would like to see what's happening about the evolution of hierarchy coordinates in power transmission networks. From the analysis performed above, we find that the Feedforwardness and Oderability remain "1" while the Treeness varies in a specific band. And the hierarchy evolution of different power networks has different features. A discussion to all these findings needs to be addressed further:

Firstly, in each scenario, it seems only Treeness varies with respect to Feedforward and Orderability. This feature is a consequence of engineering practices focused on reducing the wiring costs while keeping the system connected. In other words, the power networks are generally planar and less mesh graph. A significant characteristic is that the degree of each node is 2, which is consistent to the conclusion of [22] that the average degree for 33 European power networks is 2.8 ($\langle k \rangle=2.8$).

Secondly, it can be observed that the Treeness evolves to positive values mainly. This feature means in most cases the hierarchy directions of these networks are from top to bottom. Or from source (generator) to sink (load). Because, generally speaking, the power is transmitted from generator to load in power grid. Therefore, the hierarchy evolution is consistent with the reality. This phenomenon can be seen as the validation of the hierarchy coordinates in turn.

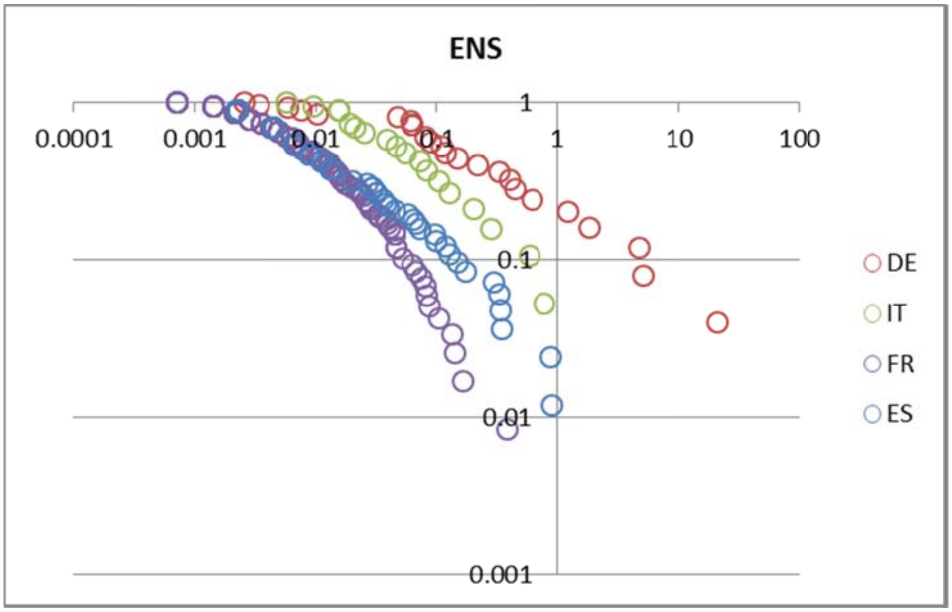

Fig. 5-11 Energy not supplied (MWh) cumulative probability distribution, normalized by the number of nodes for each network (2002 – 2013).

Furthermore, since what we mostly concern is the vulnerability property of power networks. As the method used in chapter 4, the cumulative probability distribution of the reliability data: energy not supplied (ENS), total loss of power (TLP) and restoration time (RT) of these four UCTE major power grids which can be found in [23] is used and checked. We can find that the ENS index has a most positive result to discriminate Germany network from other power networks which is shown in *Fig. 5-11*. From the figure it's observed that the cumulative ENS distribution of Germany follows a power-law, while for other networks, there is an obvious cut-off in the tail. It seems that a linkage could be built: a low variability of hierarchy implies an increased probability of accumulated major events. This linkage could extend the application of the hierarchy and vulnerability analysis of power network or even more broadly the whole complex network.

5.4. CONCLUSION

The hierarchy coordinate in morphospace is introduced to power transmission network to analyze the hierarchy evolution of four major UCTE power grids (France, Germany, Italy and Spain). The evolution is following the removing of bus in power grid according to the decreasing electrical betweenness, net-ability and random generator. The simulation results reveal that the Germany network exhibit different behavior in Treeness evolution. Based on our method proposed in chapter 4, we also learn the cumulative probability distribution of ENS, TLP and RT of these four power grids. A good coincidence occurs in ENS that the cumulative probability distribution follows a power-law distribution while for other networks have a strong cut-off in the tail. A simple explain is given to explain the reason why these phenomenon exist. However, more accurate explanation and the relationship with the cascading failure will be our future works.

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Chapter 6.

Spatial and Performance Optimality in Power Distribution Networks

Power grids, especially high voltage transmission networks, have been widely studied applying the complex network analysis approach to the electrical grid. Usually, basic topological characteristics, statistical global graph properties and vulnerability (or robustness) analysis have been thoroughly studied on many power grids in different parts of the world [1]. Especially, the vulnerability characteristic of the power grid is the main motivation for the studies. In fact topology property plays an important role in shaping the performance (e.g., effects of natural disasters or malicious attacks) of power grids [2-5]. As a result, there is an increasing interest in analyzing structural vulnerability of power grids by means of complex network methodology.

In current power systems, power plants are large to exploit economies of scales and more efficient technologies and are usually located far away from the load center. Therefore, power is transmitted from power plants to load center by high voltage transmission network, then distributed to different voltage levels to the users like homes, offices, schools, companies, and stores. Therefore, the power grid is usually divided in two main segments: transmission (high voltage) and distribution network (low voltage). Most of the scientific literature using the complex network approach applied to the power grids has focused so far on transmission grids, while little attention has been put on the distribution grids. Until to now, to our best knowledge, only [6] took distribution networks into consideration under emerging smart grid technology. As addressed by Pagani and Aiello in [6], with the development of the smart grid, the main role of high voltage transmission networks may change while the low voltage distribution networks may gain more and more importance and require a major update. Most of the research that focuses on modeling the power grids uses simple graph models with sometimes the use of basic properties such as direction and weight. However, these studies [1] miss an important characteristic of the power grid: the spatial characteristic. Spatial properties are the coordinates of the generator, transformer and, substation, the wiring direction and lengths of power cable, etc. In this section we will apply complex network method to power distribution networks since it is the part of the grid that is going to receive the most of the attention in the future and we pose special attention to the spatial aspects of the networks, since these aspects are not studied.

Another key aspect of many practical engineering problems concerns is optimization. Optimization can be applied also in the network context and usually the objective is to identify optimal network or optimal network model and the optimal flow or traffic on a network [7-9]. Optimization in power systems is also an important topic such as, the optimal dispatch of power generation [10], the optimal method for power distribution network reconfiguration [11], the optimal placement of PMUs (Phasor Measurement Units) in power networks [12] and optimal control strategy for power system facility and stability [13], which covers from the static to dynamic analysis of power systems. Two key issues should be taken into consideration in the optimization of power grids: performance and cost. For performance of power systems, the higher the performance and the lower cost, the better for the users. To assess the performance of a power system from an engineering point of view, two indexes: the Equivalent Interruption Time Related to the Installed Capacity (TIEPI) and the Equivalent Number of Interruptions Related to the Installed Capacity (NIEPI) are used [14]. To assess the cost aspect, the wiring cost is a good measure [15]. In power grids, the Euclidean length of power cables gives us a method to quantify the costs of the network taking into account its spatial properties. In this chapter we would like to identify a trade-off between the performance and the cost from a spatial network point of view.

In this chapter, a comprehensive study about the application of complex network methodology on power distribution networks from pure, extended and spatial topological point of views.

6.1. CHARACTERISTICS OF THE TOPOLOGY OF DISTRIBUTION POWER GRIDS

6.1.1. Power grid data sets

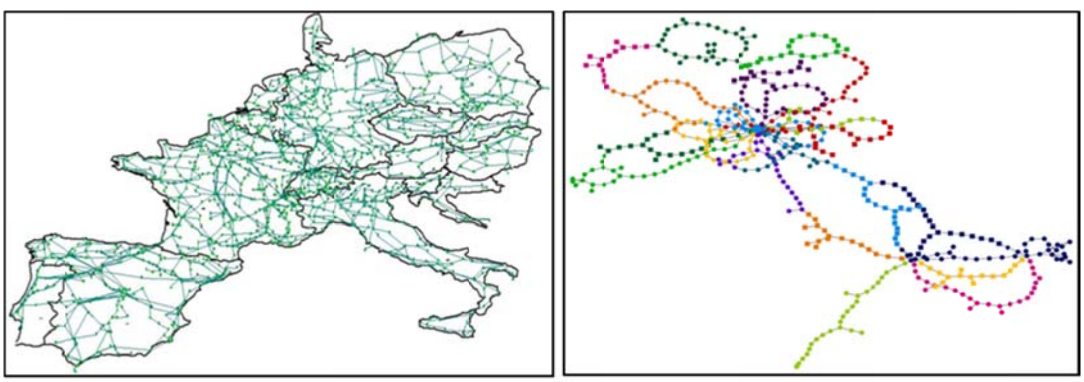
In this chapter, we will analyze two kinds of power grids as spatial network: the transmission power networks and the distribution power networks. The transmission network is a large scale interconnected bulk power transport grid. As a sample of this grid we use the European network known as Union for the Coordination of Transmission Electricity (UCTE). To analyze the distribution networks we use samples of Spain and the Netherlands. For the confidential and copyright issues, the Spanish networks are denoted as SDN1 & SDN2, while the Dutch networks are denoted as NL1 to NL 12. We emphasize that the samples used belong to real infrastructure and not to synthetic models such as IEEE-bus models. As a typical transmission network, UCTE data give us a counterpart to compare with the distribution network from topological manner. The basic information about these networks is reported in *Table 6-1*. For the networks the geographical coordinates of nodes are available which make us could model the distribution networks as spatial network, however, these are not mentioned in *Table 6-1* for obvious security and safety reasons.

Table 6-1 Basic information of distribution/transmission power networks

Network type	Number of nodes	Number of lines	Name/Geography
Transmission	2777	3762	UCTE/Europe
Distribution	519	557	SDN1/Spain
Distribution	240	263	SDN2/Spain
Distribution	451	492	NL1/The Netherlands
Distribution	473	505	NL2/The Netherlands

Distribution	241	254	NL3/The Netherlands
Distribution	287	305	NL4/The Netherlands
Distribution	221	231	NL5/The Netherlands
Distribution	193	209	NL6/The Netherlands
Distribution	957	1095	NL7/The Netherlands
Distribution	371	391	NL8/The Netherlands
Distribution	223	237	NL9/The Netherlands
Distribution	204	207	NL10/The Netherlands
Distribution	271	279	NL11/The Netherlands
Distribution	480	509	NL12/The Netherlands

In our abstraction to represent the power grid as a graph, we consider all the substations and transformers equal and are presented as nodes in graph; the cables are abstracted as edges; this type of abstractions are common in the study of power grid in the complex network framework [16]. The topology of the UCTE and SDN1 networks are shown in *Fig. 6-1*. From the map we can see that the power grids are typically a planar graph both at transmission and distribution level [6].



(a) (b)

Fig. 6-1 UCTE transmission network (a) and a sample of a distribution network (b). Differences in topology are obvious.

6.1.2. Topological metrics

In the complex network methodology several metrics are used to quantify properties both from a global and local point of view. We assess a series of metrics including degree, betweenness, characteristic path length, etc. The metrics of the three power grids are reported in *Table 6-2*. Looking at the average degree one sees that the distribution grids have a very similar value which is around 2.1 while the UCTE network is definitely higher (2.7). This aspect gives us already an idea that the distribution grid tends to have a more radial-like structure since a node has only two connections to other nodes, while the transmission network presents a more meshed structure.

Table 6-2 Significant topological metrics for each network

Network	Average degree	Average betweenness	Average geodesic distance	Average clustering efficiency	Graphic density
UCTE	2.709	30147.632	22.712	0.07067	0.00098
SDN1	2.146	6142.634	24.669	0.01279	0.00414
SDN2	2.192	1785.888	15.878	0.00903	0.00917
NL1	2.213	2257.417	11.0085	0.00547	0.004917467
NL2	2.156	3798.359	17.058602	0.01592	0.004568746
NL3	2.116	1285.726	11.665777	0.00330	0.008817427
NL4	2.181	1613.948	12.726142	0.01024	0.00762652
NL5	2.118	1025.222	10.2735	0.00137	0.009625668
NL6	2.176	795.56	9.238959	0.00284	0.011334197
NL7	2.341	3595.902	9.800753	0.00769	0.002448376
NL8	2.113	2610.86	15.072021	0.00151	0.005711372
NL9	2.161	1093.296	10.800861	0.00112	0.009736194
NL10	2.049	1496.235	15.664072	0.00131	0.010093693
NL11	2.081	1693.026	14.765535	0.00075	0.007708077
NL12	2.158	2800.156	13.113497	0.00215	0.004505915

A further discussion about the topological metrics is given as following. First of all, *Fig. 6-2* shows the degree of each network where x axis denotes degree value and y axis denotes percentage. Two key features are observed, on the one hand, the degree distribution of transmission network (UCTE) is obviously different with the distribution network (Spain and the Netherlands). On the other hand, large part of degree of distribution networks are 1 and 2, while the for UCTE network, the degree value distributes more evenly from 1 to 5. Therefore, for these two kinds of network, the difference in degree focuses on distribution of the small value part (less than 5).

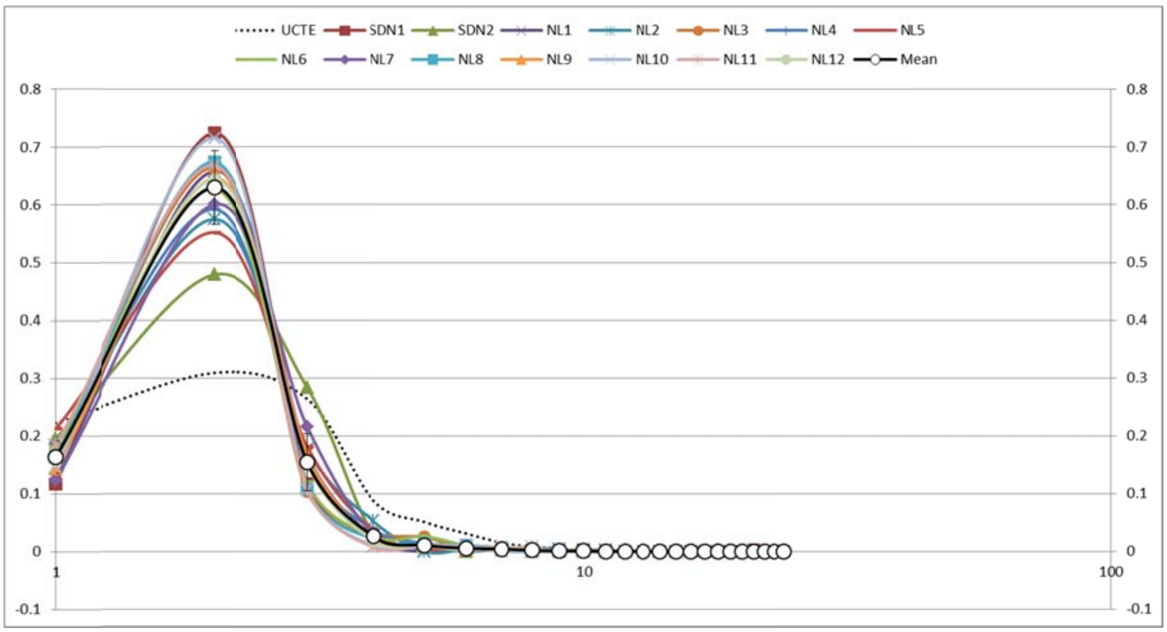


Fig. 6-2 Degree distribution of each network

The identification of the right complex network model for the power grid is one the most important task. There are three general models of networks have been intensely studied and fairly well developed so far: random, small-world and scale-free. The probability distribution of degree gives us an insight into the general properties of the network and allows us to classify it. The log-linear scale cumulative degree distribution of these networks is shown in *Fig. 6-3*. The mean and deviation are also shown in *Fig. 6-3*. One notes that the degree distribution of UCTE network follows an exponential distribution, which is in line with other papers that studied transmission power grids [1]. For other distribution networks, there isn't an obvious exponential fitting in their degree distribution. It seems that a power-law kind distribution is exhibited: most of the nodes degree is 1 or 2, only a small number of nodes have a larger degree. For example, only one node has the maximum degree which has a value of 21 in the SDN1 network. For the Dutch distribution networks there is not a definitive answer as it is suggested in [6]; some samples exhibit a power-law such as sample NL7, while others have a faster decay in the node degree distribution such as sample NL8. In other word, the rich get richer and the poor get poorer. This conclusion coincides with the result in the paper [6] and further support that the power distribution networks tend to be scale-free networks. This conclusion about scale-free network has its own valuable meaning in power grids' vulnerability analysis. Because an interesting reliability property for scale-free networks is that they exhibit high robustness to random failures, whereas they are very sensitive to targeted attacks towards hubs [17-19].

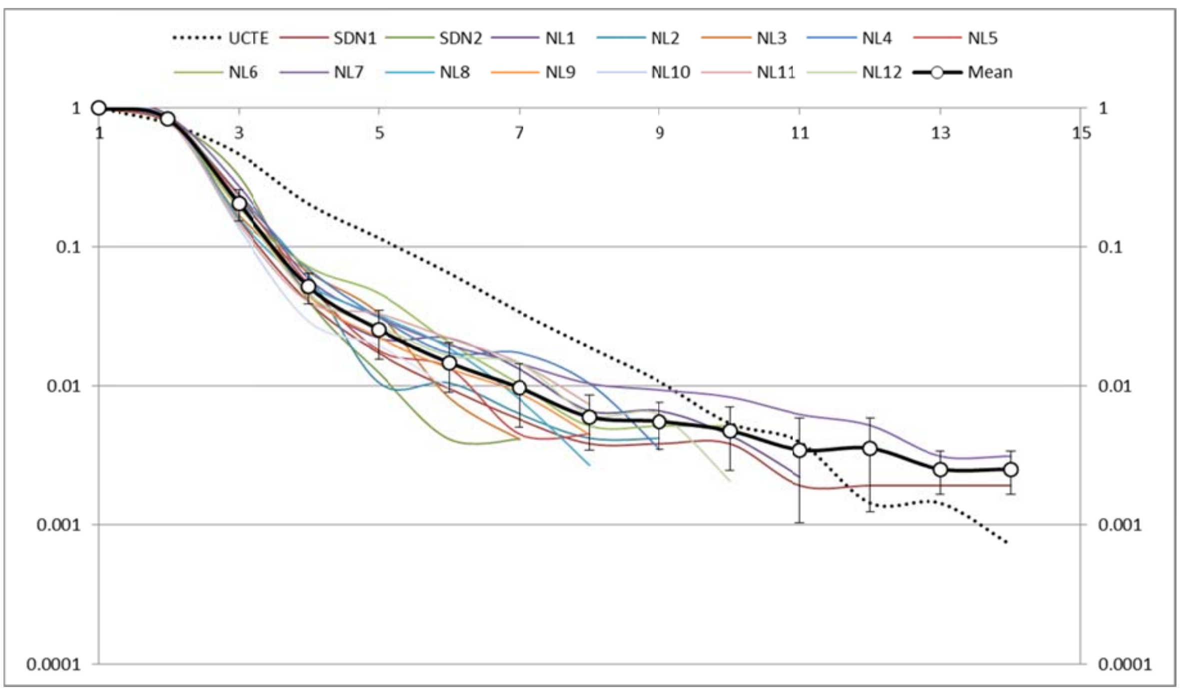


Fig. 6-3 Cumulative degree distribution of each network

Additionally, in pure topological methodology, betweenness is an important measurement to assess how a node is central in a network. It gives an information of the node importance in the physical transmission [18][19]. The betweenness property of each power distribution network is studied and *Fig. 6-4* reports the normalized cumulative betweenness distribution of each network in log-linear scale. The mean value of normalized betweenness and its deviation are shown in *Fig. 6-5*. The x axis is the normalized betweenness value which normalized to their maximum values of the each network.

Generally speaking, there is an exponential distribution with a bit of decay at the tail for each distribution network. While for transmission network UCTE, an exponential distribution is exhibited. It worth to point out that a very faster decay exists in SDN1 compared to other networks which means the nodes with very high values of betweenness are less likely to be present in the network. The decay can also be interpreted from an engineering perspective: power distribution network is quite radial and hierarchical so that the paths tend to follow the few noes admissible by the relative simple topology [6].

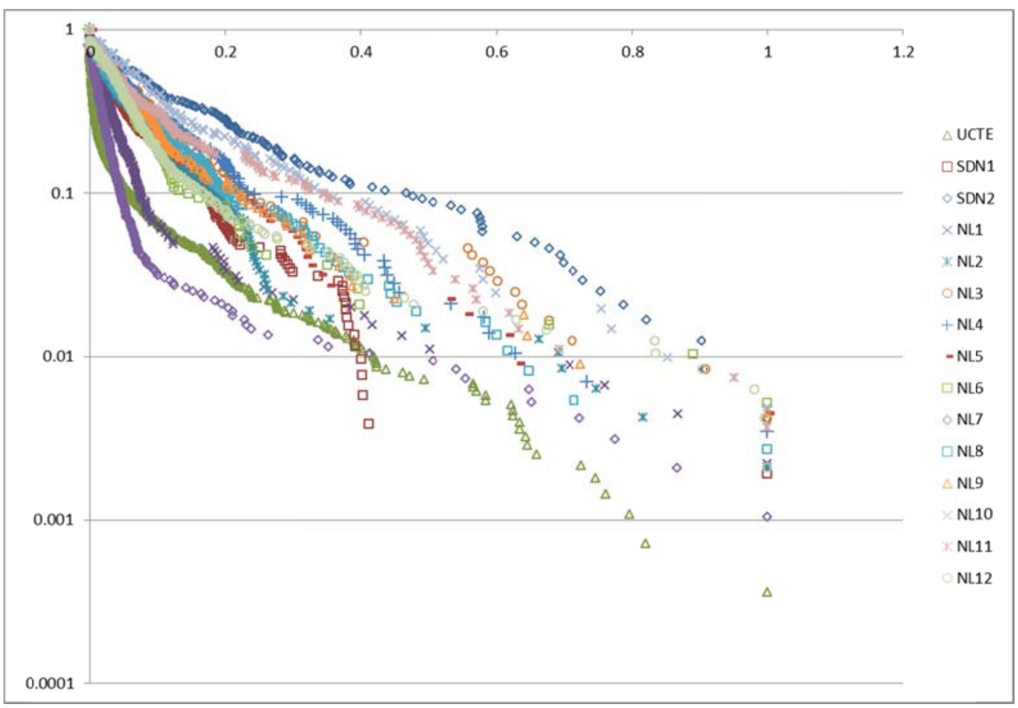


Fig. 6-4 Normalized cumulative betweenness distribution of each network.

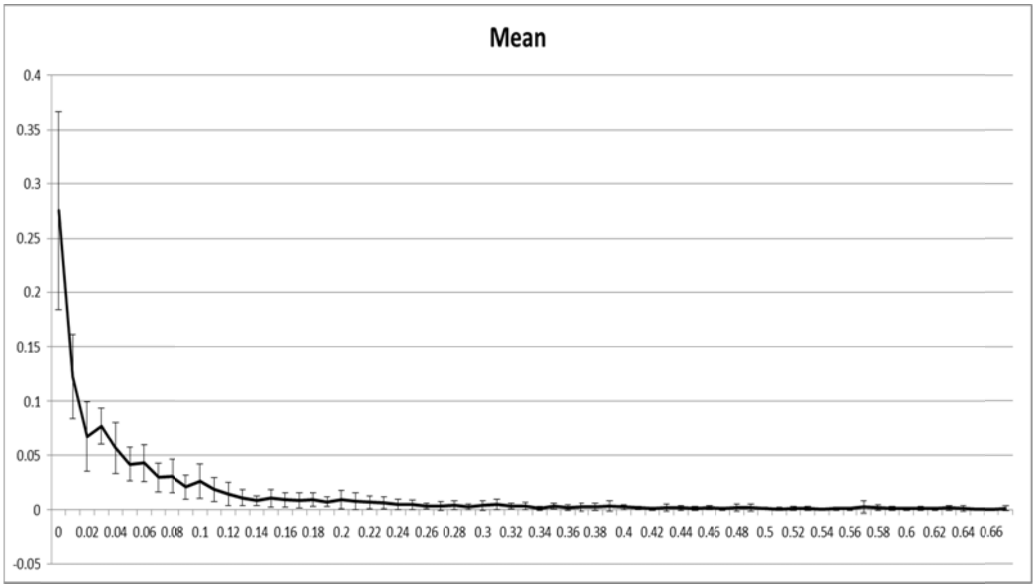


Fig. 6-5 Mean and deviation of betweenness of the power networks analyzed.

Furthermore, network motifs which are defined as recurrent and statistically significant sub-graphs or patterns [20] have encode important local properties of networks. Each of these sub-graphs defined by a particular pattern of interactions between vertices reflects the structural design principles of complex networks. From the comparison of degree and betweenness above, one can see that there exist some differences between each network. In order to dig into these differences, the network motifs property of each network is examined and reported in the *Fig. 6-6*. From the figure we can see that the percentage of sub-graph #1 in the SDN1 distribution network is larger than all the other network samples considered. The only sample that has a similar value for the considered motifs is the biggest sample (in term of order and size) of the Dutch set (i.e., NL7). In general, for the motifs analyzed, one notes that only two motifs appear significant: sub-graph #1 and sub-graph #3 which reinforce the presence of a radial-like structure. It means that the SDN1 network is more radical than other networks since it has largest value of sub-graph #1 which in turn to explain the existence of the a faster decay in the cumulative betweenness distribution plot.

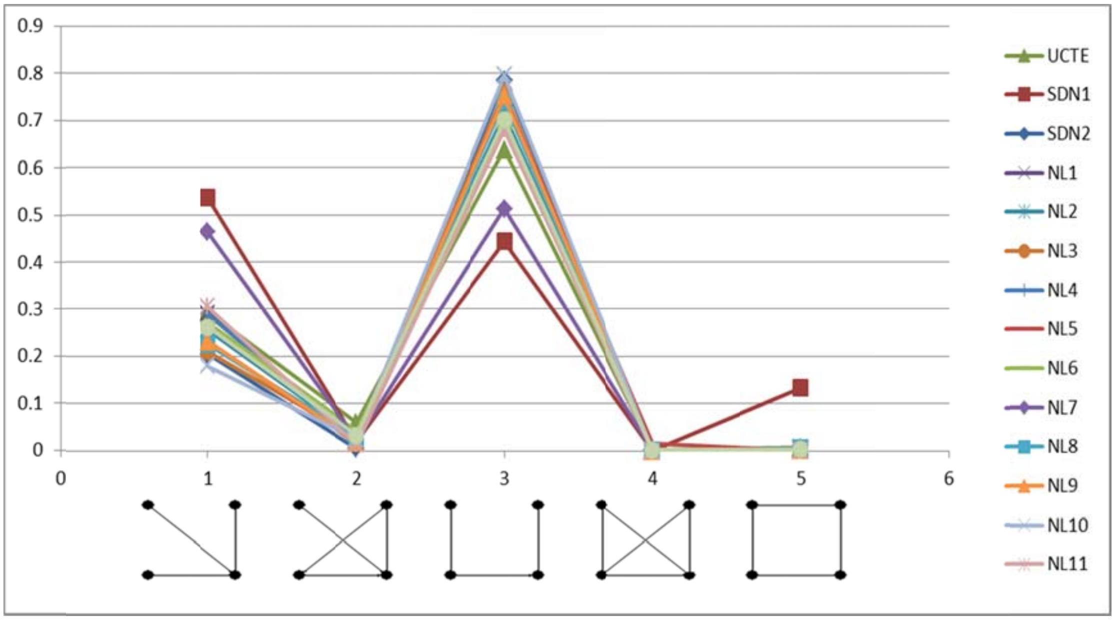


Fig. 6-6 Motifs property of each network.

On the summary, looking at *Table 6-2*, we find interesting to compare the two Iberian distribution networks with the Dutch ones. In particular given the number of nodes and edges we consider fair to compare SDN1 network with NL12 and SDN2 with NL9. First of all despite almost the same average degree SDN1 has an average geodesic distance that is double the amount of NL12. A similar trend is actually present in betweenness too: the sample of SDN1 has a much higher average betweenness compared to NL12, almost twice as big. On the other hand, the SDN1 network has a higher clustering coefficient than the Dutch one. The same tendency is found on the comparison between SDN2 and NL9. SDN2 needs on average a path that is 50% longer compared to NL9, and again the betweenness of the Iberian network is higher than the Dutch (70% increase). The same pattern found in the clustering coefficient for the previous pair applies: SDN2 has higher local clustering, almost double the NL9 sample.

From the examinations about the pure topological metrics, it worth to point that on the one hand, distribution network has its own properties compared to transmission network; on the other hand, even in distribution network, there exhibits difference among each other. The question is raised that how this difference influences the performance of distribution networks? In order to answer this problem, the spatial property, spatial constraints and optimality will be taken into consideration in the following section.

6.2. OPTIMALITY AND SPATIAL CONSTRAINTS

6.2.1. Spatial property of power grids

After checking the pure topological metrics of different kind of power networks, we would like to study the power grids from spatial network point of view. Considering the spatial aspects, our power grids data provide us with detail about the geographical position of the nodes and the length of each power cable. First of all, the branch distance (length) distribution of each network is reported in *Fig. 6-7*. And the cumulative distribution of branch distance of each network is analyzed and reported in *Fig. 6-8* in log-log scale.

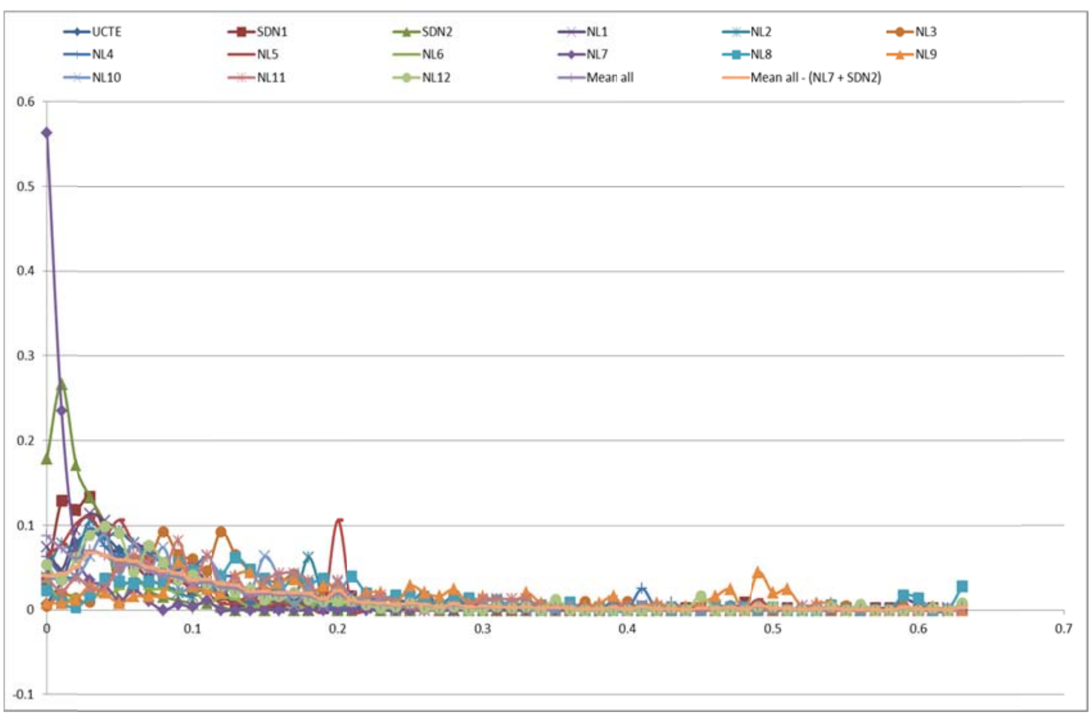


Fig. 6-7 Normalized branch distance (length) distribution of each network.

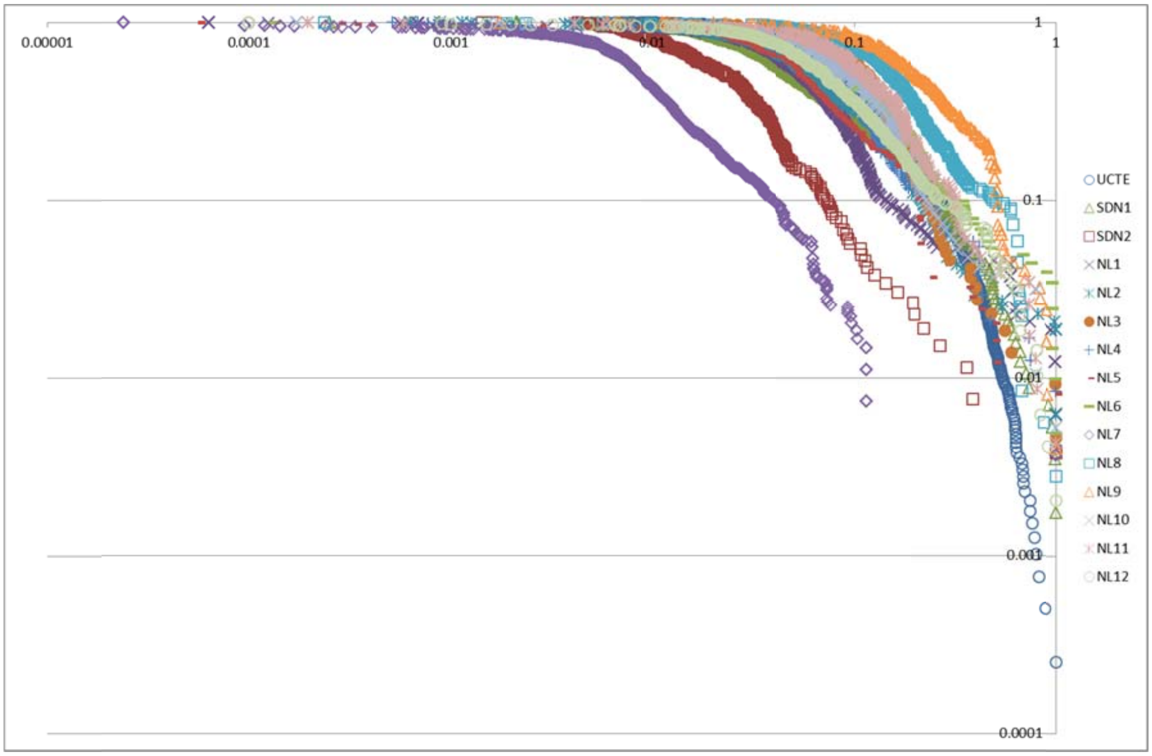


Fig. 6-8 Cumulative probability distribution of real lengths (normalized).

Here the branch length is normalized to their maximum of each network. In the first glimpse, from *Fig. 6-7*, the SDN2 and NL7 have a different distribution of the branch length. In the meantime, the SDN2 and NL7 also have a cumulative distribution that at least in the central part is closer to a power-law distribution exhibited in *Fig. 6-8* because a straight fitting exist in alone the tail. While for UCTE, SDN1 and other Dutch networks, one sees an overall distribution that looks exponential, especially having consistent exponential effects in the tails of the distribution. For better understanding this observation, *Fig. 6-9* just shows the branch length distribution of UCTE, SDN2, NL7, Mean all and Mean all except SDN2 and NL7. It's observed that the SDN2 and NL7 are obviously share different distribution of branch length in small values part (less than 0.05) that discriminate them among all networks.

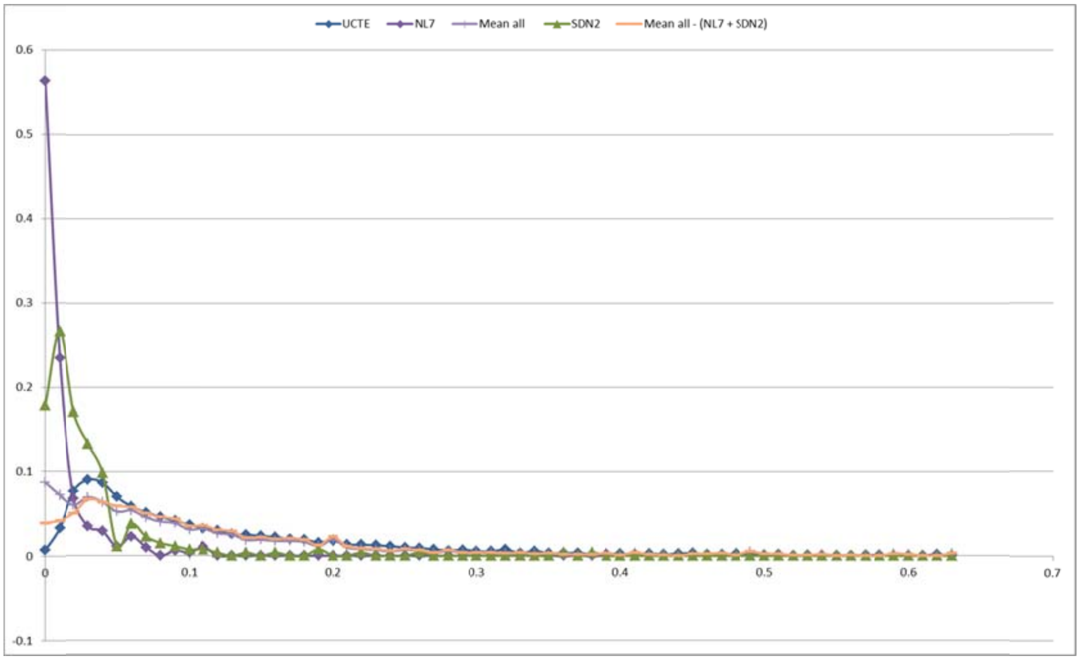


Fig. 6-9 Branch distance (length) distribution of some specific networks.

Together with the pure topological metrics analysis above, there is an interesting pattern for SDN2, NL7 distribution networks with other networks. The power-law tendency of cumulative degree distribution supports a scale-free network conclusion if treating the distribution network as an undirected graph. While when considering the spatial properties of the networks, the two groups networks have different cumulative branch distance distribution. The consequent key issue is that what we can find through this phenomenon. Obliviously, the spatial property and constraints could influence the branch wiring, while the branch wiring would influence the branch length distribution. Since it’s noticed that there exists significant difference in the branch length distribution. Therefore, we can derive that the branch wiring plays an important role in the spatial model of power distribution networks. Because how the topology influences the performance is the main motivation of applying complex network methodology to power systems. In the next section, the role of branch wring will be further investigated from the performance (or the behavior) optimality point of view in power networks.

6.2.2. Optimality property of power grids

To further study the role of branch wiring, we adopt the methods used by Ahn *et al.* [15] to rearrange the position of nodes and the endpoints of edges by using a shuffling mechanism applied to the fifteen networks. Two shuffling methods are used:

1) Edge exchange (EE) shuffling in which vertices of randomly selected two edges exchange their partner vertices;

2) Vertex swapping (VS) shuffling in which two randomly chosen vertices simply exchange their positions while preserving all the connections.

Inspired by the method used in [15], we apply a Monte-Carlo (MC) scheme using Metropolis algorithm and controlled by a given temperature T is used for our numerical simulation. The fully random shuffling of network using either EE or VS method corresponds to the MC simulation at T = ∞ dented as EE(inf) and VS(inf). And the simulated annealing technique is used to get the optimal value denoted as EE(0).

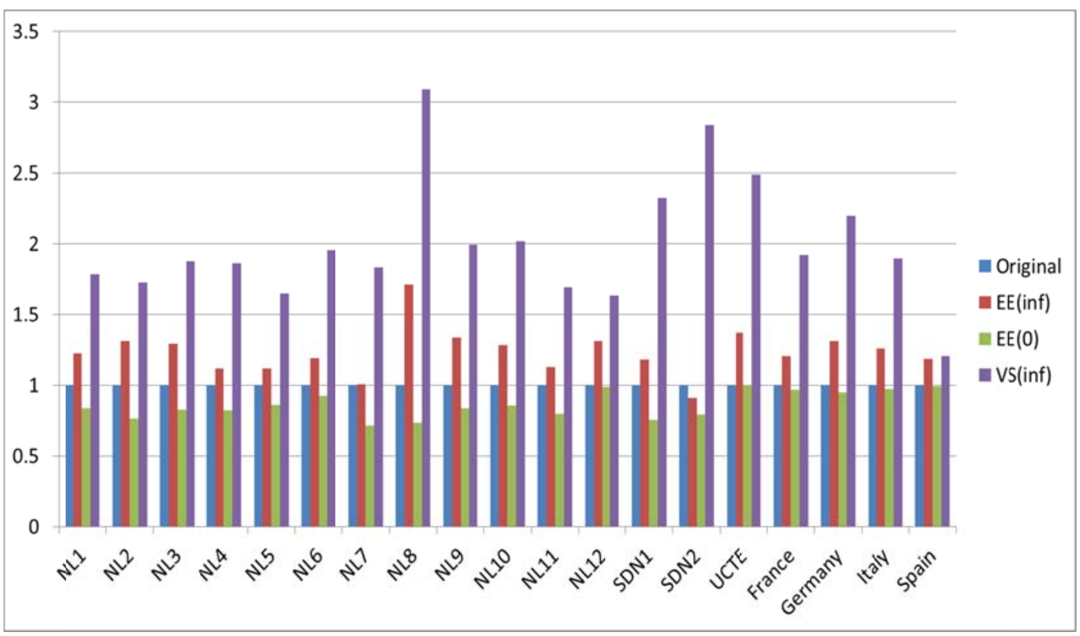


Fig. 6-10 Normalized wiring costs (physical distances) of networks obtained by fully random vertex and edge shuffling.

The simulation results for these networks are reported in *Fig. 6-10*. The EE(inf), EE(0) and VS(inf) are all normalized to the original wiring length of each network. From the *Fig. 6-10* we can see that the original distance of UCTE transmission network is nearly approaching the optimal one. While in the other two distribution networks the original lengths are not optimal. And the difference arises again, the EE(inf) value in SDN1 network is less than its original length. On the contrary, in SDN2 network, the situation is opposite. It means, in SDN2 network, there exist a couple of branches have long distance which also be embodied in the power-law distribution of cumulative branch distance. While in SDN1, the exponential decay means the upper constraint of branch distance.

The Dutch samples have the same behavior as the Iberian ones; the optimality is not achieved in the majority of the samples. Only few samples (i.e., NL3, NL6, NL10, NL11 and NL12) have the current and wiring distance similar to the one of the optimal situation. Generally, the optimality is achieved by those samples that are more spatially compact that are the network expands in a relatively small geography and therefore the original distance is limited.

On the summary, the simulation confirms our hypothesis that the branch wiring will influence the performance optimality of power network. It worth to denoted that here we use the wiring cost (branch length) as our Hamiltonian.

6.2.3. Evolution of optimality in power grids

Without loss of generality, we also would like to see the evolution of optimality of a specific power network in a large time scale. Based on this consideration, the French 400KV power transmission network from 1966 to 2000 is collected and investigated. *Table 6-3* reports the basic information of the series power transmission networks analyzed.

Table 6-3 Basic information of French power network from 1966 to 2000

Year	Number of nodes	Number of lines
1966	14	17
1970	21	26

1976	34	41
1980	51	61
1986	96	124
1990	124	165
1996	140	186
2000	149	197

The topology evolution of French transmission networks are illustrated in *Fig. 6-11*.

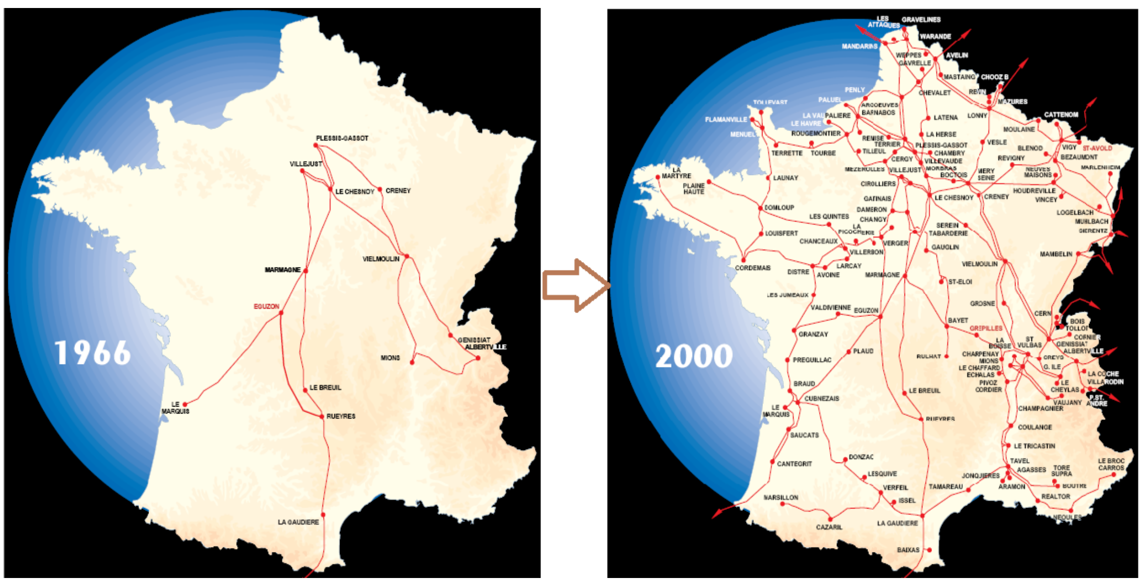


Fig. 6-11 Illustration of the evolution of French power transmission network.

The EE shuffling and VS shuffling procedures are used again, and the Monte-Carlo simulation is performed one by one to each year's network. The simulation results are shown in *Fig. 6-12*. From the figure we can see that the original distances of the network are approaching to the optimal one which means the power transmission network will gain better capacity with its network evolution. In the meantime, the difference between the original distance and EE(inf) is increasing with the evolution which supports again the performance improvement.

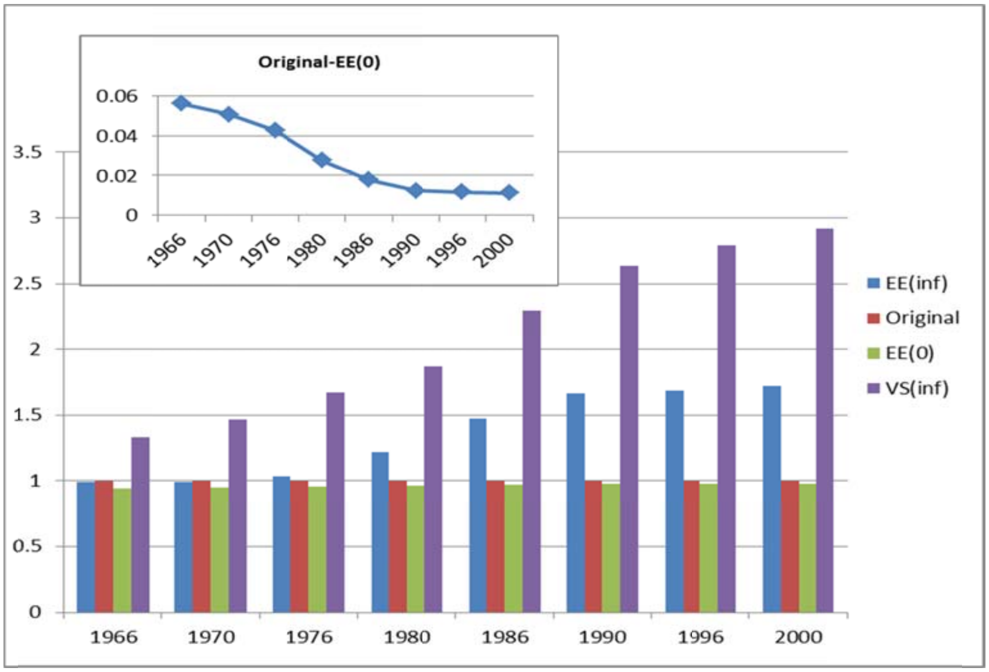


Fig. 6-12 The evolution of optimality with EE(0) for French transmission network. Inset: detail of the evolution as a sigmoid function of time.

6.2.4. Spatial constraints

Up to now, we have enough evidence to say that the spatial model of power distribution networks could give us a novel perspective to find out the network footprint. Thus, it is interesting to correlate the spatial properties of the network with the performance of the power distribution network from the engineering point of view. As we noted in the previous section the topologies of the SDN1 and SDN2 networks are radial-like, resembling a tree with some loops. This structure is very similar to a case for a high density of loops in real optimal network: the veins in leaves or insect wings [7]. Therefore, we would like to examine the properties of this structure including the cumulative distribution of the nodes in each loop and the cumulative distribution of the nodes in each antenna. The results for the Iberian networks and Dutch networks are reported in *Fig. 6-13* for antenna and *Fig. 6-14* for loop under log-linear scale. Here the x axis denotes the antenna numbers which are normalized to their node number of each network. The normalization operation is due to the consideration of network size.

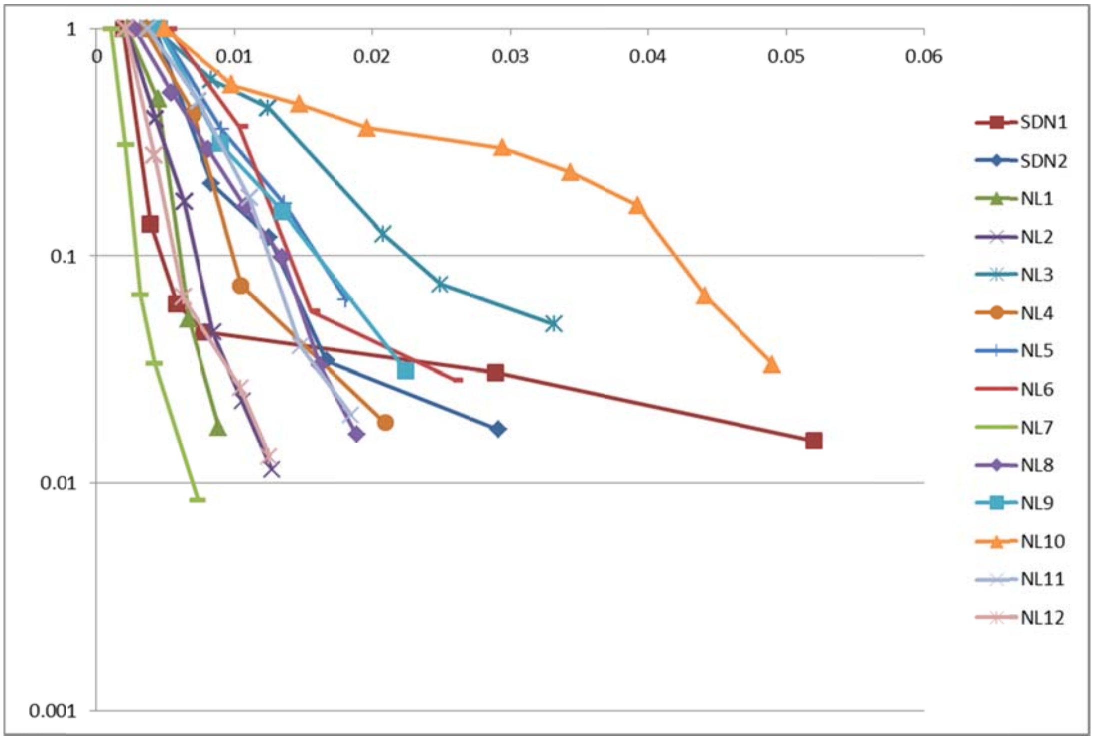


Fig. 6-13 Antennas in distribution networks.

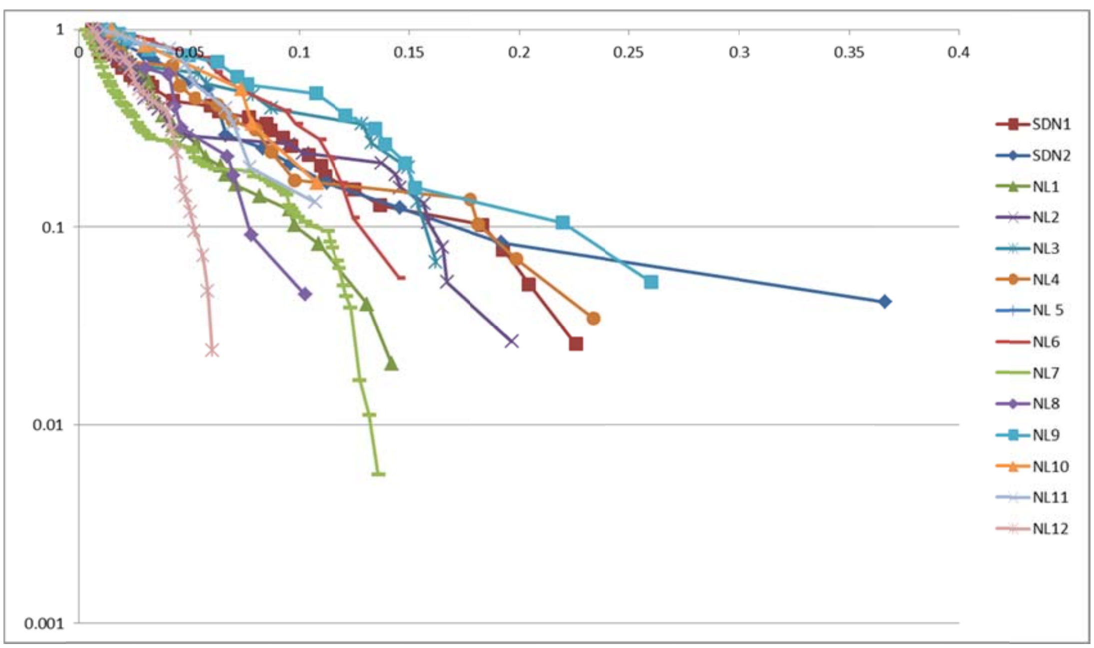


Fig. 6-14 Loops in distribution networks.

In the SDN2 network most of the antennas contain less than 4 nodes, whereas for the SDN1 network there exist some antennas which contain more nodes even up to 27. The Dutch samples behave in this respect closer to the SDN2 network since the samples have in general antenna structures that do not contain many nodes. For the antenna with less than 5 nodes, the cumulative distribution values are very similar for both the Dutch samples, SDN1 and SDN2. The difference part focuses on the nodes large than 5. Dutch sample NL7 has higher probability of finding more than 5 nodes organized in an antenna structure compared to the other samples; however there are no antenna configurations with more than 10 nodes for that sample. Considering loops structures, SDN2 contains loops involving considerably fewer nodes compared to the SDN1 network. In other words, based on the analysis above we can say that the SDN2 distribution network is more fragmentation and homogenization: most of the antennas with fewer nodes like 1 or 2, and most of the loops contains within 10 nodes, compared to the SDN1 distribution network.

The Dutch samples seem divided into two categories for the aspects considering loops: almost half of the samples have loops containing few than 50 nodes and actually a substantial amount of samples that do not have even 30 nodes in loops structures. The other samples (NL7, NL2, NL4, NL9, NL1) have loops involving an higher number of nodes. It is interesting to note that there is no correlation in the number of nodes and edges of the network and the size of the loops formed in that network. In this last set of samples both the high and small networks in terms of order are present.

Which spatial topology is better for power distribution network in case of performance? The performance here should quantify the distribution systems are operated under normal condition: the voltage in a safe range, the active and reactive power balancing, etc. The service quality index will be used to quantify the performance of two typical distribution network SDN1 and SDN2 to find out the relationship between topology and performance.

6.3. RELIABILITY

As mentioned above, we would like to build a linkage between the topology property and performance for distribution network. The precondition is the collection of real malfunctions data. In Iberia countries, two indexes: the Equivalent Interruption Time Related to the Installed Capacity (TIEPI) and the Equivalent Number of Interruptions Related to the Installed Capacity (NIEPI) are used to quantify the service quality of distribution network. The

TIEPI is used to quantify the average time during which the supply to a customer is interrupted [14]:

$$\mathrm{TIEPI} = \frac{\sum_i P_{ri} \times r_i}{P_{rT}} \tag{6.1}$$

where P_{ri} is the sum of the rating of all interrupted medium-voltage/low-voltage transformers plus the contracted power of all interrupted medium-voltage and high-voltage customers. P_{rT} is the total rating of all medium-voltage/low-voltage transformers plus the total contracted power of all medium-voltage and high-voltage customers connected to the system.

NIEPI is used to quantify the average number of supply interruptions [14]:

$$\mathrm{NIEPI} = \frac{\sum_i P_{ri}}{P_{rT}} \tag{6.2}$$

where P_{ri} is the sum of the rating of all interrupted medium-voltage/low-voltage transformers plus the contracted power of all interrupted medium-voltage and high-voltage customers. P_{rT} is the total rating of all medium-voltage/low-voltage transformers plus the total contracted power of all medium-voltage and high-voltage customers connected to the system.

Based on the dataset that we can get from the DSOs, here only the TIEPI can be used to compare the performance of SDN1 and SDN2. The statistical chart is shown in *Fig. 6-15*.

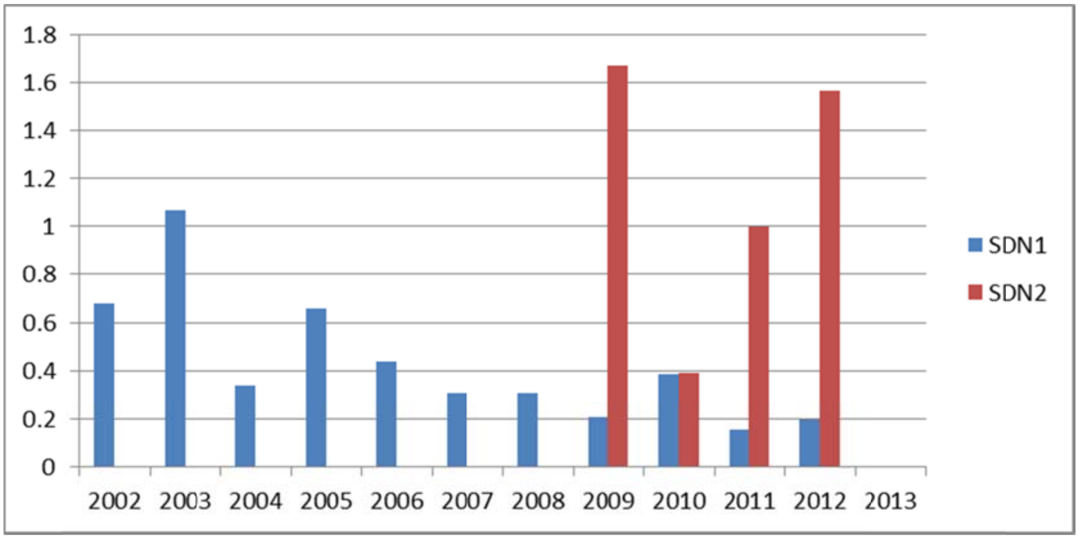


Fig. 6-15 Evolution of TIEPI values for SDN1 and SDN2 distribution networks.

From *Fig. 6-15* we can see that the evolution of the equivalent time of interruption for the optimal (SDN1) and non-optimal (SDN2) distribution networks, suggesting a significant increase in TIEPI values for the latter. Based on this observation, it seems that if the wiring cost of a network achieves its optimal then its resilience to the malfunctions is higher than the one that can't get its optimal wiring cost. The conclusion is quit meaningful because it seems the first time to build the linkage between spatial properties of power distribution networks with their real malfunctions indexes.

6.4. CONCLUSION

In this chapter, a comprehensive analysis to real power distribution networks (SDN1, SDN2 and the Netherlands) from complex network point of view is given. Since present researches focus on power transmission networks, our works extends the application of complex network theory to power systems. Another contribution of this chapter to power networks analysis using complex methodology is involving spatial network conception. A spatial model is proposed to the power distribution network and its specifics like: branch length, loop and antenna are analyzed and some interest conclusions have been proposed. In order to further analyze the performance of power distribution network, the wiring cost (length) is use as the object of the optimization, the edge shuffling and vertex shuffling methods are used to analyze their optimality. The Monte-Carlo (MC) scheme is also adopted to increase the accuracy. It's noticed that there exist some differences in performance optimality of distribution networks. However, the difference is just in wiring cost or economic point of view. What we more care about is the reliability analysis of power systems. Therefore, on the one hand, the spatial constraints of each power distribution network are studied and we try to use it to explain the difference mentioned above. On the other hand, the reliability data of SDN1 and SDN2 networks are collected and compared. More meaningful conclusion is found that if the wiring cost of a network achieves its optimal then its resilience to the malfunctions is higher than the one that can't get its optimal wiring cost. Although this result is not strong enough, we would dig into the reason for this problem in our future works.

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Chapter 7.

Conclusion

Complex network theory has been widely used to analyze the power networks from basic topological property to statistic robustness analysis and dynamic resilience property. However, to our best knowledge, there are still many problems need to be investigated and addressed. This dissertation has paid attention on the application of complex science and complex network methodology in power system analysis from different aspects:

Firstly, an extended topological methodology was proposed which involving specific characteristics of power systems in pure topological method to make traditional complex network approach closer to the reality. Based on this extended methodology, three new metrics (entropy degree, electrical betweenness and net-ability) were proposed and used to analyze the vulnerability of power networks. A simplified UCTE bulk power transmission network is used as an example to show how these metrics to spot the importance of components in power grid. In the meantime, as fundamental outputs, our proposed extended metrics will also be used in our following researches.

Secondly, a first attempt is given to build a linkage between topological measures and empirical data of power systems. Here the empirical data can be interpreted as the grids′ realistic behavior (i.e., malfunctions and major events). The entropy degree and electrical betweenness were applied to four major power transmission networks (Germany + Italy, France + Spain), and their cumulative distribution of malfunctions from 2002 onwards were investigated also. A meaningful and significant linkage between structural measures and the real dynamical output (i.e., major events) of a grid is built though still weakly.

Thirdly, we compared and evaluated the evolution of hierarchy for some real power transmission networks when buses are attacked selectively in decreasing order of some topologically and electrically defined values. It seems that hierarchy increases as the network is being attacked and a low variability of hierarchy implies an increased probability of accumulated major events. This conclusion extends the application of hierarchy conception to vulnerability analysis of power systems or even the whole complex network research.

Last but not least, complex network methodology was extended to power distribution networks. The pure topological properties of some real distribution networks of Spain and the Netherlands were studied. Furthermore, the spatial network model was built up for these networks and their spatial properties were also analyzed. In order to investigate the relationship between performance and topology, the edge shuffling and vertex shuffling method were used to analyze the wiring cost and the performance optimality. In the meantime, the real malfunctions data was used to verify our simulation results aforementioned.

Although we tried to cover a whole picture of applying complex network in the emerging power system vulnerability analysis, and many aspects and characteristics of power systems have been revealed from a new perspective as complex systems, a lot more extensive features can be exploited using similar method developed from complex network theory, and those are considered as a promising future work. Our future works are summarized as the following:

- Although in this thesis, the PTDF (or power flow equivalent) is introduced into traditional pure topological method. And the power grid is not just abstracted as simple undirected graph but the flow based flow. Our research is still in the static analysis scope. The future object should be involving dynamic features in our study. For example, batch of papers have addressed that the Kuramoto oscillators applied in complex network to analyze its synchronization property. A natural thought is that using this kind of oscillator to replace the synchronous generator to simplify the synchronization stability problem of power systems. Therefore, if we can introduce dynamic features into the application of complex network methodology to power systems, the models or the metrics based on complex network theory will be closer to power system reality.

- How to connect the topological (or extended) metrics with the empirical data (malfunctions data is used in this thesis) is an interesting and promising work which is worth of more attention to be paid. On the one hand, this linkage can prove the correctness and validity of using complex network theory to power systems. On the other hand, this linkage would help us to discriminate the vulnerability form component to the whole network in power networks. Although this dissertation proposed a linkage between extend topological metrics with the malfunctions data of UCTE major power networks (France, Germany, Italy and Spain). More types of dataset and more different real power networks are needed to verify our proposed method. Or even more novel methods are needed.
- Because in this thesis we have found something about the relationship between hierarchy and reliability of power networks. Which means hierarchy will affect the spread of failure in the network. Therefore, we have the reason to pay more attention on the studying of this mechanism. A new cascading model involving this hierarchy coordinates could be as a first start.
- Modeling power grids as spatial network is a new perspective applying complex network method in power systems. Accept the results addressed in the thesis, more works could be done such as how the space constraint influence the wiring of a network so that influence its performance.