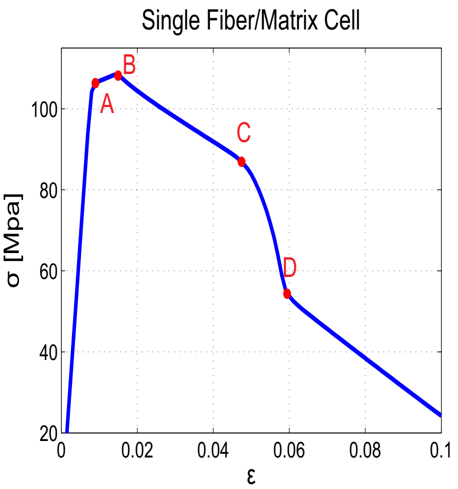
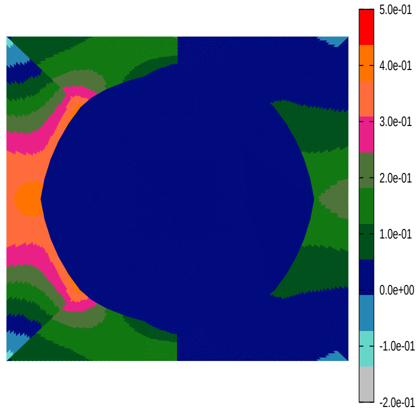
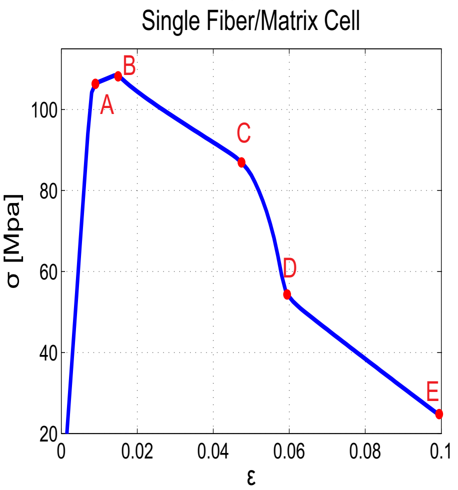


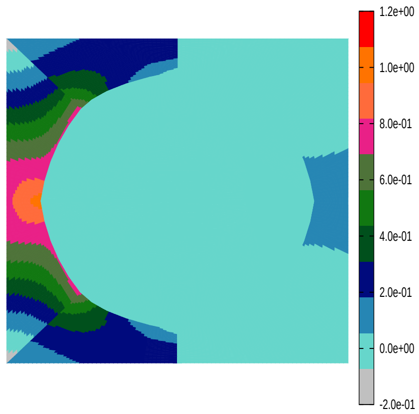

(d)




(d)




(e)




(e)

Figure 6.20. Crack path for the single fiber RUC, $G_{IC,2}$ curve.

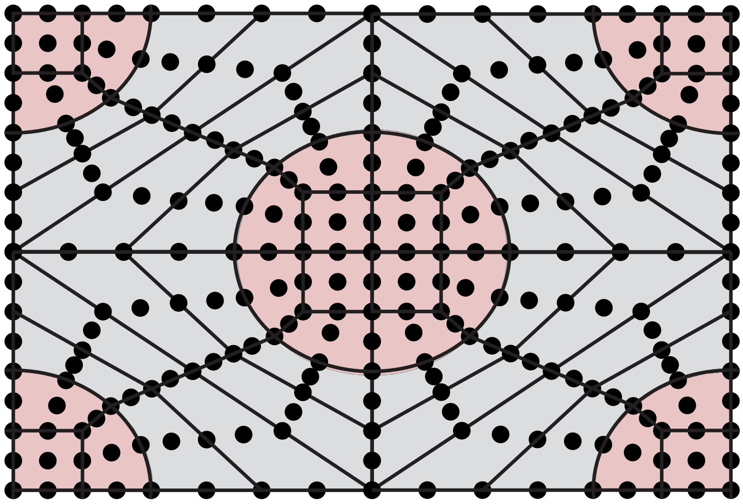


Figure 6.21. Hexagonally packed cell: mesh 2.

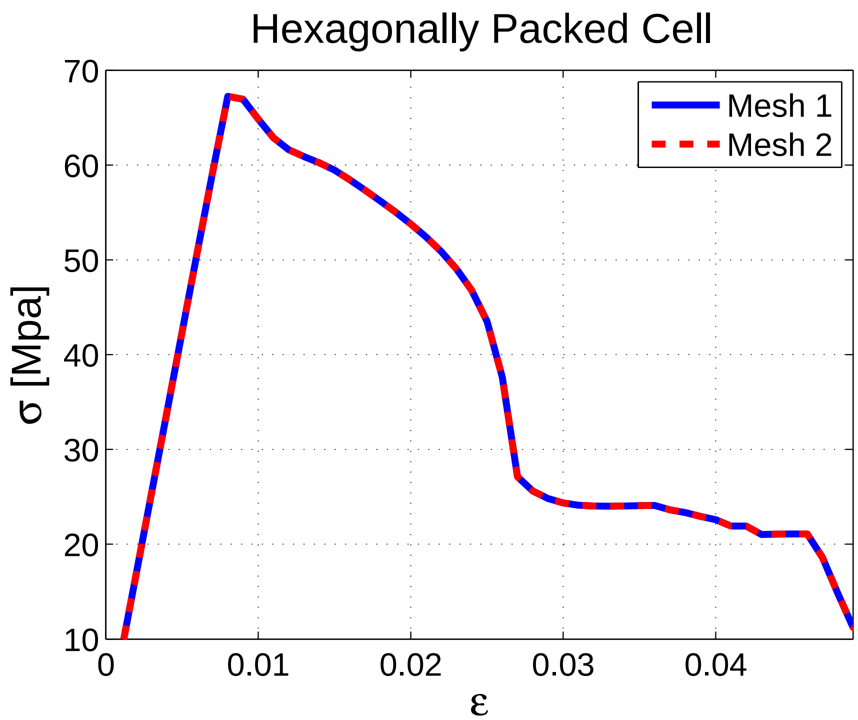



Figure 6.22. Stress/strain curves comparison for the progressive failure analysis of two hexagonally packed cell in a mesh objective way.

# Chapter 7

# Conclusions

In this work two different multi-scale approaches have been proposed for the analysis of fiber-reinforced composite structures. In chapters 3, 4 and 5 results were evaluated in terms of displacements, stress/strain fields and failure indexes. Different structural problems were discussed, including fiber-matrix cells, laminated beams and composite C-shaped beams. The failure analysis has been conduced applying different criteria for the failure initiation and the crack band method for the progressive failure analysis as explained in Chapter 6. Comparisons with Solid models from different commercial codes were proposed (ANSYS, Patran/Nastran); for the two-scale analysis approach the 1D CUF has been included in Abaqus environment to model the micro-scale. The results obtained suggest that:

1. multiscale techniques have to be taken into account if the failure mechanism of composite structures wants to be analyzed. Detailed models would be extremely helpful in improving the design of composite structures reducing the test activity costs; however a compromise between accuracy of results and computational cost has to be reached. Despite the increasing development in computer hardware, the computational effort of these methods is still prohibitive for extensive applications. The reduction of the computational time and cost required to perform failure analysis is still a challenging task;

2. the 1D CUF is an extremely powerful tool to investigate the behavior of composite structures. Higher-order elements allow a refined description of the stress and strain states at different levels with a reduced computational cost. A small number of elements on the cross-section is enough to obtain the solid-like accuracy;

3. the 1D nature of the fibers makes easy to model properly their orthotropic behavior through 1D elements, properties are defined in the beam axis direction;

4. in the framework of the 1D CUF, the LE models proved to be the best accuracy/computational cost choice to facilitate the identification of failure according to the employed criteria;

5. based on the 1D CUF with the LE polynomials the CW approach has been developed for the modelling of a fiber-reinforced composite structure. This approach is a concurrent method that allows to simultaneously model different length scales in the same model;

6. the proposed CW approach offers significant improvements in detecting the mechanical behavior of laminated structures in particular when stress fields around fiber and matrix cells have to be accurately computed;

7. in the CW approach, cells can be opportunely included in order to refine the model in areas that were considered critical following preliminary analyses; as a general guideline, the CW approach should be adopted in a global-to-local analysis scenario where results from globally refined models are exploited to evaluate the most critical areas of a given structure and where locally refined models are then employed to obtain accurate stress fields in those critical areas;

8. Critical zones can be identified introducing integral quantities. Subdomains can be part of the macro- or the micro-structure. Evaluations on 3D subdomains have been proposed nevertheless, the same approach can be extended to 1D- and 2D-regions;

9. within the CW approach, the failure analysis can be performed directly on the components. Conducing progressive failure analysis in a mesh objective way, it is possible to predict the crack evolution without knowing the path in advance and without introducing different types of finite elements;

10. computational advantages from CUF 1D can be even more evident when iterative strategies have to be introduced to perform the progressive failure analysis (hierarchical multiscale techniques);

11. a two-scale analysis has been developed using the CW approach at the microscale to perform linear analysis in a multiscale scheme. This approach is of particular interest for the analysis of structures with complex shapes since the macroscale can be modelled through a commercial code, nevertheless the 1D CUF can also used to model the macroscale. The progressive failure analysis can be extended to the macroscale to compute the failure propagation in the global structure;

12. it has been proved that the present 1D formulation is extremely advantageous in terms of computational costs if compared with solid models.

# List of Figures

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