

Figure 6.20. Crack path for the single fiber RUC,  $G_{IC,2}$  curve.

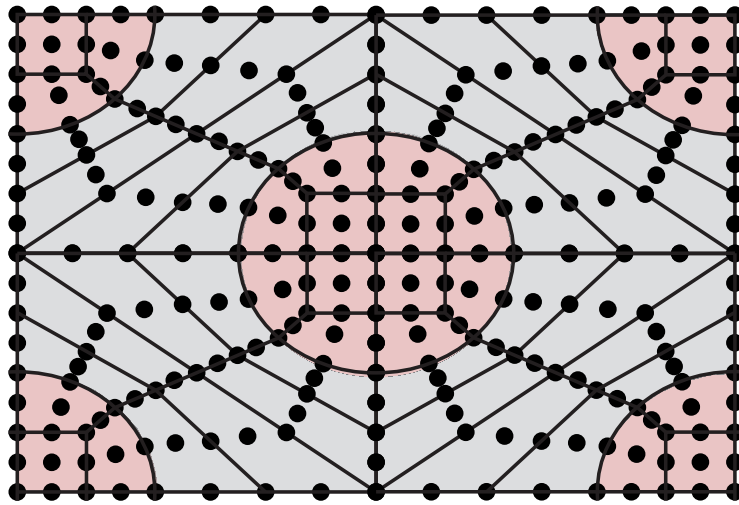


Figure 6.21. Hexagonally packed cell: mesh 2.

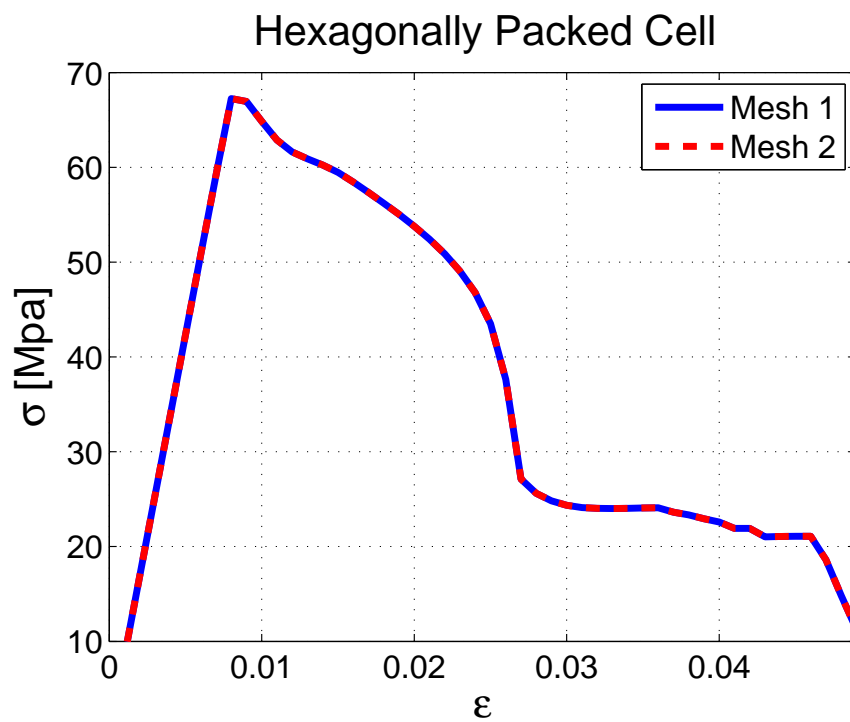


Figure 6.22. Stress/strain curves comparison for the progressive failure analysis of two hexagonally packed cells in a mesh objective way.

## Chapter 7

# Conclusions

In this work two different multi-scale approaches have been proposed for the analysis of fiber-reinforced composite structures. In chapters 3, 4 and 5 results were evaluated in terms of displacements, stress/strain fields and failure indexes. Different structural problems were discussed, including fiber-matrix cells, laminated beams and composite C-shaped beams. The failure analysis has been conducted applying different criteria for the failure initiation and the crack band method for the progressive failure analysis as explained in Chapter 6. Comparisons with Solid models from different commercial codes were proposed (ANSYS, Patran/Nastran); for the two-scale analysis approach the 1D CUF has been included in Abaqus environment to model the micro-scale. The results obtained suggest that:

1. multiscale techniques have to be taken into account if the failure mechanism of composite structures wants to be analyzed. Detailed models would be extremely helpful in improving the design of composite structures reducing the test activity costs; however a compromise between accuracy of results and computational cost has to be reached. Despite the increasing development in computer hardware, the computational effort of these methods is still prohibitive for extensive applications. The reduction of the computational time and cost required to perform failure analysis is still a challenging task;
2. the 1D CUF is an extremely powerful tool to investigate the behavior of composite structures. Higher-order elements allow a refined description of the stress and strain states at different levels with a reduced computational cost. A small number of elements on the cross-section is enough to obtain the solid-like accuracy;
3. the 1D nature of the fibers makes easy to model properly their orthotropic behavior through 1D elements, properties are defined in the beam axis direction;
4. in the framework of the 1D CUF, the LE models proved to be the best accuracy/computational cost choice to facilitate the identification of failure according to the employed criteria;

5. based on the 1D CUF with the LE polynomials the CW approach has been developed for the modelling of a fiber-reinforced composite structure. This approach is a concurrent method that allows to simultaneously model different length scales in the same model;
6. the proposed CW approach offers significant improvements in detecting the mechanical behavior of laminated structures in particular when stress fields around fiber and matrix cells have to be accurately computed;
7. in the CW approach, cells can be opportunely included in order to refine the model in areas that were considered critical following preliminary analyses; as a general guideline, the CW approach should be adopted in a global-to-local analysis scenario where results from globally refined models are exploited to evaluate the most critical areas of a given structure and where locally refined models are then employed to obtain accurate stress fields in those critical areas;
8. Critical zones can be identified introducing integral quantities. Subdomains can be part of the macro- or the micro-structure. Evaluations on 3D subdomains have been proposed nevertheless, the same approach can be extended to 1D- and 2D-regions;
9. within the CW approach, the failure analysis can be performed directly on the components. Conducting progressive failure analysis in a mesh objective way, it is possible to predict the crack evolution without knowing the path in advance and without introducing different types of finite elements;
10. computational advantages from CUF 1D can be even more evident when iterative strategies have to be introduced to perform the progressive failure analysis (hierarchical multiscale techniques);
11. a two-scale analysis has been developed using the CW approach at the microscale to perform linear analysis in a multiscale scheme. This approach is of particular interest for the analysis of structures with complex shapes since the macroscale can be modelled through a commercial code, nevertheless the 1D CUF can also be used to model the macroscale. The progressive failure analysis can be extended to the macroscale to compute the failure propagation in the global structure;
12. it has been proved that the present 1D formulation is extremely advantageous in terms of computational costs if compared with solid models.

# List of Figures

1.1	Utilisation of composite material in military airplanes: (a) F-15 (early 1970s), 2% composites, (b) F/A-18 A/B (mid-1970s), 10% composites, (c) AV-8B Harrier (early 1980s), 27% composites, (d) F/A-18 E/F (mid-1990s), 22% composites. . . . .	2
1.2	Evolution in use of composite structures in Airbus. . . . .	3
1.3	Composite structural weight development. . . . .	3
1.4	Composite Parts in the A400M. . . . .	4
1.5	Use of composite materials in the A350XWB airframe. . . . .	4
1.6	Boeing 787 Material Distribution. . . . .	5
1.7	Motivations of the present thesis. . . . .	6
2.1	Coordinate frame. . . . .	9
2.2	Physical (x,y,z) and material (1,2,3) reference systems. . . . .	12
2.3	Composite layup. . . . .	12
2.4	Cross-Section elements in actual geometry. . . . .	15
2.5	Cross-Section elements in the natural frames. . . . .	15
2.6	Beam-axis shape functions. . . . .	19
2.7	TE assembly technique for a multicomponent structure. . . . .	22
2.8	LE assembly technique for a multicomponent structure. . . . .	22
2.9	Assembly technique for a TE case study, N=2. . . . .	24
2.10	Assembly technique for a LE case study, 1 L4 element. . . . .	25
2.11	Global stiffness matrix for the TE, LE case studies with 1 B3 element in the y-direction. . . . .	26
2.12	K, stiffness matrix obtained through the 1D CUF. . . . .	26
3.1	Component-Wise approach to simultaneously model layers, fibers and matrices. . . . .	30
3.2	Component-wise approach to simultaneously model layers, fibers and matrices. . . . .	31
3.3	Model descriptions: 1 L9, 4 L9, 16 L9. . . . .	33
3.4	Stress distribution for the homogeneous cell: (a) Axial Stress $\sigma_{yy}$ at $x = h/2, y = 0$ , (b) Shear stress $\sigma_{yz}$ at $x = h/2, y = 0$ , (c) Axial Stress $\sigma_{yy}$ at $x = h/2, y = L/2$ , (d) Shear Stress $\sigma_{yz}$ at $x = h/2, y = L/2$ . . . . .	35

3.5	CW cell geometry. . . . .	37
3.6	LE Element distribution: 12 nine-point (L9) elements and 8 six-point (L6) elements. . . . .	37
3.7	Axial stress at the clamped cross-section. . . . .	39
3.8	Shear stress at the clamped cross-section. . . . .	39
3.9	Axial $\varepsilon_{yy}$ and shear $\varepsilon_{yz}, \varepsilon_{yx}$ strain at $L = 0$ for the Solid and LE CW cell models. . . . .	40
3.10	Double cell geometry. . . . .	41
3.11	Double cell case study. . . . .	41
3.12	L9 + L6 distribution of the double cell cross-section. . . . .	43
3.13	Shear stress $\sigma_{yz}$ at the clamped cross-section. Double cell Solid and CW models. . . . .	45
3.14	Geometry of the laminated plate. . . . .	46
3.15	Different modeling approaches for the laminate. . . . .	47
3.16	Axial stress $\sigma_{yy}$ along $z$ at $x = 0.3, y = 0.0$ . . . . .	49
3.17	Shear stress, $\sigma_{yx}$ , distribution above the cross-section at $y = 0$ , laminated beam, LE models. . . . .	50
3.18	Geometry of the C-shaped cross-section. . . . .	52
3.19	Description of the modeling approaches for the C-shaped beam. . . . .	53
3.20	Deformed cross-sections of the Model 1. . . . .	55
3.21	Transverse displacement distribution above the free-tip cross-section via Model 4, first loading case. . . . .	55
3.22	Stress distributions above the C-shaped cross section at $y = 0$ via model 4, first loading case. . . . .	56
3.23	Deformed configuration of the C-shaped beam via model 2, second loading case. . . . .	57
4.1	Typical component interfaces for composite laminates. . . . .	59
4.2	1D-, 2D- and 3D-subdomains for the evaluation of integral failure parameters. . . . .	59
4.3	3D Post-processing for the evaluation of integral parameters through the 1D LE CUF. . . . .	61
4.4	(a) 8 node hexahedral element coordinates in the natural reference frame. (b) 8 node element sub-domain in a 1D LE CUF model. . . . .	64
4.5	(a) 20 node hexahedral element coordinates in the natural reference frame. (b) 20 node element sub-domain in a 1D LE CUF model. . . . .	65
4.6	(a) 27 node hexahedral element coordinates in the natural reference frame. (b) 27 node element sub-domain in a 1D LE CUF model. . . . .	66
4.7	CW and Solid Models for the homogeneous assessment. . . . .	70
4.8	Strain energy distribution along the y-axis. . . . .	72
4.9	Component-Wise approach for volume evaluations. . . . .	72
4.10	Loading configurations. . . . .	73
4.11	First subvolume distributions. . . . .	74
4.12	Second subvolume distributions. . . . .	74
4.13	Matrix subvolumes near the free tip. . . . .	76

4.14	Maximum Stress (MS) and Tsai-Wu (TW) Failure indexes over the single cell clamped cross-section under bending loading. . . . .	79
4.15	Component-Wise approach for subvolume evaluations. . . . .	81
4.16	First laminated beam mode - laminate (a). . . . .	81
4.17	Laminate beam structure with one fiber/matrix cell included under torsion loading. . . . .	82
4.18	Axial stress distribution at the clamped cross section for the laminate beam (one fiber/matrix cell included) under bending and torsion loadings. . . . .	83
4.19	Second laminated beam model - laminate (b). . . . .	85
4.20	Laminate beam structure with eight fiber/matrix cells included under bending loading. . . . .	85
5.1	Two scales analysis obtained using the 1D CUF at the microscale. . . . .	89
5.2	Microscale dimensions and reference system for a generic RUC. . . . .	90
5.3	PBC side notations. . . . .	92
5.4	PBC applied on a square homogeneous RUC, 1 L9 (cross-section) and 1 B3 (beam axis) elements. . . . .	92
5.5	Displacement field (u,v,w) on the single fiber/matrix cell due to the PBC. . . . .	93
5.6	Single Fiber/Matrix RUC (20 L9) and Hexagonally Packed cell (40 L9) meshes. . . . .	94
5.7	Notched one layer laminate geometry. . . . .	95
5.8	Macroscale boundary conditions for the notched laminate model. . . . .	95
5.9	Abaqus C3D8 solid mesh. . . . .	96
5.10	Maximum principal stress distribution: multiscale analysis applied at the notched laminate case study. . . . .	97
5.11	Maximum principal stress distribution obtained through the homogenized model for the notched laminate case. . . . .	97
5.12	Maximum principal stress distribution in the notched are: multiscale analysis. . . . .	98
5.13	Maximum principal stress distribution in the notched area: homogenized model. . . . .	98
5.14	Open hole plate under tension. . . . .	99
5.15	Open Hole laminate solid macroscale mesh. . . . .	100
5.16	$\sigma_{11}$ obtained through a multiscale analysis with the single fiber/matrix cell at the microscale (a) and through the homogenized model (b). . . . .	101
5.17	$\sigma_{22}$ obtained through a multiscale analysis with the single fiber/matrix cell at the microscale (a) and through the homogenized model (b). . . . .	102
5.18	$\sigma_{12}$ obtained through a multiscale analysis with the single fiber/matrix cell at the microscale(a) and through the homogenized model (b). . . . .	103
6.1	Failure phenomena [Camañho, Dávila, Pinho and Remmers: Mechanical response of Composites]. . . . .	105
6.2	Reference system for the failure analysis. . . . .	105
6.3	Stress-strain diagram for the fracture process. . . . .	111
6.4	Stress-strain diagram for the fracture process. . . . .	112

6.5	$\sigma - \delta'$ diagram for the fracture process. . . . .	112
6.6	Algorithm used to implement the Crack-Band method in the 1D CUF formulation. . . . .	113
6.7	Evaluation of the characteristic length, $l_c$ . . . . .	114
6.8	Plate cross-section. . . . .	115
6.9	Failure Indexes CW single cell models at $y = 0$ . . . . .	123
6.10	Failure Indexes CW double cell models at $y = 0$ . . . . .	124
6.11	Three layers laminate with a fiber/matrix cell inclusion in the first ply. . . . .	125
6.12	Maximum stress and Tsai-Wu criteria FI distributions in the cell subdomain in the 1st ply of the three layers laminate. Bending load applied. . . . .	125
6.13	Maximum stress and Tsai-Wu criteria FI distributions in the cell subdomain in the 1st ply of the three layers laminate. Torsion load applied. . . . .	125
6.14	Isotropic square cell for a mesh objectivity preliminary assessment. . . . .	126
6.15	Model descriptions for the mesh objectivity preliminary assessment: 9 L9, 25 L9, 45 L9. . . . .	126
6.16	Stress/strain curve for the mesh objectivity preliminary assessment. . . . .	127
6.17	Single fiber RUC progressive failure analysis boundary conditions. . . . .	127
6.18	Stress/strain curve for the progressive failure of the single fiber RUC varying $G_{IC}$ . . . . .	128
6.19	Crack path for the single fiber RUC, $G_{IC,2}$ curve. . . . .	129
6.20	Crack path for the single fiber RUC, $G_{IC,2}$ curve. . . . .	130
6.21	Hexagonally packed cell: mesh 2. . . . .	131
6.22	Stress/strain curves comparison for the progressive failure analysis of two hexagonally packed cell in a mesh objective way. . . . .	131



# List of Tables

2.1	L3 cross-section element point natural coordinates. . . . .	16
2.2	L4 cross-section element point natural coordinates. . . . .	16
2.3	L9 cross-section element point natural coordinates. . . . .	16
2.4	Shape function coefficients for B2 elements. . . . .	18
2.5	Shape function coefficients for B3 elements. . . . .	18
2.6	Shape function coefficients for B4 elements. . . . .	19
3.1	Material properties and failure coefficients. . . . .	32
3.2	Displacement of the single cell homogeneous model at the loading point. . .	34
3.3	Displacement for the single cell CW model at the loading point. . . . .	34
3.4	Stress values $[N/mm^2]$ for the single cell CW model at Point B,D ( $y = L/2$ ) and Point B',D' ( $y = 0$ ). . . . .	36
3.5	Strain values for the CW cell model at $y = 0$ . . . . .	36
3.6	Displacement values for the single cell model. . . . .	38
3.7	Stress values for the single cell model. . . . .	38
3.8	Displacement of the double cell CW model at $[a/2, L, h/2]$ . . . . .	42
3.9	Stress values $[N/mm^2]$ for the double cell CW model at Point E,F ( $y = L/2$ ) and Point E',F' ( $y = 0$ ). . . . .	42
3.10	Displacement values for the double cell model. . . . .	43
3.11	Stress values for the double cell model. . . . .	44
3.12	Transverse displacement, at $[b/2, L, 0]$ , and axial stress, at $[0.5, 0, -0.2]$ , of the laminate. . . . .	48
3.13	Shear stress, $\sigma_{xy}$ MPa, at two different points of the laminate, A $[0.8, 0, 0]$ and B $[0.55, 0, -0.2]$ , LE models. . . . .	49
3.14	Cross-Section dimensions of the C-shaped cross-section. . . . .	52
3.15	Vertical displacement, $u_z \times 10^2$ mm, at the loading point of the C-shaped beam, first loading case. . . . .	54
3.16	Axial stress, $\sigma_{yy} \times 10^2$ MPa, at $[a/2, 0, -h/2]$ , C-shaped beam, first loading case. . . . .	54
3.17	Vertical displacement, $u_z$ [mm], at $[a, L, h/2]$ , C-shaped beam, second loading case. . . . .	54
4.1	Brick8 element point natural coordinates. . . . .	63

4.2	Brick20 element point natural coordinates. . . . .	67
4.3	Brick27 element point natural coordinates. . . . .	69
4.4	Strain energy distribution in homogeneous beam subvolumes. . . . .	71
4.5	Strain Energy in beam subvolumes. . . . .	75
4.6	Strain Energy the matrix subvolumes. . . . .	75
4.7	Strain Energy in the fiber subvolumes. . . . .	77
4.8	Strain Energy in fiber and matrix subvolumes. . . . .	77
4.9	Local Effect on matrix due to the bending loads. . . . .	78
4.10	Failure index integrated over the subvolumes normalized with respect to the number of subvolumes. . . . .	80
4.11	Strain energy distribution in a laminate beam under torsion $F = 1$ N load. . . . .	82
4.12	Strain energy distribution in a laminate beam under bending $F = 5$ N load. . . . .	82
4.13	Strain energy distribution in a fiber/matrix cell included in a laminate beam under d torsion $F = 1$ N loads. . . . .	84
4.14	Strain energy distribution in a fiber/matrix cell included in a laminate beam under bending $F = 5$ N loads. . . . .	84
4.15	MS Failure index integrated over the fiber,matrix and whole cell subvolumes under bending $F = 5$ N and torsion $F = 1$ N loads. . . . .	84
4.16	Strain energy distribution in a laminate beam with 8-fiber/matrix cell in- cluded under bending $F = 5$ N. . . . .	86
4.17	CW vs Solid. Total energies. . . . .	86
4.18	MS Failure index integrated over the fibers, matrix and whole cells subvol- umes under bending $F = 5$ N. . . . .	86
5.1	Notched laminate dimensions. . . . .	95
6.1	Orthotropic material properties. . . . .	115
6.2	Failure indexes at $x = b/2, y = L/10$ , thin plate. . . . .	116
6.3	Failure indexes at $x = 0, y = L/10$ , thin plate. . . . .	116
6.4	Failure indexes for the isotropic beam at $L/2$ . . . . .	116
6.5	Material properties and failure coefficients. . . . .	117
6.6	Failure Index for the single cell CW model Point B,D and Point B',D': Maximum Stress criterion. . . . .	118
6.7	Failure Index for the single cell CW model: Maximum Strain criterion. . . . .	119
6.8	Failure Index for the double cell CW model: Maximum Stress criterion. . . . .	120
6.9	Failure Index for the double cell CW model: Maximum Strain criterion. . . . .	120
6.10	Isotropic square cell dimensions . . . . .	121
6.11	DOFs for the isotropic preliminary assessment in the mesh objective failure analysis. . . . .	121
6.12	Material properties fort the analysis of the single fiber/matrix RUC . . . . .	121

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