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Design, Implementation and Testing of Advanced Control Laws for Fixed-wing UAVs

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ABSTRACT

The present PhD thesis addresses the problem of the control of small fixed-wing Unmanned Aerial Vehicles (UAVs). In the scientific community much research is dedicated to the study of suitable control laws for this category of aircraft. This interest is motivated by the several applications that these platforms can perform and by their peculiarities as dynamical systems. In fact, small UAVs are characterized by highly nonlinear behavior, strong coupling between longitudinal and latero-directional planes, and high sensitivity to external disturbances and to parametric uncertainties. Furthermore, the challenge is increased by the limited space and weight available for the onboard electronics. The aim of this PhD thesis is to provide a valid confrontation among three different control techniques and to introduce an innovative autopilot configuration suitable for the unmanned aircraft field.

Three advanced controllers for fixed-wing unmanned aircraft vehicles are designed and implemented: PID with H_∞ robust approach, $\mathcal{L}_1$ adaptive controller and nonlinear back-stepping controller. All of them are analyzed from the theoretical point of view and validated through numerical simulations with a mathematical UAV model. One is implemented on a microcontroller board, validated through hardware simulations and tested in flight.

The PID with H_∞ robust approach is used for the definition of the gains of a commercial autopilot. The proposed technique combines traditional PID control with an H_∞ loop shaping method to assess the robustness characteristics achievable with simple PID gains. It is demonstrated that this hybrid approach provides a promising solution to the problem of tuning commercial autopilots for UAVs. Nevertheless, it is clear that a tradeoff between robustness and performance is necessary when dealing with this standard control technique.

The robustness problem is effectively solved by the adoption of an $\mathcal{L}_1$ adaptive controller for complete aircraft control. In particular, the $\mathcal{L}_1$ logic here adopted is based on piecewise constant adaptive laws with an adaptation rate compatible with the sampling rate of an autopilot board CPU. The control scheme includes an $\mathcal{L}_1$ adaptive controller for the inner loop, while PID gains take care of the outer loop. The global controller is tuned on a linear decoupled aircraft model. It is demonstrated that the achieved configuration guarantees satisfying performance also when applied to a complete nonlinear model affected by uncertainties and

parametric perturbations.

The third controller implemented is based on an existing nonlinear backstepping technique. A scheme for longitudinal and latero-directional control based on the combination of PID for the outer loop and backstepping for the inner loop is proposed. Satisfying results are achieved also when the nonlinear aircraft model is perturbed by parametric uncertainties. A confrontation among the three controllers shows that $\mathcal{L}_1$ and backstepping are comparable in terms of nominal and robust performance, with an advantage for $\mathcal{L}_1$, while the PID is always inferior.

The backstepping controller is chosen for being implemented and tested on a real fixed-wing RC aircraft. Hardware-in-the-loop simulations validate its real-time control capability on the complete nonlinear model of the aircraft adopted for the tests, inclusive of sensors noise. An innovative microcontroller technology is employed as core of the autopilot system, it interfaces with sensors and servos in order to handle input/output operations and it performs the control law computation. Preliminary ground tests validate the suitability of the autopilot configuration. A limited number of flight tests is performed. Promising results are obtained for the control of longitudinal states, while latero-directional control still needs major improvements.

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NOMENCLATURE

A_{lon}, A_{lat} State matrices

B_{lon}, B_{lat} Input matrices

$C(s)$ Plant controller transfer function

C_L, C_Y Aerodynamic coefficients

$C_{L\alpha}$ Lift aerodynamic derivative

$C_{l\delta_a}$, $C_{l\delta_e}$, $C_{l\delta_r}$ Roll moment control derivatives

$C_{l\beta}$, $C_{l\dot{\beta}}$, C_{lp}, C_{lr} Roll moment aerodynamic derivatives

$C_{m\delta_a}$, $C_{m\delta_e}$, $C_{m\delta_r}$ Pitch moment control derivatives

C_{m0}, $C_{m\alpha}$, $C_{m\dot{\alpha}}$, C_{mq} Pitch moment aerodynamic derivatives

$C_{n\delta_a}$, $C_{n\delta_e}$, $C_{n\delta_r}$ Yaw moment control derivatives

$C_{n\beta}$, $C_{n\dot{\beta}}$, C_{np}, C_{nr} Yaw moment aerodynamic derivatives

$\boldsymbol{F}$ Total force vector [N]

$F_B(X_B, Y_B, Z_B)$ Generic Body axes coordinate system

$F_E(X_E, Y_E, Z_E)$ ECEF axes coordinate system

$F_N(X_N, Y_N, Z_N)$ NED axes coordinate system

$F_W(X_W, Y_W, Z_W)$ Wind axes coordinate system

F_x, F_y, F_z Components of the force vector $\boldsymbol{F}$ along body axes [N]

$G(s)$ Plant transfer function

I Aircraft inertia tensor [kg· m^2]

$K_P,\ K_I,\ K_D$ PID gains

K_{mult} Multiplicative gain for the root locus plot

$Lift$ Lift force [N]

M_P Overshoot

$M_x,\ M_y,\ M_z$ Components of the moment vector $\boldsymbol{M}$ about body axes [N· m]

$\boldsymbol{M}$ Total moment vector [N·m]

$\boldsymbol{M}(s)$ LFT matrix transfer function

T Engine thrust [N]

$\boldsymbol{V}$ Aircraft linear velocity vector [m/s]

X, Y, Z Aerodynamic and propulsion forces along body axes [N]

$X_{aero}, Y_{aero}, Z_{aero}$ Aerodynamic forces along body axes [N]

b Wingspan [m]

c Wing mean aerodynamic chord [m]

$g = 9.81$ m/s^2 Gravity acceleration

$g_2,\ g_3$ Gravity acceleration components [m/s^2]

$k_{\alpha,1},\ k_{\alpha,2},\ k_{\beta,1},\ k_{\beta,2}$ Backstepping controller gains

m Aircraft mass [kg]

$\dot{p},\ \dot{q},\ \dot{r}$ Components of the angular acceleration vector $\dot{\boldsymbol{\omega}}$ [rad/s^2]

$\hat{p},\ \hat{q},\ \hat{r}$ Nondimensional angular velocities about body axes

$p,\ q,\ r$ Components of the angular velocity vector $\boldsymbol{\omega}$ [rad/s]

t_S Settling time [s]

t_r Rise time [s]

$\boldsymbol{u}_{lat}$ Latero-directional plane control vector

$\boldsymbol{u}_{lon}$ Longitudinal plane control vector

$u,\ v,\ w$ Components of the linear velocity vector $\boldsymbol{V}$ along body axes [m/s]

$u_c(t)$ Generic control signal

$u_{ad}(t)$ Adaptive control signal

$\boldsymbol{x}_{lat}$ Latero-directional plane state vector

$\boldsymbol{x}_{lon}$ Longitudinal plane state vector

$\hat{x}(t)$ Estimation of the state signal

$x(t)$ State signal

$\boldsymbol{\Delta}(s)$ Unstructured uncertainties transfer function

$[\Psi],\ [\Theta],\ [\Phi]$ Elementary rotation matrices associated with general Euler angles

$\Psi,\ \Theta,\ \Phi$ General Euler angles [rad]

α Angle of attack [rad]

β Sideslip angle [rad]

γ Ramp angle [rad]

δ_a Aileron command [rad]

δ_e Elevator command [rad]

δ_{th} Throttle command

θ Pitch angle [rad]

μ Structured singular value

ϕ Roll angle [rad]

ψ Yaw or heading angle [rad]

$\dot{\boldsymbol{\omega}}$ Aircraft angular acceleration vector about body axes [rad/s^2]

$\boldsymbol{\omega}$ Aircraft angular velocity vector about body axes [rad/s]

ACKNOWLEDGMENTS

1. PREFACE

The present PhD thesis is realized within the Department of Mechanical and Aerospace Engineering (DIMEAS) of the Politecnico di Torino for the fulfillment of the PhD degree in Aerospace Engineering. The research project was mainly carried out within the Unmanned Aerial Vehicles (UAVs) research group of this department and for a total of nine months at the University of Denver Unmanned Systems Research Institute (DU2SRI).

Both of these universities are active in a wide range of research projects related to the unmanned aircraft technology. The first one, thanks to its aerospace background, focuses on unmanned aircraft design and development with an emphasis on their Guidance, Navigation and Control (GNC) systems. Examples of the activities include the realization of a small fixed-wing aircraft for civil surveillance missions, the design of path planning and guidance algorithms and the implementation of control logic to a self-developed autopilot. The DU2SRI is managed by the Department of Electrical and Computer Engineering and by the Department of Computer Science. Its focal points are robotics, automation, computational intelligence and distributed intelligence multi agent systems. Some of the on-going projects include the realization of a sense-and-avoid RADAR-based sensor system, the design of autonomous take-off and landing logic for rotorcrafts and the implementation of a control and communication protocol for decentralized multi-robot team formation.

The combination of the expertise from these research groups allowed the realization this PhD thesis. The aim of the project is the design and the implementation of three advanced control laws for fixed-wing unmanned aircraft vehicles: PID with H_∞ robust approach, $\mathcal{L}_1$ adaptive controller and nonlinear backstepping controller. The idea is to provide a valid confrontation among three different techniques that might help to guide the selection of a suitable control law for small fixed-wing UAV. Among the controllers, the backstepping method is selected for being implemented on a microcontroller and tested in flight on a real UAV. This is relevant because, as it will be explained in Chapter 2, the large majority of the flying autopilots for fixed-wing UAVs still relies on PIDs. Furthermore, it is innovative because the number of backstepping controllers guaranteeing longitudinal and latero-directional control for fixed-wing aircraft which have actually flown is even more restricted.

The approach in the realization of the PhD project was the following. The three controllers were designed separately and sequentially. For each of them a bibliographic study was initially performed in order to understand the state of the art and to identify the innovative contributions. The adopted controllers were chosen taking into account three parameters: suitability and benefit in the application to UAV autopilots, interest of the scientific community, possibility to introduce an innovation. For two of them, PID and $\mathcal{L}_1$, applicative exercises were initially performed to acquire confidence with the control techniques and to assess their performance, see for instance Appendix A. In the preliminary phase a Sliding Mode Controller (SMC) was also evaluated based on the wing rock motion case [1]. It was then disregarded because of its high-frequency switching nature that might generate the chattering phenomenon, an oscillation in the control action incompatible with the control surfaces adopted.

Once each controller was identified, the following phase consisted in the study and in the definition of its mathematical basis. The theoretical foundation of the control law was generally taken from existing work, but it was elaborated and combined in such a way to guarantee an innovative contribution. The software implementation and the realization of numerical simulations were used to validate the proposed approach. All controllers were tested for the MH850 UAV, see Section 3.6.1. The same aircraft configuration was employed to guarantee easy confrontation: complex maneuvers and robustness to uncertainty in the model parameters were assessed.

As it will be explained in Section 6.3.2, the backstepping controller was selected for the implementation on a real fixed-wing model aircraft. The definition of the backstepping approach together with the first steps towards implementation were carried out at the DU2SRI. The adoption of an innovative microcontroller technology allowed its real time validation through hardware-in-the-loop simulations (HIL), see Section 6.5. A considerable part of the PhD project was dedicated to the integration of the controller on the aircraft and to the interface among all subsystems for the realization of a complete autopilot system, see Chapter 7. Ground tests were initially carried out to verify the functioning of the autopilot system and to verify the correct action of the backstepping controller. Different practical problems were solved in this phase and the control law was adjusted to deal with real implementation. Finally, preliminary flight tests were carried out at the DU2SRI. Unfortunately, some logistic and technical difficulties limited the number of tests to a handful.

The thesis is structured in the following way. Chapter 2 introduces the UAV technology and defines the control problem addressed in this work. The solutions here proposed are illustrated with an emphasis on their contribution to the state of the art. Chapter 3

describes the mathematical linear and nonlinear aircraft models employed for the definition and the testing of the controllers. The vehicles adopted for the simulations and their physical characteristics are also introduced. Chapter 4 and Chapter 5 deal, respectively, with the design and the numerical simulations of the H_∞-based PID controller and of the $\mathcal{L}_1$ adaptive controller. In Chapter 6 the backstepping design is presented together with the results of the main software simulations. Here a comparison among the three controllers is proposed and, following the decision to implement the backstepping controller, hardware simulations are introduced to validate its real time implementability on a microcontroller. Chapter 7 describes the preparation for the flight tests. This include the validation through software and hardware simulations of the control law for the selected RC aircraft model. The physical integration of the controller on the vehicle is presented, the problems encountered and the adopted solutions are illustrated. The results of preliminary ground and flight tests are proposed. Finally, Chapter 8 draws the conclusions and suggests possible improvements to be carried out during future work. Two appendices can be found at the end of the thesis. The first one describes the applicative exercise of $\mathcal{L}_1$ to the wing rock phenomenon, the second one illustrates the connection schemes of the autopilot system.

2. INTRODUCTION

This chapter introduces the main features of the unmanned aircraft technology and describes the state of the art relative to the control of these vehicles. The proposed solutions which will be analyzed in this thesis are illustrated underlining their advantages and drawbacks, emphasizing their contribution to the existing work.

2.1 Overview of UAV technology

UAV stands for Unmanned Aerial Vehicle. This acronym comprehends a broad range of *powered, aerial vehicles that do not carry any human operator, use aerodynamic forces to provide vehicle lift, can fly autonomously or be piloted remotely, can be expendable or recoverable, and can carry a lethal or nonlethal payload. Ballistic or semi-ballistic vehicles, cruise missiles, and artillery projectiles are not considered UAVs* [2].

The interest for this category of vehicles has its origins in the military environment during the Cold War. Within the United States Air Force, the necessity of avoiding risky surveillance missions over the enemy territory became evident during the Korean War and after the 1960 shooting down of Gary Power's U2 above the Soviet Union [3]. This requirement gave birth to the first generation of reconnaissance drones, among them the most significant was the Ryan Model 147 Lightning Bug which was widely employed in the Vietnam conflict [4]. On the other side of the Iron Curtain the Soviet Union carried out unmanned reconnaissance missions with a family of drones from the Tupolev design bureau, the first being the Tu-123 Yastreb introduced into active service in 1964 [5], see Fig. 2.1.

Meanwhile in Europe, British and West German forces started operating the Canadian designed and jointly produced Canadair CL-89, later adopted also by Italian and French armies. In the following years a variety of projects with increasing levels of autonomy and capability were conceived, but the application of unmanned vehicles experienced alternating periods according to the current status of rival technologies, strategic vision, political doctrine and geopolitical framework. The major event which imposed UAVs as important tools for the military field was the 1990 Gulf War where their operational ability in combat situations

Fig. 2.1: Tupolev Tu-143 reconnaissance drone [9].

was proved [6]. The primary UAV employed was the Pioneer, a vehicle developed by the U.S. together with the Israel Aircraft Industries [7]. The end of the Cold War strongly reduced military expenditures for strategic reconnaissance and forced to find inexpensive alternatives [8]. Nowadays major air forces worldwide rely on the UAV technology for a wide range of missions, many of them being for reconnaissance, target acquisition and communication support. It is estimated that more than 50 countries are currently developing more than 1000 Unmanned Aerial Vehicles [5]. Among them much emphasis is now given to small portable systems, easy to deploy and maintain, relatively inexpensive and so expendable.

The development of UAVs for civil applications started only 20 years ago, supported by the know-how achieved in the military field at the cost of large investments. One of the key programs that promoted the use of UAVs for commercial science applications began in 1994 and was managed by NASA with the collaboration of several industries and universities [10]. The ERAST (Environmental Research Aircraft and Sensor Technology) program laid the foundations for the commercial applications of sensor and communication technologies found in modern civil UAVs. Reduced size and weight of sensors, increased computational power of microcontroller boards, reduced cost of electronic components and high-density energy sources are now allowing the civil UAV market to thrive. The players in this field are not only large industries or government organizations: small companies, research centers and universities have the possibility to propose creative and effective solutions for determined applications. A serious limitation to the full growth of the civil UAV market is the national airspace authority regulation [11]. Many restrictions are still imposed to the flight in public airspace, in particular where densely populated [12]. A gradual relaxation of the requirements is in progress thanks to an increase of reliability of the technologies adopted and to a more accurate assessment of the risks involved. Some of the most popular uses include [8][13]:

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- wildfire detection

- pollution monitoring

- event security

- traffic monitoring

- law enforcement

- disaster relief

- search and rescue

- pipeline and transmission line inspection

- meteorology, see Fig. 2.2

- remote aerial mapping

Fig. 2.2: Aerosonde Mark 4.7 UAV for Antartic climate studies [14].

Both military and civilian missions where UAVs are widely employed can be classified as D^3: Dull, Dirty and Dangerous. Dull missions require long flying time and require minimum crew intervention, with physical fatigue and loss of concentration being the major problems. The emphasis on the task accomplishment is more on the payload features and less on the pilot skills. Surveillance and reconnaissance are typical cases. Dirty are the missions where the aircraft has to fly into an environment hazardous for the health of the crew, because

of possible exposure to nuclear, biological or chemical agents. Unmanned aircraft are easier to decontaminate compared to a traditional ones. Some scientific or military missions fall within this category. Finally, Dangerous are the missions where the life of the crew is in put in danger because of hostile environment, adverse weather or risky maneuvers.

The most obvious advantage of employing UAVs relies in the absence of the human body and its weaknesses from the aircraft. The problems related to human factor are eliminated or transferred to the ground, where they can be handled in a more comfortable way. For instance, tired operators can be simply replaced after a normal shift. Another advantage of UAVs is in benefits for the aircraft design. The maximum g-load is imposed by the aircraft structure and not by the crew resistance, allowing the aircraft to perform more aggressive maneuvers. All the systems dedicated to the crew can be removed: the weight and the volume available for the payload increase or, as an alternative, the overall aircraft size decreases. The aircraft shape can be optimized for the mission and this results in better aerodynamics and a more stealthy profile. As the size is reduced, noise and sight impact decrease, as well as emissions. These features are vital in environmental missions where the disturbance to inhabitants or animals must be minimized. Finally, for the same mission profile, buying and operating a UAV is economically more favorable than operating an equivalent manned aircraft. It is estimated that the cost of a UAV and its control station is 40% to 80% the cost of the corresponding manned aircraft [15]. Similarly the operating costs are calculated to be around 40%.

Any typical UAV mission requires the aircraft to be part of a more complex system, generally referred as Unmanned Aircraft System (UAS). This definition includes a variety of subsystems, the most important being the aircraft itself, the Ground Control Station (GCS) and the operators. The operators fulfill the tasks that once belonged to the onboard crew: piloting the aircraft, following air traffic regulation restrictions, tactical navigation, communication and payload operation. An open issue is whether the person responsible for the conduct of the UAV should be a former aircraft pilot or a specifically trained operator [16][17]. The interface between the operators and the aircraft is the Ground Control Station, see Fig. 2.3. Here the large amount of data about the UAV mission are presented in a optimal way to guarantee maximum situation awareness [18][19][20]. These information include the status of the onboard systems, the aircraft position, neighbor air traffic and payload data. At the same time the GCS allows the ground crew to remotely operate the aircraft and its payload in order to accomplish the designated mission.

The advancements in three technological fields played a key role in the UAV success. The first one is the payload technology which is the core of the UAV system. The accomplishment

Fig. 2.3: Example of GCS from Aeronautics Defense Systems [21].

of the mission strongly depends on this subsystem and its technical specifications deeply influence the aircraft configuration. The miniaturization, reduction in cost and increase of performance of electronic components allow the realization of payload systems that can equally, if not better, fulfill the task once performed by the aircraft crew, Fig. 2.4. The second is the communication technology which allows a constant stream of information to flow from and to the aircraft. The relocation of the aircraft crew requires a large amount of data to be broadcasted in real time over large distances. From the UAV these include vital information about the state of the aircraft and its payload; from the RCS these are the commands to the vehicle and to its payload. The communication layer is a feature of unmanned systems which in general requires a level of complexity, and so weight, cost and power, much higher compared to normal aircraft.

Finally, the third and most characteristic element is automation. Generally automation is defined as *the automatically controlled operation of an apparatus, a process, or a system by mechanical or electronic devices that take the place of human organs of observation, decision, and effort* [23]. Different levels of automation exist according to the interaction between the human and the machine. A classification shared between [24] and [25] lists ten levels, from level 1 where the computer offers no assistance and the human takes all decision and actions, to level 10 where the computer decides everything, Table 2.1.

These concepts were born in the 1970s within the robotic research field that was booming in that period. They can be applied nowadays to unmanned systems which can be fully considered as robotic systems. In fact, the definition of UAV implies that some level of

Fig. 2.4: Example of EO/IR payload from Controp [22].

Tab. 2.1: Levels of autonomy

Level	Description
10	The computer decides everything
9	The computer informs the human only if it decides to
8	The computer informs the human only if asked
7	The computer executes automatically, then informs the human
6	The computer allows the human to veto its decision
5	The computer asks for approval
4	The computer suggests one alternative
3	The computer narrows the selection down to few alternatives
2	The computer offers a complete set of decision alternatives
1	The computer offers no assistance, the human takes all decisions

autonomy is achieved. Remotely Piloted Vehicles (RPV) or model aircraft do not belong to this category. The term drone, currently much abused, indicates a category of vehicles which can perform pre-programmed tasks with the autonomy limited to the flight phase. There is no interaction with the human agent and no decisional ability. The first unmanned aircraft systems developed in the 1960s for reconnaissance mission were actually drones. They were launched on a pre-programmed path with a sequence of data to be acquired, these information were recovered only at the end of the flight [15]. On the contrary, current UAVs are more and more conceived with onboard systems that replicate and substitute, on different levels, human capabilities and intelligence. Interaction with the human operator is constant and guaranteed through the communication system. The level of autonomy for each phase of the UAV mission is defined in the design process and it depends on the complexity of the system considered.

The benefits provided by the automation of some processes are several. First of all the workload for the ground crew can be drastically reduced. This increases the focus on the tasks and improves the global performance and the mission probability of success. Furthermore, a reduction in costs can be achieved as a smaller number of man-hours is required. Another benefit appears in case of fault, the system can detect it and act in order to minimize its impact on the mission. In particular in case of weak or disturbed/jammed radio link between aircraft and GCS, an appropriate level of autonomy could be fundamental [26].

The core technology at the basis of automation in the UAV field is the autopilot system. An autopilot is a device able to define and impose the commands that an aircraft has to maintain in order to follow a desired flight condition, determined according to the mission requirements. Autopilots are present on commercial airliners as relief to the crew workload, but on an unmanned aircraft they become an essential part of the aircraft Flight Control System. Guided by an autopilot system, an unmanned aircraft is required to fly between a series of waypoints, pre-determined or updated in real time. This assignment process is called Navigation. The definition of the aircraft flight parameters requested to approach these waypoints is the Guidance. It varies according to the mission considered, the aircraft properties and the payload features. The Control process is responsible for maintaining an aircraft attitude that guarantees the respect of the defined flight conditions.

As it will be discussed in detail in the next section, much effort is put in the definition of an appropriate logic for the aircraft control.

2.2   Control problem and proposed solutions

The interest in the problem of finding suitable control laws for UAVs is growing in response to the recognition that these platforms will soon be performing missions in many civilian applications. The wide range of possible missions is strongly stimulating the development of unmanned aerial vehicles very different in size and configuration. A significant part of the research is dedicated to the design of adequate onboard controllers. A particular interest, especially in civilian applications, is for mini fixed-wing aircraft which are cheap to build and to maintain, easily deployable, crashable and have low kinetic energy in case of impact. According to a classification proposed in [11] the mini-UAV category includes aircraft with a weight between 0.2 kg and 2.4 kg. The drawback of this configuration is in its flight dynamics behavior: highly nonlinear, strongly coupled between the longitudinal and latero-directional planes, very sensible to external disturbances and parametric uncertainties. Furthermore, the reduced dimensions of the fuselage limit the available space and weight for onboard electronics and the dedicated power supply. These issues motivate why small fixed-wing UAVs represent an interesting platform for testing advanced control techniques. The aim is to meet the always more demanding requirements for flight maneuvering and mission accomplishment.

Recent surveys illustrate the current technologies available for autopilot systems and describe the control laws commonly employed, see [27] and [28]. The use of PID gains is still a popular approach in practice, in particular when dealing with commercial off-the-shelf autopilots such as the MicroPilot MP Series [29]. This method guarantees simple implementation and low computational effort. The designer has adequate control over the system response and a clear understanding of the control action. In detail, starting with proportional gain and adding integral and derivative terms, the designers can obtain a zero steady state error and a fast time response for a step input reference. The tuning of the PID gains can be performed with many non-heuristic methods, as explained in [30] and [31]. In the last twenty years, some researchers have focused their attention on the development of automatic tuning and adaptation techniques for the definition of the PID gains. They eliminate graphical, heuristic and trial and error procedures to verify the robustness of the selected parameters.

One drawback of the simple PID approach is the inability to deal with the flight envelope that might be required in most non-trivial mission profiles. As the performance of a PID controller decreases far from the design point, gain scheduling is a common approach to extend the validity of this technique to the whole flight envelope. Another disadvantage of

traditional PIDs is that they do not guarantee enough robustness to the extent of model parametric uncertainties which can occur in small fixed-wing UAVs.

Therefore, researchers are currently developing nonlinear, adaptive and robust control laws able to theoretically guarantee satisfying performance over a large flight envelope also in presence of uncertainties. For instance the authors of [32] propose a nonlinear model predictive control for fixed-wing UAV path tracking, [33] investigates the feasibility of H_2 and H_∞ autopilots for longitudinal UAV control and [34] presents a combined adaptive control law based on shunting method and passification for an UAV autopilot homing guidance system. Nevertheless, the constraints imposed by real-time implementation often make these algorithms unsuitable for the limited computational platforms available for small scale UAVs. As an example, the controller proposed in [32] is successfully implemented in a dedicated onboard computer installed on an experimental fixed-wing UAV and tested with real-time hardware-in-the-loop simulations. The authors, however, underline the need for a compromise between smooth convergence and computational performance in the determination of the receding horizon size.

High computational requirements, complex algorithms and the necessity to smoothly combine high-level intelligent tasks with low-level input/output routines are the main obstacles. The miniaturization and reduction in cost of micro-controllers, together with their increase in performance, see [35] and [36], is now enabling researchers to implement unmanned aircraft driven by self-developed control laws. Whereas several examples have been published for rotorcraft, an excellent survey is [37], there are relatively few for fixed-wing aircraft. The diffusion of frameworks for control law development (e.g., [38]-[41]) has helped to reduce the barriers to successful implementation. Two examples are [42], where a neural network adaptive controller is used for the transition from horizontal flight to hover, and [43], where a nonlinear dynamic inversion approach is used for formation flight.

Within this context three autopilot configurations are here proposed and compared. The first one combines traditional PID control with an H_∞ loop shaping approach to assess the robustness characteristics achievable with the PID technique. As commonly done, the PIDs are applied to the control loops of a linearized and decoupled aircraft model. In contrast to the PID method, two configurations based on advanced control laws are also presented here: the first one relies on the $\mathcal{L}_1$ adaptive controller and the second one on the nonlinear backstepping controller. In these cases, emphasis is given to the design of the control laws and to their real-time implementation capability. Differently from many related studies, the possibility to implement the proposed solutions on microcontroller boards allows to actually exploit their benefits on a fixed-wing UAV. Both of these approaches share the same structure,

the advanced control law takes care of the inner loop variables, while outer loop variables are controlled via PIDs. This choice allows the designer to maintain a clear understanding of the control action, it limits the required computational power and eases the implementation procedure.

The full PID controller is proposed because it still represents a potentially relevant technique in modern autopilots for UAVs, especially when combined with the H_∞ approach. One aim of the present thesis is to emphasizes the implementability of the proposed controllers, and the PID is unbeatable from this point of view. Furthermore, it is interesting to observe its performance when compared to more advanced control laws which involve higher computational cost.

The choice of $\mathcal{L}_1$ is motivated by the need of an adaptive controller able to handle the presence of model uncertainties due to the platform variations which occur during the flight tests, such as the variation in the payload mass and its position. The simple structure, the reduced presence of oscillations in the implementation and a reasonable computation effort make the $\mathcal{L}_1$ adaptive controller an ideal candidate as autopilot control law. Moreover, this adaptive algorithm guarantees bounded inputs and outputs, uniform transient response and steady-state tracking [44].

The backstepping controller is chosen for its ability to deal with nonlinearities. Unlike traditional linear control techniques, such as LQ or feedback linearization, a nonlinear control law applied to nonlinear aircraft dynamics guarantees satisfying performance over the whole flight envelope [45]. With backstepping control design, useful nonlinearities are maintained and additional nonlinear damping terms can be introduced to increase robustness to model errors or to improve transient performance [46]. Furthermore, as backstepping belongs to the Lyapunov family of techniques it has guaranteed convergence of the tracking error and asymptotic stability [47].

2.3   PID with H_∞ related work and contribution

Traditional PID gains design commonly requires a compromise between system robustness and performance. The employed design methods are generally based on Ziegler-Nichols theory [48], H_∞ approach [49][50] and gain-phase margin technique [51]. Recently some researchers have focused their attention on the development of techniques that combine identification and control within loop shaping approach. Some of the advantages are illustrated in [52], where the authors have demonstrated that the application of H_∞ loop shaping methods give a satisfying controller, if one exists, and the obtained closed-loop system is robustly sta-

ble. The idea behind H_∞ loop shaping is to improve robustness with the design of a controller that minimizes the signal transmission from disturbances and measurement noise. Alternatively, convex optimization techniques can be used to tune the controller. These methods are based on frequency loop shaping theory (sensitivity function and its complementary function) and specifications are usually given in the form of a desired loop shaping function. In [53] a method that integrates identification and PID parameters tuning is presented. The system uncertainty is modeled considering a multiplicative uncertainty structure.

Many robust stability and performance problems can be considered in the H_∞ framework and a complete theory for control systems synthesis exists. However, the order of the optimal control is high, comparable to the order of the plant. Some authors have considered the problem of synthesizing PID controllers and they propose a parametric approach based on the generalization of the Hermite-Biehler theorem. When applying this method for a given fixed proportional gain, the set of stabilizing gains is obtained by the intersection of linear inequalities. In [54] a computational characterization of all stabilizing PID controllers for an arbitrary plant is provided. This H_∞ optimal design, usually carried out by force optimization search, is computationally very time-consuming. For this reason the work described in [55] proposes a computationally efficient procedure for H_∞ PID optimal design instead of brute force search. This method reveals some important structural properties of H_∞ PID controllers, however it is validated only for Single Input Single Output (SISO) systems. The authors of [56] have proposed a set of simple closed formulas for the explicit computation of the parameters in finite terms. They eliminate graphical, heuristic and trial and error procedures to verify the robustness of the selected parameters.

The method here illustrated follows an hybrid approach based on H_∞ loop shaping theory where nominal stability is guaranteed using root locus method [57], as proposed in [56]. In this case, root locus is applied to identify the desired value of the PID derivative gain and zeros. These are later employed to define the proportional and integrative gains. Gains are chosen on one hand to obtain adequate robust stability and performance, on the other hand to optimize closed-loop performance in terms of step response and waypoints tracking. Besides, specifications for PIDs design are chosen to guarantee that the weighted H_∞ norm of the interconnection system matrix is less than a specified constant value. The relevance of the proposed method is theoretical, since a robust solution is provided for the PID design with standard optimization method and practical, since the computational effort is limited. As the final purpose of this mixed approach is the tuning of an autopilot for fixed-wing aircraft control, some simulations illustrate the compliance to the defined performance requirements. It is verified that an adequate mission oriented design is a key feature to obtain satisfactory

flying and handling qualities and good closed-loop response with an acceptable stability margin. In order to achieve them full robustness can not be guaranteed.

2.4 $\mathcal{L}_1$ related work and contribution

A variety of adaptive control techniques have been proposed for the derivation of autopilot inner loop control. Researchers in [58] implement a two-loop controller where the inner loop is a dynamic inversion controller with an adaptive neural network and the outer loop is a LQR controller. Similarly, [59] presents the implementation of an adaptive neural network controller for autonomous flight. However, traditional model-based adaptive controllers may not be applicable since they are generally useful on the condition that the system dynamics are linear-in-the-parameters. In [60] the authors illustrate a complex adaptive controller based on neural networks using backstepping technique. The main feature of the work of [60] is that the adaptive controller is designed assuming that all of the nonlinear functions of the system have uncertainties, the neural network weights are adjusted adaptively via Lyapunov theory. Similarly, [61] derives an adaptive backstepping approach for the longitudinal aircraft control and a Lyapunov analysis of the stability properties of the closed loop system was considered.

The $\mathcal{L}_1$ adaptive control methodology addresses some of the problems exhibited by these traditional adaptive control approaches. It provides fast and robust adaptation simultaneously leading the system input and output signals to the desired transient performance, in addition it guarantees steady-state tracking. The decoupling between fast adaptation and robustness is achieved with the introduction of a low-pass filter on the adaptive control signal. This key element can be designed based on robustness and performance specifications. The complete theory behind the $\mathcal{L}_1$ technique is presented in [65] and [44].

This very recent control logic has been the object of the attention of many research projects that are trying to build $\mathcal{L}_1$-based autopilot systems for UAVs. Many of them are carried out by the collaboration of different research groups with the creators of the controller, Naira Hovakimyan and Changyu Cao. In one of the first works, the authors of [62] propose an $\mathcal{L}_1$ adaptive pitch controller for mini UAV and validate the proposed algorithm in experimental flight tests. A drawback of [62] is the implementation and validation of a single loop, a pitch attitude hold. In [63] the $\mathcal{L}_1$ controller is employed for the control of an aircraft dedicated to collect biological samples. The $\mathcal{L}_1$ adaptivity is used to compensate for the altered aircraft dynamics caused by the installation of the sampling instruments. One of the most complete works about $\mathcal{L}_1$ is presented in [64]. Here the inner loop of a commer-

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cial autopilot is equipped with $\mathcal{L}_1$ controller for path following taking into account model uncertainties and environmental disturbances. Extensive flight tests validate the proposed approach.

In relation with the existing work previously discussed, the present project validates an $\mathcal{L}_1$ controller applied to a complete nonlinear UAV aircraft model inclusive of model uncertainties and unmodeled dynamics. The control law is designed and tuned starting from a linear decoupled state space model which was derived about a specific flight condition. The contribution resides in the demonstration that, using a low fidelity linear model significantly different from the nonlinear model, the $\mathcal{L}_1$ controller is robust to model changes and a gain retuning is not required. Moreover, the effectiveness of the tuned controller is demonstrated for different aircraft configurations. In fact, uncertainties related to mass properties together with variations in longitudinal stability derivatives are considered. Another important advantage is that both the outer loop PID gains tuned on the linear case do not require an additional tuning.

2.5 Backstepping related work and contribution

A variety of projects illustrates the application of backstepping to fixed-wing aircraft flight control. Unlike with rotorcraft, the difficulty for fixed-wing aircraft is in arranging the equations of motion into the required cascade form. In the literature there are several examples where backstepping is independently dedicated to longitudinal or latero-directional control. For instance, [66] presents an adaptive backstepping control law for angle of attack tracking, [67] uses adaptive backstepping for UAV velocity and flight path angle control and [68] combines $\mathcal{L}_1$ adaptive methodology with backstepping for longitudinal control of a multi-axis thrust vectoring fighter aircraft. As mentioned before, [61] adopts adaptive backstepping for the longitudinal aircraft control. UAV trajectory tracking with adaptive backstepping is described in [69], where velocity and roll angle are controlled. The path-following problem is addressed in [70], where the roll angle command is generated through backstepping with parameters adaptation, hardware-in-the-loop (HIL) simulations validate the results.

There are few examples of complete aircraft control. In [71], outer loop variables, incidence, sideslip and roll angles, are controlled by adaptive backstepping with neural networks through body-axes angular rates. In [72], constrained adaptive backstepping with neural adaptation laws is employed for tracking angle of attack, wind-axes roll rate and total velocity while sideslip is maintained at zero.

In contrast to existing work, in this thesis an autopilot configuration for longitudinal and

latero-directional fixed-wing UAV control based on the backstepping technique is presented. The objective is double: the adaptation of an existing backstepping controller with the aim of generating an autopilot configuration suitable for mini-UAVs; its real-time implementation on a microcontroller board. The inner loop variables angle of attack, sideslip angle and wind-axes roll rate are controlled via the backstepping approach described in [74]. This method is designed for general aircraft maneuvering within the whole flight envelope. Nonlinear natural-stabilizing aerodynamic loads are included and employed by the controller. This approach is different from feedback linearization, where these forces are first modeled and then canceled. Less dynamic outer loop variables, velocity, altitude and heading, are controlled by PID gains. The main purpose of this work, in fact, is to provide a starting framework for the actual employment of backstepping control on flying UAVs. Adaptation and a more advanced outer loop design is beyond the scope of this thesis.

A constant in all the approaches summarized above is the combination of backstepping controller with complex adaptive laws. The benefits of combining nonlinear control with adaptation are clear, but the problems of real-time implementation are considerable. Aside from [70], none of the implementations described above has been performed on micro-controllers suitable for small UAVs. The algorithm described in [75], based on adaptive backstepping for directional control in presence of crosswind, is currently being implemented. This effort is aided by the limited number of controlled variables and by the simplicity of the adaptation approach. The only application of the backstepping controller on a flying fixed-wing unmanned aircraft is presented in [76]. There, basic roll and pitch angles hold and trimming tasks are achieved through adaptive backstepping implemented on a Procerus Kestrel autopilot. In the present thesis an innovative use of microprocessor technology based on cutting-edge transistor computers is employed to support the controller implementation [77]. The combination of this tool with the proposed control layout strongly facilitates the passage from theoretical simulation to practical application. In fact, the proposed control scheme is validated through HIL simulations, real-time operation is demonstrated with satisfying flight performances

3. FIXED-WING AIRCRAFT MATHEMATICAL MODEL

The model of the aircraft is defined by three sets of differential equations governing the aircraft dynamics. They describe the forces and moments acting on the aircraft, and its orientation state with respect to a reference frame [78]. In this chapter the full nonlinear model is presented together with the simplified linear model valid about a steady flight condition. Both of them will be employed in this project. Some tools necessary to fully understand the equations of motions are introduced first, these include the main reference frames and the Euler angles. Finally, the aircraft adopted for the simulations and for the flight tests are introduced, in particular the MH850 UAV which represents the testbed for all the controllers proposed in this thesis.

3.1 Reference frames

A reference frame is a set of axes employed as coordinate system in order to represent the position and the orientation of a dynamical system, in this case the aircraft. A large variety of reference frames exist in the aeronautic field, an introduction to the ones used in this work is necessary for sake of clarity.

3.1.1 Generic body axes

Generic body axes have the origin in the aircraft center of gravity. They are defined as follows: X_B and Z_B lie in the aircraft plane of symmetry, with X_B generally parallel to the fuselage reference line and directed towards the aircraft nose, Z_B is directed from the upper to the lower surface of the wing airfoil; the Y_B axis is selected so that the coordinate frame is right-handed, see Fig. 3.1. Generic body axes are fixed with respect to the aircraft, the moments of inertia calculated in this reference frame do not change during the motion.

3.1.2 Wind axes

The wind axes reference system has the origin in the aircraft center of gravity and it is defined as follows: the longitudinal axis X_W is aligned with the direction of the airspeed vector $\mathbf{V}$
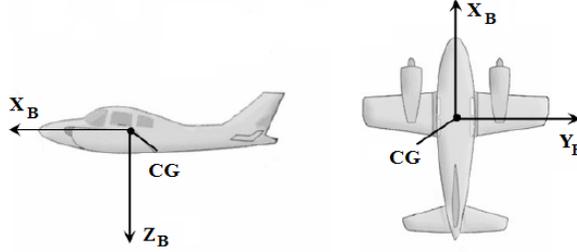
Fig. 3.1: Generic body axes

in absence of wind, the lateral axis Y_W is orthogonal to X_W and oriented from left to right with respect to the center of mass trajectory, Z_W completes the right handed frame by lying in the plane of symmetry of the aircraft, directed from the upper to the lower wing airfoil surface, see Fig. 3.2. The direction of wind axes changes with respect to the aircraft as, while maneuvering, the angle of attack and the sideslip angle change.
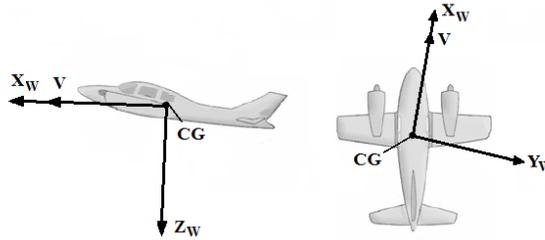
Fig. 3.2: Wind axes

3.1.3 NED axes

North-East-Down (NED) axes are centered in the aircraft center of gravity. The vertical axis Z_N is directed along the local gravity acceleration vector, X_N points towards north, Y_N points towards east. The X_N and Y_N axes belong to a plane parallel to another plane tangent to the Earth surface at zero altitude. Fig. 3.3 shows NED axes orientation with respect to an Earth Centered Earth Fixed (ECEF) reference frame (X_E, Y_E, Z_E).
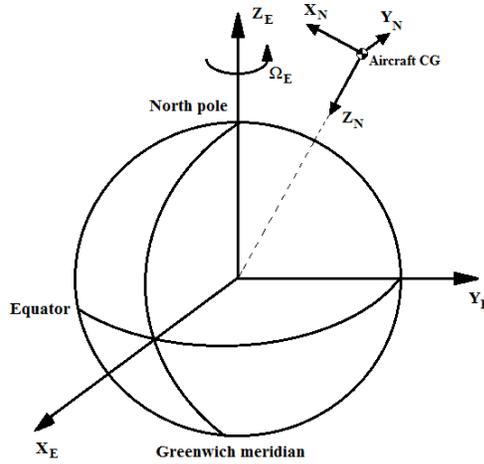
Fig. 3.3: NED and ECEF axes orientation

3.1.4   ECEF axes

The origin of the Earth Centered Earth Fixed (ECEF) system is the earth center of mass. The Z_E axis points towards the North Pole. The direction of X_E is determined by the intersection of the plane defined by the Greenwich meridian and the equatorial plane. The axis Y_E completes the right handed reference frame, it lies in the equatorial plane and points 90 degrees east of the X_E direction, see Fig. 3.3. The ECEF reference system rotates together with the earth about the Z_E axis with an angular speed $\Omega_E = 7.272 \cdot 10^{-5}$ rad/s .

3.2   Euler angles

The Euler angles are a tool used to define the orientation of a reference frame with respect to another one. The Euler angles Ψ, Θ, Φ are the parameters representing three independent angular rotations necessary to align two reference frames. For instance, taking into consideration the coordinate systems $F_1(X_1, Y_1, Z_1)$ and $F_2(X_2, Y_2, Z_2)$ represented in Fig. 3.4, the sequence Ψ, Θ, Φ aligns F_2 to F_1. The rotations are performed in sequence and the order of the rotations is fixed.

Euler angles are also employed to define the transformation of the components of a generic vector between two reference frames. The components of a vector which undergoes a single rotation are calculated using the elementary rotation matrix. In fact, assuming $[R_{X_1},$ $R_{Y_1},\ R_{Z_1}]^T$ to be a generic vector of the F_1 coordinate system, the relationship with the
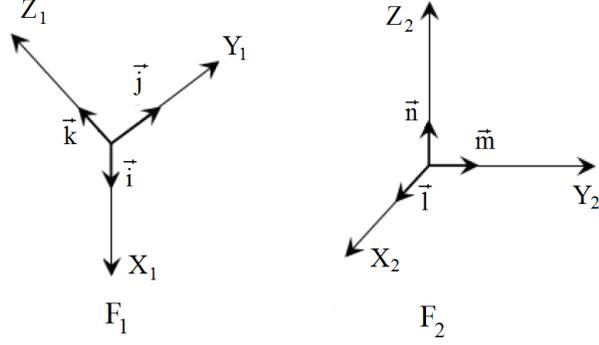
Fig. 3.4: Two generic reference frames

corresponding vector $[R_{X_2}, R_{Y_2}, R_{Z_2}]^T$ in the final coordinate system F_2 is

$$
\begin{bmatrix} R_{X_2} \\ R_{Y_2} \\ R_{Z_2} \end{bmatrix} = [\Psi][\Theta][\Phi] \begin{bmatrix} R_{X_1} \\ R_{Y_1} \\ R_{Z_1} \end{bmatrix} = [T_{21}] \begin{bmatrix} R_{X_1} \\ R_{Y_1} \\ R_{Z_1} \end{bmatrix}
$$

where $[T_{21}]$ is the complete transformation matrix and $[\Psi]$, $[\Theta]$, $[\Phi]$ are the elementary rotation matrices:

$$
[\Psi] = \begin{bmatrix} \cos\Psi & -\sin\Psi & 0 \\ \sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad [\Theta] = \begin{bmatrix} \cos\Theta & 0 & \sin\Theta \\ 0 & 1 & 0 \\ -\sin\Theta & 0 & \cos\Theta \end{bmatrix}, \quad [\Phi] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \sin\Phi & \cos\Phi \end{bmatrix}
$$

As the elementary matrices $[\Psi]$, $[\Theta]$, $[\Phi]$ are orthogonal, also $[T_{21}]$ is orthogonal. Thus $[T_{21}]^{-1} = [T_{12}] = [T_{21}]^T$, which yields to define the inverse transformation as

$$
\begin{bmatrix} R_{X_1} \\ R_{Y_1} \\ R_{Z_1} \end{bmatrix} = [T_{21}]^T \begin{bmatrix} R_{X_2} \\ R_{Y_2} \\ R_{Z_2} \end{bmatrix}
$$

3.3 Notable Euler angles

Two of the most important Euler angles that will be used in the present work are here illustrated. They represent the rotations necessary to align some of the frames illustrated in Section 3.1.

3.3.1 Body axes - Wind axes

In order to align the wind axes reference frame F_W to generic body axes reference frame F_B two rotations are necessary. The first rotation is of magnitude $-\beta$ about Z_W and the second one about Y_B equal to α. The corresponding Euler angles are

$$\Psi = -\beta \quad \Theta = \alpha$$

where α is called angle of attack and β is the sideslip angle, see Fig. 3.5. The analytical definition of the aerodynamic angles α and β is

$$\alpha = \arctan \frac{w}{u}$$
$$\beta = \arcsin \frac{v}{V} \tag{3.1}$$

with

$$V = |\boldsymbol{V}| = \sqrt{u^2 + v^2 + w^2}$$

where u, v, w are the components of the velocity vector $\mathbf{V}$ along the X_B, Y_B and Z_B axes respectively.

The two rotations allow the definition of the angular velocities in wind axes $\boldsymbol{\omega}_w = (p_w, q_w, r_w)^T$ as

$$\boldsymbol{\omega}_w = R_{wb}\boldsymbol{\omega} \tag{3.2}$$

where R_{wb} is the rotation matrix

$$R_{wb} = \begin{bmatrix} \cos\alpha\cos\beta & \sin\beta & \sin\alpha\cos\beta \\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$
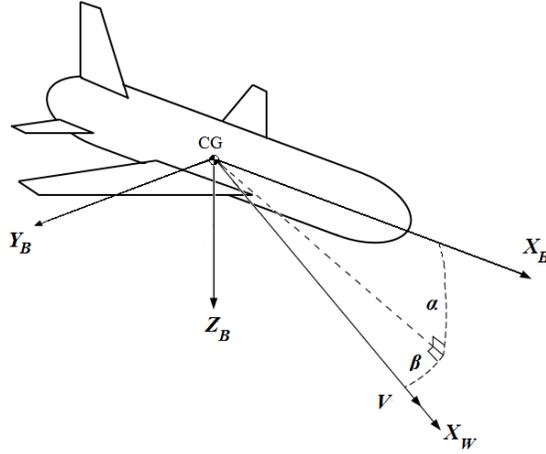
Fig. 3.5: Euler angles for body axes - wind axes rotation

and $\boldsymbol{\omega} = (p, q, r)^T$ is the vector containing the angular velocity components about the body axes.

3.3.2 Body axes - NED axes

The rotations necessary to align NED axes F_N reference frame to a generic body axes reference frame F_B are three

$$\Psi = \psi \quad \Theta = \theta \quad \Phi = \phi$$

An illustration of the rotation procedure is represented in Fig. 3.6. These angles have a
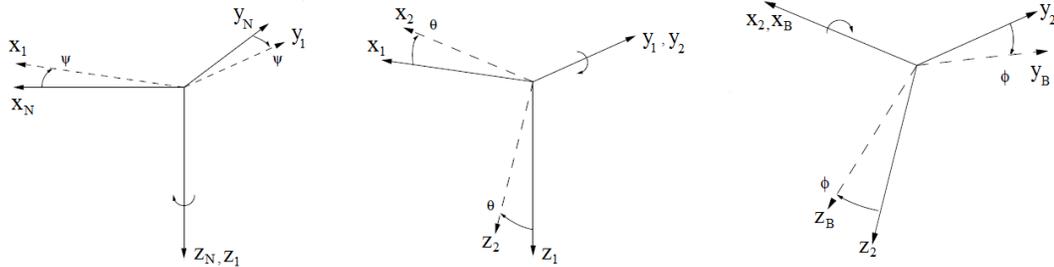
Fig. 3.6: Sequence of rotations to align F_N to F_B

physical meaning which is very relevant in the study of the aircraft navigation:

- ψ is the yaw or heading angle, it represents the angle between the nose of the aircraft and the North Pole. By neglecting the effects of magnetic declination and magnetic deviation this can be assumed as the angle indicated by the aircraft compass;

- θ is the pitch angle, it represents the angle between the nose of the aircraft and the horizontal plane;

- ϕ is the roll angle, it represents the lateral inclination of the aircraft with respect to the horizontal plane.

3.4   Nonlinear mathematical model

The nonlinear aircraft model is built starting from the dynamic and kinematic equations of motion of the aircraft. There are three vectorial equations: the forces equation, the moments equation and the attitude equation.

3.4.1   Forces equation

The forces equation is based on Newton's second law of motion and it describes the evolution in time of the aircraft velocity. It is built by describing the equilibrium among all forces acting on the aircraft along the axes of the generic body coordinate system F_B. In scalar form the equations are

$$
\begin{cases}
F_x = m \left(\dot{u} + qw - rv \right) \\
F_y = m \left(\dot{v} + ru - pw \right) \\
F_z = m \left(\dot{w} + pv - qu \right)
\end{cases}
\tag{3.3}
$$

where m is the aircraft mass and $\boldsymbol{\omega} = (p, q, r)^T$ is the vector of the angular velocity components about the body axes. The force vector $\boldsymbol{F} = (F_x, F_y, F_z)^T$ contains the sum of all forces acting on the aircraft center of mass along the X_B, Y_B and Z_B axes, including aerodynamics forces, gravity and engine thrust. It is possible to group the three equilibriums of forces in a single equation through the vectorial symbology

$$
m\dot{\boldsymbol{V}} = \boldsymbol{F} - \boldsymbol{\omega} \times m\boldsymbol{V}
\tag{3.4}
$$

3.4.2 Moments equation

The equilibrium of the moments acting about the three generic body axes allows writing in scalar form the three moments equations

$$
\begin{cases}
L = \dot{p}I_{xx} - \dot{r}I_{xz} - pqI_{xz} + qr(I_{zz} - I_{yy}) \\
M = \dot{q}I_{yy} + pr(I_{xx} - I_{zz}) + (p^2 - r^2)I_{xz} \\
N = \dot{r}I_{zz} - \dot{p}Ixz + pq(I_{yy} - I_{xx}) + qrI_{xz}
\end{cases}
\tag{3.5}
$$

The three total moments L, M and N act respectively about the X_B, Y_B and Z_B axes, and can be grouped in the vector $\boldsymbol{M} = (L, M, N)^T$. The generic I_{ij}, with $i, j = \{x, y, z\}$, represents an element of the aircraft inertia tensor

$$
I = \begin{bmatrix}
I_{xx} & 0 & -I_{xz} \\
0 & I_{yy} & 0 \\
-I_{xz} & 0 & I_{zz}
\end{bmatrix}
$$

The tensor is symmetric and has some null elements thanks to the aircraft symmetry in the longitudinal plane. In vectorial form the final equation can be written as

$$
I\dot{\boldsymbol{\omega}} = \boldsymbol{M} - \boldsymbol{\omega} \times I\boldsymbol{\omega}
\tag{3.6}
$$

where $\dot{\boldsymbol{\omega}} = (\dot{p}, \dot{q}, \dot{r})^T$ is the vector containing the derivatives of the angular velocity components about body axes.

3.4.3 Attitude equation

The attitude equation describes the behavior of the aircraft attitude with time and it is based on the kinematic equations of motion. The equation is obtained by expressing the derivatives in time of the Euler angles. In this case we are considering the Euler angles between body and NED axes, their dynamics are expressed by

$$
\begin{cases}
\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\
\dot{\theta} = q\cos\phi - r\sin\phi \\
\dot{\psi} = q\sin\phi/\cos\theta + r\cos\phi/\cos\theta
\end{cases}
\tag{3.7}
$$

By defining the vector $\boldsymbol{\Phi} = (\phi, \theta, \psi)^T$ and calling $[\mathrm{E}(\boldsymbol{\Phi})]$ the rotation matrix

$$[E(\boldsymbol{\Phi})] = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix}$$

it is possible to write the attitude equation as

$$\dot{\boldsymbol{\Phi}} = [E(\boldsymbol{\Phi})]\boldsymbol{\omega} \tag{3.8}$$

Note that the matrix $[\mathrm{E}(\boldsymbol{\Phi})]$ does not depend on the angle ψ and it is singular when $\theta = \pm$ 90 degrees.

It is also useful to write down the equations that define the angular velocities about body axes according to the Euler angles and their derivatives

$$\begin{cases} p = \dot{\phi} - \dot{\psi}\sin\theta \\ q = \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi \\ r = \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi \end{cases} \tag{3.9}$$

3.5 Linear mathematical model

The nonlinear equations of motion have a general validity within the flight envelope, with the mentioned singularity for $\theta = \pm$ 90 degrees. They provide a powerful tool for studying the performance and the dynamic behavior of the aircraft. In some occasions the accuracy provided by the nonlinear model might not be necessary, and a lighter mathematical model would be preferable. Linearizing the equations of motion about a steady flight condition results in two low order linear systems more comfortable to handle and with good accuracy in the surroundings of the defined point.

The approach employed to obtain the linear model is based on the small-disturbance theory [79]. It is assumed that the motion of the aircraft is composed of two contributions, the steady condition plus a small perturbation. All variables appearing in the nonlinear model of Equations (3.3), (3.5), (3.7) and (3.9) are written in such form. With some mathematical manipulation, based on the assumptions that flight conditions are symmetric, propulsive forces are constant, and that perturbations are small, it is possible to obtain the reduced equations which describe the linear disturbance dynamics.

The resulting variations of forces and moments can be expressed as Taylor expansions which depend only on the variation of selected states or controls, relevant from the physical point of view. The approach is based on the assumption that aerodynamic forces and moments can be expressed as linear functions of the instantaneous values of the perturbation variables. It is important to note that the choice of physically relevant states or controls allows the decoupling of the longitudinal and latero-directional motions. In the longitudinal F_x, F_z and M equations the considered longitudinal states are u, w, θ and q and the controls are elevator deflection δ_e and throttle δ_{th}. In the latero-directional F_y, L and N equations the latero-directional states are v, ϕ, ψ, p and r and the controls are aileron deflection δ_a and rudder deflection δ_r. The two motions can be studied separately, the command inputs in one plane will only affect the states in that plane.

Combining the linearized forces and moments equations with the linearized kinematic equations, it is possible, for each plane, to build a state space representation

$$\dot{\boldsymbol{x}}_{lon} = A_{lon}\boldsymbol{x}_{lon} + B_{lon}\boldsymbol{u}_{lon} \tag{3.10a}$$

$$\dot{\boldsymbol{x}}_{lat} = A_{lat}\boldsymbol{x}_{lat} + B_{lat}\boldsymbol{u}_{lat} \tag{3.10b}$$

where $A_{lon} \in \mathbb{R}^{4\times4}$ and $A_{lat} \in \mathbb{R}^{5\times5}$ are the state matrices, $B_{lon} \in \mathbb{R}^{4\times2}$ and $B_{lat} \in \mathbb{R}^{5\times2}$ are the input matrices. The state vectors representing the perturbations are

$$\boldsymbol{x}_{lon} = \begin{bmatrix} u \\ w \\ \theta \\ q \end{bmatrix} \qquad \boldsymbol{x}_{lat} = \begin{bmatrix} v \\ \phi \\ \psi \\ p \\ r \end{bmatrix}$$

while the control vectors are

$$\boldsymbol{u}_{lon} = \begin{bmatrix} \delta_e \\ \delta_{th} \end{bmatrix} \qquad \boldsymbol{u}_{lat} = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$

Note that in the longitudinal plane it can be physically meaningful to have as state the angle of attack α. This is achieved by substituting the vertical velocity w dynamics thanks to the relationship

$$\dot{\alpha} = \frac{\dot{w}}{V_0}$$

where V_0 is the total linear velocity at steady state. Furthermore, in control applications it might be necessary to regulate the value of the altitude h. The altitude dynamics can simply be modeled as

$$\dot{h} = V_0 \sin \gamma = V_0 \sin (\theta - \alpha)$$

where $\gamma = \theta - \alpha$ is the ramp angle, the angle between the total velocity vector and the horizon. By applying the small-disturbance theory the sine of an angle can be identified with the angle itself, so that

$$\dot{h} = V_0 \theta - V_0 \alpha$$

In this cases the longitudinal state vector becomes

$$\boldsymbol{x}_{lon} = \begin{bmatrix} u \\ \alpha \\ \theta \\ q \\ h \end{bmatrix} \tag{3.11}$$

and the new matrices of the state space representation will have a larger size, $A_{lon} \in \mathbb{R}^{5\times5}$ and $B_{lon} \in \mathbb{R}^{5\times2}$. It is interesting to observe that the addition of the altitude equation adds a null eigenvalue to the state matrix A_{lon}. In fact, altitude motion does not contribute to the aircraft dynamic behavior.

Similarly, in the latero-directional plane it can be useful to express the sideslip angle β. This is done by replacing the v dynamics with the β dynamics thanks to

$$\dot{\beta} = \frac{\dot{v}}{V_0}$$

As result the latero-directional state vector becomes

$$\boldsymbol{x}_{lat} = \begin{bmatrix} \beta \\ \phi \\ \psi \\ p \\ r \end{bmatrix} \tag{3.12}$$

and the size of the new state space matrices remains unchanged.

3.6 Aircraft models

The possibility to carry out the present project within two different research groups and the necessity to employ several simulation tools forced the adoption of three different aircraft models: the MH850 UAV, the Cessna 172P aircraft and the Ultrastick 25e RC aircraft.

The Politecnico di Torino UAV research group employs the MH850 UAV. Its mathematical model is used as testbed for the simulated implementation of all the proposed control laws. The performance confrontation among the three controllers is carried out with this aircraft model.

The necessity to perform an initial assessment of the backstepping controller ability to work in real time when implemented on a microcontroller board requires the adoption of the Cessna 172P aircraft nonlinear model. This model is present in FlightGear flight simulator and it is employed for the hardware-in-the-loop simulations tests.

Finally, the University of Denver Unmanned Systems Research Institute uses the Ultrastick 25e RC aircraft. The final aim of the activity in Denver is the flight testing of the backstepping controller with the Ultrastick 25e. An accurate nonlinear model of this aircraft is initially employed to validate the controller in real-life conditions and to tune the controller gains accordingly. The controller is implemented on a microcontroller board and integrated with the required sensors on the aircraft. Preliminary ground and flight tests with the Ultrastick 25e are carried out.

The adoption of many controllers, aircraft models and simulation tools may generate some confusion. Therefore Table 3.1 is added for sake of clarity. In any case, the possibility to successfully test the backstepping controller for many different platforms and flight conditions is a demonstration of its universal ability to control fixed-wing UAVs.

Tab. 3.1: Employed controllers and aircraft models

Aircraft	H_∞	$\mathcal{L}_1$	Backstepping
MH850 linear model	X	X	-
MH850 nonlinear model	X	X	X
C172P nonlinear model	-	-	X
Ultrastick 25e nonlinear model	-	-	X
Ultrastick 25e aircraft	-	-	X

3.6.1 MH850 UAV model

The MH850 aircraft is a mini-UAV developed by MAVTech s.r.l. spin-off company and by the Department of Mechanical and Aerospace Engineering (DIMEAS) of Politecnico di Torino. The MH850 is a man-portable high performance UAV designed with a focus on light weight and long endurance characteristics and with a special attention to stability and handling qualities. The primary mission of the UAV is surveillance of protected areas, monitoring of geological events and support to rescue missions.

The MH850, see Fig. 3.7, is characterized by tailless configuration, electric propulsion and non-movable vertical fins at wingtips [80] - [82]. The wingspan is 85 cm, the approximate mass 1 kg and the cruise speed 14 m/s. Aircraft aerodynamic control is achieved with elevons: they control longitudinal motion when symmetrically deflected and latero-directional motion when asymmetrically deflected. The aircraft structure is designed to be rugged and crashable, the fuselage is realized in sintered nylon, the wings are made of Expanded PolyPropylene (EPP) foam and the fins at wingtips in carbon fiber. Take-off is performed by hand launch or through a catapult, when the mission is over the aircraft lands on its belly. The main data for the MH850 UAV are reported in Table 3.2. An analysis of the decoupled linear models of Equations (3.10a) and (3.10b) allows calculating the M850 dynamical modes, see Table 3.3. At the considered airspeed (14 m/s) just the spiral mode is slightly unstable having a real positive pole equal to 0.0061.

In order to build the aircraft mathematical models previously described it is necessary to estimate the aerodynamic and control derivatives of the vehicle. This is performed thanks to a software tool available at the DIMEAS and called Aircraft Configuration Interface (ACI), see Fig. 3.8. ACI is a Java-based interface that handles input/output operations for the main

Fig. 3.7: MH850 UAV

Tab. 3.2: MH850 UAV characteristics

Parameter	Value
Length	0.45 m
Wingspan	0.85 m
Wing surface	0.25 m^2
Max. weight	0.98 kg
Payload	0.1 kg
Max. power	160 W
Cruise speed	14 m/s
Endurance	50 min

Tab. 3.3: MH850 UAV modes

	Natural Frequency	Damping	Period	Halving/Doubling* time
Longitudinal				
Short period	17.06 rad/s	0.48	0.42 s	0.09 s
Phugoid	0.89 rad/s	0.08	7.02 s	10.28 s
Latero-directional				
Dutch roll	6.15 rad/s	0.13	1.03 s	0.89 s
Roll	-	1	-	0.05 s
Spiral*	-	1	-	69.24 s

Fortran elaboration package. The Fortran application performs the aerodynamic analysis of the lifting surfaces according to the extended lifting line theory, 2D aerodynamics of airfoils is provided as input in tabular format. Fuselage aerodynamics is achieved by superposition of potential flow, friction drag and cross-flow effects. The overall aerodynamic configuration is obtained by adding all the separate contributions. The aerodynamic coupling of the fuselage and the empennages with the wing is also considered. Aircraft control derivatives are computed with the lifting line theory, stability and damping derivatives are also evaluated. Among other properties, the Fortran application is also able to estimate neutral and maneuvering points, elevator and stick force gradients, hinge moments, mass distribution and moments of inertia.
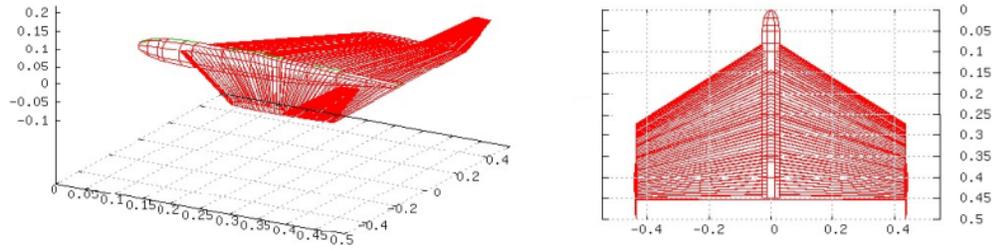
Fig. 3.8: MH850 UAV model in ACI software tool

The accessory software package Endran, which includes a complete motor model, realizes a performance analysis for the obtained aircraft model. The analysis includes h-V diagram,

level flight range-endurance limitations, climbing and turning flight. Flight tests have validated the estimated performance with a good degree of accuracy. The aircraft model obtained with ACI is therefore thought to be reliable enough to guarantee a satisfying understanding of the aircraft dynamical characteristics. This allows to perform with confidence an initial assessment of the controlled motion and to have a preliminary tuning the controller gains before the flight.

3.6.2   MH850 UAV actuators model

Small UAVs require compact, lightweight and responsive actuators. The actuators for these applications evolved consistently during last years, increasing their performances and becoming more suitable to UAV applications. Servos are small boxes that contain a DC electric motor, gears with an output shaft, a position-sensing mechanism, and a control circuitry. Most servos require a power supply between 4.8 V and 6.0 V. The higher the voltage, the faster the servo will move and the more torque it will have. A standard RC radio receiver sends Pulse Width Modulation (PWM) signals to the servo. The electronics inside the servo translate the width of the pulse into a position. When the servo is commanded to rotate, the motor is powered until the potentiometer reaches the value corresponding to the commanded position. The length of the pulse indicates the position to take. Nominally, when the pulse width is 0.6-2.4 ms the servo angular position is ∓45 deg. A pulse width of 1.5 ms sets the servo to central position. Increasing the pulse width by 10 μs results in about a degree of movement on the output shaft. The servo expects a pulse every 20 ms in order to gain correct information about the angle.

An analog sub-micro servo produced by GWS, see Fig. 3.9 is studied to identify typical properties of traditional servos employed in small UAVs applications. The analog servo model GWS IQ-100 at 6 V produces a torque of 0.084 Nm and has a time response to 60 degrees equal to 0.09 seconds. Its weight is only 5.5 grams. Static and dynamical performances are analyzed, and, in particular, an estimate of the system transfer function is performed. This is later introduced in the MH850 aircraft model that will be used as test for the controllers proposed in this project. The data for this analysis are based on a previous work where wind tunnel experiments tested the behavior of various servo configurations at different airspeed and frequency inputs. More details about the experimental setup are available in [83].

The servo is installed on a wing and tested at four different airspeeds: 0, 2.5, 5 and 7.5 m/s. Large frequency sweeps ranging from 0.1 to 4 Hz over a time interval up to 150 s are sent as input, they represent approximately 50% of the full stick range. The decomposition of a video sequence recorded at 25 Hz, sampling time interval 0.04 s, allows storing a time

Fig. 3.9: GWS IQ-100 analog servo.

domain input and output series of points. Fig. 3.10 illustrates a time series for 7.5 m/s where input and output data are normalized and the average value is subtracted. In order to reduce the noise influence on the model, a first order Butterworth lowpass filter is applied to the data with a cut off frequency equal to half the Nyquist frequency. The study is performed at $V = 7.5$ m/s as this speed is closer to the flight conditions encountered by the MH850 UAV.
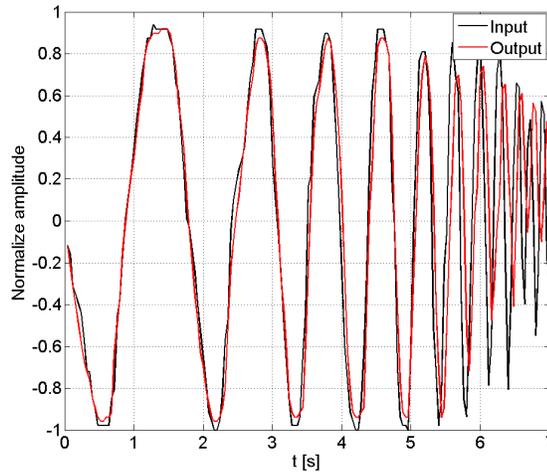
Fig. 3.10: Time domain normalized input and output series for $V = 7.5$ m/s.

The experimental time series is elaborated with Matlab to estimate the system transfer function. The selected approach relies on the Prediction-Error Minimization Method (PEM). This algorithm estimates a discrete-time state space model using the subspace method, then it refines it by minimizing the prediction error generated from an optimally determined predictor [84]. In this case the error is numerically minimized through the scalar cost function

$$V_N\left(G(z), H(z)\right) = \sum_{t=1}^{N} err^2(t)$$

where $err(t)$ is the vector containing the error calculated for each of the N time steps. The higher the value of N, the more accurate the prediction is. For a linear SISO model the error is proportional to the difference between the measured output $y(t)$ and the predicted output $G(z)u_c(t)$

$$err(t) = H^{-1}(z)\left(y(t) - G(z)u_c(t)\right)$$

Note that z is the discrete variable, $G(z)$ and $H(z)$ are the transfer functions of the estimator and $u_c(t)$ is the input. A state space model which fits the experimental data is estimated and the related continuous time transfer function $S(s)$ is easily computable as

$$S(s) = \frac{9.311s + 8.241}{s^2 + 21.99s + 53.97}$$

Fig. 3.11 shows the Bode plot of the servo transfer function.
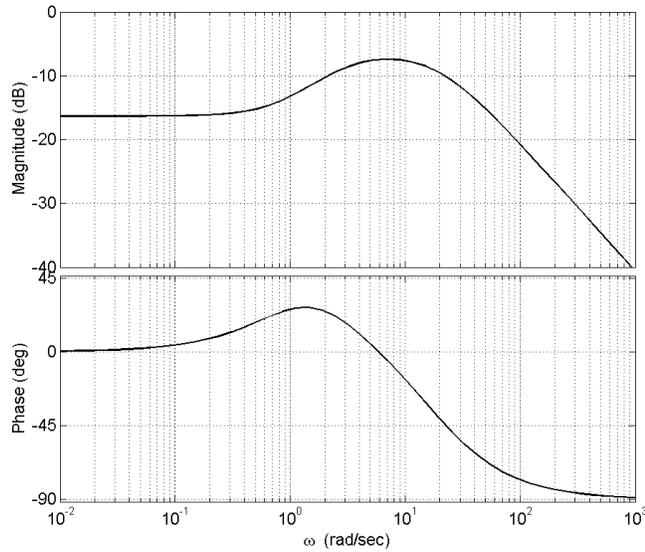
Fig. 3.11: Bode plot for the servo transfer function

3.6.3 C172P aircraft model

The Cessna 172P, see Fig. 3.12, is a single combustion engine aircraft with standard config-
uration including high-wing and fixed tricycle landing gear. Technical data are summarized
in Table 3.4. The aircraft is powered by a Lycoming O-320-D2J engine able to produce 160
hp and to guarantee a cruise speed of 55 m/s. The control surfaces include aileron, elevator
and rudder. The choice of this vehicle is motivated by two reasons: i) it is a popular aircraft
with much technical data available; and ii) the aircraft is available in FlightGear simulator
which is employed for preliminary hardware-in-the-loop simulations.

Fig. 3.12: Cessna 172P aircraft

Tab. 3.4: C172P aircraft characteristics

Parameter	Value
Length	8.3 m
Wingspan	10.9 m
Wing surface	16.1 m^2
Max. weight	1100 kg
Max. power	117.7 kW
Cruise speed	55 m/s

FlightGear is a freeware open-source flight simulator [85]. Developed by volunteers around

the world, it offers to academic developers an experienced tool to test their aircraft models and control laws, see for instance [86] - [88]. FlightGear version 2.6.0 is used and the JSBSim flight dynamics library is employed for the C172P. JSBSim is an open source flight dynamics model defining the six-degree of freedom equations which characterize the aircraft motion [89].

3.6.4   Ultrastick 25e aircraft model

The Ultrastick 25e is the aircraft chosen for testing the backstepping controller into flight, see Fig. 3.13. The main obstacle in testing a controller is the difference between the theoretical aircraft model and the real-life model. Unmodeled dynamics, high order nonlinear terms, parametric uncertainties and external disturbances can introduce unexpected behaviors which can affect the controller performance. The backstepping controller ability to deal with nonlinearities proves to be an advantage in these situations, but having a reliable and accurate mathematical model of the aircraft is still a necessary requirement. In fact, the possibility to perform an accurate tuning of the controller gains in simulation allows to foresee the controlled behavior of the aircraft. This considerably reduces the risk and time of the flight tests.

Fig. 3.13: Ultrastick 25e aircraft

A mathematical model based just on theoretical assumptions, even if very detailed, can hardly reach the level of accuracy required. A model based on experimental tests is preferred. The work described in [90] and [91] deals with this issue, the authors describe the procedure to experimentally identify the model of the Ultrastick 25e aircraft. The aircraft has traditional configuration, electrical propulsion and control is achieved through elevator, ailerons and

rudder. Its characteristics are summarized in Table 3.5.

Tab. 3.5: Ultrastick 25e aircraft characteristics

Parameter	Value
Length	1.05 m
Wingspan	1.27 m
Wing surface	1.27 m^2
Max. weight	1.8 kg
Payload	0.3 kg
Max. power	500 W
Cruise speed	17 m/s

The procedure consists in a preliminary modeling based on wind tunnel tests, the obtained baseline model is used to define flight tests where frequency domain system identification is used to build a definitive model. The result of this work is accessible at [92]. Among other tools offered by this website, one of the most relevant is a Simulink model of the aircraft. This is available in linear and nonlinear form, it embodies an accurate model of the aircraft dynamics, including trim conditions, actuators transfer functions and electric motor behavior. Furthermore, sensors noise from experimental data is added to the aircraft state variables, an optional wind/gust simulation tool is available.

Considering the completeness and accuracy of the work illustrated and taking into account other elements such as aircraft size, performance and availability on the market, the Ultrastick 25e is chosen as the aircraft model for testing the backstepping controller.

Finally, it is interesting to observe that the characteristics of the employed aircraft models differ considerably not only in terms of configuration, absolute dimension, weight and power. Relative values of the C172P, such as power-to-weight ratio and wing loading, are poorer than those of the large majority of small UAVs, see Table 3.6. Testing the controller on a less performing platform helps to prove its universality and to identify its limits.

Tab. 3.6: MH850, C172P and Ultrastick 25e aircraft specific properties for standard flight conditions

Aircraft	Wing loading [kg/m^2]	Power-to-weight ratio [W/kg]
MH850	3.6	178
C172P	54.2	134
Ultrastick 25e	5.2	308

4. H_∞ ROBUST APPROACH TO PID DESIGN

As illustrated in Section 2 this chapter deals with the design of a PID controller based on a robust H_∞ approach. The aim is to apply a novel robust technique to a traditional method still popular in the design of autopilots for UAVs. After a brief introduction to PID and H_∞ principles, the mathematical formulation of the problem is presented. A series of simulations and a practical application case validate the proposed approach, finally some conclusions are drawn based on the obtained results.

4.1 Introduction to PID technique

The PID technique was developed in the first decades of the 20th century for automatic ship steering. The first theoretical formulation from Minorsky [93] was published in 1922 and applied with successful results to U.S. Navy ships. The early mechanical implementation soon led to electronic analog controllers that determined the success of PIDs in industrial applications. Their benefits, as illustrated in Section 2, made this approach very popular in a large variety of applications, including aviation where it still thrives nowadays.

The basic principle of PIDs is the regulation of a feedback signal, in particular of the error between the measured and the desired state. The PID gains adjust the control action according to the properties of the error signal $err(t)$. The starting point is a simple proportional gain K_P which determines the control action $u_c(t)$ as

$$u_c(t) = K_P \cdot err(t)$$

The limits of adopting only a proportional gain are the possible steady-state offset in response to a constant reference and the inability to reject a constant disturbance. Furthermore, an excessively large value of K_P might lead to instability, in particular when dealing with high-order systems.

In to order to eliminate the steady state error the integral gain K_I is commonly adopted to support the proportional action. Its contribution is proportional through the gain K_I to the integral of the error from the initial time to the considered moment t

$$u_c(t) = K_I \cdot \int_0^t err(\tau)\,\mathrm{d}\tau$$

The integral contribution has the advantage of taking into account the past values of the error. For this reason, it is possible to have a control action different from zero also when the error is zero. The addition of the integral term to a simple proportional controller improves the tracking of the steady state response. The drawback of increasing K_I is a slower response for equal overshoot, or a rise in overshoot for an unchanged response velocity.

Finally, a derivative term is added to improve stability by increasing the damping of the response thanks to an anticipatory behavior. The rate of change of the signal is used so that its contribution to the total control action is proportional to the gain K_D

$$u_c(t) = K_D \cdot \frac{\mathrm{d}err(t)}{\mathrm{d}t}$$

The sum of all the three contributions yields to the proportional-integral-derivative controller, a linear combination of the error, its integral and its derivative in time

$$u_c(t) = K_P \cdot err(t) + K_I \cdot \int_0^t err(\tau)\,\mathrm{d}\tau + K_D \cdot \frac{\mathrm{d}err(t)}{\mathrm{d}t}$$

The PID control formulation in the frequency domain is

$$U_C(s) = K_P + \frac{K_I}{s} + K_D \cdot s \tag{4.1}$$

4.2 Introduction to H_∞ approach

The H_∞ control problem was initially formulated in 1981 by Zames [94] as an alternative to the 1960s Linear Quadratic Gaussian (LQG) theory used for targeting robust control. Within LQG theory uncertainties are modeled as a white noise Gaussian process which is added to the system as an extra input vector. This approach is valid when modeling measurement noise but can not be adopted when dealing with parameter uncertainties. The early H_∞ frequency domain formulations, which originated high-dimensional optimal or near-optimal controllers, were quickly replaced in the late 1980s by simpler time-domain approaches that accelerated the research on this topic [95]. Among others, major contributions to H_∞ theory are the works from Doyle, for instance [96], where the structured singular value tool is introduced for testing robust stability, or [97], where the first solution to a general MIMO H_∞ optimal control problem is proposed. Very relevant is also the research from Francis, in [98] he gives a detailed treatment of the theory for H_∞ control. More related to the work here

proposed is the contribution of Glover and McFarlane who in [99] and [100] introduced the H_∞ loop-shaping design method which provides systematic procedures for obtaining sensible controllers that meet performance objectives and guarantee robustness against model uncertainty and unmeasured disturbances. An accurate treatment of H_∞ design, also considering the case of dynamic output feedback, is [101].

Nowadays, after the publication of many papers and books on the subject, H_∞ is one of the most effective approaches to target multivariable robust control. Its combination with loop shaping provides a tool that can be effectively applied to flight control laws design with satisfying results. An advantage over classical control is its simplicity of application to robust problems involving multivariable systems with cross-coupling between channels, which is typical of fixed-wing aircraft behavior.

The origin of the name comes from the H_∞ norm, as H_∞ is the space where the norm is defined. This space has a rational transform and is bounded in the right half plane comprehensive of the imaginary axis. The norm represents the maximum absolute value of the signal transform in the $j\omega$ axis. Its definition for a scalar signal u_c is

$$\|u_c\|_\infty = \sup_\omega |U_C(j\omega)|$$

In this case, it can be also demonstrated that the H_∞ norm is the maximum of the absolute value of the function over the whole right plane. In the multivariable case, for instance for a vector or a matrix $G(s)$, the H_∞ norm is defined as

$$\|G(j\omega)\|_\infty = \sup_\omega \overline{\sigma}\left[G(j\omega)\right]$$

where $\overline{\sigma}(\omega)$ is the maximum singular value of the argument of the matrix for every value of ω. As a matter of fact, the H_∞ norm of a input-output dynamic operator can be simply computed as follows: for SISO systems it is the maximum absolute value in frequency of its transfer function, for MIMO systems it is the maximum absolute singular value of the transfer matrix.

The idea behind H_∞ approach is to address the issue of worst-case controller design for linear plants subject to unknown additive disturbances and plant uncertainties. The H_∞ norm is a tool used to measure the effect of an exogenous input, which is the disturbance acting on the system, over an output of the system. It is required that in all conditions the influence of the disturbance on the designated output is minimum. This requirement is generally expressed imposing that the H_∞ norm of a transfer matrix connecting the input to the output is bounded under a defined threshold. Finding an appropriate controller respecting this condition is the aim of the H_∞ design problem.

4.3 Problem formulation and proposed control design

Typically, the objective of a control design problem expresses requirements in terms of disturbances attenuation, effects of the measurement noise and robustness to model uncertainties. The described robust approach targets these issues with a method that can be considered in the class of H_∞ loop shaping. A graphical procedure, based on root locus method, is applied to evaluate preliminary PID controller gains in order to satisfy nominal stability. Finally, these parameters are tuned to comply with nominal performance, robust stability and robust performance requirements by following H_∞ theory.

This approach is applied to a fixed-wing UAV linear model, where decoupling between longitudinal and latero-directional planes exists. Note that the linear model with the states defined in Equation (3.11) and Equation (3.12) is employed. In the longitudinal plane the linear speed u is controlled by elevator δ_e, while altitude h is controlled by throttle δ_{th}, see Fig. 4.1. Three PID controllers have to be designed. There are two feedbacks on the speed loop, pitch angle θ and speed u, and one on the altitude loop, altitude h. The state vector of the system in this plane, in addition to θ, u and h, includes the angle of attack α and the pitch angular velocity q. In the latero-directional plane, Fig. 4.2, the ailerons δ_a control the heading angle through two feedbacks on roll and heading angles, respectively ϕ and ψ. Two PID loops need to be configured. In this plane the state vector of the system is defined by ϕ, ψ, the sideslip angle β, roll angular velocity p and yaw angular velocity r.
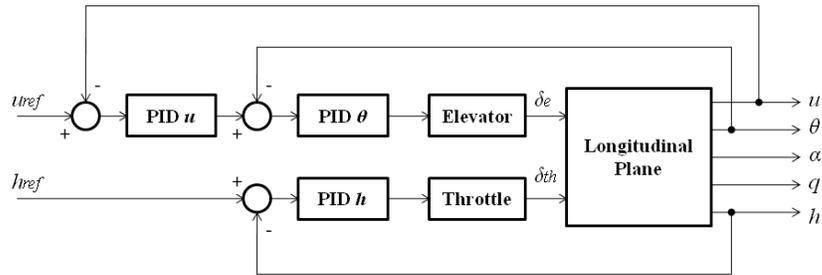
Fig. 4.1: Longitudinal control scheme
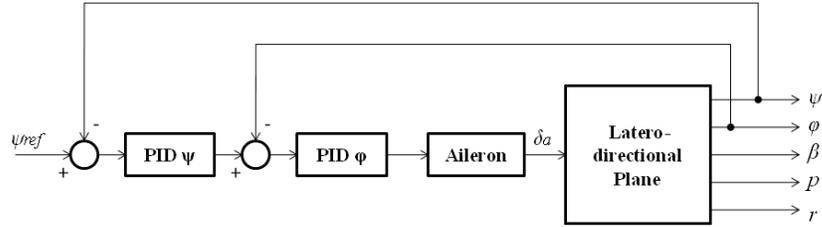
Fig. 4.2: Latero-directional control scheme

The first requirement to be achieved is the nominal stability of the controlled system. This is obtained if all transfer functions from all input-output pairs have poles with negative real part. For each loop nominal stability is assessed in the time domain by analyzing the closed-loop response to a step input. Rise time, settling time, overshoot and steady state error are the parameters taken into account.

The root locus method allows plotting the position of the poles of a closed-loop transfer function at the changing of a parameter K_{mult}. K_{mult} is the gain of $C(s)$, the controller of the plant $G(s)$ having $\mathbf{R}(s)$ as input and $\mathbf{Y}(s)$ as output. The importance of the root locus method in control system theory lies in the fact that the location of the system poles determines the system stability and transient response. Therefore, the desired performance of a control system can be obtained by changing only the static gain. The closed-loop transfer function is

$$\frac{\mathbf{Y}(s)}{\mathbf{R}(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)}$$

and its poles can be found solving the characteristic equation $1 + C(s)G(s) = 0$. If $L(s)$ is a transfer function proportional to $C(s)G(s)$ through K_{mult}, the characteristic equation can be written as

$$1 + K_{mult}L(s) = 0 \tag{4.2}$$

The root locus curve represents the position of all the possible solutions of Equation (4.2), as K_{mult} changes from 0 (cross sign) to ∞ (circle sign), see Fig 4.3. Therefore, the root locus plot identifies, according to the value of K_{mult}, the position of the poles of the closed-loop transfer function, and thus the system stability.

Root locus method is applied to define the PID controller gains K_P, K_I, K_D. PID zeros z_1 and z_2 can be obtained imposing equal to zero the numerator of the PID controller transfer
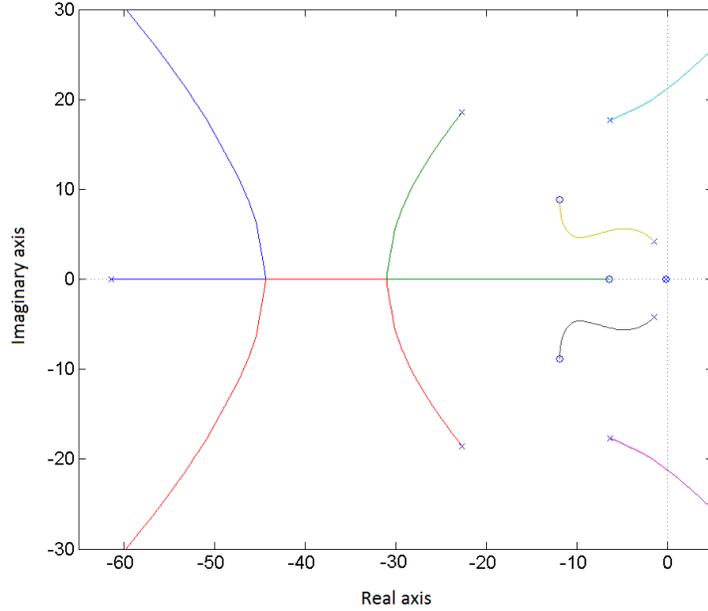
Fig. 4.3: Root locus plot

function of Equation (4.1), $K_D s^2 + K_P s + K_I = 0$. The application of Vieta's formula to this quadratic expression yields to the following relationship between the zeros and the coefficients of the equation

$$\begin{cases} z_1 + z_2 = -\frac{K_P}{K_D} \\ z_1 \cdot z_2 = \frac{K_I}{K_D} \end{cases}$$

from which

$$\begin{cases} K_P = -K_D \cdot (z_1 + z_2) \\ K_I = K_D \cdot (z_1 \cdot z_2) \end{cases}$$

The derivative gain K_D coincides with K_{mult}. Therefore the design of a PID controller requires the definition of three parameters: the two PID zeros z_1 and z_2 and the gain K_{mult}. The zeros are chosen arbitrarily with negative real part in order to increase the stability of the system. The root locus method is applied to define a value of K_{mult} that satisfies the closed-loop step response requirements in terms of rise time t_r, settling time t_S and overshoot

M_P. These target parameters can be calculated as function of damping ratio ζ and natural frequency ω_n of the system according to the definitions of [102]-[104]

$$
\begin{cases}
t_r = \dfrac{1}{\omega_n\sqrt{1-\zeta^2}}\left(\pi - \arctan\left(\dfrac{\sqrt{1-\zeta^2}}{\zeta}\right)\right) \\
t_S = -\dfrac{\ln(tolerance fraction\%)}{\zeta\omega_n} \\
M_P = \exp^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}
\end{cases}
\tag{4.3}
$$

This trial and error procedure can be repeated until a satisfying, but yet not definitive, closed-loop step response is attained. As the variation of each gain may have different and conflicting effects on the closed-loop response, a compromise has to be found.

At the same time performance and robust characteristics of the system need to be taken into account. In order to assess nominal performance, robust stability and robust performance both the longitudinal and latero-directional schemes of Fig. 4.1 and Fig. 4.2 have to be reshaped into the robust control scheme of Fig. 4.4.
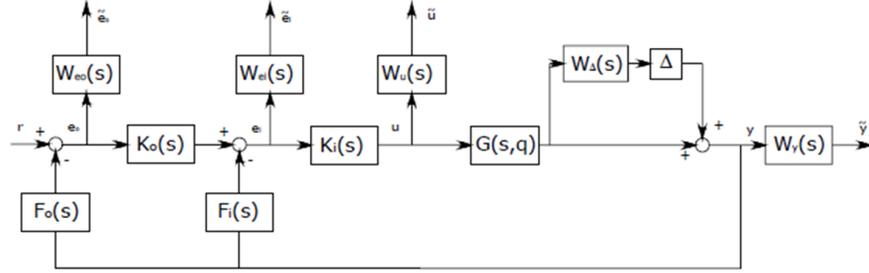
Fig. 4.4: Robust control scheme

This includes disturbances, errors and some weighting functions necessary to evaluate the compliance with robust requirements. Unstructured uncertainties $\boldsymbol{\Delta}(s)$ are modeled as $\boldsymbol{\Delta}(s) = \boldsymbol{W_\Delta}(s)\Delta$ [105]. They include linearization errors and neglected system dynamics. The block $\boldsymbol{W_\Delta}(s)$ represents the unstructured uncertainties error distribution, it is modeled as a diagonal matrix

$$
\boldsymbol{W_\Delta}(s) =
\begin{bmatrix}
w_\Delta(s) & 0 & \dots \\
0 & w_\Delta(s) & \dots \\
\dots & \dots & \dots
\end{bmatrix}
$$

where the diagonal elements contain the error distribution along the frequency range. This is assumed to be low at low frequencies, where the linearization approximations appear, and high at high frequencies where the aircraft fast dynamics are neglected. The selected transfer function is

$$w_\Delta(s) = \frac{s + \omega_{LF}}{s + \omega_{HF}}$$

with $\omega_{LF} = 1$ rad/s and $\omega_{HF} = 200$ rad/s. The block Δ is unknown but it must respect the condition $\|\Delta\|_\infty < 1$ [106].

$W_{eo}(s)$ and $W_{ei}(s)$ represent, respectively, the errors of the outer and inner loops. The aim of having no steady state error for step input command means that, in the frequency domain, the gain must be null for $s = 0$. This is achieved by defining the error transfer functions as high-pass filters with a cutoff frequency equal to the considered loop bandwidth. $W_u(s)$ and $W_y(s)$ are the weighting functions on the input and output signals. The first poses a limit on the available control action, for example the maximum elevons deflection, the second on the value of the outputs, for example the angle of attack has to be smaller than the stall angle. In the scheme of Fig. 4.4 $\boldsymbol{K_o}(s)$ and $\boldsymbol{K_i}(s)$ are, respectively, the outer and inner loop PID controllers, $\boldsymbol{F_o}(s)$ and $\boldsymbol{F_i}(s)$ are, respectively, the outer and inner loop closing matrices.

The next step requires the introduction of Linear Fractional Transformations (LFT) [107]. According to [106] the system of Fig. 4.4 can be redrawn into a general LFT interconnection, Fig. 4.5.(a). Here Δ represents all the possible uncertainties, $\boldsymbol{P}(s)$ is the interconnection matrix, $K(s)$ is the controller, $\boldsymbol{u_c}$ is the control signal, $\boldsymbol{y}$ is the measurement, $\boldsymbol{w}$ is a vector signal including noise, $\boldsymbol{z}$ is a vector signal including all the controlled states, $\boldsymbol{u_{c\Delta}}$ and $\boldsymbol{y_\Delta}$ are the input and output signals given by the uncertainties.

Thanks to the formulation of Fig. 4.5.(a) the plant, the controller, the performance weights and the uncertainties are grouped into a unified framework. The plant and the controller can still be enclosed into a unique matrix $\boldsymbol{M}(s)$ computing a lower LFT. The result is the $M - \Delta$ framework, Fig. 4.5.(b), a popular configuration for robust design and analysis of uncertain feedback systems. The matrix $\boldsymbol{M}(s)$ can be partitioned into four blocks according to the input and output dimensions, so that

$$\begin{bmatrix} \boldsymbol{y_\Delta} \\ \boldsymbol{z} \end{bmatrix} = \begin{bmatrix} \boldsymbol{M}_{11}(s) & \boldsymbol{M}_{12}(s) \\ \boldsymbol{M}_{21}(s) & \boldsymbol{M}_{22}(s) \end{bmatrix} \begin{bmatrix} \boldsymbol{u_{c\Delta}} \\ \boldsymbol{w} \end{bmatrix} = \boldsymbol{M}(s) \begin{bmatrix} \boldsymbol{u_{c\Delta}} \\ \boldsymbol{w} \end{bmatrix}$$

The matrix $\boldsymbol{M}(s)$ is the starting point for the analysis of nominal performance, robust
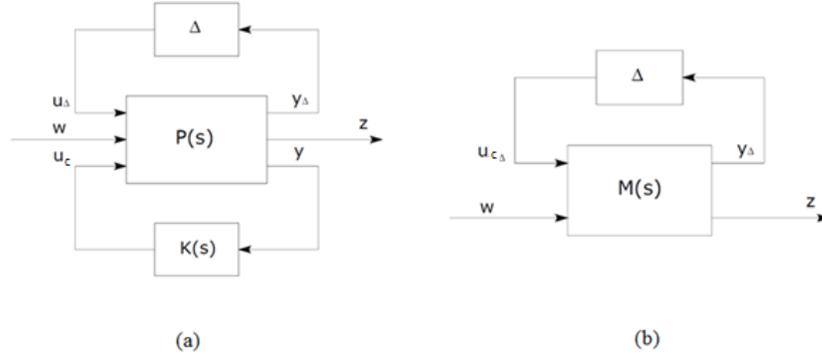
Fig. 4.5: $P - K - \Delta$ framework (a) and $M - \Delta$ framework (b)

stability and robust performance through the use of small gain theorem [106]. For a constant $\gamma > 0$ the system of Fig. 4.5.(b) is well-posed and internally stable for all $\boldsymbol{\Delta}(s)$ if and only if

$$\|\boldsymbol{M}(s)\|_\infty \leq \gamma \quad \text{with} \quad \|\Delta\|_\infty \leq \frac{1}{\gamma}$$

The small gain condition is sufficient to guarantee internal stability even if uncertainties are nonlinear and time-varying.

Nominal performance is achieved if, for the nominal model $\boldsymbol{M}(s)$ ($\Delta = 0$), the energy of the output $\boldsymbol{z}$, due to energy-bounded disturbances, is also bounded. A condition easier to verify is obtained assuming stable the interconnection of Fig. 4.5.(b). As the submatrix $\boldsymbol{M_{22}}(s)$ links $\boldsymbol{z}$ with $\boldsymbol{w}$, it can be stated that nominal performance is attained if

$$\|\boldsymbol{M}_{22}(s)\|_\infty \leq 1$$

Robust stability is achieved when all the members of the family of systems including uncertainties are nominally stable. Assuming $\boldsymbol{M}(s)$ internally stable, the resulting robust stability condition is

$$\|\boldsymbol{M}_{11}(s)\|_\infty \leq 1$$

Robust performance is guaranteed when all the members of the family of systems with uncertainties achieve nominal performance. In other words, the output z should have bounded energy for all energy-bounded disturbances and for all models in the set, this is expressed as

$$\|\boldsymbol{z}(s)\|_2 \leq 1 \quad \forall\{\boldsymbol{w} \in \mathcal{L}_2, \|\boldsymbol{w}(s)\|_2 \leq 1\}, \quad \forall\boldsymbol{\Delta}(s) \mid \|\boldsymbol{\Delta}(s)\|_\infty \leq 1$$

The verification of this requirement passes through the μ analysis of the frequency response of the full matrix $\boldsymbol{M}(s)$. In [106] it is shown that an uncertain system is robustly stable and satisfies (even if only partially) H_∞ performance for all $\boldsymbol{\Delta}(s)$ if, and only if, the structured singular value μ_Δ of the corresponding interconnection model is no greater than one. The frequency dependent structured singular value with respect to the uncertainty is defined as

$$\mu_\Delta\left(\boldsymbol{M}(s)\right) = \frac{1}{\min\left\{\overline{\sigma}\left(\boldsymbol{\Delta}(s)\right) : \boldsymbol{\Delta}(s) \in \Delta, \det\left(1 - \boldsymbol{M}(s)\boldsymbol{\Delta}(s)\right) = 0\right\}}$$

where $\overline{\sigma}$ represents the largest singular value and $\Delta \subset \mathbb{C}^{n \times n}$ is defined as

$$\Delta = \left\{diag\left[\delta_1\boldsymbol{I}_{r1}, \ldots, \delta_S\boldsymbol{I}_{rs}, \Delta_1, \ldots, \Delta_F\right] : \delta_i \in \mathbb{C}, \Delta_j \in \mathbb{C}^{mj \times mj}\right\}$$

with $\|\delta_i\|_\infty < 1$; $r_1, \ldots, r_S$ and $m_1, \ldots, m_F$ are positive integers. As a matter of fact the robust performance condition is

$$\mu_\Delta\left(\boldsymbol{M}_{11}(s)\right) \leq 1$$

The computation of μ_Δ is a difficult nonconvex problem and research was mainly concentrated in establishing its upper and lower boundaries. The upper boundary, which is of interest in this case, can be efficiently computed by solving a complex optimization problem. However, the gap between μ_Δ and its upper boundary can be arbitrarily large, therefore only a sufficient condition for robust stability can be achieved and checked with structured singular value. A robust controller design based on μ analysis is less conservative than classical robust H_∞ design.

4.4   Practical application and simulation results

The PID gains defined with the presented robust control strategy are applied to a multi-loop commercial autopilot integrated on the MH850 UAV. The selected system is the MicroPilot MP2028, a multi-loop PID autopilot designed for fully autonomous operations. Its capabilities include airspeed hold, altitude hold, turn coordination, GPS waypoint navigation and autonomous launch and recovery. Its compact dimensions (100 x 40 x 20 mm) and low weight (28 g) makes it suitable for mini-UAV applications. The MicroPilot MP2028, represented in Fig. 4.6, offers several control loops [108] as shown in Fig. 4.7.

A linear decoupled model of the MH850 aircraft, see Equation (3.10a) and Equation (3.10b), is created. The state vectors, as already mentioned, follow the representations of Equation (3.11) and Equation (3.12) where α, h and β are included. The mathematical

Fig. 4.6: MicroPilot MP2028 autopilot [109]

Fig. 4.7: MP2028 scheme as implemented in the simulator [108]

model of the plant include actuators: the throttle is described with a first order transfer function while elevons with the second order transfer function defined in Section 3.6.2. As stated in the previous section, five PID gains need to be designed, three for the longitudinal plane and two for the latero-directional plane. The controller definition process is handled with MathWorks Matlab software environment and its Robust Control Toolbox.

The application of the root locus procedure and the resulting PID tuning allows closing longitudinal and latero-directional control loops with step response characteristics that satisfy the MH850 specifications of Equation (4.3)

$$
\begin{cases}
t_r \cong 2s \\
t_S \cong 60s \\
M_P \cong 55\%
\end{cases}
\tag{4.4}
$$

Unfortunately the resulting PID gains need to be adjusted. In fact, a contrast between closed-loop response characteristics and robust specifications emerges. Initially priority is given to robust specifications requirements, a decay on the closed-loop response is considered acceptable as long as nominal stability is maintained. As a consequence, the PID gains are modified with a trial and error procedure with the intention of satisfying nominal performance, robust stability and robust performance. From the images of Fig. 4.8, that refer to the longitudinal plane, it is clear that not all requirements can be achieved even accepting, for instance, a slow step response in the pitch from airspeed loop. The defined unitary boundaries are crossed in two cases out of three.

Similar considerations can be drawn for the latero-directional plane, see Fig. 4.9, even if here results are more satisfactory. In fact, by comparing robust stability, nominal performance and robust performance plots, it can be seen that the latero-directional plane is more compliant with requirements in terms of maximum value achieved and width of frequency interval crossing. The obtained PID gains are summarized in Table 4.1 and Table 4.2.

The aircraft behavior with the assigned gains is simulated using a Fortran based software which implements a commercial multi-loop PID autopilot, Micropilot MP2028 (Fig. 4.6) or equivalent. Its capabilities include attitude, airspeed and altitude hold, turn coordination and sequential GPS waypoint navigation. The simulator has been tested and experimentally validated using flight test data [110][111]. The aircraft rigid body model is detailed in terms of propulsive, aerodynamic and inertial actions. The propulsion system is modeled with propeller, DC motor and batteries and its aerodynamics is implemented using the blade element theory corrected for inflow effects. Blade airfoil aerodynamics is generated with a
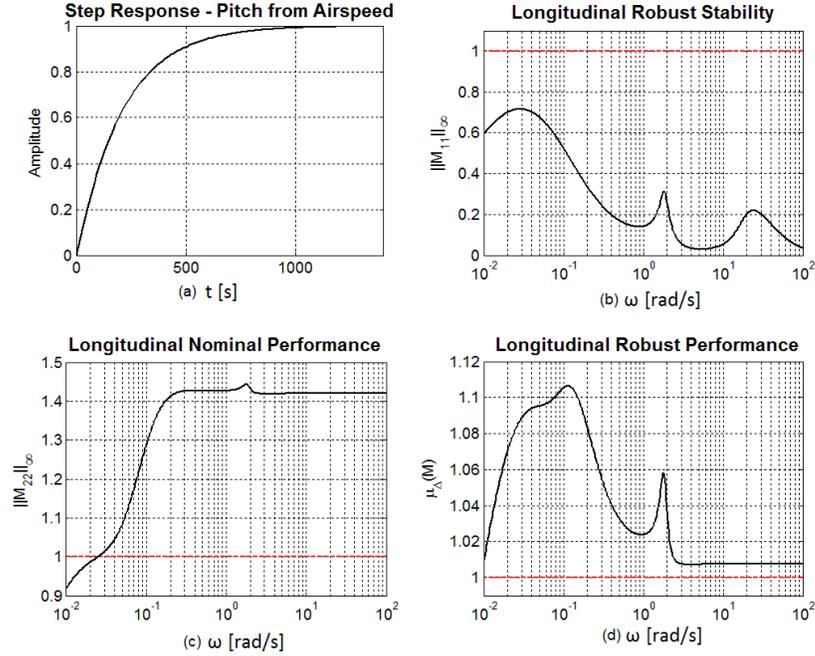
Fig. 4.8: Longitudinal plane characteristics, gains optimized for requirements

Reynolds number dependent database. The DC motor is parametrized with no-load and stall current, nominal voltage and stall torque. The simulator allows a post-flight analysis as it stores in memory the assigned commands and the aircraft states along the flight path. The use of such simulation environment plays a key role in the reduction of expensive and risky flight tests.

The path chosen for testing the aircraft behavior is a 500 m side square with a constant 100 m altitude, cruise speed is 15 m/s. The simulator of the MP2028 autopilot is set with the flyto navigation mode with a 1 Hz GPS update rate. In this mode the aircraft is flown to the waypoint maintaining altitude and airspeed, the desired heading is set with no attempt to travel in straight line between two points [108].

The results obtained with these PID gains are not adequate, as visible from Table 4.3 and Fig. 4.10. Table 4.3 indicates the cumulative error of some navigation parameters. For a general parameter k in a simulation with N steps the cumulative error ec is defined as

$$ec = \sum_{i=1}^{N} |K_i - k_i|$$
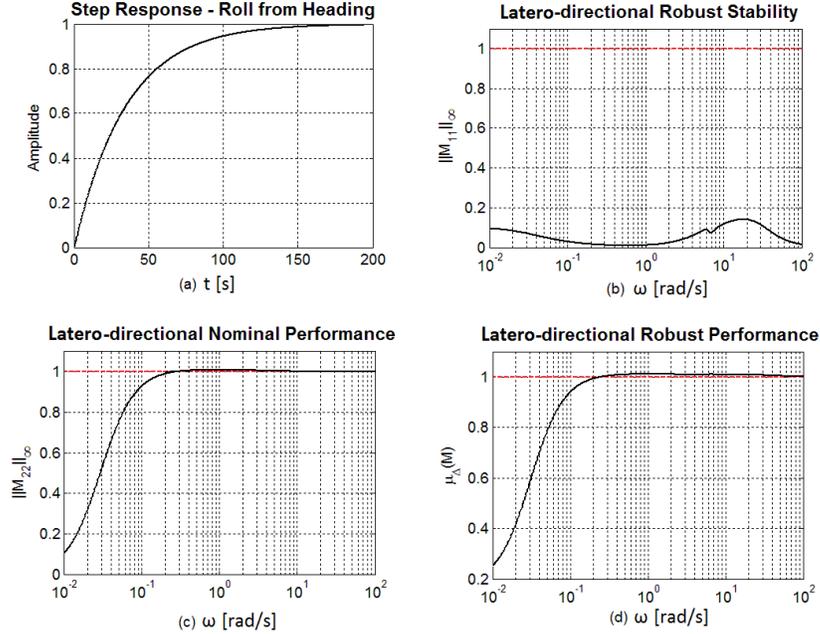
Fig. 4.9: Latero-directional plane characteristics, gains optimized for requirements

where k_i is the parameter at the ith step and K_i is the reference value of the same parameter at that step.

Fig. 4.10 represents the 3D view of the path with respect to the defined waypoints. The aircraft passes through the first waypoint which is in line with the starting point, the following waypoints are missed with an error which has an order of magnitude comparable to the path dimension. Remarkable oscillations on the longitudinal plane are also observable. This undesired performance is due to a slow response in the latero-directional plane and to an underdamped behavior in the longitudinal plane. Oscillations in the commands are significant, in particular for throttle and elevator which control altitude and speed, see Fig. 4.11.

As the flight trajectory proves to be unsatisfactory, nominal performance, robust stability and robust performance conditions are relaxed. Gains are now tuned with the purpose of improving the aircraft behavior along the path without forgetting the other requirements. These results are plotted in Fig. 4.12 and Fig. 4.13.

The shortening in the step response settling time coincides with a worsening in the compliance with robust requirements. In the longitudinal plane two peaks are observable at $10^{-1.8}$ rad/s and $10^{1.3}$ rad/s which correspond to the natural frequencies of the underdamped poles

Tab. 4.1: Longitudinal plane gains optimized for requirements

	K_P	K_I	K_D
Elevator from Pitch	-0.04	-0.25	-0.002
Pitch from Airspeed	-0.0002	-0.0001	-0.0001
Throttle from Altitude	0.055	0.0005	0

Tab. 4.2: Latero-directional plane gains optimized for requirements

	K_P	K_I	K_D
Aileron from Roll	0.08	0	0
Roll from Heading	0.05	0	0

Tab. 4.3: Cumulative error, gains optimized for requirements

Parameter	Cumulative Error
Cross track	7 289 412 m
Heading angle	390 846 deg
Altitude	66 860 m
True airspeed	88 135 m/s
Pitch angle	3 275 deg
Roll angle	59 deg
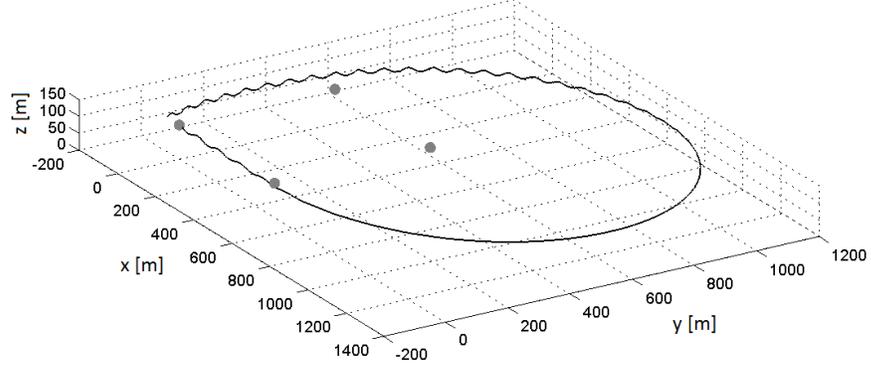
Fig. 4.10: 3D trajectory view, gains optimized for requirements
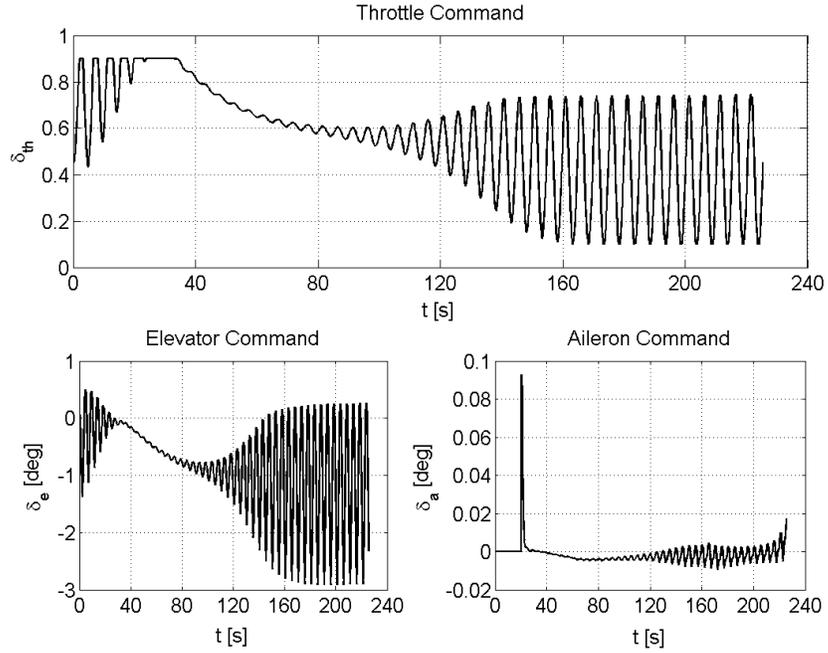
Fig. 4.11: Aircraft commands, gains optimized for requirements
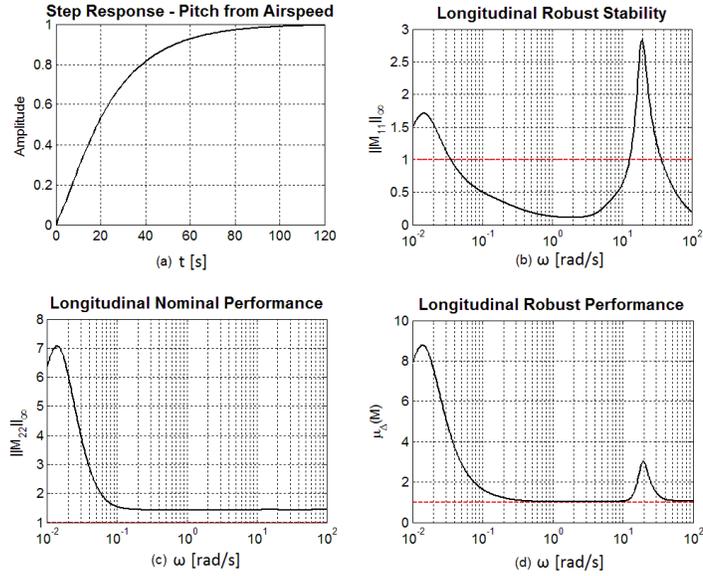
Fig. 4.12: Longitudinal plane characteristics, gains optimized for performance

of the closed-loop system. The latero-directional plane still proposes better results, both in terms of step response settling time and respect of the unitary threshold. A peak is observable at $10^{0.8}$ rad/s where a pole with 0.085 damping is present. The new PID gains are indicated in Table 4.4 and Table 4.5. A general increase in the absolute value of the gains is remarked, except in the case of the throttle from altitude loop.

Tab. 4.4: Longitudinal plane gains optimized for performance

	K_P	K_I	K_D
Elevator from Pitch	-0.3	-3.25	-0.01
Pitch from Airspeed	-0.0021	-0.00087	-0.0015
Throttle from Altitude	0.019	0.0002	0.01

If the simulations are performed over the same path of the previous case, the cumulative errors of Table 4.6 are now obtained. The percentage of the new value with respect to the previous value is indicated between brackets. Except from the roll angle, a considerable
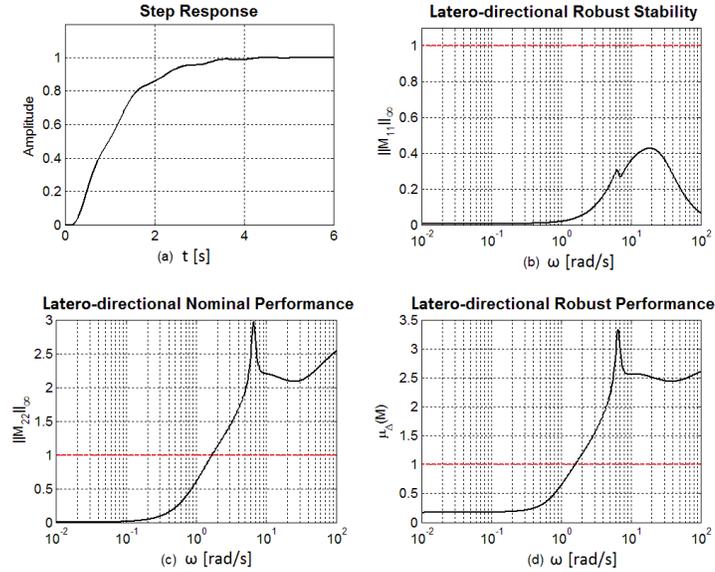
Fig. 4.13: Latero-directional plane characteristics, gains optimized for performance

Tab. 4.5: Latero-directional plane gains optimized for performance

	K_P	K_I	K_D
Aileron from Roll	0.12	0.0005	0.001
Roll from Heading	1.5	0.005	0.01

improvement of the cumulative errors is achieved. The roll angle unsatisfying result is moti-vated by the four turns performed by the aircraft where error is accumulated; in the previous case, in fact, a single long turn was executed. Improvements are visible from Fig. 4.14, the new path is smooth and regular, no altitude oscillations are recorded and waypoints are reached with precision. Fig. 4.15 illustrates the commands, during the turns also an elevator action is observable.

A good match between reference and actual flight parameters is guaranteed, see Fig. 4.16. During the initial transition to trimmed flight the aircraft loses altitude and gains speed, finally the error cancels out as steady flight condition is reached.

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Tab. 4.6: Cumulative error, gains optimized for performance

Parameter	Cumulative Error
Cross track	216 651 m (3%)
Heading angle	167 540 deg (42%)
Altitude	9 315 m (14%)
True airspeed	2 955 m/s (3%)
Pitch angle	7 deg (0.2%)
Roll angle	439 deg (744%)
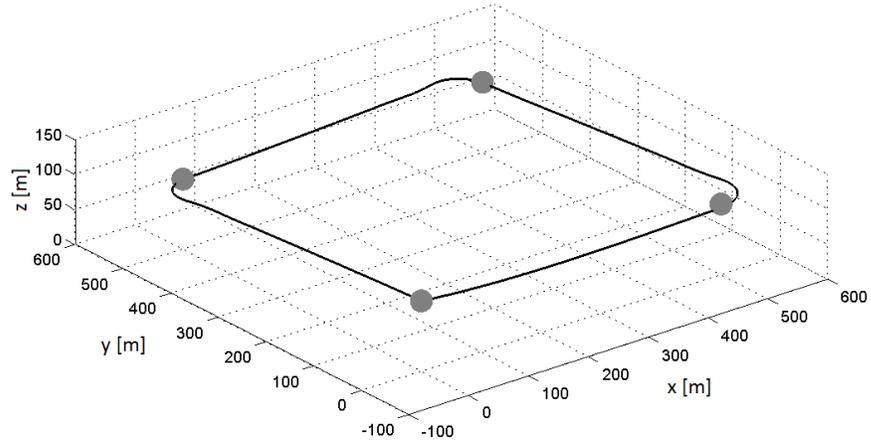
Fig. 4.14: 3D trajectory view, gains optimized for performance
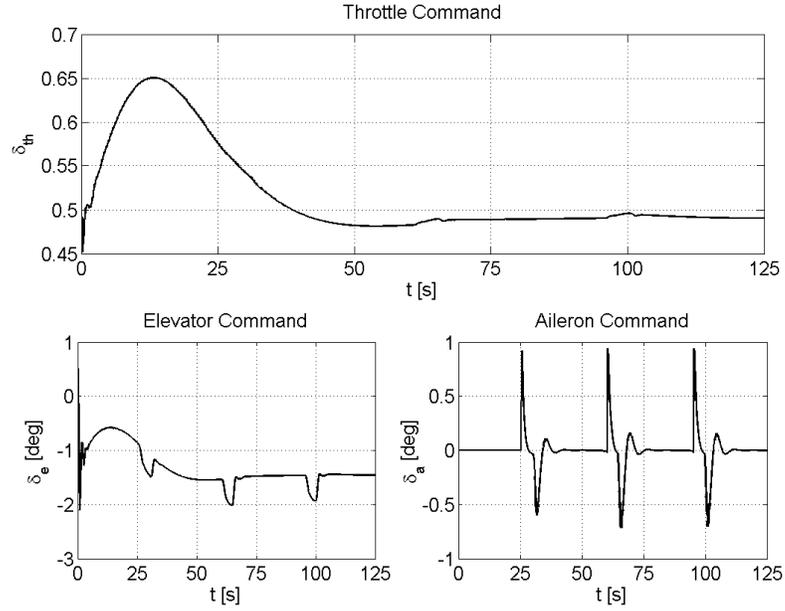
Fig. 4.15: Aircraft commands, gains optimized for performance
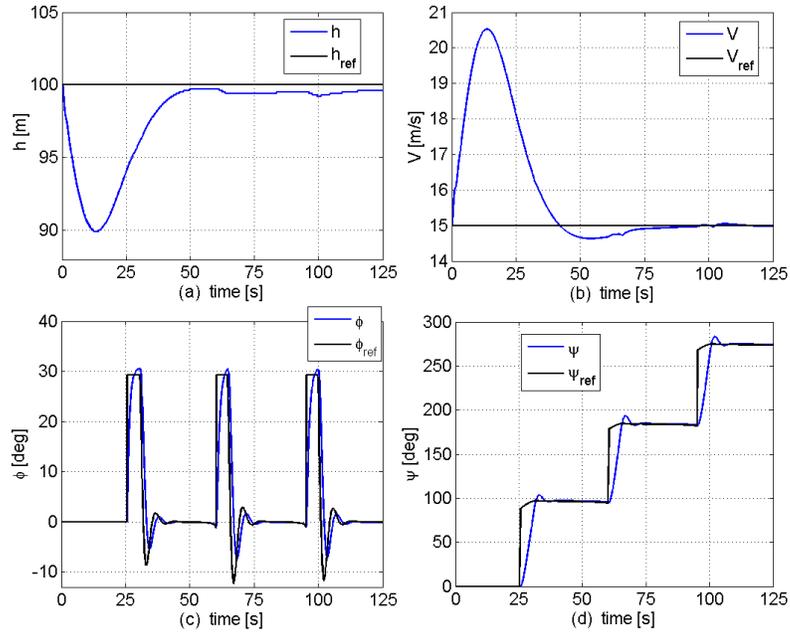
Fig. 4.16: Flight parameters responses, gains optimized for performance

4.5   Conclusions

In this chapter the problem of designing a multi-loop PID controller for a commercial UAV autopilot was tackled. The criterion for the definition of the PID gains was not merely based on the aircraft performance, but it also considered the robustness of the resulting system. In fact, sensor noise, parametric uncertainties and external disturbances are major issues affecting flight control systems of small UAVs.

A graphical method and a robust a posteriori control validation were proposed. PID gains were determined by a tradeoff between step response characteristics and robust performance. The design process was divided into two steps: initially the preliminary parameters of the controller were estimated with the root locus method, later the definitive PID gains were defined according to robust specifications.

The effectiveness of this approach was crosschecked with a comprehensive simulation tool which includes the aircraft mathematical model and the principal control functions of the autopilot. Results show that this hybrid technique provides a promising solution to the problem of tuning commercial autopilots for UAVs with the aim of enforcing robustness to plant variations and external disturbances. Nevertheless, it is clear that a tradeoff between robustness and performance is necessary when dealing with such a simple and traditional control technique. In the next chapter a controller specifically designed to target robustness issues is presented.

5. $\mathcal{L}_1$ ADAPTIVE CONTROLLER

The need for robustness to unmodeled dynamics and parametric model uncertainties is paramount when controlling the flight of Unmanned Aerial Vehicles. In Chapter 4 it is demonstrated that traditional PID control technique struggles to guarantee at the same time robustness and appropriate flight qualities. It is possible to find a good compromise, but more advanced control laws that satisfy both requirements are currently available. One of them is the $\mathcal{L}_1$ adaptive controller. In this chapter the application of an $\mathcal{L}_1$ adaptive controller to the MH850 test case is presented. In particular, it is demonstrated that the proposed control design can stabilize the nonlinear system, even if the controller parameters are selected starting from a decoupled linear model. A brief introduction to the $\mathcal{L}_1$ control is followed by the problem formulation and by the application example where its robustness to parameters variation is also tested. In Appendix A a further example of the application of $\mathcal{L}_1$ to aircraft control is presented.

5.1 Introduction to $\mathcal{L}_1$ adaptive controller

The $\mathcal{L}_1$ adaptive controller is a recent technique. It was first presented by its creators C. Cao and N. Hovakimyan at a conference in 2006 [112] and further formalized and expanded in the following years. The $\mathcal{L}_1$ controller is a modification of the Model Reference Adaptive Controller (MRAC), a technique which was first developed by the MIT in the late 1950s [113] for solving the problem of adaptive flight control. In those years, as explained in [114], there was no deep theoretical knowledge of the approaches that were implemented, and much was based on empirical results. The introduction of Lyapunov stability theory into adaptive control, see for instance [115], and in particular the definition of the condition for global stability introduced in [116], led to a better understanding of the adaptive approach. In the 1980s the theory behind adaptive control was completed, but the applications were limited to some specific commercial products. The need for a controller able to handle reconfiguration and damage control for air vehicles, and so to limit the costly validation and verification procedures, gave in the years 2000s a second life to adaptive control thanks to

a joint program among NASA, U.S. Air Force, academia and industry. $\mathcal{L}_1$ has its origins in this new interest for adaptive control techniques.

The main feature of $\mathcal{L}_1$ is the ability to guarantee robustness and transient performance with fast adaptation. The ability to quickly adapt to model changes, sudden failures and plant uncertainties is crucial for an adaptive controller, but usually this requirement is conflicting with the need to avoid poor robustness or destabilization. The $\mathcal{L}_1$ controller architecture allows decoupling the design for robustness and adaptation. In this case, fast adaptation is beneficial not only for performance but also for robustness. The design for robustness is achieved with classical and robust control methods. Fast adaptation is an advantage because error norms are inversely proportional to the square root of the adaptation rates, their limit is imposed only by high-frequency noise and by the computational power of the operating device. In the $\mathcal{L}_1$ controller the presence of a filter in the control action is crucial and it represents the main feature to distinguish it from the MRAC approach. This filter, in fact, is responsible for the tradeoff between performance and robustness. It is also employed to avoid high frequency in the control signal and to shape nominal response, its design can be handled by linear theory. The filter and the largest value of the unknown parameters are included in certain transfer functions that need to satisfy a condition on their $\mathcal{L}_1$ norm. The name $\mathcal{L}_1$ adaptive controller comes from this condition.

5.2 Problem formulation and proposed control approach

The problem of finding a control architecture able to deal with the high level of uncertainty characterizing small UAVs is the challenge here proposed for the $\mathcal{L}_1$ adaptive controller. In particular, in this section uncertain system input gain and time- and state-dependent unknown nonlinearities are introduced as major source of uncertainties. Furthermore, it is assumed that only a linear decoupled model is available for the controller design. The scope is to verify the ability of the so tuned $\mathcal{L}_1$ controller to handle a complete nonlinear aircraft model.

The dynamics of the considered system are the described by

$$\begin{cases} \dot{x}(t) = A_m x(t) + B_m \omega_u u_{ad}(t) + f(x(t), z(t), t), & x(0) = x_0 \\ \dot{x}_z(t) = g(x_z(t), x(t), t), & x_z(0) = x_{z0} \\ z(t) = g_0(x_z(t), t) \\ y(t) = C x(t) \end{cases} \tag{5.1}$$

which represents a general linear system affected by uncertainties. The known matrix $A_m \in$

$\mathbb{R}^{n \times n}$ is Hurwitz and specifies the desired dynamics of the closed-loop system, $B_m \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{m \times n}$ are known constant matrices that guarantee the controllability and observability of the system. This system includes $\omega_u \in \mathbb{R}^{m \times m}$, the uncertain system input gain matrix, $z(t) \in \mathbb{R}^p$ and $x_z(t) \in \mathbb{R}^l$, respectively the output and state vector of internal unmodeled dynamics, and the unknown nonlinear functions $f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^p \mapsto \mathbb{R}^n$, $g_0 : \mathbb{R} \times \mathbb{R}^l \mapsto \mathbb{R}^p$ and $g : \mathbb{R} \times \mathbb{R}^l \times \mathbb{R}^n \mapsto \mathbb{R}^n$. Another form of writing the first line of Equation (5.1) generates the system

$$
\begin{cases}
\dot{x}(t) = A_m x(t) + B_m(\omega_u u_{ad}(t) + f_1(x(t), z(t), t)) + B_{um} f_2(x(t), z(t), t), \ \ x(0) = x_0 \\
\dot{x}_z(t) = g(x_z(t), x(t), t), \ \ x_z(0) = x_{z0} \\
z(t) = g_0(x_z(t), t) \\
y(t) = C x(t)
\end{cases}
$$

$$(5.2)$$

where $B_{um} \in \mathbb{R}^{n \times (n-m)}$ is a constant matrix so that $B_m^T B_{um} = 0$ and the rank of $B = [B_m, B_{um}]$ is n. The unknown nonlinear functions $f_1 : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^p \mapsto \mathbb{R}^m$ and $f_2 : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^p \mapsto \mathbb{R}^{(n-m)}$ satisfy the condition

$$
\begin{bmatrix}
f_1(x(t), z(t), t) \\
f_2(x(t), z(t), t)
\end{bmatrix}
= B^{-1} f(x(t), z(t), t)
$$

The matched component of the uncertainties is represented by $f_1(\cdot)$, while the unmatched components by $B_{um} f_2(\cdot)$.

The $\mathcal{L}_1$ adaptive controller here applied is extensively described in the book of the creators of the controller [44]. The adaptive law, as explained in [117] and [118], is piecewise constant and guarantees fast estimation. The main feature is that the adaptation rate can be associated with the sampling rate of an autopilot board CPU. Moreover, this adaptive algorithm guarantees bounded inputs and outputs, uniform transient response and steady-state tracking. This extension of the $\mathcal{L}_1$ controller was applied to the NASA AirSTAR [119] and to the Boeing X-48B [120] aircraft. A key feature of this controller, as explained in the previous works, is that the hardware interface is executed at a lower rate (about $10 - 100$ Hz) than the control algorithm (about $100 - 1000$ Hz), therefore demanding insignificant CPU power. Thus, the computational power can be dedicated to fast adaptation.

Before defining the control law some assumptions on the system of Equation (5.2) are necessary:

- Assumption 1: The H_∞ norm of the unknown nonlinear functions f_1 and f_2 is bounded;

- Assumption 2: The functions f_1 and f_2 satisfy semiglobal Lipschitz condition;

- Assumption 3: The x_z dynamics are BIBO stable with respect to both initial conditions x_{z0} and input $x(t)$;

- Assumption 4: The system input gain matrix ω_u is assumed to be an unknown non-singular strictly row-diagonally dominant matrix with know signum of its elements. It belongs to a known compact convex set Ω so that $\omega_u \in \Omega \subset \mathbb{R}^{m \times m}$, and the nominal value $\omega_{u0} \in \Omega$ is known;

- Assumption 5: The transmission zeros of the transfer matrix $H_m(s) = C\left(s\mathbb{I} - A_m\right)^{-1} B_m$ lie in the open left half plane.

Further details for these assumptions and their complete formulations are illustrated in [44]. The proposed control law adopts as state predictor

$$
\begin{cases}
\dot{\hat{x}}(t) = A_m \hat{x}(t) + B_m(\omega_{u0} u_{ad}(t) + \hat{\sigma}_1(t)) + B_{um}\hat{\sigma}_2(t), \ \ \hat{x}(0) = x_0 \\
\hat{y}(t) = C\hat{x}(t)
\end{cases}
$$

where $\hat{\sigma}_1(t) \in \mathbb{R}^m$ and $\hat{\sigma}_2(t) \in \mathbb{R}^{n-m}$ are the adaptive estimates. The piecewise-constant adaptation laws are

$$
\begin{bmatrix}
\hat{\sigma}_1(t) \\
\hat{\sigma}_2(t)
\end{bmatrix}
= -
\begin{bmatrix}
\mathbb{I}_m & 0 \\
0 & \mathbb{I}_{n-m}
\end{bmatrix}
B^{-1}\Phi^{-1}(T_s)\mu(iT_s)
\tag{5.3}
$$

for $i = 0, 1, 2, ...$, and $t \in [iT_s, (i+1)T_s]$, where $T_s > 0$ is the adaptation sampling time associated with the sampling rate of the CPU installed on the autopilot board. In Equation (5.3) also

$$
\Phi(T_s) = A_m^{-1}(e^{A_m T_s} - \mathbb{I}_n), \ \ \in \mathbb{R}^{n \times n}
$$

$$
\mu(iT_s) = e^{A_m T_s}\tilde{x}(iT_s),
$$

appear, where $\tilde{x}(t) = \hat{x}(t) - x(t)$ is the error between the predicted state and the system state.

Finally, the last element of the controller is the control law defined as

$$
u_{ad}(t) = -K\chi(t)
$$

where $K \in \mathbb{R}^{m \times m}$ is a feedback matrix. Calling s the complex argument resulting from the Laplace transform of the corresponding time domain signal, it is possible to define

$$\chi(s) = D(s)\hat{\eta}(s)$$
$$\hat{\eta}(t) = \omega_{u0}u_{ad}(t) + \hat{\eta}_1(t) + \hat{\eta}_{2m}(t) - r_g(t)$$
$$\hat{\eta}_1(t) = \hat{\sigma}_1(t)$$
$$\hat{\eta}_{2m}(s) = H_m^{-1}(s)H_{um}(s)\hat{\sigma}_2(s)$$
$$r_g(s) = K_g(s)r(s)$$

$D(s)$ is a proper stable transfer matrix of dimension $m \times m$, $r(t)$ is the reference signal that the state $x(t)$ needs to follow. The choice of K and $D(s)$ requires that the low-pass filter $C(s)$

$$C(s) = \omega_u KD(s)\left(\mathbb{I}_m + \omega_u KD(s)\right)^{-1}$$

for all $\omega_u \in \Omega$, is strictly proper stable with DC gain $C(0) = \mathbb{I}_m$. Furthermore, it is required that $C(s)H_m^{-1}$ is a proper stable transfer matrix. The transfer functions $H_m(s)$ and $H_{um}(s)$ are calculated starting from the matrices of the systems (5.1) and (5.2)

$$H_m(s) = C(s\mathbb{I}_n - A_m)^{-1}B_m$$
$$H_{um}(s) = C(s\mathbb{I}_n - A_m)^{-1}B_{um}$$

while the feedforward prefilter $K_g(s)$ is chosen as the constant matrix $K_g = -(CA_m^{-1}B_m)^{-1}$ to achieve decoupling among the signals. In fact, this allows the diagonal elements of the desired transfer matrix $M(s) = C(s\mathbb{I}_n - A_m)^{-1}B_mK_g$ to have DC gain equal to one and the off-diagonal elements to have zero DC gain. The scheme of the adopted $\mathcal{L}_1$ controller is represented in Fig. 5.1.

5.3   Implementation and simulation results

The $\mathcal{L}_1$ adaptive controller is now designed on the linear model of the aircraft which separates the longitudinal and latero-directional dynamics. Final tests will be carried out on the nonlinear model which better represents the behavior of the real aircraft.

The $\mathcal{L}_1$ controller is assigned to the control of the inner loop variables that need a faster response and are more prone to be affected by unmodeled uncertainties. These include the pitch angle θ, the longitudinal component of the airspeed u and the roll angle ϕ. Altitude h and heading ψ outer navigation loops are instead controlled by simple PIDs. Fig. 5.2
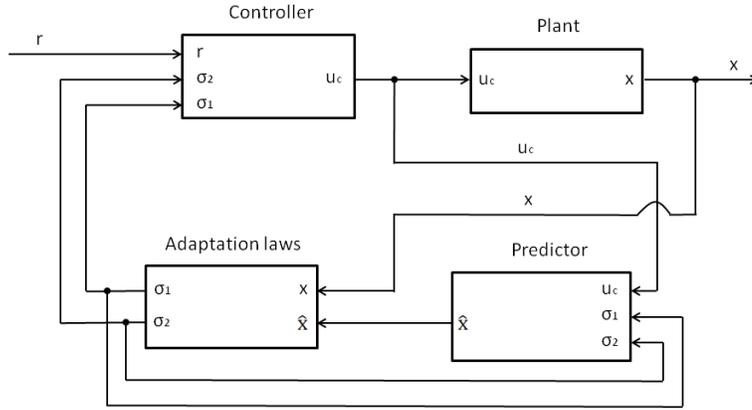
Fig. 5.1: $\mathcal{L}_1$ controller scheme

represents the global control scheme and highlights the control actions and the controlled variables. In the scheme the symbol $\bar{y}$ represents all the longitudinal and latero-directional states as they will be defined in Equations (5.4) and (5.5).
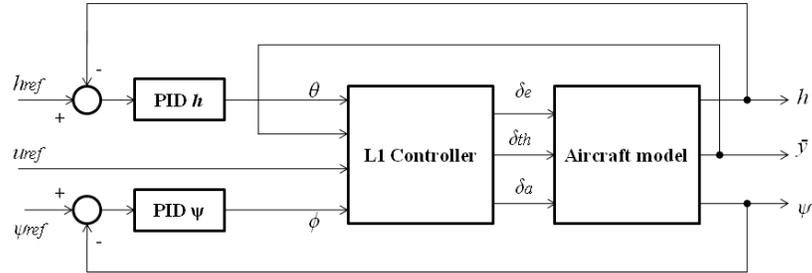
Fig. 5.2: $\mathcal{L}_1$ with PID global controller scheme

5.3.1   Linear case design and simulation results

The $\mathcal{L}_1$ adaptive controller parameters are designed on the linear decoupled model of the MH850 aircraft. Reference flight conditions for building the model are speed $u_0 = 15$ m/s, altitude $h_0 = 100$ m, angle of attack $\alpha_0 = 4.1$ deg, pitch angle $\theta_0 = 4.1$ deg (the ramp angle γ_0 is considered zero), $\beta_0 = 0$ deg, $\phi_0 = 0$ deg, $\psi_0 = 0$ deg and no body axes angular velocities. The linearization of the equations of motion results in the decoupling of the longitudinal and latero-directional planes, each of them modeled with standard continuous time-invariant

66

state space representation, see Equations (3.10a) and (3.10b). The aircraft linear model for the simulations is the one with the states represented in Equations (3.11) and (3.12), they include aerodynamic angles α and β and the outer loop variables h and ψ. However, the outer loop variables are controlled via PID gains, so the $\mathcal{L}_1$ adaptive controller is designed on a reduced-order linear model where h and ψ do not appear.

The resulting longitudinal state space elements are

$$
\begin{aligned}
x_{lon}(t) &= [u,\ \alpha,\ \theta,\ q]^T \in \mathbb{R}^{n_{lon}}, \\
u_{c\,lon}(t) &= [\delta_{th}, \delta_e]^T \in \mathbb{R}^{m_{lon}}, \\
A_{lon} &\in \mathbb{R}^{n_{lon} \times n_{lon}}, \\
B_{lon} &\in \mathbb{R}^{n_{lon} \times m_{lon}}, \\
C_{lon} &\in \mathbb{R}^{m_{lon} \times n_{lon}},
\end{aligned}
\tag{5.4}
$$

with $n_{lon} = 4$ and $m_{lon} = 2$. The state matrix A_{lon} is Hurwitz. As already illustrated in Section 3.6.1, the short period mode has natural frequency $\omega_{SP} = 17.06$ rad/s and damping $\zeta_{SP} = 0.48$, while phugoid mode has natural frequency $\omega_{PH} = 0.89$ rad/s and damping $\zeta_{PH} = 0.08$. The latero-directional state space elements for control design are

$$
\begin{aligned}
x_{lat}(t) &= [v,\ p,\ r,\ \phi]^T \in \mathbb{R}^{n_{lat}}, \\
u_{c\,lat}(t) &= [\delta_a] \in \mathbb{R}^{m_{lat}}, \\
A_{lat} &\in \mathbb{R}^{n_{lat} \times n_{lat}}, \\
B_{lat} &\in \mathbb{R}^{n_{lat} \times m_{lat}}, \\
C_{lat} &\in \mathbb{R}^{m_{lat} \times n_{lat}},
\end{aligned}
\tag{5.5}
$$

with $n_{lat} = 4$ and $m_{lat} = 1$, the only command is the aileron as the MH850 has rudderless configuration. The state matrix A_{lat} has one real and negative eigenvalue corresponding to a stable roll mode, one real and positive eigenvalue showing a slightly unstable spiral mode, and a couple of complex conjugate eigenvalues for the Dutch Roll characterized by natural frequency $\omega_{DR} = 6.15$ rad/s and damping $\zeta_{DR} = 0.13$.

The matrix A_m of Equation (5.2) is achieved by applying a pole placement design to both the A_{lon} and A_{lat} matrices of Equations (5.4) and (5.5). This procedure defines the gain matrix k_m so that the state feedback control $u_m(t) = -k_m x(t)$ generates the Hurwitz matrix $A_m = A - Bk_m$ with desired eigenvalues. The desired dynamics are chosen to optimize the closed-loop response in terms of rise time, settling time and overshoot. In the longitudinal plane two oscillating modes are designed, one with high natural frequency and damping ($\omega_1 = 8$ rad/s and $\zeta_1 = 0.92$) representing the short period, another representing

the phugoid with lower natural frequency and damping ($\omega_2 = 3$ rad/s and $\zeta_2 = 0.8$). In the latero-directional plane two real negative eigenvalues are chosen ($\lambda_1 = -6$ and $\lambda_2 = -10$), the desired oscillating mode representing the dutch roll is defined by $\omega_3 = 5$ rad/s and $\zeta_3 = 0.5$.

The total control action u_c is the combination of the state feedback control u_m and the adaptive control u_{ad}

$$u_c(t) = -k_m x(t) + u_{ad}(t)$$

The adaptive control u_{ad} is defined following the $\mathcal{L}_1$ approach described in Section 5.2. This procedure is performed on both the longitudinal and latero-directional models when considered affected by uncertain system input gain and time- and state-dependent unknown nonlinearities, see Equation (5.2). The adaptation time step T_s is chosen equal to 0.001 seconds. Filter bandwidths are chosen as 60 rad/s in the longitudinal plane for both the matched and unmatched signals, while in the latero-directional plane the values selected are 12 rad/s for the matched signal and 8 rad/s for the unmatched. The complete control scheme includes two PID controllers dedicated to altitude and heading angle control. As the hybrid approach described in Chapter 4 results of difficult application to the considered structure, the PIDs are tuned manually taking in consideration the specifications expressed in Equation (4.4). The value of the PID gains are summarized in Table 5.1. The controller synthesis and all simulations are performed using MathWorks Matlab/Simulink software environment.

Tab. 5.1: Outer loop PID gains

	K_P	K_I	K_D
Pitch from Altitude	0.0345	0.0061	0.016
Roll from Heading	0.55	0	0

The first set of simulations imposes a step reference signal to the three navigation-related variables u, h and ψ. Results for the inner loop variables directly controlled with $\mathcal{L}_1$ are illustrated in Fig. 5.3. The corresponding outer loop controlled by the combination of $\mathcal{L}_1$ and PID gains is represented in Fig. 5.4. These results show the ability of the controller configuration to handle with satisfying performance a maneuver where a contemporary variation of all variables is performed. Inner loop variables, in particular the speed, demonstrate a good tracking despite the abrupt transition imposed by the step inputs. Coupling between speed u and pitch angle θ is caused by the fact that both elevator δ_e and throttle δ_{th} contribute

to their control, see Fig. 5.5. As we are dealing with a linear model there is no coupling between longitudinal and latero-directional planes. Outer loop variables, in particular ψ, show satisfying response to step input. The response for h is partially penalized by the longitudinal coupling with the speed control. The image of Fig. 5.5 shows that the deflection of the control surfaces remains well under the saturation limits, while full throttle is achieved for few seconds after the moment of transition.
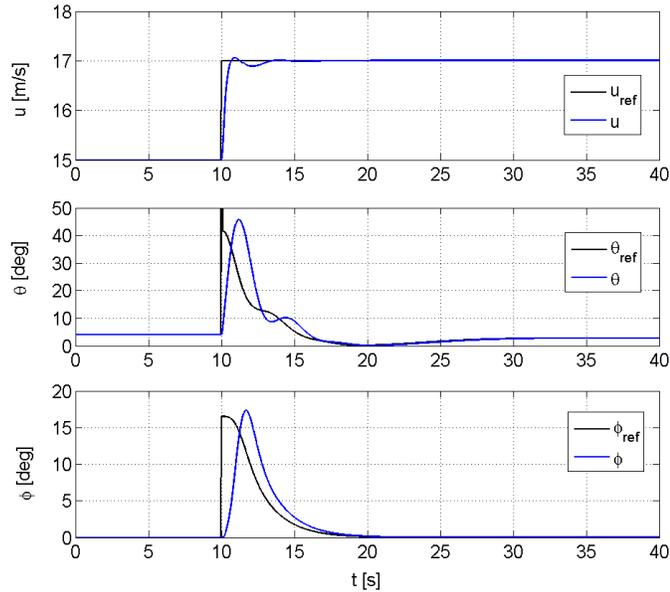
Fig. 5.3: Linear model inner loop variables

5.3.2   Nonlinear case simulation results

The controller, tuned for the decoupled linear model, is validated considering the complete nonlinear model obtained from the aircraft equations of motion described in Section 3.4. The same reference states of the linear case are imposed, starting from the same equilibrium conditions.

Inner loop results appear in Fig. 5.6. Tracking capabilities are maintained despite some mild initial oscillations in the longitudinal variables which fade after few seconds. A light coupling between longitudinal and latero-directional states is observable just after the step time because the nonlinear aircraft model does not separate their motions. Similarly, no real difference between linear and nonlinear models is observable in the outer loop variables of Fig.
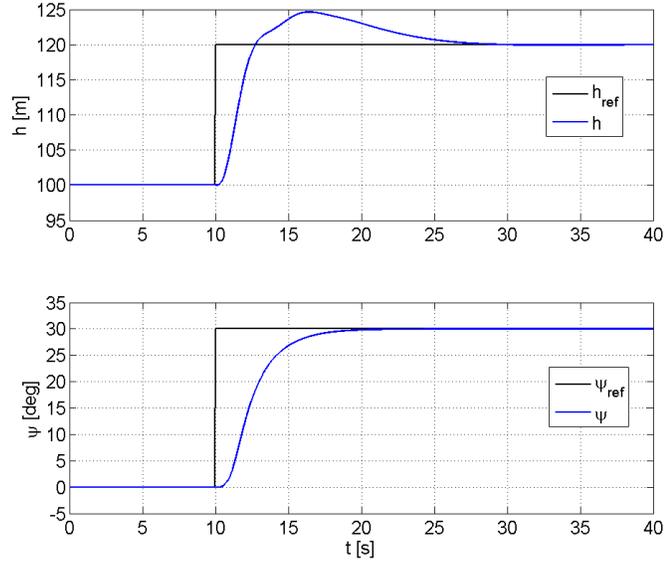
Fig. 5.4: Linear model outer loop variables
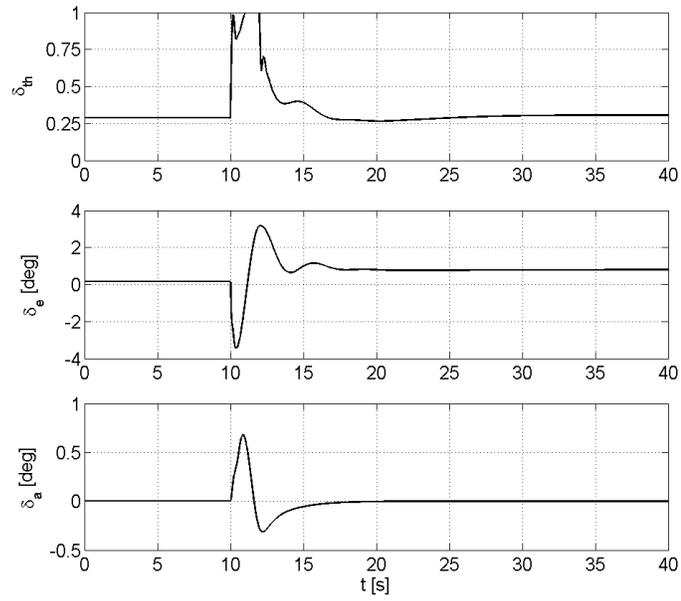
Fig. 5.5: Linear model commands

5.7. On the contrary, the controller seems to better perform on the nonlinear model. Note that here the longitudinal variable h shows minimal initial oscillations thanks to a proper PID initialization at starting time. The oscillations problem in the nonlinear case, in fact, is caused by an incorrect estimation of the initial longitudinal commands from the $\mathcal{L}_1$ controller. This is motivated by the different physical meaning of linear and nonlinear aircraft models. In the linear case the controller acts on the already trimmed aircraft which is flying in an equilibrium state. Therefore, the commands computed by $\mathcal{L}_1$ are the variation about the trim commands, Fig. 5.5 shows the total commands originated by these two contributions. In the nonlinear case, instead, the $\mathcal{L}_1$ controller needs to compute the total commands. Because of this, starting from initial estimates, it requires a couple of seconds to achieve trimmed flight, see Fig. 5.8. In this time interval the aircraft responds to the incorrect commands in an undesired way, until the controller, damping these motions, forces the aircraft into the desired steady state. Note that the latero-directional plane is already trimmed. In fact, as the initial conditions are described by symmetric flight with no angular velocities, the initial estimated command $\delta_a = 0$ deg already guarantees trimmed flight. Except from the oscillatory interval, the nonlinear commands are comparable to linear commands in terms of behavior.
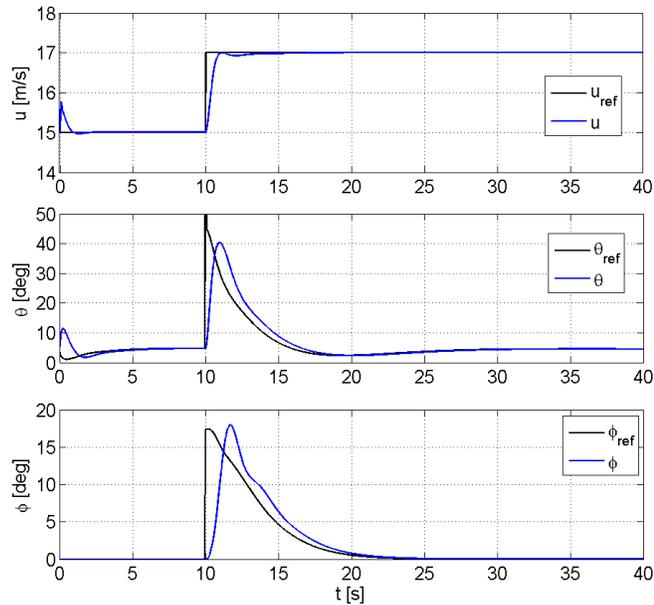
Fig. 5.6: Nonlinear model inner loop variables
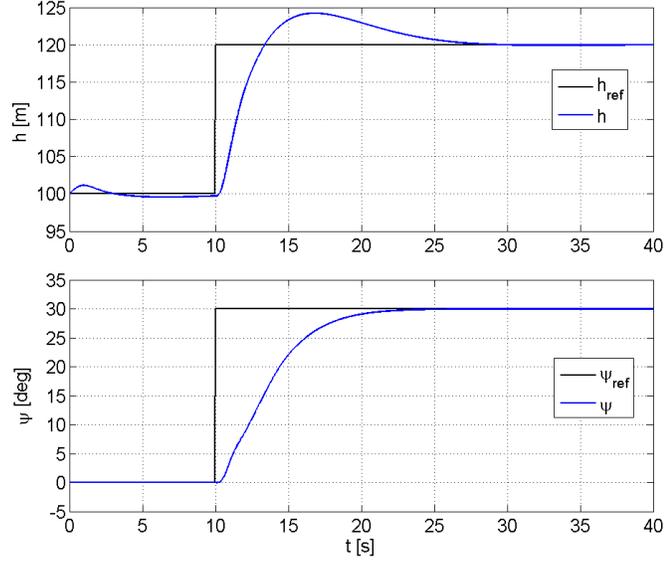
Fig. 5.7: Nonlinear model outer loop variables
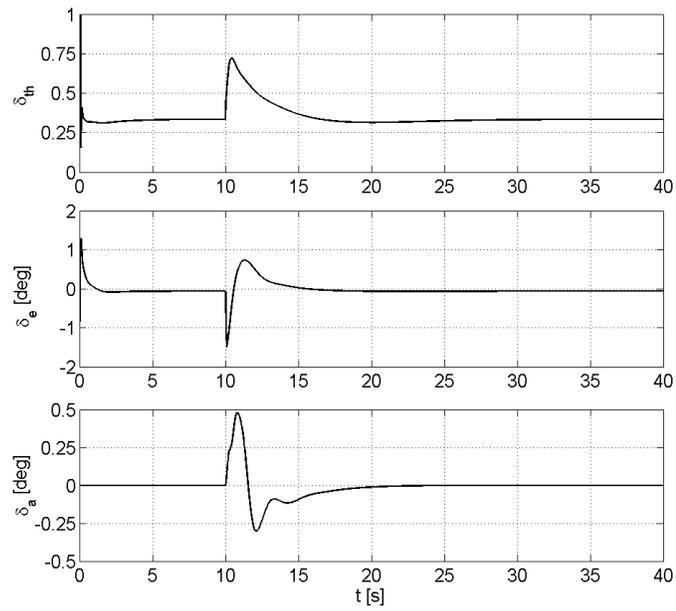
Fig. 5.8: Nonlinear model commands

5.3.3 Parametric robustness validation

A validation of the control scheme robustness to aircraft parameters uncertainties is performed with two test cases. In both of them significant variations in aircraft mass, inertia and static margin are considered. The altered parameters are introduced in the nonlinear aircraft model while the controller settings remain unchanged from the nominal case. Case 1 contemplates a heavier aircraft, with higher inertia and with a reduced static margin so that the derivatives $C_{m\alpha}$ and $C_{m\delta_e}$ are weaker. In Case 2 the aircraft is lighter, has lower inertia and its center of gravity is moved forward so that the magnitude of the derivatives is higher. In both cases the variation in m, I, $C_{m\alpha}$ and $C_{m\delta_e}$ is 30% from the nominal value. The adopted parameters are summarized in Table 5.2.

Tab. 5.2: Parameters for the parametric robustness validation cases

Parameter	Nominal	Case 1	Case 2
m [kg]	0.9	1.17	0.819
I [kg $\cdot$ m^2]	$\begin{bmatrix} 0.0109 & 0 & -4.6e^{-5} \\ 0 & 0.0119 & 0 \\ -4.6e^{-5} & 0 & 0.0223 \end{bmatrix}$	$\begin{bmatrix} 0.0142 & 0 & -5.98e^{-5} \\ 0 & 0.0155 & 0 \\ -5.98e^{-5} & 0 & 0.0291 \end{bmatrix}$	$\begin{bmatrix} 0.0076 & 0 & -3.22e^{-5} \\ 0 & 0.0083 & 0 \\ -3.22e^{-5} & 0 & 0.0156 \end{bmatrix}$
$C_{m\alpha}$	-0.3434	-0.2404	-0.4464
$C_{m\delta_e}$	-0.7317	-0.5122	-0.9512

The results of the simulations performed with the altered parameters are represented in Fig. 5.9, Fig. 5.10 and Fig. 5.11. For all variables no substantial difference is noticeable. However, it is interesting to observe the steady state value for the longitudinal variable θ. The alteration of the mass and of the derivatives $C_{m\alpha}$ and $C_{m\delta_e}$ results in a different pitch angle at equilibrium. As a consequence, trim elevator deflection δ_e changes too.

5.4 Conclusions

In this chapter it is demonstrated that a combination of PID and $\mathcal{L}_1$ adaptive controller is a suitable control logic for UAV autopilots. The PIDs take care of outer loop variables altitude and heading while $\mathcal{L}_1$ controls inner loop states. In particular, an $\mathcal{L}_1$ approach based on piecewise constant adaptation laws is selected in order to guarantee adaptation rates compatible with an autopilot board CPU and low computational effort.

It is illustrated how the tuning of the controller on a linear aircraft model is sufficient
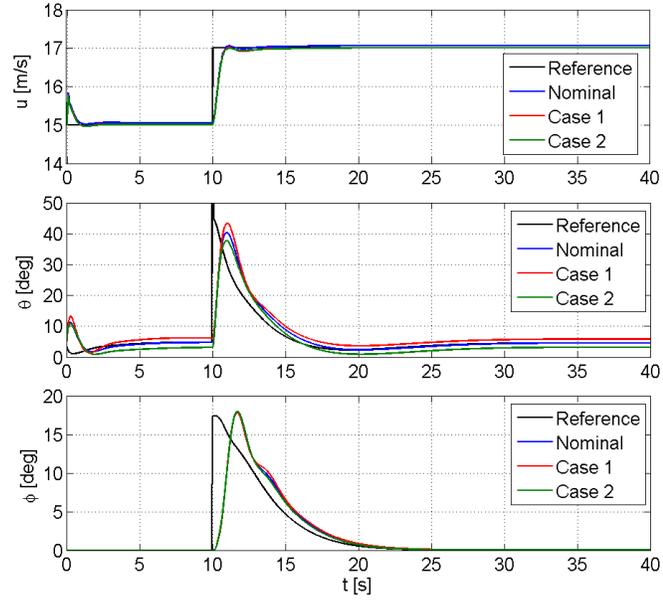
Fig. 5.9: Nonlinear model inner loop variables with parametric uncertainties
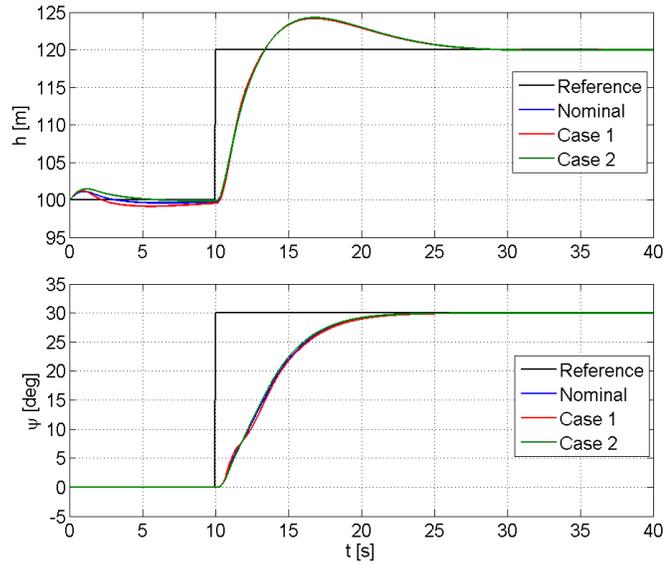
Fig. 5.10: Nonlinear model outer loop variables with parametric uncertainties
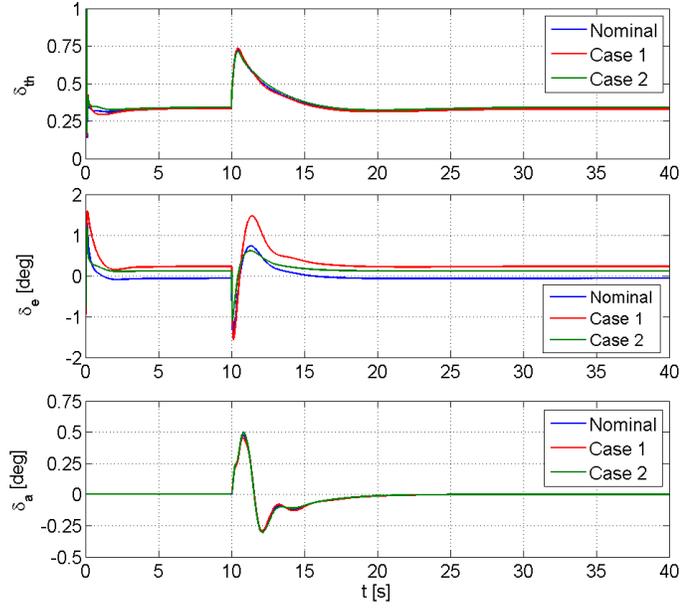
Fig. 5.11: Nonlinear model commands with parametric uncertainties

to guarantee satisfying flight performance also for the high-fidelity nonlinear model. This is relevant because a simplified linear model of the aircraft is in general easily available and easier to build and run. Furthermore, the adaptive nature of $\mathcal{L}_1$ guarantees a minimum variation of the controlled states also when the aircraft model is subject to large parametric uncertainties, which is typical of unmanned aircraft.

6. BACKSTEPPING NONLINEAR CONTROLLER

This chapter deals with the application of a backstepping nonlinear technique to the longitudinal and latero-directional control of the inner loop of a fixed-wing UAV. As explained in Section 2, this kind of approach has seen limited examples in the literature, and even less are the projects where real implementation was performed. Here the proposed method is briefly explained from the theoretical point of view and extensively validated with various software simulation cases. Furthermore, the designed controller is implemented on a microcontroller board and tested through hardware simulations.

6.1   Introduction to backstepping nonlinear controller

The concept of backstepping controller was developed in the early 1990s starting from the ideas of Tsinias [121], Byrnes and Isidori [122], Sontag and Sussmann [123], Kokotović and Sussmann [124], and Saberi, Kokotović and Sussmann [125]. The major contributions from these researchers who were investigating nonlinear feedback control theory were summarized and formalized by Kokotović in 1992 [126]. Three years later a book dealing with the complete theory for uncertain nonlinear systems was published by the same author together with Kstić and Kanellakopoulos [46]. Here a thorough treatment of the backstepping approach, including adaptivity, is presented.

The name backstepping derives from the recursive nature of the controller, the design process starts from the inner scalar function and "steps back" towards the external control input moving through a sequence of integrations. Each of these nonlinear differential equations governing the system is considered as a subsystem to be controlled [47]. The backstepping controller is designed so that the external control input controls in cascade all the subsystems, step by step, from the outer subsystem to the inner subsystem. Each subsystem is stabilized about the origin by a virtual control input which is the state of the differential equation higher in order. This technique requires that the equations describing the system dynamics may assume a certain structure, called strict-feedback form or, the more general, pure-feedback form [46]. An example of pure-feedback form is

$$
\begin{cases}
\dot{x} = f(x) + g(x)\xi_1 \\
\dot{\xi}_1 = f_1(x, \xi_1, \xi_2) \\
\dot{\xi}_2 = f_2(x, \xi_1, \xi_2, \xi_3) \\
\vdots \\
\dot{\xi}_{k-1} = f_{k-1}(x, \xi_1, ..., \xi_k) \\
\dot{\xi}_k = f_k(x, \xi_1, ..., \xi_k, u_c).
\end{cases}
\tag{6.1}
$$

In this form $x \in \mathbb{R}^n$ is the state vector and $\xi_{1,...,k}$ are scalars denoting other states of the system; the functions f_i $(i = 1, ..., k)$ are nonlinear and they depend only on x and on the states ξ_j $(j = 1, ..., i+1)$, i.e., they depend at most on the state variable of the upper order subsystem. The scalar u_c is the external controller of the global system; each subsystem represented by the state ξ_l $(l = 1, ..., k-1)$ is controlled by the virtual control input ξ_{l+1}. Table 6.1 summarizes the control sequence of the backstepping controller action for the system example of Equation (6.1).

Tab. 6.1: Control sequence for the system of Equation (6.1)

Step	Control	Directly controlled	Controlled in cascade
1	u_c	ξ_k	$\xi_{k-1}, \xi_{k-2}, \cdots, \xi_2, \xi_1, x$
2	ξ_k	ξ_{k-1}	$\xi_{k-2}, \cdots, \xi_2, \xi_1, x$
3	$\vdots$	$\vdots$	$\vdots$
4	ξ_3	ξ_2	ξ_1, x
5	ξ_2	ξ_1	x
6	ξ_1	x	-

6.2 Problem formulation and proposed control approach

One of main obstacles to the application of backstepping technique to fixed-wing aircraft lies in the condition imposed to the structure of the equations governing the system. As mentioned above, a backstepping controller may be derived and applied provided that the

equations can be arranged in the pure-feedback form of Equation (6.1). The system composed by Equations (3.4)-(3.6)-(3.8), as such, does not have this property. Since forces and moments in $\mathbf{F}$ and $\mathbf{M}$ are, in general, function of the states $\mathbf{V}$ and $\boldsymbol{\omega}$ and of the aerodynamic angles and control actions, the cascade form is not respected. Nevertheless, under some assumptions it is possible to convert the equations of motion into a suitable form for a limited number of aircraft states: angle of attack α, sideslip angle β and wind-axes roll rate p_w, see Fig. 6.1. The proposed approach was initially formulated in [73] and later proposed with some modifications in [74].
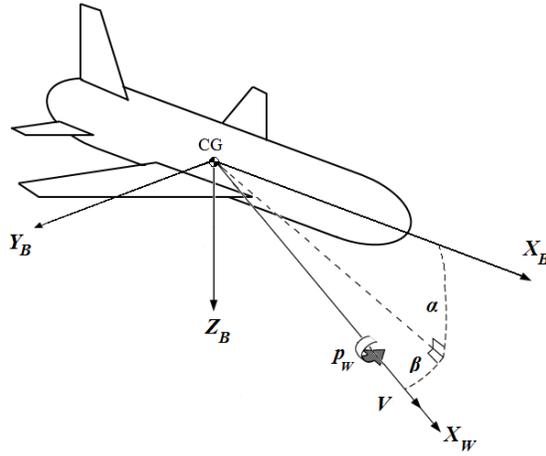
Fig. 6.1: Controlled variables and reference axes

6.2.1 Shaping of the equations of motion

The aim is to design a controller so that $\alpha = \alpha^{ref}$, $p_w = p_w^{ref}$ and $\beta = 0$. Control over angle of attack and roll rate is essential to determine, respectively, the longitudinal behavior and the flight direction of the aircraft. A null sideslip angle is desired to achieve symmetric flight during cruise and to reduce aerodynamic drag. In order to further reduce the amount of β generated during roll maneuvers, a velocity vector roll is adopted. This consists in a roll motion about the X_W axis with, ideally, α and β remaining constant. The choice of controlling p_w instead of p is motivated by the fact that, at high angles of attack, a roll maneuver about body axis X_B turns α into β, and vice versa.

The following assumptions are formulated:

- **Assumption 1**: The deflection of control surfaces only generates a variation in mo-

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ments, the variation in forces is small enough to be neglected.

- **Assumption 2**: Lift and side force coefficients, C_L and C_Y, only depend on aerodynamic angles and not on aerodynamic angle rates of change: $C_L = C_L(\alpha)$, $C_Y = C_Y(\beta)$. Derivatives $C_{L\dot\alpha}$ and $C_{Y\dot\beta}$ can be neglected.

The first assumption is reasonable for aircraft with traditional configuration, so that control surfaces are far from the aircraft center of gravity [78]. The deflection of a control surface generates forces and, as a consequence, moments. The comparison between the control derivative for a force and the one for the resulting moment shows that, in general, the moment derivative has same order of magnitude or is larger. In fact, its definition includes, among other terms, the product between the force derivative and the distance of the control surface from the center of gravity. Furthermore, the addition of a reference lever-arm distance in the moment mathematical formulation, see for instance Equation (6.16), increases the moment contribution with respect to the force contribution. Once the trim condition is achieved, the control deflections for maneuver are minimal, reducing to a negligible value the variation of forces so produced.

Assumption 2 is considered valid in steady flight or during smooth maneuvers. In fact, the disregarded aerodynamic derivatives $C_{L\dot\alpha}$ and $C_{Y\dot\beta}$ are originated by the delay in the pressure distribution of the unsteady flow to adjust to sudden attitude variation. Assumption 2 is on the conservative side as it targets progressive maneuvers, the ability of backstepping to control aggressive flight will be demonstrated.

The differential equations governing the variation in time of the controlled variables α, β and p_w are now obtained. The definition of the aerodynamic angles α and β is expressed in Equation (3.1). The equations relating their derivatives with the angular velocities and α and β themselves are

$$
\begin{aligned}
\dot\alpha &= q - (p\cos\alpha + r\sin\alpha)\tan\beta + \frac{Z_{aero}\cos\alpha - (X_{aero} + T)\sin\alpha + mg_2}{mV\cos\beta} \\
\dot\beta &= p\sin\alpha - r\cos\alpha + \frac{Y_{aero} - T\cos\alpha\sin\beta + mg_3}{mV}
\end{aligned}
\tag{6.2}
$$

where T is the engine thrust and X_{aero}, Y_{aero}, Z_{aero} are the aerodynamic forces in body axes. The gravity acceleration components g_2 and g_3 are

$$
\begin{aligned}
g_2 &= g(\cos\alpha\cos\theta\cos\phi + \sin\alpha\sin\theta) \\
g_3 &= g(\cos\beta\cos\theta\sin\phi + \sin\beta\cos\alpha\sin\theta - \sin\alpha\sin\beta\cos\theta\cos\phi)
\end{aligned}
\tag{6.3}
$$

where $g = 9.81$ m/s^2 is the gravity acceleration. Equation (6.2) can be written in a more compact and meaningful form. The relationship

$$Lift = X_{aero} \sin \alpha - Z_{aero} \cos \alpha$$

is used to include the lift force $Lift$ in the $\dot{\alpha}$ equation. Thanks to Equation (3.2) wind-axes angular rates are introduced in $\dot{\alpha}$ and $\dot{\beta}$ dynamics, the result is

$$
\begin{aligned}
\dot{\alpha} &= \frac{q_w}{\cos \beta} + \frac{-Lift - T \sin \alpha + mg_2}{mV \cos \beta} \\
\dot{\beta} &= -r_w + \frac{Y_{aero} - T \cos \alpha \sin \beta + mg_3}{mV}
\end{aligned}
\tag{6.4}
$$

The backstepping controller is designed to directly control the wind-axes angular velocities $\boldsymbol{\omega}_w$ through the control vector $\mathbf{u}_c = (u_1, u_2, u_3)^T$. Therefore, the dynamics of the wind-axes angular velocities are described by the relationship $\dot{\boldsymbol{\omega}}_w = \mathbf{u}_c$. Combining this formulation with Equation (6.4) gives

$$
\begin{cases}
\dot{p}_w = u_1 \\
\dot{\alpha} = \dfrac{q_w}{\cos \beta} + \dfrac{-Lift - T \sin \alpha + mg_2}{mV \cos \beta} \\
\dot{q}_w = u_2 \\
\dot{\beta} = -r_w + \dfrac{Y_{aero} - T \cos \alpha \sin \beta + mg_3}{mV} \\
\dot{r}_w = u_3
\end{cases}
\tag{6.5}
$$

Note that the lift force $Lift$ depends on the angle of attack through the $C_L = C_L(\alpha)$ coefficient and the side force Y_{aero} on the sideslip angle through the $C_Y = C_Y(\beta)$ coefficient. The thrust T is considered independent from the aerodynamic angles.

6.2.2 Backstepping controller design

In order to simplify the controller design, an additional set of assumptions is proposed:

- **Assumption 3**: The time derivatives of speed V, altitude h and heading ψ can be neglected as they have a slower rate of change compared to the controlled variables α, β and p_w.

- **Assumption 4**: Actuators have rapid enough dynamics, thus they can be ignored in the design process.

Assumptions 3 is mainly valid for cruise flight and progressive maneuvers. Here a controlled variation in the aircraft equilibrium state has a primary effect on the faster dynamics characterizing the attitude, and a secondary effect on the slow-changing variables defining the navigation. Finally, Assumption 4 is very common and generally reasonable, provided that Assumptions 2 and 3 are respected.

Equation (6.5) is not suitable for the application of a total backstepping controller because the cascade form is not respected, in particular due to the presence of β in the α dynamics, and vice versa. However by separating its dynamics as

$$\dot{p}_w = u_1 \tag{6.6}$$

$$\begin{cases} \dot{\alpha} = \dfrac{q_w}{\cos\beta} + \dfrac{-Lift - T\sin\alpha + mg_2}{mV\cos\beta} \\[2mm] \dot{q}_w = u_2 \end{cases} \tag{6.7}$$

$$\begin{cases} \dot{\beta} = -r_w + \dfrac{Y_{aero} - T\cos\alpha\sin\beta + mg_3}{mV} \\[2mm] \dot{r}_w = u_3 \end{cases} \tag{6.8}$$

three sub-controllers stabilizing the desired states α, β and p_w can be defined. A cross-coupling exists. In fact, it is possible to observe the presence of β in the α dynamics and, at the same time, the presence of α in the β dynamics. During the individual sub-control design mathematics imposes β constant in the α controller, and α constant in the β controller. Instead, when dealing with the simultaneous control action on the three variables, this assumption is disregarded because not physically realistic and not necessary, as it will be shown later. Because of this coupling, the computation of a control action considers, at each moment, the value of the state controlled by another control action. For instance, the control law defining u_2 is evaluated with the instantaneous value of β controlled by u_3. This occurrence is beneficial when dealing with maneuvers where strong coupling between longitudinal and latero-directional planes exists.

A simple proportional controller is chosen for p_w, Equation (6.6), while the cascade form of Equation (6.7) and Equation (6.8) allows the application of backstepping controller for α and β. Note that Equations (6.7) and (6.8) have the similar structure

$$\begin{cases} \dot{\omega}_1 = f(\omega_1, y) + \omega_2 \\[2mm] \dot{\omega}_2 = u_w \end{cases} \tag{6.9}$$

A single backstepping controller designed for Equation (6.9) is suitable for Equations (6.7) and (6.8). As it is preferable to have the origin as the desired equilibrium point, a change of

variables is defined

$$x_1 = \omega_1 - H$$
$$x_2 = \omega_2 + f(H, y)$$
$$\Omega(x_1) = f(x_1 + H, y) - f(H, y)$$

where H is the reference value for the controlled variable. The resulting dynamics are

$$\begin{cases} \dot{x}_1 = \Omega(x_1) + x_2 \\ \dot{x}_2 = u_w \end{cases} \tag{6.10}$$

The external control input u_w controls x_2 that, in cascade, acts as virtual control to stabilize x_1. Table 6.2 summarizes the relationships between the variables used in the new and in the original systems. The functions $f_\alpha(\alpha, y_\alpha)$ and $f_\beta(\beta, y_\beta)$ are

$$f_\alpha(\alpha, y_\alpha) = \frac{-Lift - T\sin\alpha + mg_2}{mV}$$
$$f_\beta(\beta, y_\beta) = \frac{Y_{aero} - T\cos\alpha\sin\beta + mg_3}{mV}$$

As fully demonstrated in [74] through Lyapunov stability theory, a simple globally stabilizing control law for the system of Equation (6.10) is

$$u_w = -k_u(x_2 + \Psi(x_1)) \tag{6.11}$$

if, for all $x_1 \neq 0$, a constant k_u exists so that

$$k_u \geq \frac{\Omega(x_1)}{x_1}$$

The function $\Psi(x_1)$ is built so that $\Psi(x_1) = -x_2^{ref}$, where x_2^{ref} is the desired value for the state x_2 acting as virtual control input for the subsystem x_1. This choice guarantees asymptotic stability for the subsystem x_1

$$\dot{W}(x_1)\big|_{x_2=x_2^{ref}} = (\Omega(x_1) - \Psi(x_1))\, x_1 < 0, \quad x_1 \neq 0$$

having chosen as temporary control Lyapunov function

$$W(x_1) = \frac{1}{2}x_1^2$$

Tab. 6.2: Change of variable relationships

General system	Longitudinal	Latero-directional
ω_1	α	β
ω_2	$\dfrac{q_w}{\cos \beta}$	$-r_w$
u_w	$\dfrac{u_2}{\cos \beta}$	$-u_3$
y	β	α
$f(\omega_1, y)$	$\dfrac{f_\alpha(\alpha, y_\alpha)}{\cos \beta}$	$f_\beta(\beta, y_\beta)$
H	α^{ref}	0
x_1	$\alpha - \alpha^{ref}$	β
x_2	$\dfrac{q_w}{\cos \beta} + \dfrac{f_\alpha(\alpha^{ref}, y_\alpha)}{\cos \beta}$	$-r_w + f_\beta(0, y_\beta)$
$\Omega(x_1)$	$\dfrac{f_\alpha(\alpha, y_\alpha)}{\cos \beta} - \dfrac{f_\alpha(\alpha^{ref}, y_\alpha)}{\cos \beta}$	$f_\beta(\beta, y_\beta) - f_\beta(0, y_\beta)$

Furthermore it can be demonstrated that $\Psi'(x_1)$, the time derivative of $\Psi(x_1)$, is bounded

$$0 < \Psi'(x_1) < k_u$$

The global control Lyapunov function used to define the control law of Equation (6.11) is

$$V_L = \int_0^{x_1} -\Psi'(y)\left(\Omega(y) - \Psi(y)\right) dy + \frac{1}{2}\left(x_2 + \Psi(x_1)\right)^2$$

which satisfies

$$\dot{V}_L = -\Psi'(x_1)\left(\Omega(x_1) - \Psi(x_1)\right)^2 - \left(k_u - \Psi'(x_1)\right)\widetilde{x}_2^2$$

where $\widetilde{x}_2 = x_2 - x_2^{ref}$.

A linear control is chosen assigning $\Psi(x_1) = k_1 x_1$ so that

$$u_w = -k_2(x_2 + k_1 x_1)$$

with $k_2 > k_1 > \max\{0, k_u\}$. For $k_2 > 2k_1$ the controller is optimal as it minimizes the cost function

$$\int_0^\infty \left(k_1 (\Omega(x1) - k_1 x_1)^2 + \left(\frac{k_2}{2} - k_1 \right) (x_2 + k_1 x_1)^2 + \frac{u_w^2}{2k_2} \right) dt$$

Using the relationships of Table 6.2, the control laws for the systems of Equations (6.7) and (6.8) are obtained

$$
\begin{aligned}
u_2 &= -k_{\alpha,2} \left(q_w + k_{\alpha,1} \left(\alpha - \alpha^{ref} \right) \cos \beta + f_\alpha(\alpha^{ref}, y_\alpha) \right) \\
u_3 &= k_{\beta,2} \left(-r_w + k_{\beta,1} \beta + f_\beta(0, y_\beta) \right)
\end{aligned}
\tag{6.12}
$$

with

$$
\begin{aligned}
k_{\alpha,2} &> 2k_{\alpha,1}, \quad k_{\alpha,1} > \max\{0, k_\alpha\} \\
k_{\beta,2} &> 2k_{\beta,1}, \quad k_{\beta,1} > \max\{0, k_\beta\}
\end{aligned}
\tag{6.13}
$$

where

$$
\begin{aligned}
k_\alpha &= \max_{\alpha, y_\alpha} \frac{\partial f_\alpha(\alpha, y_\alpha)}{\partial \alpha} \\
k_\beta &= \max_{\beta, y_\beta} \frac{f_\beta(\beta, y_\beta) - f_\beta(0, y_\beta)}{\beta}
\end{aligned}
$$

Finally, a proportional control is adopted for p_w

$$u_1 = k_{p_w} \left(p_w^{ref} - p_w \right), \quad k_{pw} > 0 \tag{6.14}$$

The relation between control inputs and wind-axes angular accelerations is defined by $\mathbf{u_c} = (u_1, u_2, u_3)^T = \dot{\boldsymbol{\omega}}_w$. Angular accelerations are the result of the variation in moments originated primarily by the deflection of aircraft control surfaces. The vector of deflections $\boldsymbol{\delta}$ is obtained from the moment equation (Equation (3.6)) rearranged

$$\mathbf{M}(\boldsymbol{\delta}) = I \left(R_{wb}^T \mathbf{u_c} + \dot{R}_{wb}^T \boldsymbol{\omega}_w \right) + \boldsymbol{\omega} \times I \boldsymbol{\omega} \tag{6.15}$$

To calculate $\boldsymbol{\delta}$, a control strategy matching the controlled variables with the aircraft control surfaces must be defined.

6.2.3   Control strategy

The controller described above stabilizes three variables related to the attitude of the aircraft. A global autopilot configuration capable of controlling speed V, altitude h and heading

ψ is required. In real-life implementation, these variables could be easily measured with, respectively, a pitot tube, a barometric pressure sensor and magnetometer. The control strategy is defined as follows: the backstepping controller acts on α, β and p_w in the inner loop, three PID controllers act on V, h and ψ in the outer loop. This approach separates the fast dynamics, characterizing aircraft attitude, from the slower dynamics, characterizing aircraft navigation. The prompt response of the backstepping controller is necessary when dealing with fast-changing inner loop variables. These, in fact, are of prime importance for the aircraft safety. For instance, an immediate control of α for a UAV affected by vertical gust could prevent the stall and dangerous flight regimes. Consistent with Assumption 3, slower variation of the navigation variables can be successfully handled using traditional PID technique. PID gains are tuned manually following a trial and error approach. The goal is optimizing the response in terms of values of overshoot, rise time, settling time and ringing compatible with aircraft behavior, see Equation (4.3).

The PID controlling the speed feeds the backstepping controller with the desired angle of attack, while the PID controlling the heading defines the desired roll rate. These values are limited in magnitude in order to avoid the request of a motion incompatible with the aircraft dynamics during sudden maneuvers. In particular, standard values for the imposed saturation are the stall angle of attack for α^{ref} and typical roll rate for p^{ref}. Note that in the outer loop the desired roll rate is expressed in body axes (p^{ref}), the conversion to wind axes (p_w^{ref}) is performed with Equation (3.2). The control surfaces employed are the elevator δ_e, the aileron δ_a and the rudder δ_r. According to Assumption 1 these only generate a variation in moments and not in forces. The deflection vector $\boldsymbol{\delta} = (\delta_e, \delta_a, \delta_r)^T$ is obtained substituting in Equation (6.15) the most general expressions of the moments

$$
\begin{aligned}
L(\delta_a, \delta_e, \delta_r) &= \frac{1}{2}\rho V^2 Sb \left(C_{l\beta}\beta + C_{l\dot{\beta}}\dot{\beta} + C_{lp}\hat{p} + C_{lr}\hat{r} + C_{l\delta_a}\delta_a + C_{l\delta_e}\delta_e + C_{l\delta_r}\delta_r \right) \\
M(\delta_a, \delta_e, \delta_r) &= \frac{1}{2}\rho V^2 Sc \left(C_{m0} + C_{m\alpha}\alpha + C_{m\dot{\alpha}}\dot{\alpha} + C_{mq}\hat{q} + C_{m\delta_a}\delta_a + C_{m\delta_e}\delta_e + C_{m\delta_r}\delta_r \right) \\
N(\delta_a, \delta_e, \delta_r) &= \frac{1}{2}\rho V^2 Sb \left(C_{n\beta}\beta + C_{n\dot{\beta}}\dot{\beta} + C_{np}\hat{p} + C_{nr}\hat{r} + C_{n\delta_a}\delta_a + C_{n\delta_e}\delta_e + C_{n\delta_r}\delta_r \right)
\end{aligned}
\tag{6.16}
$$

and solving the resulting linear system with three equations and three unknowns. The nondimensional angular rates $\hat{p}$, $\hat{q}$ and $\hat{r}$ calculated as

$$
\hat{p} = \frac{pb}{2V}, \quad \hat{q} = \frac{qc}{2V}, \quad \hat{r} = \frac{rb}{2V}
$$

and ρ is the air density, b is the aircraft wingspan, c the mean aerodynamic chord and S the

wing surface. The aerodynamic derivatives are C_{m0}, $C_{m\alpha}$, $C_{m\dot{\alpha}}$, C_{mq}, $C_{l\beta}$, $C_{l\dot{\beta}}$, C_{lp}, C_{lr}, $C_{n\beta}$, $C_{n\dot{\beta}}$, C_{np} and C_{nr}, while the control derivatives are $C_{m\delta_a}$, $C_{m\delta_e}$, $C_{m\delta_r}$, $C_{l\delta_a}$, $C_{l\delta_e}$, $C_{l\delta_r}$, $C_{n\delta_a}$, $C_{n\delta_e}$ and $C_{n\delta_r}$. Note that commonly the contribution of $C_{m\delta_a}$, $C_{m\delta_r}$, $C_{l\delta_e}$ and $C_{n\delta_e}$ is very small or zero. In this case the calculation of the commands is more simple: δ_e is found from the $M(\delta_e)$ equation, while δ_a and δ_r are found solving the linear system with $L(\delta_a, \delta_r)$ and $N(\delta_a, \delta_r)$. The engine thrust vector is considered aligned with the aircraft X_B axis, it does not generate moments.

The third PID controls the altitude by defining the required throttle value δ_{th} independently of the backstepping controller which, in fact, acts through angular rates. The outer loop strategy, where control surfaces (as a matter of fact the elevator) control airspeed and throttle controls altitude, is a standard autopilot mode. As explained in [127], this approach guarantees better tracking of the airspeed which is a key parameter for an unmanned aircraft. Table 6.3 summarizes the controlled variables, their commands, and the control method.

Tab. 6.3: Relationship between variables and commands

Outer loop	Inner loop	Command	Control method
V	α	δ_a, $\boldsymbol{\delta_e}$, δ_r	Backstepping + PID
h	$-$	δ_{th}	PID
ψ	p_w, β	$\boldsymbol{\delta_a}$, δ_e, $\boldsymbol{\delta_r}$	Backstepping + PID

The proposed and implemented control scheme is shown in Fig. 6.2. The resulting elevator, aileron and throttle control inputs act on the nonlinear aircraft model. The measures of the controlled states, total speed, altitude and heading angle, are the feedback variables. The differences with the corresponding reference values, V^{ref}, h^{ref} and ψ^{ref}, define the error inputs for the PIDs. The throttle command and the measured speed are given as input to the backstepping controller as required by the control law definition and for the estimation of the inner loop states.

Note, in fact, that the variables α, β and p_w, used for the definition of the inner loop error, are estimated inside the backstepping controller with good degree of accuracy, as later demonstrated in Fig. 6.4, integrating Equations (6.6)-(6.8). A support to the accurate estimation of α and β is provided by the feedback of the measures of ϕ and θ. These values appear in Equation 6.3 for the calculation of g_2 and g_3. The reason for this unconventional solution lies in the intention, later explained in Section 6.3.2, of implementing and testing
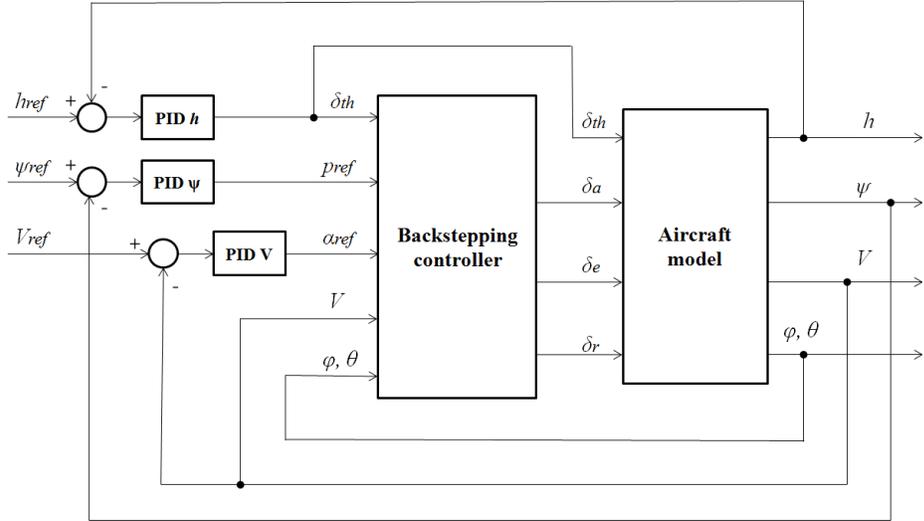
Fig. 6.2: Backstepping control strategy for fixed-wing aircraft

in flight the backstepping controller on a real aircraft. The possibility to effectively estimate these variables much simplifies the structure of the autopilot system and significantly reduces the development time and cost. The need for a measure of α and β would be undermined by the lack of affordable, reliable and compact aerodynamic angles sensors suitable for small UAVs.

6.3   Simulation results

In order to test the proposed control approach, the controller is implemented in MathWorks Matlab/Simulink and applied to the MH850 nonlinear aircraft model already employed in the previous chapters. The integration of the equations is performed through a 2nd order Heun method with 0.01 seconds time step. This low-order method is chosen as tradeoff between accuracy and computational power required. It is interesting to explain how the calculation of the commands for the MH850 rudderless configuration is performed. As already pointed out, the δ_e command is found from the $M(\delta_e)$ equation as $C_{m\delta_a} = 0$. Both L and N moments are function of the remaining command δ_a, this generates a system of two equations with one unknown which cannot be solved. It is chosen to disregard the $N(\delta_a)$ equation and to obtain δ_a from $L(\delta_a)$. This is motivated by the strong predominance of the rolling moment over the yawing moment in case of aileron deflection. In fact, for the MH850 UAV $C_{l\delta_a} \approx 10 \cdot C_{n\delta_a}$.

Within the backstepping controller only the gains of Equation (6.12) and Equation (6.14) need to be defined, their values are illustrated in Table 6.4.

Tab. 6.4: MH850 backstepping controller gains

$k_{\alpha,1}$	$k_{\alpha,2}$	$k_{\beta,1}$	$k_{\beta,2}$	k_{pw}
10	30	20	60	1

As the hybrid tuning approach described in Chapter 4 is not applicable to the considered structure, the three outer loop PIDs are tuned manually taking in consideration the specifications expressed in Equation (4.4). Their values are summarized in Table 6.5.

Tab. 6.5: MH850 outer loop PID gains

	K_P	K_I	K_D
Angle of attack from Airspeed	-0.08	-0.05	-0.045
Throttle from Altitude	0.08	0.01	0.04
Roll rate from Heading	0.16	0	1.05

The controller is tested for the same coupled maneuver proposed to the $\mathcal{L}_1$ adaptive controller in the nonlinear case, see Section 5.3.2. The only difference resides in the fact that here the controlled speed parameter is the total velocity V and not just its longitudinal component u. Reference values are defined as $V^{ref} = 17$ m/s, $h^{ref} = 120$ m and $\psi^{ref} = 30$ deg, these are typical figures expected in standard flight conditions. Longitudinal and latero-directional commands are applied at the same time. The outer loop responses are represented in Fig. 6.3. The capability of the controller to effectively control the inner loop variables with good tracking and short settling time is demonstrated. Note that although no rudder is used, the response on the heading angle ψ is still satisfying with aileron control.

Fig. 6.4 and Fig. 6.5, respectively, show the inner loop responses and the commands. Each of the inner loop plots includes the reference value, the state estimated within the backstepping controller and the real aircraft state. The accurate outer loop velocity tracking is achieved thanks to an excellent angle of attack controlled response in the inner loop. In this case the absolute value of α is bounded to 12 degrees in order to avoid near-stall conditions. Sideslip angle β shows some oscillations originating at the moment of transition. These are
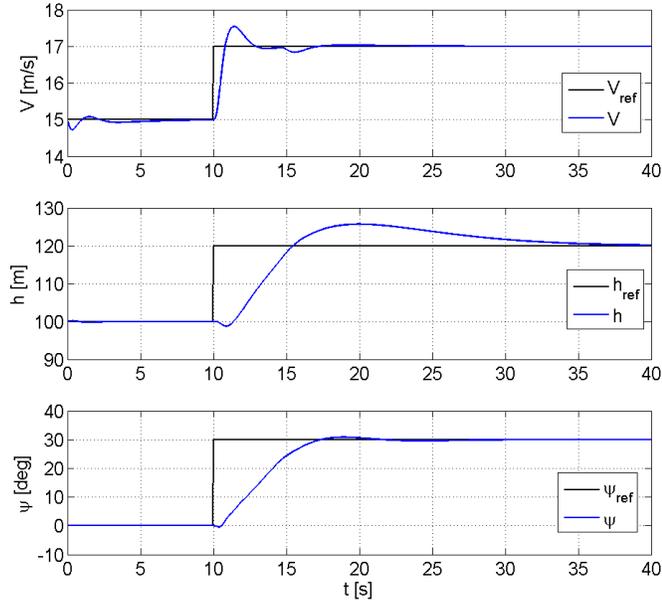
Fig. 6.3: Nonlinear model outer loop variables

generated by the magnitude of the gain necessary to achieve a satisfying tracking of p_w and therefore of ψ. The limited directional damping provided by the vertical fins at the wingtips of the MH850 is insufficient to prevent their onset. In any case the magnitude of the oscillations is minimal, less than 0.4 degrees, barely noticeable in a real aircraft application. The elevons deflections always remain within the 20 degrees maximum value, throttle saturation is measured only for few seconds after the step time.

6.3.1   Parametric robustness validation

The backstepping controller is validated by testing its performances when parametric uncertainties are introduced in the nonlinear aircraft model. The same two cases already proposed for the $\mathcal{L}_1$ controller are considered. Significant variations in aircraft mass, inertia and static margin are adopted, the altered parameters were summarized in Table 5.2.

The comparison with the nominal case is analyzed. In the outer loop response of Fig. 6.6 V and ψ tracking, which are controlled through backstepping, remain almost unchanged despite the considerable variation of the parameters. A slightly higher oscillatory behavior in V is observable for Case 1 motivated by a lower pitch damping. The altitude response, controlled with throttle through PID, suffers stronger variations from the nominal case. As
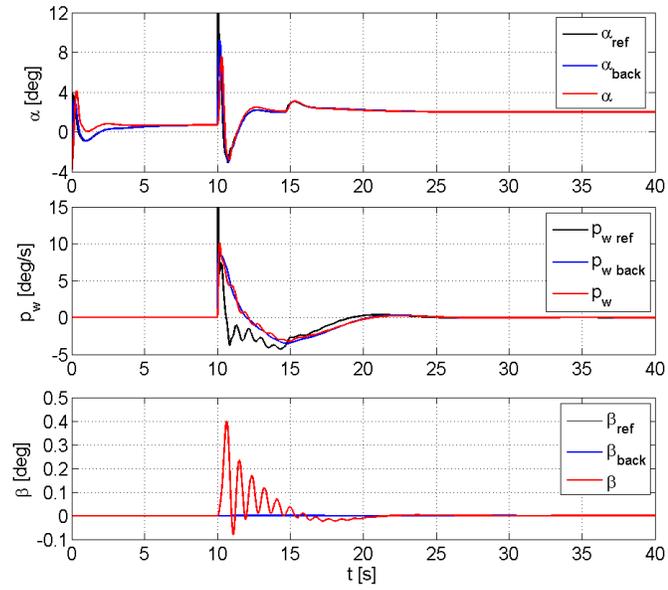
Fig. 6.4: Nonlinear model inner loop variables

Fig. 6.5: Nonlinear model commands

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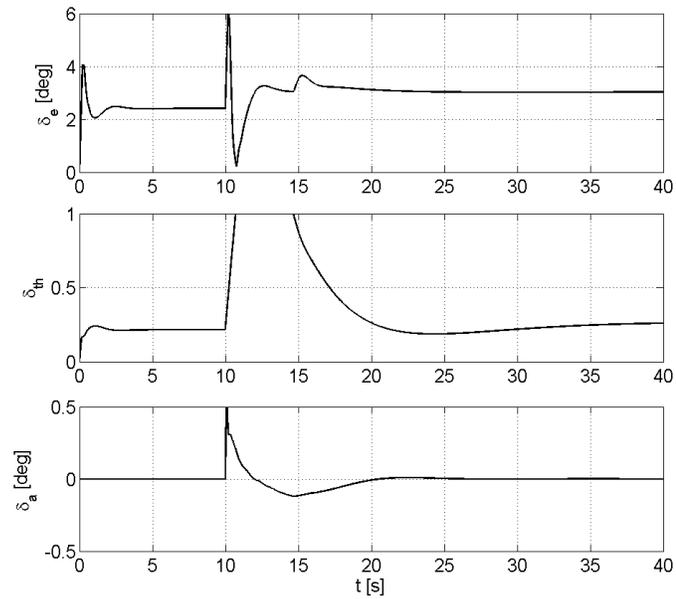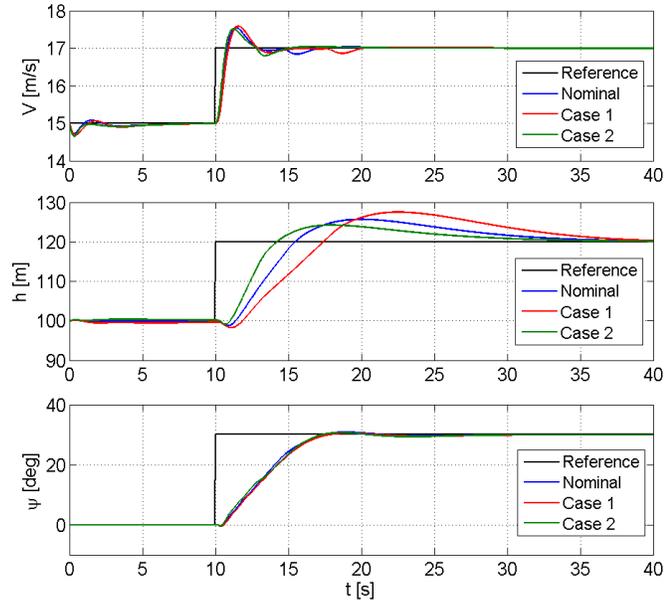
Fig. 6.6: Nonlinear model outer loop variables with parametric uncertainties
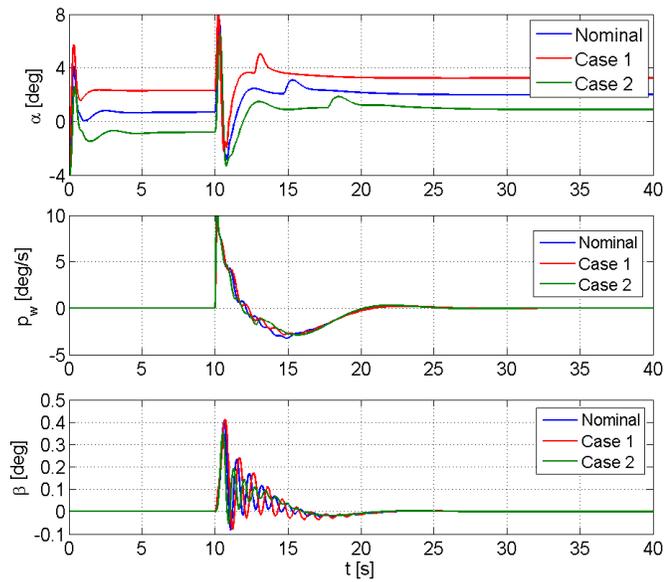
Fig. 6.7: Nonlinear model inner loop variables with parametric uncertainties
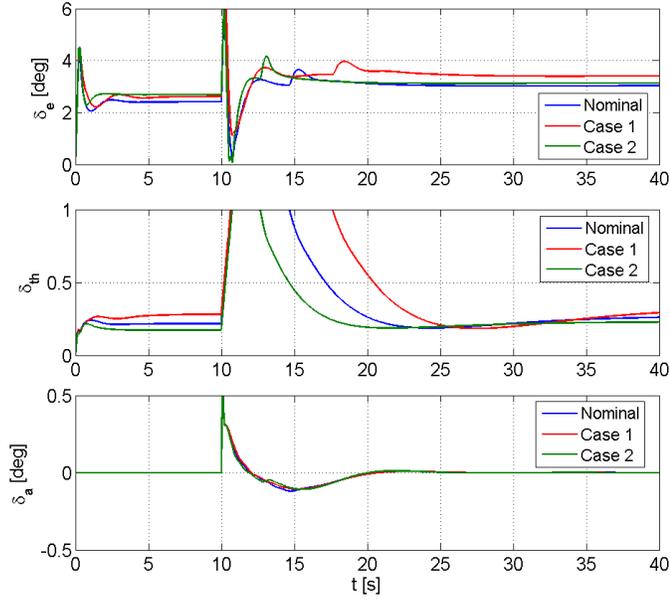
*Fig. 6.8:* Nonlinear model commands with parametric uncertainties

expected the aircraft with higher mass and inertia has a slower response to step input, higher overshoot and settling time. In the inner loop, Fig. 6.7, lateral oscillations are emphasized in Case 1. It is interesting to observe how the α trim condition changes in the two cases. The commands plot of Fig. 6.8 confirms that full throttle command is required longer for the heavier aircraft of Case 1. The backstepping controller proves to be robust to substantial parametric uncertainties as it guarantees satisfying performances in all the examined cases.

6.3.2 Confrontation among backstepping, $\mathcal{L}_1$ and PID controllers

The three controllers proposed in this project are compared in their outer loop performances for the nominal case, see Fig. 6.9. The comparison of the responses indicates that $\mathcal{L}_1$ is the best performer on the longitudinal plane, while backstepping guarantees better ψ control. The PID has worst performance in the V and ψ control, while the h response is equivalent to the one from backstepping because the same gains for altitude control are employed. Note, in fact, that in this case the nonlinear model is tested and the PID gains designed in Chapter 4 need to be adjusted in order to accommodate for this. As PID and backstepping controllers share the throttle to altitude loop, the gains tuned for the backstepping already represent a satisfying choice.

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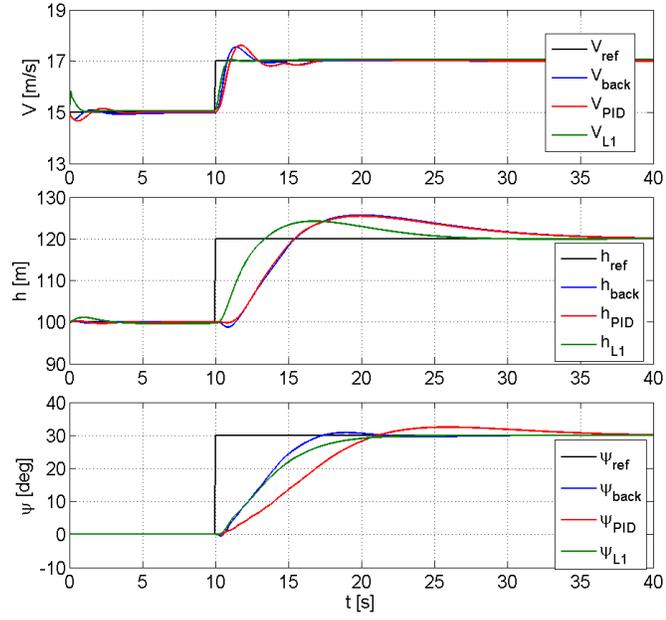
Fig. 6.9: Outer loop variables confrontation among the proposed controllers
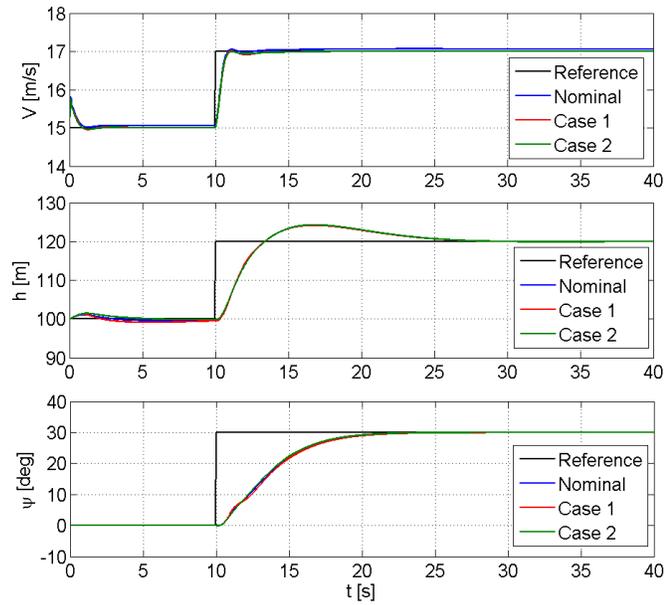
Fig. 6.10: $\mathcal{L}_1$ controlled navigation variables with parametric uncertainties

It is also interesting to see how the three controllers behave when then nonlinear aircraft model is affected by parametric uncertainties related to its dynamical properties, see Table 5.2. The test was already proposed for $\mathcal{L}_1$ and backstepping, the main results for the navigation variables are here summarized and the PID case is included. Fig. 6.10 illustrates the results for $\mathcal{L}_1$, in the previous section the backstepping case was analyzed, see Fig. 6.6. The adaptive nature of $\mathcal{L}_1$ guarantees minimal differences among the considered cases. Backstepping is comparable but penalized by the control of the altitude performed with the throttle only via PID. The PID technique, in fact, proves not to be robust enough to handle large variations in aircraft model parameters. According to Fig. 6.11 the aircraft loses directional control, it accelerates while quickly losing altitude. It crashes to the ground in less than 10 seconds. Despite backstepping and PID controllers were almost equivalent in the nominal case, it is clear that a traditional PID configuration is not able to deal with substantial changes in the aircraft parameters.
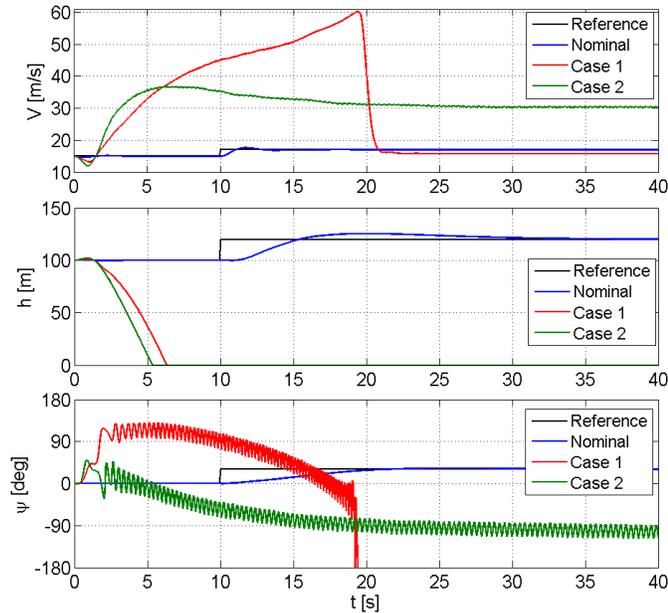
Fig. 6.11: PID controlled navigation variables with parametric uncertainties

The idea behind the proposed thesis is to actually implement and test a controller on a fixed-wing UAV. Among the three proposed controllers one needs to be chosen. The PID is discarded for two reasons. First of all it is widely employed and thus its implementation would not represent any innovative contribution. Furthermore, one of its well known

limitations is here demonstrated: it delivers unsuitable performance in presence of plant uncertainties. When comparing $\mathcal{L}_1$ and backstepping similar performance is observable in the nominal case, with $\mathcal{L}_1$ proving to cope slightly better with uncertainties thanks to its adaptive nature. In recent years many publications deal with $\mathcal{L}_1$ implementation but, as explained in Chapter 2, very few, if not none, are the real-life examples of backstepping controller for fixed-wing aircraft contemporary longitudinal and latero-directional control. Its implementation would represent a novel contribution to the development of this controller. Furthermore, the proposed backstepping control scheme is simple enough to guarantee easy implementability without sacrificing performance. The idea behind this backstepping approach, in fact, is to provide a starting framework for the actual employment of backstepping control technique on microcontrollers for small UAVs. Adaptation and a more advanced outer loop design is beyond the scope of this thesis. For these reasons backstepping is chosen for implementation.

6.4   C172P SIL simulation results

The discretization of the controller is the first necessary step towards real-time implementation. A different approach is applied to test the controller code which will be later implemented on the microcontroller board, software-in-the-loop (SIL) simulations. The control law is implemented in C code and applied to the Cessna 172P model present in the FlightGear simulator.
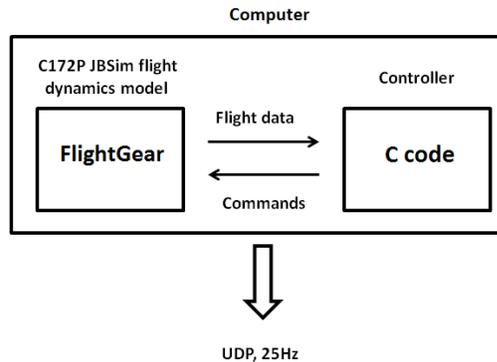
Fig. 6.12: FlightGear SIL layout

The data transfer between the C application and FlightGear is performed through User Datagram Protocol (UDP), the layout is illustrated in Fig. 6.12. With UDP, an application can send messages to another host on an Internet Protocol (IP) network without prior com-

munications to set transmission channels or data paths. As UDP does not perform checks on transmission errors, it is considered an unreliable protocol. However, when the physical and data link layers are reliable, like in this case, its performance improve considerably. UDP is suitable for uses where error checking and correction is either not necessary or performed within the application. FlightGear provides the value of the feedback variables, the controller returns the commands δ_e, δ_a, δ_r and δ_{th}. A 25 Hz frequency is chosen in order to guarantee a consistent data rate compatible with real sensors. Integration is performed with a 2nd order Heun method with 0.01 seconds (100 Hz) time step.
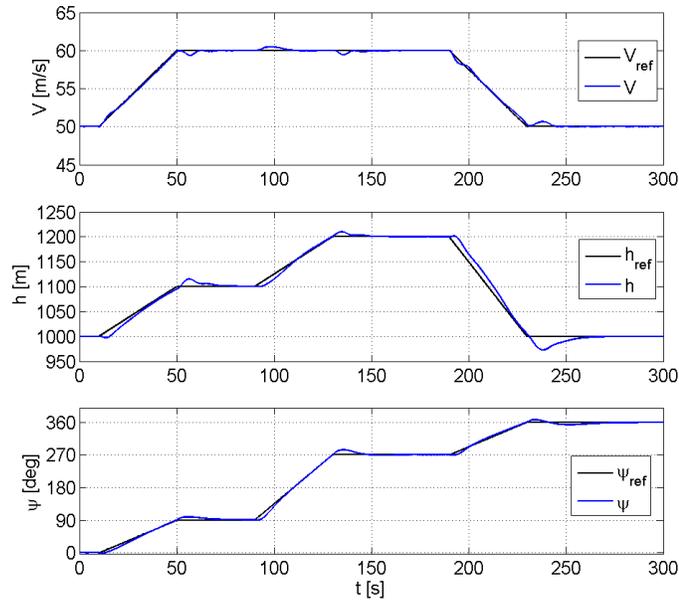
Fig. 6.13: FlightGear SIL simulated maneuver for C172P

In Fig. 6.13 the results of a complex maneuver are shown. The aircraft is requested first to climb and turn while accelerating, then to maintain the speed while climbing and turning more aggressively, finally to decelerate while performing another turn and rapidly losing altitude. The different nature of the reference to be followed is motivated by the different responses expected from the two airplanes: aggressive for the UAV and progressive for the Cessna. A more aggressive request to the C172P, for instance a higher climbing rate, would still result in zero steady-state altitude error but with a larger error in the climbing phase. This is not due to a problem with the controller, but by the lack of power of the C172P. Despite these differences, the same controller configuration is demonstrated to work

for aircraft different in size and configuration with adequate results.
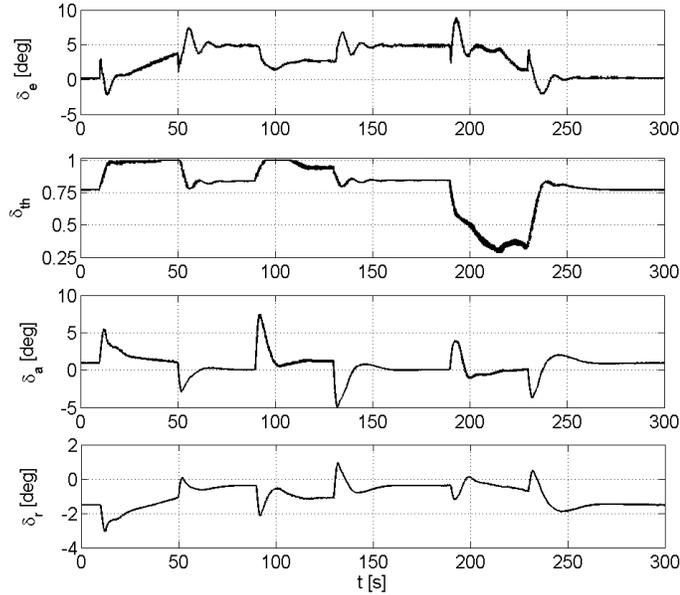
Fig. 6.14: FlightGear SIL simulated maneuver commands for C172P

All variables are tracked with good accuracy in every phase of the maneuver. Speed tracking performs the best, the quick response is guaranteed by the choice of using the elevator instead of the throttle for its control. Similarly, the heading angle shows good results despite some mild overshoot. The altitude response is penalized by some overshoot/undershoot and some mild oscillations in the settling phase, note also the difference in slope between reference and actual values. The slower engine response and the low power-to-weight ratio are responsible for this. In this project, envisaging the UAV application, priority is given to the speed which is a sensible parameter to avoid stall. Finally, it is interesting to observe how the changes in altitude affect the speed. The corresponding commands are illustrated in Fig. 6.14. The surfaces deflections always remain well within the saturation limits, around 20 degrees for elevator and aileron, 16 degrees for the rudder. The motor instead goes full throttle during the climbing phases. The gains used in this simulation are summarized in Table 6.6 and Table 6.7.

Tab. 6.6: C172P backstepping controller gains

$k_{\alpha,1}$	$k_{\alpha,2}$	$k_{\beta,1}$	$k_{\beta,2}$	k_{pw}
6	18	5	15	1

Tab. 6.7: C172P outer loop PID gains

	K_P	K_I	K_D
Angle of attack from Airspeed	-0.0105	-0.0021	0
Throttle from Altitude	0.012	0.001	0.0355
Roll rate from Heading	0.29	0.011	2.1

6.5   C172P HIL simulation results

After testing the controller in discrete time using software-in-the-loop approach, the next natural step is the hardware-in-the-loop simulation (HIL). The controller is implemented on a control board and tested in real time with the simulated aircraft model.
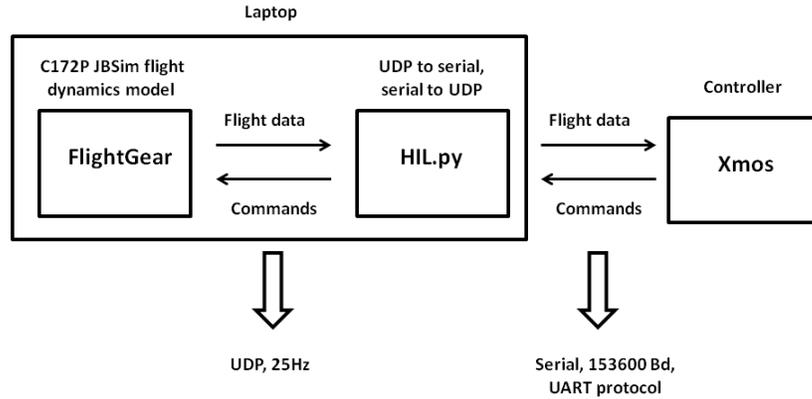
Fig. 6.15: FlightGear HIL layout

The HIL scheme is represented in Fig. 6.15 and its real implementation in Fig. 6.16. The simulator for the HIL is always FlightGear, its settings remain unchanged. This time the simulator does not communicate directly with the controller. A Python application,

HIL.py in Fig. 6.16, is used as a bridge between the simulator and the board. Its role is to capture flight data arriving from FlightGear through UDP and send them over serial protocol to the board; at the same time it acquires the serial commands from the board and sends them to FlightGear via UDP. The UDP data rate is maintained at 25 Hz. The application also stores all the data in transit into two text files. The Python application is necessary as direct serial communication between FlightGear and the board, even if theoretically possible, results unsuccessful. A baudrate of 153600 Bd is chosen to maximize the data transmission speed and to avoid overlap of send and receive tasks. The controller computation time is on average slightly less than 0.004 seconds. This result is obtained thanks to the 2nd order integration method and to the simplicity of the operations performed by the control law.

Fig. 6.16: FlightGear HIL setup

The controller is implemented on the XMOS XK-1A board, see Fig. 6.17. A breakout board for USB to serial conversion is placed between the XMOS board and the laptop. A detail of the HIL cables connection is represented in Fig. 6.18.

The XMOS board is a technology produced by XMOS Ltd [128]. This board is characterized by a multi-core multi-thread processor able to perform eight real-time tasks in parallel. Its parallel computing ability is essential for unmanned applications where high level tasks, for instance the control logic, have to be combined with low level assignments, such as I/O [77]. An advantage in using the XMOS technology is the ease of programming the board. The language for the XMOS board is called XC. This language, even if not too different from C, shows some additional commands for the management of ports and pins. Furthermore,

Fig. 6.17: XMOS XK-1A board
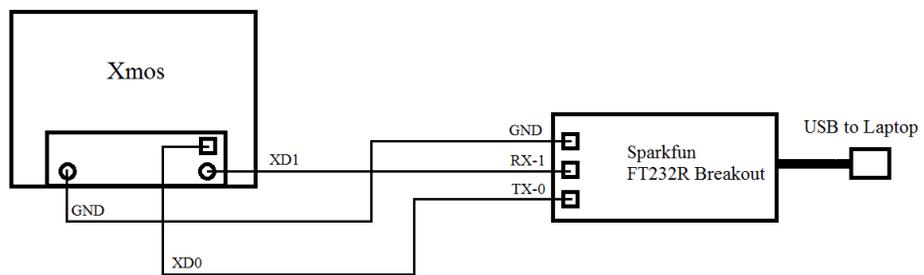
Fig. 6.18: HIL hardware connections

the major difference is the impossibility to handle floating point variables in the .xc files. This problem is overcome including .c files in the project. With a careful design of the C code, it is possible to use most of the files of section 6.4 to build the XC project. Some have to undergo minor corrections to adapt to the new control scheme. The board low cost, limited weight (19 g) and dimensions (50 x 50 mm) make it suitable for small UAV applications.

HIL simulations are performed with the same reference variables tested for the SIL simulations, backstepping and PID gains are unchanged. Fig. 6.19 validates the real-time implementation, the tracking achieved with the microcontroller is accurate and virtually identical to what is obtained in the software simulated case. The commands for this simulation are represented in Fig. 6.20, an excellent matching with the commands from the SIL simulation is evident.
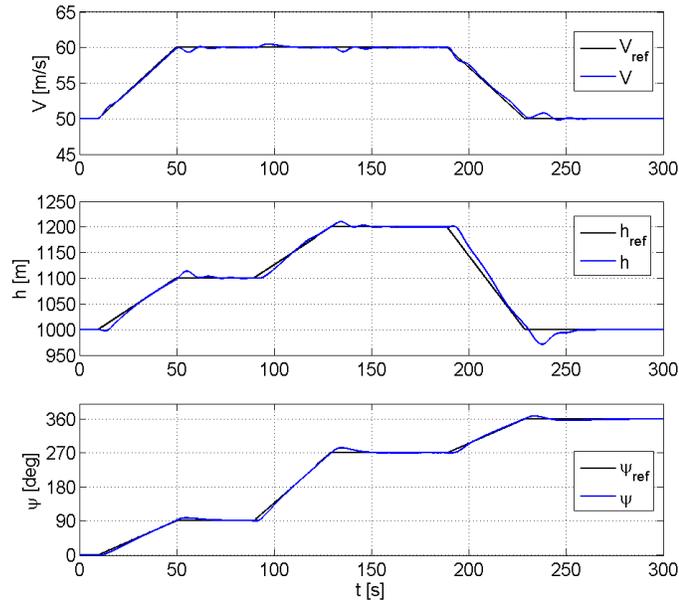
Fig. 6.19: FlightGear HIL simulated maneuver for C172P

6.6   Conclusions

In this chapter an autopilot configuration combining nonlinear backstepping control with traditional PID technique is presented. The backstepping controller is employed to stabilize fast inner loop variables characterizing the aircraft attitude, while PID gains control slower changing navigation variables.

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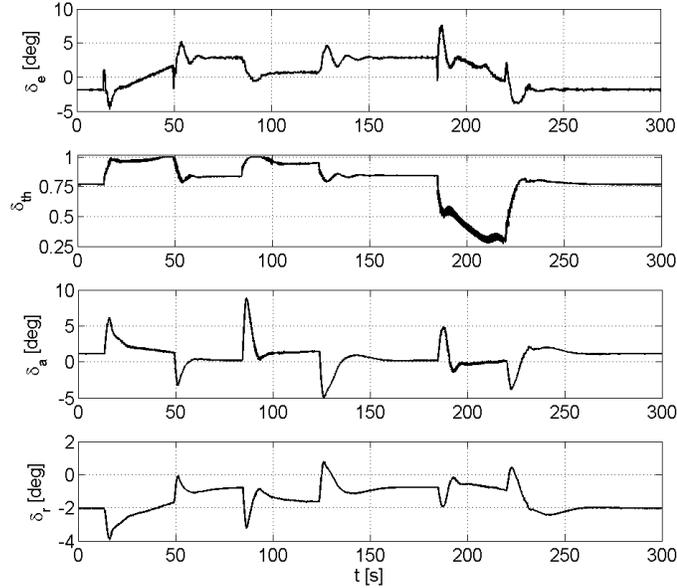
Fig. 6.20: FlightGear HIL simulated maneuver commands for C172P

Backstepping method is chosen for its ability to deal with the nonlinearities that characterize small fixed-wing UAVs dynamics. This method requires a fairly rich knowledge of the aircraft characteristics, but in return it ensures good performance over a large flight envelope. The adopted backstepping approach guarantees simultaneous control of the longitudinal and latero-directional planes. Through numerical simulations it is demonstrated that the proposed solution satisfactorily controls aircraft different in size and configuration, also in presence of large parametric uncertainties. If compared with the other proposed control approaches, backstepping is definitely superior to PID in terms of performance and robustness, and comparable with $\mathcal{L}_1$. Despite some conservative assumptions in the design process target smooth and progressive maneuvers, it is demonstrated that, aircraft allowing, aggressive flight is achievable. Complex maneuvers characterized by severe coupling are performed with little tracking error. The simple solution adopted differs from the standard adaptive backstepping approach, but it guarantees simple implementation and low computational power without loss of efficacy or robustness. In fact, an innovative real-time implementation on an autopilot board is also demonstrated for a complex maneuver and its performance is satisfying. The control strategy described herein is believed to be implementable on any microcontroller board suitable for small UAVs application.

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7. EXPERIMENTS AND FLIGHT TESTS

The previous chapter illustrates the design of the adopted backstepping approach, its implementation and the results of the simulations that prove its ability to control different aircraft configurations, also when working in real time on a microcontroller board. This chapter deals in detail with the preparation of the flight tests. The controller is integrated on a real aircraft, the following sections explain the setup defined, the problems encountered and the solutions proposed.

7.1   Sensors noise model

The backstepping strategy acts on three feedback variables, velocity V, altitude h and heading ψ, three sensors able to measure these parameters are thus necessary. Note, in fact, that the inner loop variables α, β and p_w are accurately estimated inside the backstepping controller, as it was demonstrated in Fig. 6.4. This unconventional but effective approach is adopted to maintain simple the physical integration of the controller on the aircraft and because of the lack of affordable, reliable and small aerodynamic angles sensors. The feedback of ϕ and θ must be provided as well to improve the inner loop states estimation. In this section the methods used to measure the outer loop variables are introduced. The data obtained by the sensors are used to characterize the noise spectrum which is included in the Simulink aircraft model in order the make the simulation as real as possible. In fact, the noise is expected to have great influence on the controller performance. The statistical properties of the sensors noise are analyzed, Gaussian white noise is added in Simulink to disturb the measurement.

7.1.1   Velocity measurement

The total velocity V is a key parameter as it strongly influences the aircraft states through is quadratic presence in the forces and moments acting on the airplane. In the controller structure here proposed the velocity is considered as a navigation outer loop variable, but as visible from Equation (6.5) it also directly affects the attitude of the aircraft. Achieving

a good measurement of the velocity is essential to achieve a good performance of the controller. The pitot tube method is chosen for its good precision and for its high sampling rate compatible with the controller frequency action. GPS is dismissed as not able to provide accurate data with high frequency.

The pitot principle of functioning is represented in Fig. 7.1. A pitot probe collects the total and static pressures encountered by the flying aircraft, these are carried to the Freescale Semiconductors MPXV7002 transducer [129] through two silicon tubes. The transducer outputs an analog voltage proportional to the difference between total and static pressure, this voltage is digitalized with a Linear Technology LTC 1865 16-bit Analog to Digital Converter (ADC) [130]. The digital voltage is read by the XMOS board which first converts the voltage value into pressure difference, as the pressure difference is equal to the dynamic pressure the airspeed is easily retrieved.
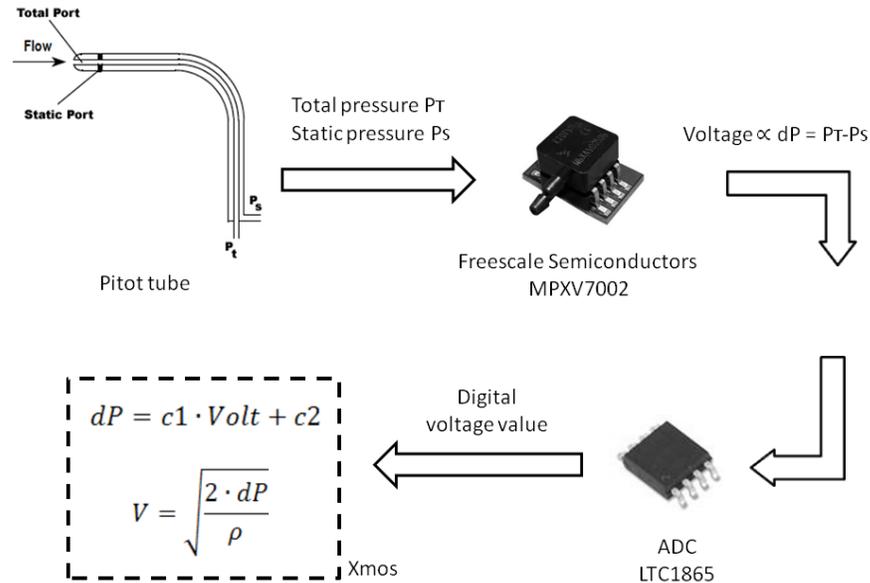
Fig. 7.1: Pitot airspeed measurement scheme

The law relating the voltage $Volt$ with the pressure difference dP is assumed linear with very limited error and the coefficients $c1$ and $c2$ are obtained through wind tunnel calibration, see Fig. 7.2. As a consequence the relationship between pressure difference and freestream velocity is quadratic. In this case the linear law is defined by
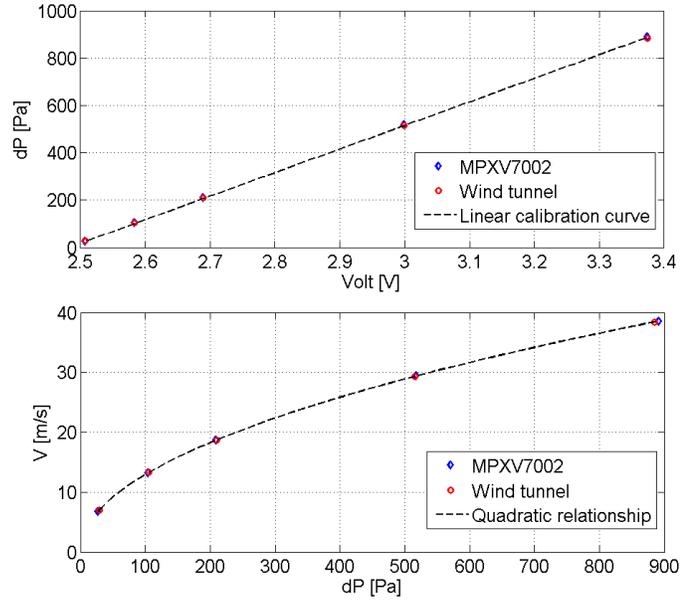
Fig. 7.2: Pitot calibration curve, comparison of data from pitot and wind tunnel

$$dP = 995.9 \cdot Volt - 2489.75$$

The wind tunnel tests were performed at the Politecnico di Torino fluid dynamics test facility. Five measurements at different velocity were performed, each one lasting for 20 seconds. The numbers plotted on Fig. 7.2 represent the mean values for each measurement. As each time series is characterized by noise, the adoption of a filter is necessary. Further details about the filtering operation will be introduced in Section 7.2. The ADC is able to sample data with a frequency up to 125 kHz when one channel is used. In this case the sampling frequency is limited to 20 Hz to match the slowest sensor, the barometric pressure sensor.

7.1.2 Altitude measurement

The measurement of the altitude is performed through a barometric pressure sensor, in particular by a Bosch BMP085 transducer [131] mounted on a Sparkfun breakout board [132], see Fig. 7.3. Considering the data frequency, range and precision required and the limited weight and space budget, other options such as GPS or laser and ultrasonic sensors were excluded.

Fig. 7.3: Bosch BMP085 barometric sensor on Sparkfun breakout board

The estimation of the altitude h relies on the measure of the local air pressure p through the formula

$$h = 44330 \cdot \left(1 - \left(\frac{p}{p_0} \right)^{0.1903} \right) \tag{7.1}$$

where p_0 is the equivalent pressure at sea level for the moment and the location considered. This parameter is generally available through weather station websites or can be calculated inverting the formula of Equation (7.1) in a location with known altitude after a single pressure measurement. In order to be independent from this value that can be unavailable or change during the day, the parameter given to the controller will be the height above the airfield (AGL, above ground level altitude) and not the absolute altitude. A first measurement is performed and stored when the control board is powered on the ground at the beginning of each flight, the AGL is obtained subtracting this value to future measurements.

The barometric pressure sensor has 4 operating modes, each characterized by a combination of sampling frequency and precision: the higher the precision, the lower the sampling frequency. The operating mode characterized by a 0.4 m root mean square noise error and 20 Hz sampling frequency is chosen as it offers the best compromise. The only faster frequency available, 22 Hz, comes with a typical 0.5 m error; the improvement of sampling speed is not enough to justify its choice. Like the pitot, this sensor is affected by noise and its output need to be filtered, more details will be added in Section 7.2.

7.1.3 Attitude measurement

The measurement of the attitude angles ϕ, θ and ψ is performed with a Vectornav VN-100 IMU chip [133] mounted on a development board, see Fig. 7.4. This sensor incorporates a 3-axis magnetometer, a 3-axis accelerometer and 3-axis gyroscope with extended Kalman

filter. Its outputs include the aircraft attitude expressed as Euler angles or quaternions, linear accelerations, angular rates or magnetic local field. In the considered case the angles of interest are obtained from the string containing the aircraft Euler angles, as represented in Fig. 7.5. The string is an output of the VN-100 sensor, it is received and analyzed by the XMOS board. For the measurement of ψ, this IMU is preferred to a simple compass because of its high measurement quality, the existence of an extended Kalman filter provides a smooth and noise-free output. Similarly to the other sensors the IMU is set to output data with a 20 Hz frequency.

Fig. 7.4: Vectornav VN-100 development board

Fig. 7.5: Example of Vectornav VN-100 string for attitude in Euler angles

7.2 Kalman filter

The presence of noise in the velocity and altitude measurements forces the inclusion of a filtering operation before feeding these values to the controller. This helps to prevent an oscillatory response which could be amplified by the presence of a derivative term in the outer loop PID control. A Kalman filter is thus implemented for the data coming from pitot and barometric pressure sensors. The IMU already includes an extended Kalman filter. The Kalman filter sequence for each sampling operation i is summarized in Equation (7.2)

$$p_i = p_{i-1} + Q$$
$$K = \frac{p_i}{p_i + R}$$
$$x_i = x_{i-1} + K \cdot (\widetilde{x}_i - x_{i-1}) \tag{7.2}$$
$$p_i = (1 - K) \cdot p_i$$

Here x_i is the filtered variable, $\widetilde{x}_i$ is the measured variable and p_i is the estimated error. K is the Kalman gain, Q is the process noise variance and R is the sensor noise variance. The value of R can be found by experimental tests, the value of Q is more difficult to estimate and it can be used as a parameter for tuning the intensity of the filter action.

Table 7.1 illustrates the values of R and Q chosen for both sensors. Moreover it indicates the standard deviation σ, with and without Kalman filter, from a mean value representing the trim condition. The improvement achieved with the filter is remarkable. Fig. 7.6 shows an example of filtering action on a randomly varying altitude measurement. It is possible to see how the filter was tuned in order to reduce the oscillations without introducing a considerable delay in the filtered time series.

Tab. 7.1: Kalman filter parameters

Measurement	R	Q	σ	σ_{Kalman}
V	0.029	0.0005	0.17 m/s	0.05 m/s
h	0.19	0.0008	0.44 m	0.16 m

7.3 Ultrastick 25e simulation results

Simulations are performed merging the Simulink model of the backstepping controller with the aircraft Simulink model available from the University of Minnesota UAV research group, see Section 3.6.4. The resultant system is integrated in discrete time with a time step of 0.01 seconds, equal to the controller action frequency, Heun method is still used. Sensors noise and a Kalman filter are modeled as described in the previous sections. The Simulink model is built so that the trim commands are already an input for the aircraft. Because of that, the backstepping controller, instead of calculating the absolute commands, calculates the commands variation to be added to the trim commands.
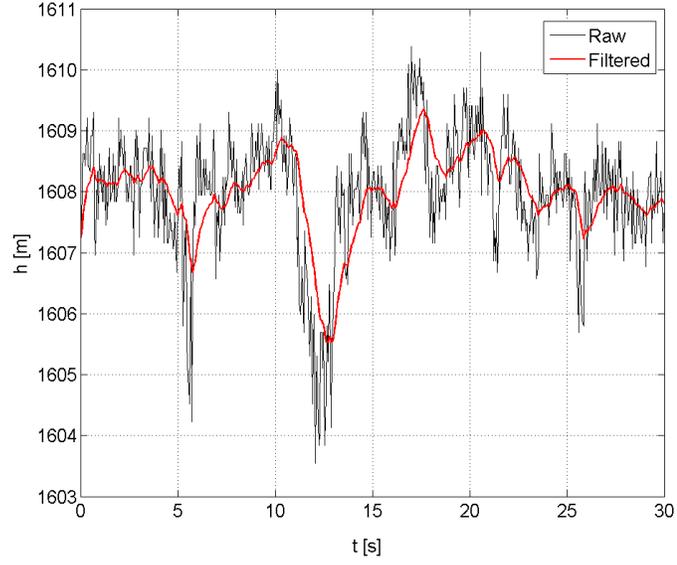
Fig. 7.6: Kalman filtering action on a noisy altitude measurement
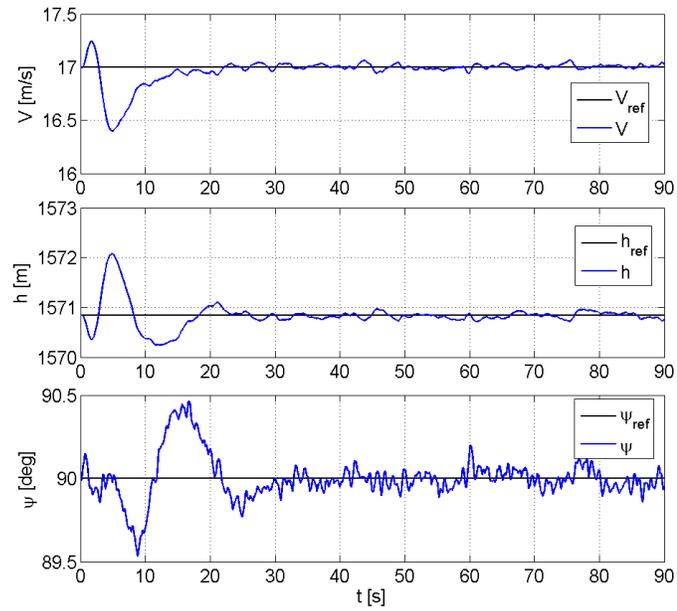
Fig. 7.7: Simulink outer loop response for Ultrastick 25e, trim conditions hold

Cruise conditions are chosen as starting values: V=17 m/s, h=1571 m and ψ=0 deg. The value for h corresponds to the altitude of the Denver research facility where initial tests are performed. It is important to keep the high altitude in consideration, for this value of h the air density is around 15% lower than sea level. Controlled results appear in Fig. 7.7 and 7.8. The first simulation asks the aircraft to maintain the cruise conditions. This represents one of the first flight tests that will be performed. As the aircraft is already trimmed for the cruise conditions, the controller has basically to adjust the commands so that they quickly return to the trim values after the usual initial oscillatory behavior. Fig. 7.7 shows that the aircraft is successfully brought back to trim conditions. An average time of 20 seconds is necessary, some minor noise-induced oscillations persist.
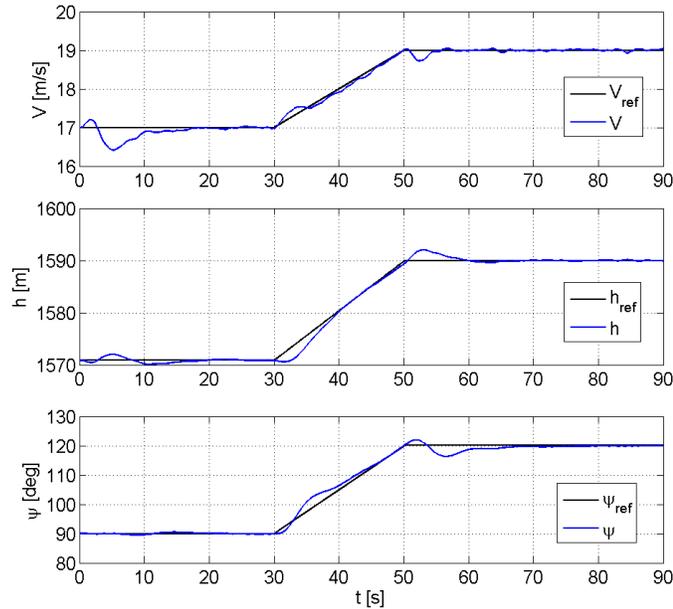
Fig. 7.8: Simulink outer loop response for Ultrastick 25e, maneuver

Fig. 7.8 shows the results when a coupled maneuver is requested. Small oscillations appear in the velocity, mostly because the derivative gain of the velocity is disturbed by the noise. The altitude response has a minor lag and overshoot when following the reference value during the slope segment, steady state value is successfully achieved. All states, in particular the heading angle ψ, suffer from some mild oscillations at the beginning and at the end of the ramp generated by cross-coupling between longitudinal and latero-directional planes. The corresponding commands are illustrated in Fig. 7.9. In spite of the filtering

action, the noise still has a strong influence on the elevator command because of the derivative gain in the outer loop PID on V. A reduction of this gain would reduce the damping and so generate an oscillatory behavior, the proposed solution is the best compromise between the two conflicting requirements. The throttle is less affected by noise as the derivative gain of the PID on h can be drastically reduced without the onset of oscillations. The gains for this configuration are illustrated in Table 7.2 and 7.3.
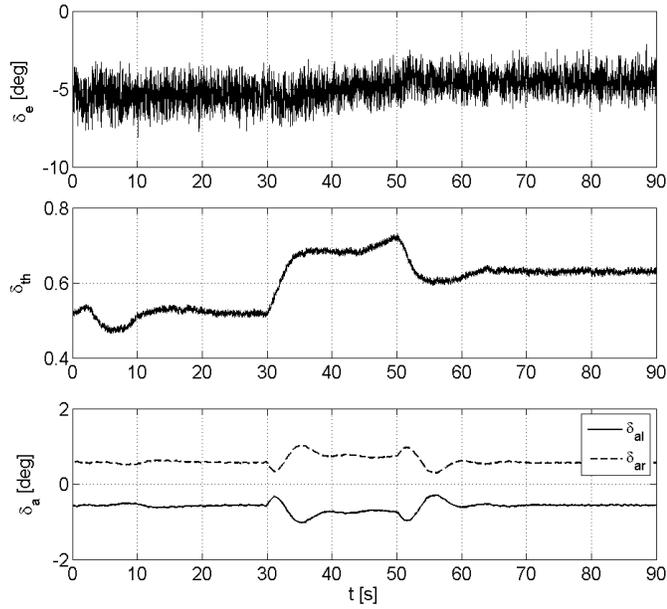
Fig. 7.9: Simulink commands for Ultrastick 25e, maneuver

Tab. 7.2: Ultrastick Simulink backstepping controller gains

$k_{\alpha,1}$	$k_{\alpha,2}$	$k_{\beta,1}$	$k_{\beta,2}$	k_{pw}
13	39	5	15	1

PID-based outer loop is built using standard Simulink PID blocks. Note that these blocks include a low pass filter in the derivative action that can be manually adjusted. In this case, in order to better cope with the noise, the filters of the velocity and of the altitude derivative gains are changed, the constant N instead of 100 is respectively chosen as 30 and 50. As explained in [134] the filter coefficient N determines the pole location of the filter. In discrete

Tab. 7.3: Ultrastick Simulink outer loop PID gains

	K_P	K_I	K_D	N
Angle of attack from Airspeed	-0.022	-0.012	-0.0205	30
Throttle from Altitude	0.028	0.01	0.002	50
Roll rate from Heading	0.85	0.07	2.1	100

time domain and for the forward Euler integration method of the PID block, the pole z_{pole} is placed at $z_{pole} = 1 - NT_s$, where T_s is the step time.

7.4 Ultrastick 25e HIL simulation results

This section presents the results obtained integrating the XMOS board with the Simulink Ultrastick 25e aircraft model. This is the simulation that imitates the closest the expected behavior of the controller implemented on the real aircraft.

HIL is obtained following the scheme of Fig. 7.10. The states and the reference values generated by the Simulink aircraft model are sent through serial to the XMOS board using appropriate Simulink blocks, the resulting commands are sent back via serial link to Simulink. The Simulink model and the controller integrate with a 0.01 seconds time step. The data to and from the XMOS board are sent with a 0.05 seconds interval in order to simulate the sensors and servos update rate. As a consequence the controller is repeated five times for each set of input data.

In this section the same reference inputs of the pure Simulink case are proposed. The results for the trim condition hold are represented in Fig. 7.11 while Fig. 7.12 and Fig. 7.13 show respectively the states and the commands for the coupled maneuver. The comparison with the Simulink simulations allows to affirm that results are basically equivalent. The slightly higher oscillatory behavior at initial time and during transition might be caused by the slower control rate of the HIL case. Note that also PID gains need to be adjusted, see Table 7.4, in particular for the speed control.

7.5 Aircraft - Controller integration

The real-time ability of backstepping to control a high-fidelity aircraft model when running on a microcontroller has been demonstrated. This section deals with the integration of the
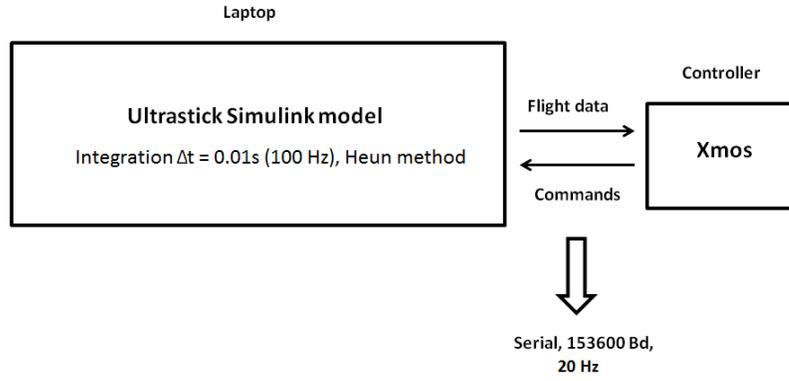
Fig. 7.10: Simulink HIL layout
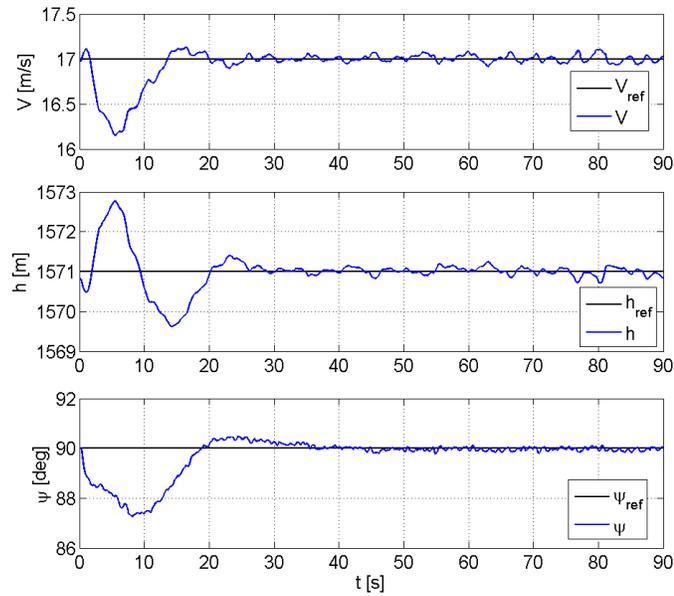
Fig. 7.11: Simulink HIL outer loop response for Ultrastick 25e, trim conditions hold
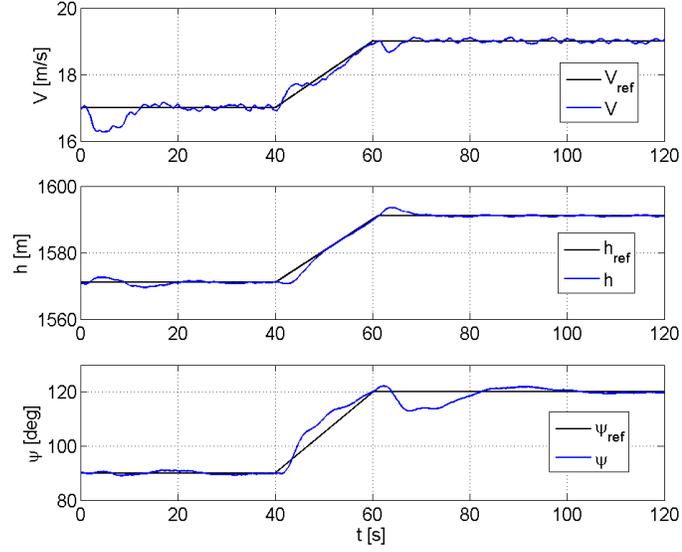
Fig. 7.12: Simulink HIL outer loop response for Ultrastick 25e, maneuver
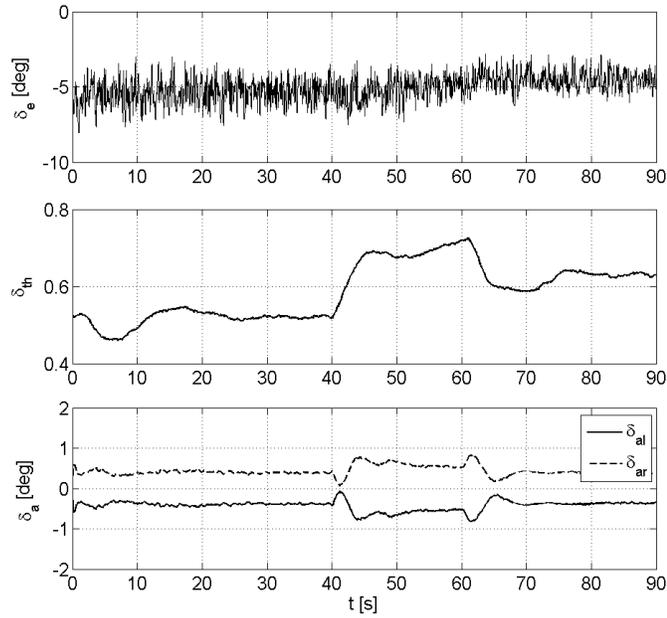
Fig. 7.13: Simulink HIL commands for Ultrastick 25e, maneuver

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Tab. 7.4: Ultrastick Simulink HIL outer loop PID gains

	K_P	K_I	K_D	N
Angle of attack from Airspeed	-0.0045	-0.007	0.023	100
Throttle from Altitude	0.03	0.005	0.005	100
Roll rate from Heading	0.4	0.065	3.2	100

controller on the Ultrastick 25e aircraft for the preparation for the flight tests.

The general autopilot configuration and the relationship of the microcontroller board with the chosen I/O devices is illustrated in Fig. 7.14. The board adopted for the in-flight control remains the XMOS XK-1A which has proved valid during the HIL simulations. The radio transmitter is the Spektrum DX8 model, while the onboard receiver is the 6-channel OrangeRX R610 Spektrum DSM2, the frequency is 2.4 GHz. The sensors were illustrated in Section 7.1. The outputs of the controller board go to the aircraft motor, model E-flite 25 BL, and to the four analog HS-225BB mini servos controlling elevator, rudder and two ailerons. Power is supplied by a 3-cell LyPo battery with 4200 mAh, the motor regulator is a brushless ESC Thunderbird 54 from Castle Creations and the propeller is a APC 12x6E. The servos and the propulsion system are the same used by the Minnesota research group in order to have matching results.

The objective of the work is autonomous flight. Still, the aircraft needs the ability to be piloted during non-controlled maneuvers (take off and landing for instance) or as a safety feature to recover from unwanted behaviors. Therefore, the radio receiver needs to be integrated with the XMOS board in order to allow manual operations and to guarantee a switch between Computer In Command (CIC) and Pilot In Command (PIC) modes. The transition is guaranteed by a discrete switch on the radio transmitter: when it is set to 0, PIC mode is active and the aircraft is flown manually, when the pilot turns it to 1, the controller takes control of the aircraft. The GEAR channel is used, a pin on the XMOS board monitors its status and, according to a signal threshold value, the board logic decides if to forward the pilot commands or to generate them through the backstepping controller. When the PIC mode is active the XMOS software monitors the pins connected to the THROTTLE, ELEVATOR, AILERON and RUDDER radio channels and forwards the received inputs to the motor and the servos. When CIC mode is active, the channels are ignored and the desired signals are directly generated by the controller.
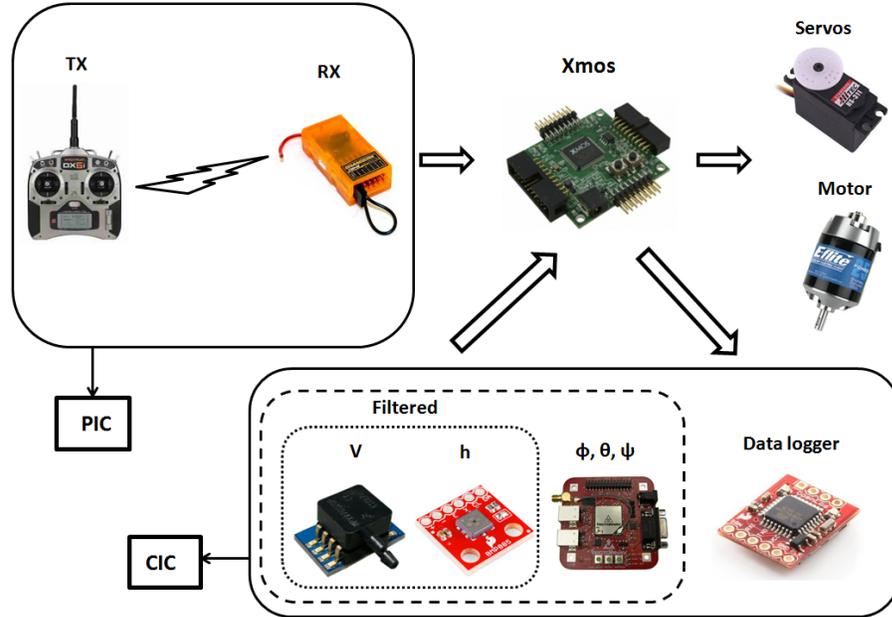
Fig. 7.14: Ultrastick 25e controller integration scheme

The signals used to control servos and motor are represented with Pulse Width Modulation (PWM), a technique to control analog components using a digital microcontroller. A square wave signal is generated. It alternates a period up (on) and period down (off), the time up is defined as the pulse width. A fast alternation of this up-down pattern simulates analog signals ranging from 0 V (off) to maximum voltage (on). Servos nominal voltage is 5 V and their update rate is 50 Hz, the basic frame period is therefore of 20 ms. A neutral position signal is represented by 1.5 ms up and 18.5 ms down. A maximum signal is achieved with 2 ms up and a minimum signal with 1 ms up. Fig. 7.15 illustrates an example of PWM signal.
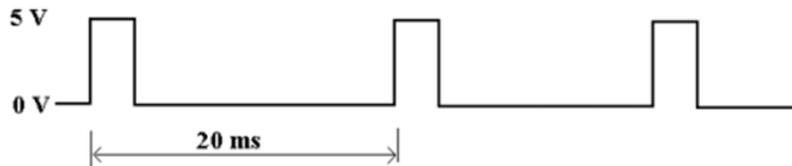
Fig. 7.15: PWM signal representation

When in autonomous mode all sensors data and command PWM are recorded for post-flight analysis on a micro-SD card. The used device is a Sparkfun OpenLog [135] as represented in Fig. 7.16. It is configured to work with a baudrate of 115200 Bd, the data of interest are saved as columns on a simple text file.
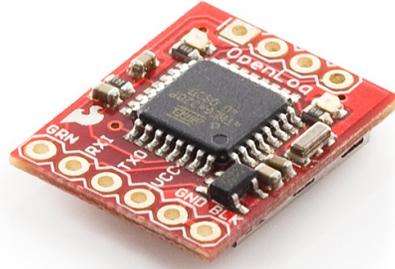
Fig. 7.16: Sparkfun OpenLog micro-SD data logger

A limit of the selected XMOS XK-1A board model became clear while integrating the subcodes that handle all the subsystems. A problem of available stack memory was often encountered and it forced to reorganize the code many times. This was beneficial as it forced to optimize the already implemented functions, but it strongly limits the development possibilities as little margin for modifications is left. The XMOS technology has proved to be reliable and suitable for the role, but the adoption of a more performing board model is suggested.

The only source of power on the aircraft is the battery with nominal 11.1 V. The battery supplies power to the electric motor and to all the onboard systems. As the servos, the RX radio and the microcontroller require a 5 V supply, a form of voltage regulation is required. An initial setup where the regulated output is provided to all utilities by a single motor controller via BEC was tested. Unfortunately, when all the servos where overloaded this resulted in a drop of voltage which caused the microcontroller to crash and the loss of radio connection. In fact, the considered ESC has maximum continuous voltage equal to 2.5 A. By observing the power budget illustrated in Table 7.5 it is clear that during challenging flight conditions when servos are employed at their maximum this value is not sufficient.

This problem was overcome by the use of two motor controllers, both connected to the main battery. One is responsible for powering the servos via BEC, the other simply acts as a DC regulator for the radio and, in cascade, the XMOS board and all sensors. All devices are

Tab. 7.5: Power budget for the onboard electronics

Component	Maximum current
Servo	4×1 A
XMOS	0.5 A (just CPU)
Data logger	6 mA
Barometric sensor	0.012 mA
Airspeed sensor	10 mA
Heading sensor	65 mA
ADC	0.85 mA
Radio receiver	50 mA
Total	> 4.6 A

grounded together. Appendix B illustrates the physical connections among all components.

The pitot tube is installed on the right wing so that the pressure ports are in the undisturbed flow ahead of the wing leading edge. The right wing is chosen in order to partially balance the reaction torque induced by the motor. In fact, as the motor spins in positive direction about the X_B axis, a negative rotation in induced to the aircraft fuselage, causing the right wing to rise and the left one to drop. The aerodynamic disturbance created by the pitot installation reduces the lift on the right wing reducing the overall rolling moment. As the motor induced rolling moment is in general counterbalanced by aileron trim, the adopted solution reduces the needed aileron deflection in steady flight. The installation of the pitot sensor is represented in Fig. 7.17, the analog transducer is placed directly inside the wing as it is preferable to run into the wing an electric cable instead of a silicon tube. This solution might introduce some noise but it considerably reduces pressure losses. The pitot tube is aligned with the aircraft fuselage. As stated by [136], generally pitot tubes can handle flows with an angle up to $\pm$ 12 degrees with respect to their axis, well beyond the values of α and β here expected.

Finally Fig. 7.18 illustrates the disposition of all electronic devices within the Ultrastick 25e bay. The IMU is tighten to a wooden support firmly blocked on the fuselage. All other

Fig. 7.17: Pitot installation on the Ultrastick 25e right wing

devices are just positioned in the free space, when flying they are wrapped in foam to reduce vibrations and mutual friction. The electric schemes for connecting all devices together are illustrated in Appendix B.

7.6   Command - Deflection correlation

In order to match the surface deflection defined by the backstepping controller with the real aircraft surface deflection, a calibration has to be performed. The idea is to identify the mathematical correlation between the PWM given to each servo and the corresponding surface deflection. In such a way, when the backstepping controller calculates a desired deflection, the conversion to the PWM for the servo is straightforward.

Using a specifically designed calibration code, discrete PWM steps from minimum (1000 microseconds) to maximum (2000 microseconds) are sent to all servos. The resulting trailing edge points are recorded on a paper sheet, the servo center of rotation is indicated too. A set of around 20 points is recorded for each control surface, a 50 microseconds interval is used. Vectors of PWMs and corresponding rotations are processed with Matlab to generate the PWM-angular displacement curve. A linear relationship is not assumable and an asymmetry
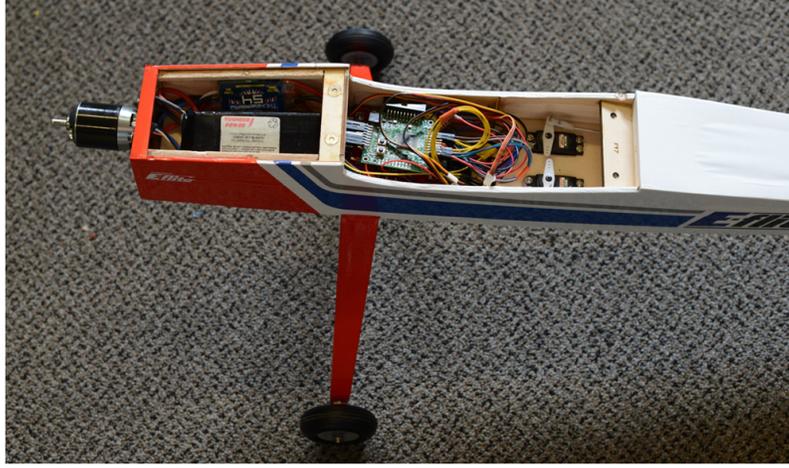
Fig. 7.18: Ultrastick 25e aircraft with sensors and microcontroller board

between positive and negative deflections is sometimes observable. Furthermore, the quality of data is affected by a degree of error due to the small rotations involved. For all these reasons, two second order polynomials are used to interpolate the rotation of the surfaces, one for each direction of deflection. Least squares method is applied. Aileron is the case displayed in Fig. 7.19, the upper part illustrates the results for negative deflections while the lower for positive deflections. The equations describing the curves for the aileron are represented in Equation (7.3). Similar curves are obtained for elevator and rudder, they are implemented in the backstepping controller XC code.

$$
\begin{cases}
-991.8 \cdot \delta_a{}^2 + 888.3 \cdot \delta_a + 1504.6 = 0 & \delta_a > 0 \\
165.5 \cdot \delta_a{}^2 + 980.5 \cdot \delta_a + 1506.6 = 0 & \delta_a < 0
\end{cases}
\tag{7.3}
$$

For the motor a similar test is performed, for each PWM value a rotational regime is measured through an optical tachometer. Tests confirm that a linear relationship can be assumed with good confidence. No throttle corresponds to the minimum PWM and full throttle to maximum PWM.

7.7 Preliminary ground tests

The configuration for the ground tests is slightly different from the one adopted for the flight. First, the propeller is not mounted for safety reasons. Second, as the aircraft is still and so the pitot would indicate zero velocity, the code is corrected to assign a fictitious input
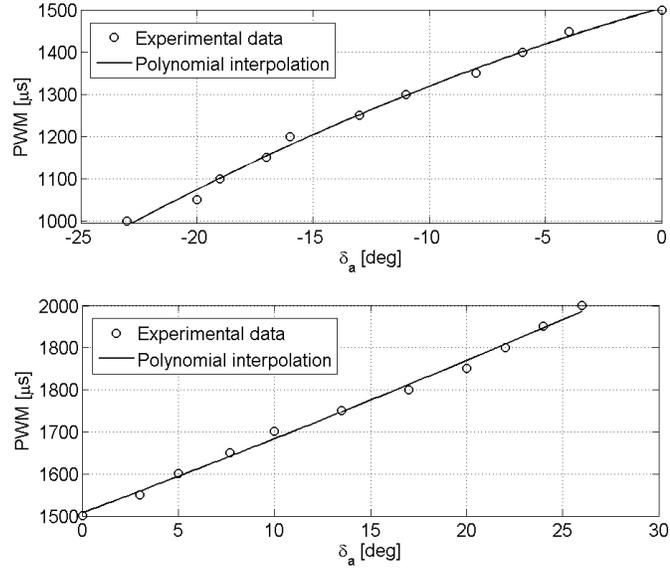
Fig. 7.19: Ultrastick 25e aileron servo calibration curves

velocity equal to the cruise speed. The aircraft is rotated and lifted to simulate heading and minimal altitude changes. Aileron and rudder commands are visible, minor throttle changes are audible. All data are recorded on micro-SD card and analyzed. In these preliminary tests the controller is asked to maintain the conditions measured when CIC mode is engaged.

Results of the ground test are shown in the following figures: Fig. 7.20 for the elevator response to speed variations, Fig. 7.21 for the altitude to throttle loop and Fig. 7.22 for heading control with latero-directional commands. The maneuver consists in turning the aircraft of 360 degrees while lifting it from the floor to maximum arm extension, the velocity is constant. All commands act to counteract perturbations. Note that the elevator peak at 15 seconds is caused by the coupling effect generated by the heading angle measuring 180 degrees. The switch between PIC and CIC modes is smooth and immediate.

7.8   Preliminary flight tests

The flight tests are performed at Cherry Creek State Park in Aurora, around 30 minutes drive from the University of Denver. The site belongs to Denver R/C Eagles and the facilities include two paved runways of 84 and 122 meters, paved taxiways and pit area. A satellite view is represented in Fig. 7.23. The airfield is located at an altitude of around 1705 meters
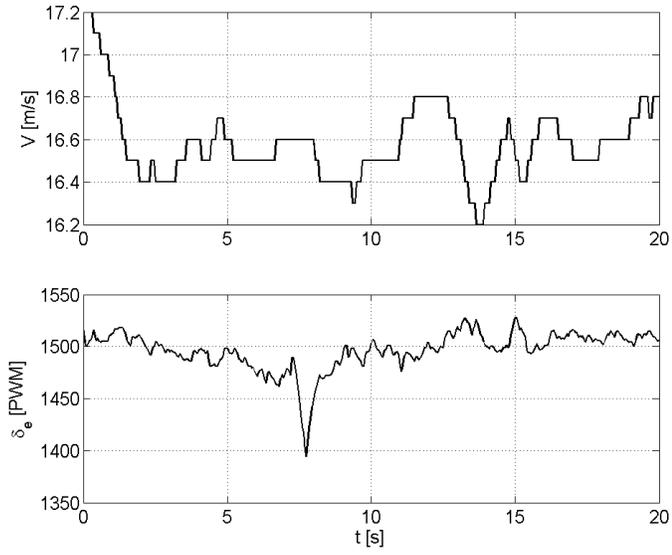
Fig. 7.20: Ultrastick 25e ground test, *V* control
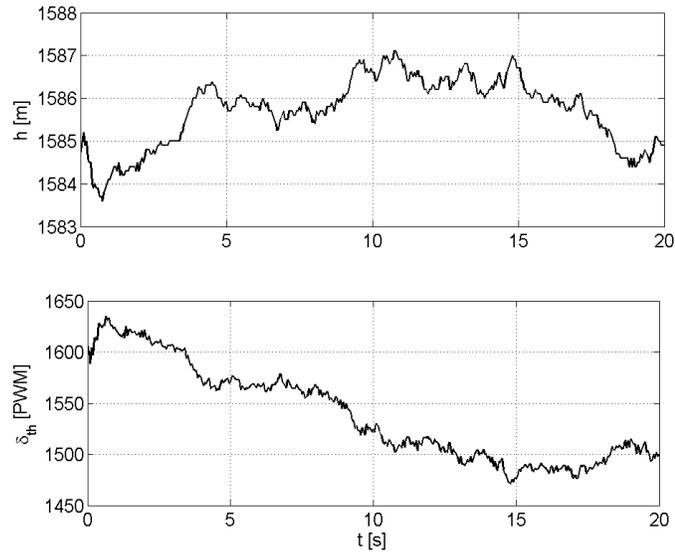
Fig. 7.21: Ultrastick 25e ground test, *h* control

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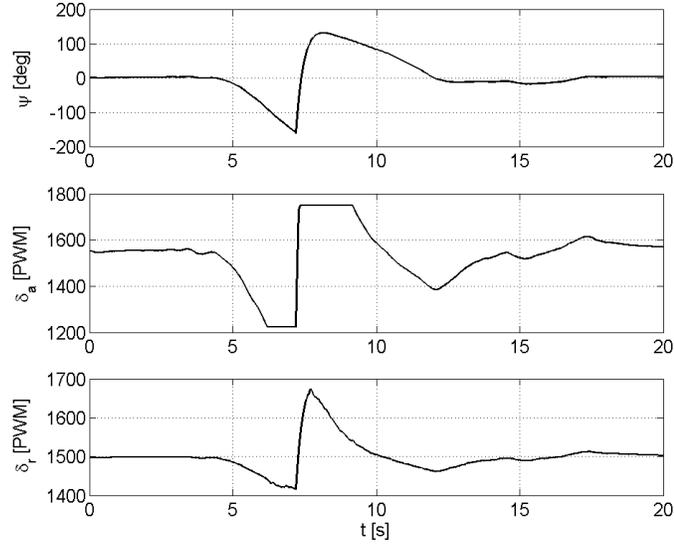
Fig. 7.22: Ultrastick 25e ground test, ψ control

above ground level. Flying altitude above the ground is limited to 400 feet (122 meters) due to the proximity with Centennial Airport. The surrounding is mainly composed of grassland, two streets run on the north and on the south side of the runway, the closest houses are 620 meters south-west of the airfield. Flight tests are performed early in the morning starting from 8 am in order to avoid wind disturbance. In optimal days the wind was acceptable for flight until around 11 am, no preferred wind direction was noticeable.

Due to a crash during the maiden flight in piloted mode, some logistics problems with spare parts orders and a software problem that took long to be found, the flight tests schedule was seriously delayed. Autonomous flight tests were performed only during the last week of the project.

Globally five flights were carried out, of these four were dedicated to latero-directional control and one to longitudinal control. The code was modified so that when backstepping is controlling the latero-directional plane with aileron and rudder, the pilot still has throttle and elevator control. The contrary happens when backstepping controls the longitudinal plane. This approach allows testing the controller behavior without controller-induced cross-coupling effects and so facilitates the initial PID tuning procedure. During the autonomous flight the pilot is asked to avoid touching the stick unless minimal corrections are required

Fig. 7.23: Airfield used for the flight tests

or dangerous situations occur. By definition a fixed-wing aircraft is a coupled system. For this reason, after the initial uncoupled tuning is performed, a more refined tuning is planned with all commands acting together.

The microcontroller code is designed so that the aircraft maintains the initial conditions that are measured when switching from PIC to CIC. These are initial velocity and altitude when the longitudinal plane is autonomously controlled, initial heading when the latero-direction plane is under autonomous control. In order to avoid abrupt transitions, the controller is designed to act about the trim conditions identified during the initial test flights.

The most promising results were obtained in the flight controlling altitude and velocity. Figures 7.25, 7.26 and 7.27 show the sensors data and the corresponding commands recorded. From Fig. 7.25 and Fig. 7.26 it is possible to observe that the aircraft is able to control both velocity and altitude. Initially the aircraft loses some altitude and thus accelerates, the controller reacts increasing the throttle and reducing (in sign) the elevator deflection. The aircraft starts to climb again and slows down closer to the target velocity. It is possible to observe from Fig. 7.26 how the throttle quickly saturates without much affecting the altitude response, a more powerful motor would have resulted in better performance. Fig. 7.27 shows

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(a) Ground preparation   (b) Flight

Fig. 7.24: Ultrastick 25e during ground preparation and in flight

that the initial heading is not maintained, despite some pilot corrections on the aileron. These corrections generate a coupling effect responsible for the oscillations in the speed and in the elevator command visible around six seconds. An adjustment in the PID tuning, in particular for the velocity control loop, is expected to guarantee considerable improvements.

Tests on the latero-directional plane did not guarantee satisfactory results. In all four tests, where slightly different gains configurations were tried, an excessive roll motion was recorded. Further tests need to be carried out to investigate the causes of the problem and to address them effectively. A likely cause might be the imprecise tuning of the gains or the inaccuracy in the PWM - δ_a correlation. In fact, during the simulations the aircraft has showed to be extremely sensible to minimal variations in the aileron command.

7.9 Conclusions

The present section illustrates the procedure adopted for implementing and testing the non-linear backstepping controller on a fixed-wing UAV. First Simulink and HIL simulations demonstrate that this backstepping approach is able to control with satisfactory performance the aircraft adopted for the flight tests. A high-fidelity model of the aircraft inclusive of real sensors noise is employed. Later, the installation and the integration of the backstepping-based autopilot on the aircraft is described. The problems encountered and the solutions adopted are explained. Finally, the results of some preliminary ground and flight tests are illustrated. Ground tests demonstrate that this autopilot configuration is suitable for the control of the aircraft. The limited number of flight tests performed indicate that a good lon-
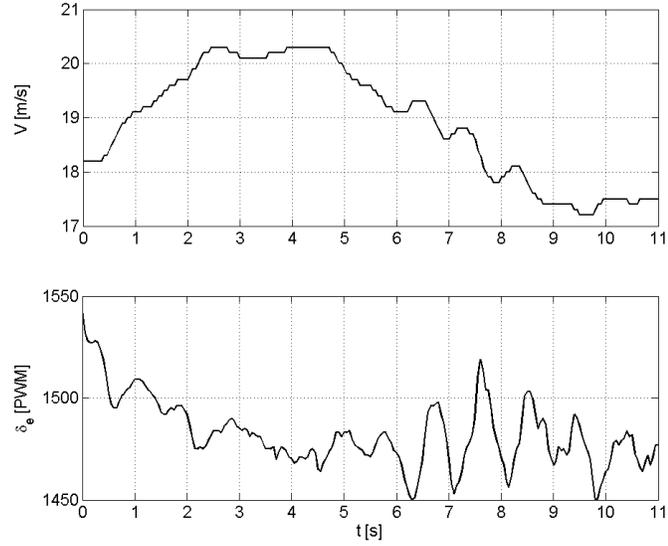
Fig. 7.25: Ultrastick 25e longitudinal flight test, V control
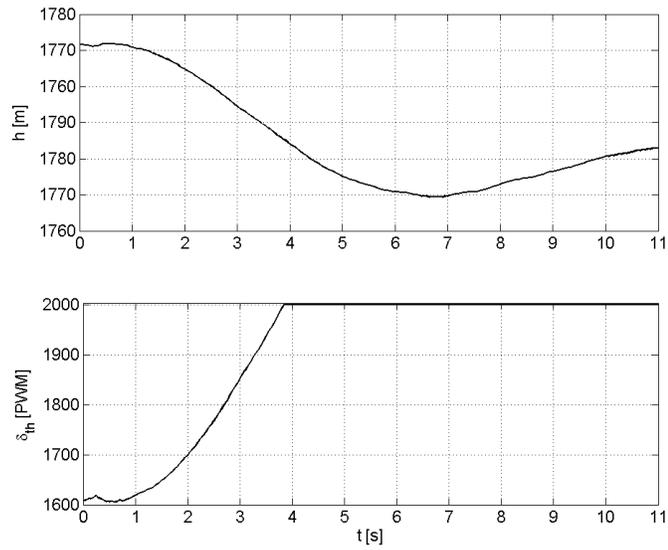
Fig. 7.26: Ultrastick 25e longitudinal flight test, h control
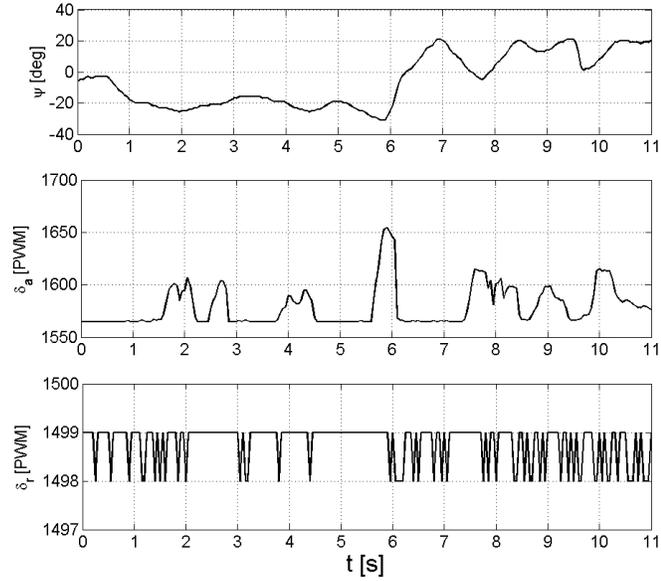
Fig. 7.27: Ultrastick 25e longitudinal flight test, latero-directional variables

gitudinal control can be easily achieved, while improvements on the latero-directional plane control are still necessary.

In order to identify these improvements a larger number of flight tests is required. A systematic analysis of the loop on ψ need to be performed to correlate the gains adjustments with the aircraft roll behavior. In early stage a comparison with simulated results might be helpful. Finally, a study to improve the precision of the correlation between theoretical and physical commands is suggested. In fact, the high responsiveness of the aircraft is affected by minimal disparities of these values.

8. CONCLUSIONS

The present PhD thesis illustrates the design and the development of three advanced control laws for unmanned aircraft vehicles: PID with H_∞ robust approach, $\mathcal{L}_1$ adaptive controller and nonlinear backstepping controller. All controllers are first illustrated in their theoretical form and then successfully applied through numerical simulations to the MH850 aircraft model. The backstepping controller is chosen for being integrated and tested on a real aircraft. Further simulations are performed in real time through a microcontroller board to asses its performance. Ground tests validate the structure of the proposed autopilot, while the limited number of flight tests indicates that a longer development is necessary to achieve a mature configuration.

The obtained results demonstrate that these control laws are suitable for the application to autopilots of unmanned aircraft. The hybrid PID tuning approach described in Chapter 4 guarantees satisfying flight performance even if this is limited about the design point. The proposed H_∞ method for the assessment of the robust characteristics is an effective tool that allows to underline the limits of the PID approach: a tradeoff between performance and robustness needs to be accepted. It was demonstrated, in fact, that the above-mentioned reference tracking results were achieved sacrificing the system robustness to sensors noise, parametric uncertainties and external disturbances. At the same time, trying to respect the H_∞-imposed boundaries strongly limited the aircraft performance.

In order to address the robustness problem in a more systematic way, the $\mathcal{L}_1$ adaptive controller was chosen, see Chapter 5. In particular, an $\mathcal{L}_1$ approach based on piecewise constant adaptation laws was selected, it guarantees adaptation rates compatible with an autopilot board CPU and low computational effort. The adaptive nature of this controller has proven to offer satisfying performance and excellent robustness to parameter uncertainties. Furthermore, it was demonstrated that this control method can be easily designed and tuned on a simplified linear aircraft model and successfully applied to a complete nonlinear model.

The backstepping controller proved too its ability to stabilize the selected aircraft model with satisfying results, see Chapter 6. Despite the presence of a pure PID control in its Throttle to Altitude loop, very good robustness to parametric uncertainty was achieved. A

comparison among all controllers demonstrated that backstepping and $\mathcal{L}_1$ are comparable in terms of nominal performance, with a slight advantage for $\mathcal{L}_1$ in the robustness test. PIDs are somehow inferior in the nominal case and completely fail in case some parametric disturbances are introduced. Considering these results, the simple structure of the proposed approach and the innovative contribution, the backstepping controller was chosen for being integrated and tested on a flying aircraft.

The backstepping controller demonstrated to be easily implementable on a microcontroller board and to guarantee satisfying performance also when running in real time. In particular, complex maneuvers with the high-fidelity model of the UAV selected for the flight were tested. The XMOS innovative microcontroller technology was introduced for the HIL simulations and it was selected as the core component of the autopilot system. The autopilot was completed integrating the XMOS board with servos, sensors and the radio system. In this phase a limit of the adopted XK-1A board became evident due to the low stack memory available. The XMOS technology has proved to be reliable and suitable for the role, but the adoption of a more performing board model is suggested. Some practical problems were overcome in the physical realization of the system. One of them was the supply of regular power to all the systems, the adoption of two DC stabilizers was the solution. Another issue, which is still open, was the correlation between theoretical and practical commands. A correlation curve based on experimental data was built.

A series of preliminary ground tests validated the autopilot configuration. The transition between piloted and controlled flight was smooth and immediate, and it guaranteed a safe flight. The backstepping controller moved the aircraft commands in such a way to counteract fictitious perturbations. Unfortunately the number of flight tests was very limited, they were not able to provide useful results. A crash in piloted mode during the maiden flight, some logistics problems with the order of the spare parts and a problem with the IMU magnetometer strongly delayed the tests schedule. The handful of performed flights indicated that good longitudinal control can be easily reached with limited development while some more work must be invested on the latero-directional plane. In particular, the adjustment of the gains and the correlation between the theoretical and physical deflection of the ailerons need to be addressed.

Future developments to the present work should target the backstepping controller and its application to flight tests. PID is a widely employed technique and $\mathcal{L}_1$, being very recent, is the object of the attention of many researchers who are still exploring its possibilities. Backstepping, instead, is a well developed technique that, unfortunately, has seen a limited number of practical applications to the fixed-wing UAV field. The approach presented in

this thesis is deliberately kept simple in order to create an initial framework for the actual implementation of backstepping to unmanned fixed-wing flight. The method adopted and the technical solutions proposed are strong basis for future improvements. As already mentioned, some adjustments to the autopilot physical configuration are suggested. Once these are completed, the backstepping controller structure can be developed including adaptivity and substituting PIDs with more advanced control techniques.

Appendices

Appendix A

EVALUATION OF AN $\mathcal{L}_1$ CONTROLLER FOR WING ROCK SUPPRESSION

This appendix illustrates the initial work done within this PhD project to become familiar with the $\mathcal{L}_1$ technique. The test case is the wing rock phenomenon on a highly-swept delta wing. The text here proposed is largely taken from an article published on the AIAA Journal of Guidance, Control, and Dynamics [137].

A.1 Introduction

The wing rock phenomenon appears on aircraft characterized by highly swept wing, leading edge extensions and slender forebody when flying at high angles of attack. It consists in an oscillatory motion about the roll axis which increases in amplitude up to a limit cycle. The appearance of wing rock is due to a nonlinear variation of the roll damping with angles of attack and sideslip. Aircraft configurations with slender forebodies are affected by wing rock because of the unsteady interaction between primary forebody vortices and lifting surfaces (leading edge extensions, wing and stabilizers). It can be encountered by high-speed civil transport and combat aircraft in their flight envelope and when this occurs their handling qualities and maneuvering capabilities are seriously degraded. For this reason the suppression of wing rock is a relevant issue, it can be achieved by changing forebody cross-section and slenderness or by the adoption of forebody vortex control techniques (boundary layer suction-blowing and/or movable forebody strakes). The action of these devices must be regulated by an appropriate control law able to handle the complexity of this phenomenon characterized by nonlinearity and parameter uncertainty.

Different techniques have been proposed, such as synergetic optimal controllers [138], fuzzy PD controllers [139] or variable phase controller [140]. Singh [141] presented a direct adaptive and neural control of the wing rock. With the adaptive control the structure of nonlinearity of the plant is considered, while in the neural control radial basis function neural

network is used. Joshi [142] presented a simple rule-based controller to suppress the limit cycle behavior of the wing rock. The rule base is constructed to be linearly separable. A simple neural controller with a single neuron is trained. This method permitted to reduce the computational effort due to parameter tuning. More recently, Liu [143] proposed a new reinforcement adaptive fuzzy control scheme, in which the adaptive channel combined with reinforcement-learning strategy is applied to tune the parameters. Similarly, Cao [144] applied an innovative adaptive and robust $\mathcal{L}_1$ controller.

The purpose of the present work is to get familiar with the $\mathcal{L}_1$ adaptive controller technique thanks to its application to the suppression of the wing rock phenomenon on a high swept wing aircraft. Adaptive control is chosen because the adaptation channel improves the performance robustness of the feedback path and reduces the tuning effort required to achieve desired closed-loop performance, particularly while operating in the presence of uncertainties and failures. The model considered in this study includes the fuselage and nonlinear uncertainties. The approach presented is similar to [144], where the nonlinearities are function of the sensor measurement errors, modeled as white noise, and a reduction of the actuator effectiveness is also considered.

A posteriori robust assessment is performed. First, to verify the scalability of the controller, some simulations are carried out considering a wide range of angles of attack and initial conditions. Second, the controller designed on the nominal model is tested on a model with perturbed parameters, so that the steady state value of the limit cycle reaches a $\pm$ 10 degrees offset from the nominal case. The proposed approach is simple, robust and stable. As demonstrated in [145]-[148] $\mathcal{L}_1$ adaptive control is capable of compensating for unexpected, unknown, severe failure events, while delivering predictable performance across the flight envelope without enforcing persistence of excitation, or resorting to gain–scheduling of the control parameters or control reconfiguration. It is shown that $\mathcal{L}_1$ adaptive control architecture has an appropriate structure for the integration of conventional frequency–domain filtering techniques, while providing at the same time a suitable framework for the analysis of the effect of these techniques in the stability and performance characteristics of the closed–loop system.

The wing rock model will be discussed in the next section, together with the definition of the experimental parameters. The most relevant design features of $\mathcal{L}_1$ adaptive control will then be recalled in Section A.3, where the most peculiar aspects related to the present application will be discussed with some more details. Some results are then proposed in Section A.4, finally a section of Conclusions summarizes the main findings.

A.2 Wing rock model

The nondimensional model describing the motion of the roll angle $\phi(t)$ was derived in [149] and [150]

$$\phi''(t)+\hat{a}_0\phi(t)+\hat{a}_1\phi'(t)+\hat{a}_2\left|\phi'(t)\right|\phi'(t)+\hat{a}_3\phi^3(t)+\hat{a}_4\phi^2(t)\phi'(t) = \phi''(t)-\hat{C}_l(\phi(t)) = 0 \quad \text{(A.1)}$$

The time derivatives are nondimensional, $\hat{a}_0, \hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4$ are the parameters relative to the experimental conditions (i.e. angle of attack, airspeed, Reynolds number and wing characteristics) and $\hat{C}_l(\phi(t))$ is the normalized rolling moment coefficient.

The term $\hat{a}_0\phi(t) + \hat{a}_3\phi^3(t)$ represents the restoring moment, it exhibits a softening of the linear stiffness $\hat{a}_0$ typical of the Duffing equation. For this reason the system is statically divergent for $\phi(t) > \sqrt{-\hat{a}_0/\hat{a}_3}$. The damping coefficient $\hat{a}_1 + \hat{a}_4\phi^2(t)$ is nonlinear and negative for $\phi(t) < \sqrt{-\hat{a}_1/\hat{a}_4}$ (Van der Pol equation). For lower roll angles the system is dynamically unstable, as $\phi(t)$ increases up to the inversion point it becomes stable. The coordinate for this dynamic stability cross-over is not coincident with limit cycle amplitude, as the stability of final state occurs when the balance between dissipation and generation of energy E is achieved

$$E \equiv \oint_\ell \hat{C}_l(\phi(t))\, d\phi = 0$$

This condition is required for a stable oscillatory limit cycle. Dynamic stability and limit cycle characteristics are also influenced by the additional damping produced by the term $\hat{a}_2|\phi'(t)|\phi'(t)$.
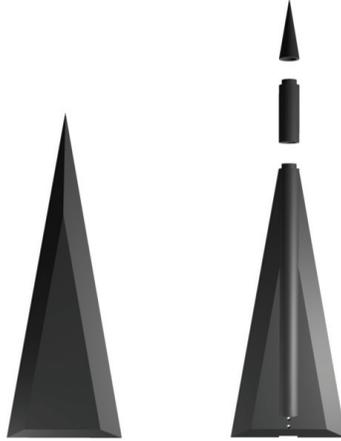

Fig. A.1: Configuration models A (left) and C (right) tested in the wind tunnel

(a) Side view   (b) Top view
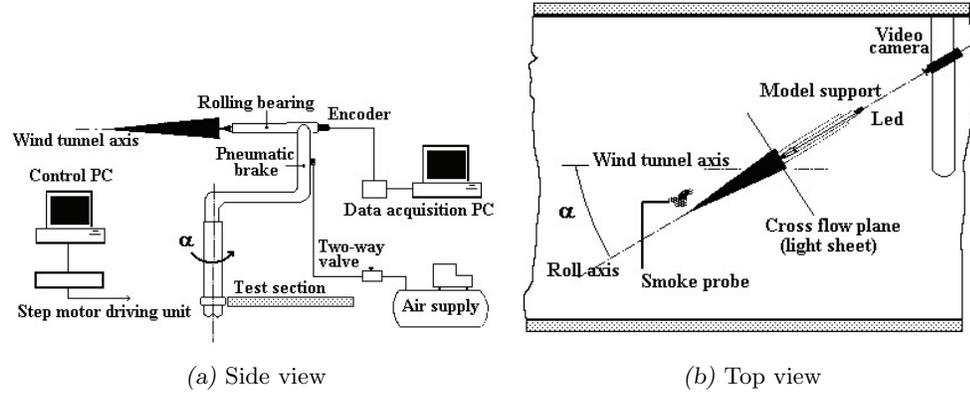
Fig. A.2: Experimental setup

The identification of the parameters $\hat{a}_i$ was performed through least squares approximation of experimental data. Extensive tests were carried out at the D3M low speed wind tunnel of Politecnico di Torino. As described in [151] different aircraft configurations were considered, among them relevance was given to a plain delta wing (model A) and to a more complex model including wing, forebody and nose tip (model C), see Fig. A.1. The wing has the following dimensions: span $b = 0.169$ m, root chord $c_r = 0.479$ m and sweep $\Lambda = 80°$. Total length of model C is $l = 0.568$ m, wing characteristics remain unchanged. The experimental setup scheme is visible in Fig. A.2. Tests included free to roll conditions with airspeed $V = 30$ m/s (Re = 950000) and angle of attack α ranging from $25°$ to $45°$.

As comprehensively explained in [149] and [150], Equation (A.1) is able to accurately describe the behavior of model A. For each α the corresponding limit cycle is reached regardless of the initial conditions up to a release roll angle $\phi_0 = 55°$. Simulations of typical roll angle time history and phase plane representation are depicted in Fig. A.3. Aerodynamic coefficients $\hat{a}_i$ for all tested angles of attack are listed in Table A.1.

Compared to model A, experimental results show that model C needs a longer transient before convergence to a limit cycle. When reached, oscillation amplitudes are smaller as if aerodynamic damping is increased by the fuselage. A singularity is present for $\alpha = 27.5°$ where wing rock is not triggered. In fact, the model settles to a non-oscillatory steady state caused by the interference between forebody and wing vortices that cancels out the hysteresis of the wing vortex normal displacement. For angles of attack greater than $37.5°$ oscillations amplitude fluctuates or the motion disappears because of the starting of wing vortices breakdown. The type of roll dynamics observed for model C is only partially described as a stable

Tab. A.1: Aerodynamic coefficients for model A

| α	$\hat{a}_{0A}$	$\hat{a}_{1A}$	$\hat{a}_{2A}$	$\hat{a}_{3A}$	$\hat{a}_{4A}$
25.0	0.00543	-0.01426	0.41336	-0.00465	0.00263
27.5	0.00594	-0.01765	0.38793	-0.00487	0.01689
30.0	0.00657	-0.02040	0.38008	-0.00537	0.02596
32.5	0.00732	-0.03104	0.53884	-0.00623	0.04189
35.0	0.00794	-0.03137	0.53455	-0.00751	0.05144
37.5	0.00914	-0.00246	0.00105	-0.01059	0.03736
40.0	0.00902	-0.01881	0.62351	-0.01187	0.06119
42.5	0.00999	-0.03219	1.5118	-0.02862	0.06867
45.0	0.01135	-0.03712	2.4252	-0.08113	0.02935

(a) Roll angle time history

(b) Phase plane representation
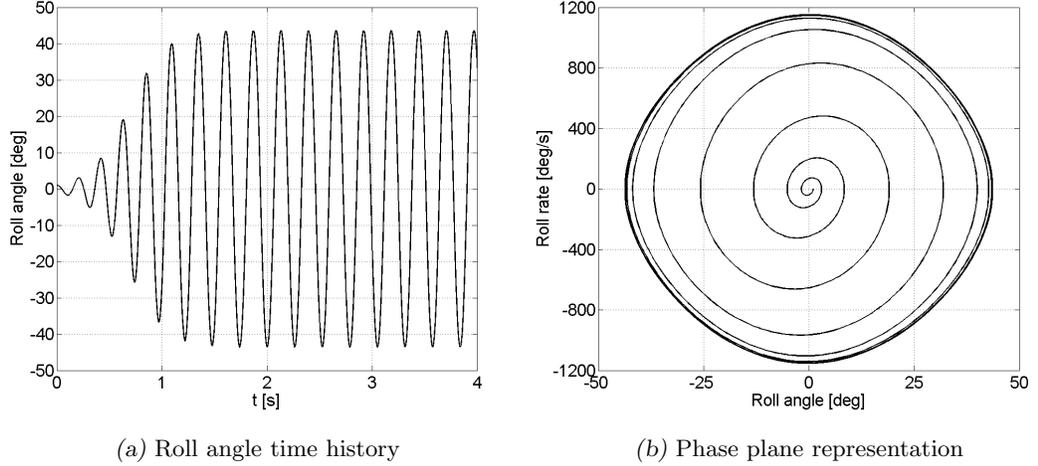
Fig. A.3: Free motion simulation for model A; $\alpha = 32.5°$, $\phi_0 = 1°$, $\dot{\phi}_0 = 0°/s$

elliptical limit cycle. Occasionally the initial release roll angle ϕ_0 prevents the build up of oscillations, in all other cases when these are triggered the limit cycle is unaffected. A steady state roll angle offset up to $\Delta\phi=20°$ can be reached due to forebody vortices asymmetry, differently steady state roll angle offset for model A was nonexistent.

Parameters a_i for model C are listed in Table A.2. Comparison with Table A.1 shows that for $\alpha \leq 35°$ coefficients representing stiffness ($\hat{a}_0$ and $\hat{a}_3$) are similar. On the contrary, the damping of the systems (identified by $\hat{a}_1$, $\hat{a}_2$ and $\hat{a}_4$) differs substantially because of the presence of fuselage in model C. Above $\alpha = 35°$ simulations obtained with the analytical model fail to accurately match experimental results. In fact, forebody vortices alter considerably the shape of the restoring moment term which can no longer be modeled as $\hat{a}_0\phi(t) + \hat{a}_3\phi^3(t)$. The steady state roll angle offset is filtered by the analytical model, with the exception of $\alpha = 42.5°$ (a non-oscillatory condition) where it is correctly guessed.

Fig. A.4 shows roll angle time history and relative phase plane representation for model C when released with angle of attack $\alpha = 32.5°$ and initial conditions $\phi_0 = 1°$, $\dot{\phi}_0 = 0°/s$. Comparison with Fig. A.3 (where same conditions were applied to model A) highlights the ability of the analytical model to capture the longer build up phase and the reduced steady state oscillations amplitude.

Tab. A.2: Aerodynamic coefficients for model C

α	$\hat{a}_{0C}$	$\hat{a}_{1C}$	$\hat{a}_{2C}$	$\hat{a}_{3C}$	$\hat{a}_{4C}$
25.0	0.00615	-0.02644	0.82603	-0.00940	0.04934
27.5	0.00310	-0.00057	1.0025	-0.01157	-1.1908
30.0	0.00523	-0.00406	0.09998	-0.00167	-0.00183
32.5	0.00729	-0.01260	0.33063	-0.00506	-0.00378
35.0	0.00591	-0.03024	1.0703	-0.00285	-0.03726
37.5	-0.00406	-0.00588	1.084	0.03646	-0.15374
40.0	0.00574	-0.00771	-0.03172	-0.01095	0.16302
42.5	-0.0040	-0.03261	2.3447	0.13848	0.90542
45.0	-0.00089	-0.02071	0.8361	0.13752	2.8685

(a) Roll angle time history

(b) Phase plane representation
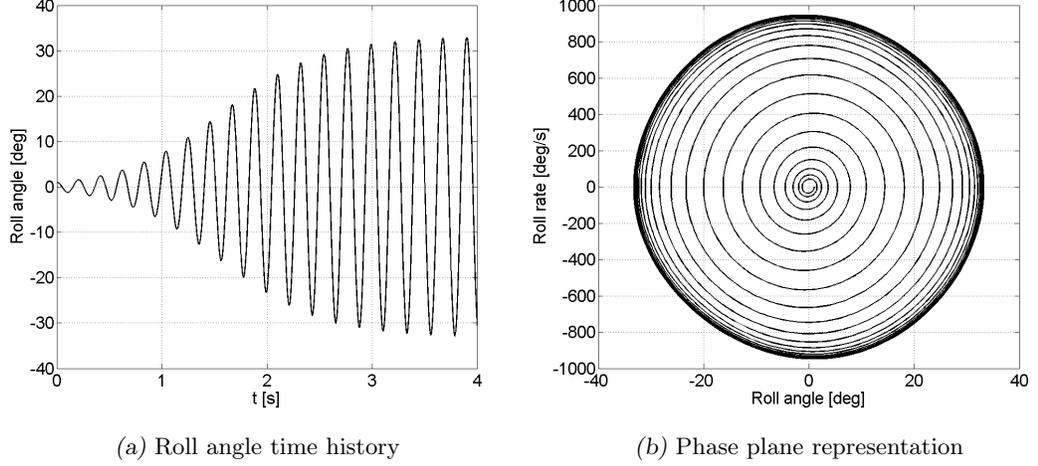
Fig. A.4: Free motion simulation for model C; $\alpha = 32.5°$, $\phi_0 = 1°$, $\dot{\phi}_0 = 0°/s$.

A.3   Controller design

The design of the $\mathcal{L}_1$ controller follows the procedure described in [44] where an example tackles the wing rock suppression problem. Introducing the reference time $t_s = b/2V$, Equation (A.1) becomes

$$\phi'' + \frac{\hat{a}_0}{t_s^2}\phi + \frac{\hat{a}_1}{t_s}\phi' + \hat{a}_2\left|\phi'\right|\phi' + \frac{\hat{a}_3}{t_s^2}\phi^3 + \frac{\hat{a}_4}{t_s}\phi^2\phi' = 0$$

including t_s in the $\hat{a}_i$ coefficients it is possible to rewrite the wing rock model equation with dimensional derivatives

$$\ddot{\phi} + a_0\phi + a_1\dot{\phi} + a_2\left|\dot{\phi}\right|\dot{\phi} + a_3\phi^3 + a_4\phi^2\dot{\phi} = 0 \tag{A.2}$$

Equation (A.2) is now written in a state space form, the state is defined as $x = [x_1, x_2]^T = [\phi, \dot{\phi}]^T$. Including disturbances $d(t)$ and the action of the controller $u(t)$ it is possible to obtain

$$\dot{x}(t) = Ax(t) + b\left(-a_2\left|x_2(t)\right|x_2(t) - a_3x_1^3(t) - a_4x_1^2(t)x_2(t) + d(t) + \omega u(t)\right), \tag{A.3}$$

where $\omega \in \mathbb{R}$ represents unknown control effectiveness and

$$A = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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Calling

$$f_0(x, t) = \left(-a_2 \, |x_2(t)| \, x_2(t) - a_3 x_1^3(t) - a_4 x_1^2(t) x_2(t) + d(t) \right)$$

Equation (A.3) can be simply written as

$$\dot{x}(t) = Ax(t) + b(f_0(x, t) + \omega u(t)) \tag{A.4}$$

The desired output $y(t)$ (the roll angle $\phi(t)$) is obtained from the state $x(t)$ through matrix c

$$y(t) = cx(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

The chosen control law $u(t) = u_{ad}(t) + u_m(t)$ is composed of two terms, a feedback control $u_m(t)$ and an adaptive part $u_{ad}(t)$. The feedback control

$$u_m(t) = -k_m^T x(t)$$

is designed so that the system of Equation (A.4) has desired closed loop dynamics. In fact, substituting $u(t) = -k_m^T x(t) + u_{ad}(t)$ in Equation (A.4) the system becomes

$$\dot{x}(t) = A_m x(t) + b(f(x, t) + \omega u_{ad}(t)) \tag{A.5}$$

where $f(x, t) = f_0(x, t) + (1 - \omega)k_m^T x$ and the matrix $A_m = A - bk_m^T$ is Hurwitz with desired poles. Assuming

$$A_m = \begin{bmatrix} 0 & 1 \\ -a_{m1} & -a_{m2} \end{bmatrix}$$

where a_{m1} and a_{m2} are the parameters chosen to define the desired closed loop dynamics, the static feedback gain k_m must be

$$k_m = \begin{bmatrix} a_{m1} - a_0 \\ a_{m2} - a_1 \end{bmatrix}$$

The design of $u_{ad}(t)$ is more complex and requires an insight of $\mathcal{L}_1$ control theory. The controller acts so that the error $\tilde{x}$ between the state $x(t)$ of the uncertain nominal plant (in our case described by Equation (A.5)) and the state $\hat{x}(t)$ of the predictor is null. The equation of the predictor is

$$\dot{\hat{x}}(t) = A_m \hat{x}(t) + b(\hat{\omega}(t) u_{ad}(t) + \hat{\theta}(t) \, \|x(t)\|_\infty + \hat{\sigma}(t))$$

with $\hat{\omega}(t)$, $\hat{\theta}(t)$ and $\hat{\sigma}(t)$ adaptive estimates. These parameters are updated through the adaptation laws

$$\dot{\hat{\theta}}(t) = \Gamma \text{Proj}(\hat{\theta}(t), -\tilde{x}^T(t)Pb\,\|x(t)\|_\infty), \quad \hat{\theta}(0) = \hat{\theta}_0,$$

$$\dot{\hat{\sigma}}(t) = \Gamma \text{Proj}(\hat{\sigma}(t), -\tilde{x}^T(t)Pb), \quad \hat{\sigma}(0) = \hat{\sigma}_0,$$

$$\dot{\hat{\omega}}(t) = \Gamma \text{Proj}(\hat{\omega}(t), -\tilde{x}^T(t)Pbu_{ad}(t)), \quad \hat{\omega}(0) = \hat{\omega}_0,$$

with $\tilde{x}(t) = \hat{x}(t) - x(t)$, $\Gamma \in \mathbb{R}^+$ is the adaptation gain, $P = P^T > 0$ is the solution of the algebraic Lyapunov equation $A_m^T P + P A_m = -Q$ where $Q = Q^T > 0$ arbitrary. The projection operator *Proj* (see Reference [44] for its definition) guarantees that adaptive estimates $\hat{\omega}(t)$, $\hat{\theta}(t)$ and $\hat{\sigma}(t)$ are bounded.

The adaptive estimates define $\hat{\eta}(t) = \hat{\omega}(t)u_{ad}(t) + \hat{\theta}(t)\,\|x(t)\|_\infty + \hat{\sigma}(t)$, let be $\hat{\eta}(s)$ its Laplace transform. The controller in the frequency domain is defined as

$$u_{ad}(s) = -kD(s)(\hat{\eta}(s) - k_g r(s)), \tag{A.6}$$

$u_{ad}(t)$ is available through inverse Laplace transform of $u_{ad}(s)$. In Equation (A.6) $k_g = -1/(cA_m^{-1}b)$, $r(s)$ is the Laplace transform of the reference signal $r(t)$; the feedback gain $k > 0$ and the strictly proper transfer function $D(s)$ must be chosen so that $\mathcal{L}_1$ norm condition is verified (see Reference [44] for further details) and to obtain the desired specifications (rise time, settling time, overshoot, etc.). A block scheme of the controller is visible in Fig. A.5.
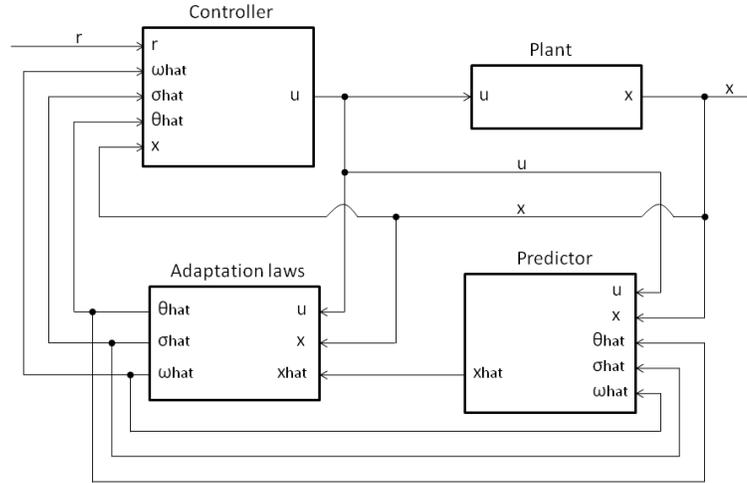

Fig. A.5: Block scheme of $\mathcal{L}_1$ controller adopted for wing rock suppression

A.4 Implementation and simulation results

The choice of the elements a_{m_1} and a_{m_2} of A_m is based on the desired closed loop response of the system. The poles of a system defined by such a matrix are obtained solving the quadratic equation

$$s^2 + a_{m_2}s + a_{m_1} = 0. \tag{A.7}$$

Assuming that the desired response needs to be stable with non-oscillatory characteristics, the chosen poles are both taken real, negative and coincident, $s_{1,2} = -10$. The corresponding quadratic equation is $s^2 + 20s + 100 = 0$, comparing it with Equation (A.7) results $a_{m1} = 100$ and $a_{m2} = 20$. Note that the natural frequency and damping of the system are thus $\omega_n = 10$ rad/s and $\zeta = 1$ as

$$A_m = \begin{bmatrix} 0 & 1 \\ -100 & -20 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}.$$

The $\mathcal{L}_1$ adaptive control methodology addresses the problems of traditional adaptive control by providing fast and robust adaptation. The value of the constant $\Gamma = 10000$ (related to the adaptation) is chosen to guarantee that the steady state error is minimized. Due to the presence of a low pass filter in the $\mathcal{L}_1$ controller architecture, high frequency oscillations are avoided and the system robustness is not reduced. Without loss of generality, it is assumed that

$$\hat{\omega}(t) \in \Omega = [\omega_l, \omega_u], \hat{\theta}(t) \in \Theta, |\ \hat{\sigma}(t)\ | \le \Delta$$

where $\omega_u > \omega_l > 0$ are respectively known upper and lower bounds, Θ is a compact convex set and Δ is a known bound for $\hat{\sigma}(t)$. The derivatives of $\hat{\theta}(t)$ and $\hat{\sigma}(t)$ are also considered bounded. The feedback gain k is chosen to guarantee that the state matrix augmented with the feedback channel (closed loop system) is Hurwitz for all $\hat{\omega}(t) \in \Omega$ and $\hat{\theta}(t) \in \Theta$. The proper transfer function $D(s)$ is chosen as in Reference [44]

$$D(s) = \frac{(s+500)(s+0.004)^2}{s(s+368)(s+0.00439)^2}.$$

A small value, $k = 10$, is taken in order to have a low input signal and so to minimize control required energy; the control effectiveness ω is assumed to be equal to 0.9, disturbances $d(t)$ are modeled as white noise.

The $\mathcal{L}_1$ adaptive controller model was built in Simulink environment. The integration is performed through a fourth-order Runge-Kutta method with a 0.001 s step time, the controller acts at each integration step.

Simulations include different initial conditions and angles of attack for both models. First a detailed analysis is performed for the case when model A and C have release angle $\alpha = 32.5°$ and initial conditions $\phi_0 = 10°$ and $\dot{\phi}_0 = 0°/s$. Fig. A.6 shows the controlled response for model A and C, in both cases the controller effectively sets to zero the roll angle and the roll rate in few seconds. No remarkable difference is observable between the two models, both systems dynamics are dominated by the control law. The nondimensional total control input $u(t)$ is very similar too, as illustrated in Fig. A.7. White noise modeled disturbances do not affect the action of the $\mathcal{L}_1$ controller.


(a) Model A   (b) Model C
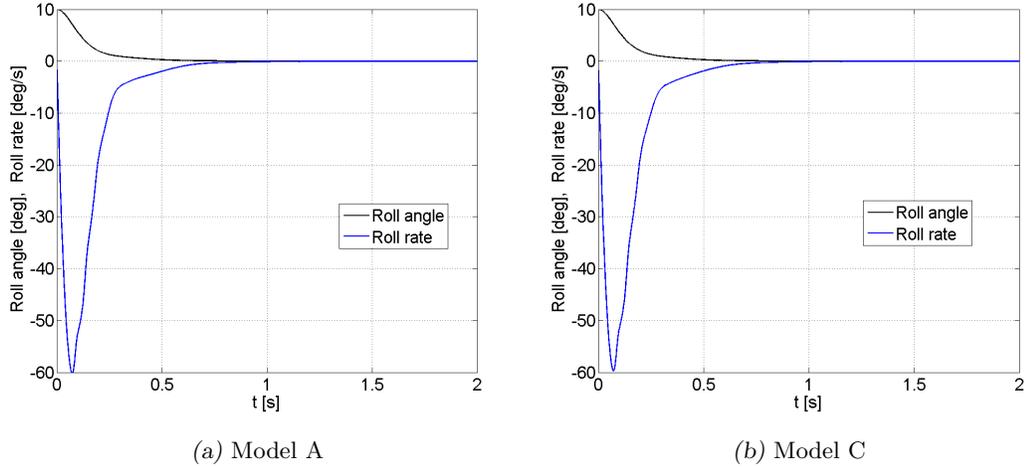
Fig. A.6: Controlled roll angle and roll rate; $\alpha = 32.5°$, $\phi_0 = 10°$, $\dot{\phi}_0 = 0°/s$.

Scalability of the controller is tested varying angle of attack and initial conditions for models A and C. Results for $\alpha = 25°, 30°, 35°, 40°$ and $\phi_0 = 10°$, $\dot{\phi}_0 = 0°/s$ appear in Fig. A.8. No remarkable difference is visible between the models, the controller suppresses wing rock independently of α. This simulation includes the condition $\alpha = 40°$ where the analytical model for configuration C fails to accurately describe the experimental behavior.

The variation of initial conditions is depicted in Fig. A.9. The reference angle of attack is maintained at $\alpha = 32.5°$ while the initial roll angle changes between $\phi_0 = -30°$ and $\phi_0 = 30°$, initial roll rate is maintained at $\dot{\phi}_0 = 0°/s$. In this interval the motion is perfectly controlled, apparently better for model C. Note that the $\mathcal{L}_1$ controller here developed is not able to suppress wing rock for model A for starting roll angles larger than $\phi_0 = \pm 32°$ when $\dot{\phi}_0 = 0°/s$. Larger initial roll angles can be achieved only when an opposite initial roll rate is applied to facilitate the controller action; for instance the starting condition $\phi_0 = 33°$ can

(a) Model A   (b) Model C
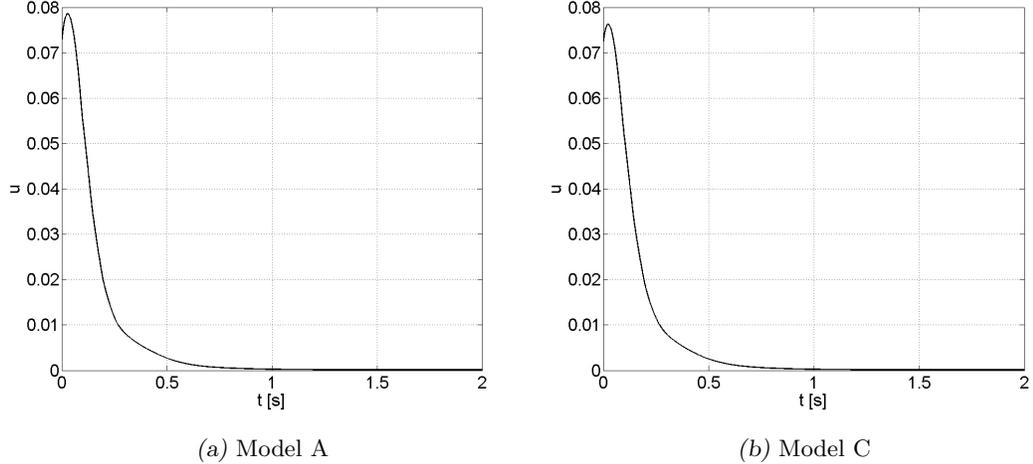
Fig. A.7: Total nondimensional control input; $\alpha = 32.5°$, $\phi_0 = 10°$, $\dot{\phi}_0 = 0°/s$.

be controlled when $\dot{\phi}_0 \leq -32°/s$. For model C the acceptable release roll angle is up to $\phi_0 = \pm 35°$ when $\dot{\phi}_0 = 0°/s$. This is motivated by the larger damping effect of the fuselage as previously noticed (see Figures A.3 and A.4).

An interesting case for testing the performance of the controller is when model C is released with $\alpha = 42.5°$ with same initial conditions as before. Fig. A.10(a) shows the roll angle behavior characterized by an oscillation offset about the value $\phi = 9.5°$. Even in this case the controller effectively cancels out roll oscillations in few seconds, with an action slower but yet similar to the previous cases.

One characteristic of wing rock is the presence of aerodynamic asymmetries, couplings and wing-body vortex interactions difficult to model, even fitting experimental data. The parameters $\hat{a}_i$ of Equation (6.5) are estimated from tests and contain a level of uncertainty $(\Delta\phi(t) = \pm 10°)$ which can alter the controller performance. The capability of the controller to handle these variations is tested perturbing separately coefficients $\hat{a}_0$ and $\hat{a}_1$. The amount of perturbation guarantees that the steady state angular value of the limit cycle in free motion reaches a $\pm 10°$ offset from the nominal case. In Equation (6.5) the parameter $\hat{a}_0$ has the meaning of linear stiffness in the restoring moment $\hat{a}_0\phi(t) + \hat{a}_3\phi^3(t)$, $\hat{a}_1$ represents a constant damping parameter of the overall damping coefficient $\hat{a}_1 + \hat{a}_4\phi^2(t)$. The analysis is performed for model C for a release angle $\alpha = 32.5°$ and initial conditions $\phi_0 = 10°$, $\dot{\phi}_0 = 0°/s$. In Fig. A.11 the effectiveness of the controller action is validated with the presence of perturbation, in particular a smaller sensitivity is shown with respect to parameter $\hat{a}_1$.

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(a) Model A   (b) Model C
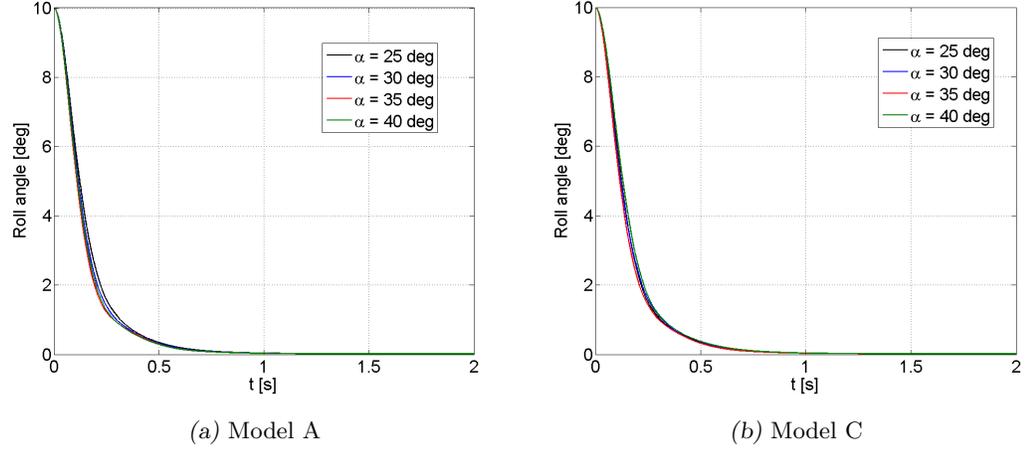
Fig. A.8: Controller action; $\alpha = 25°, 30°, 35°, 40°$, $\phi_0 = 10°$, $\dot{\phi}_0 = 0°/s$.

(a) Model A   (b) Model C
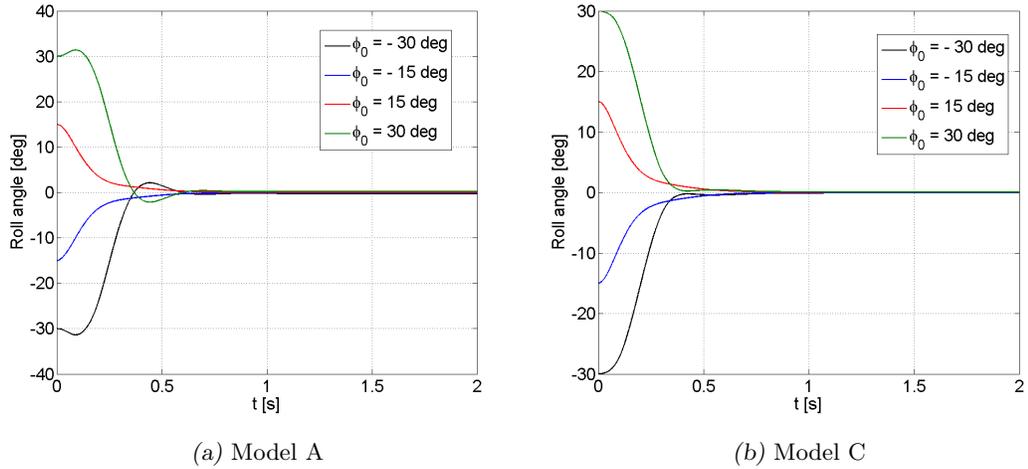
Fig. A.9: Controller action; $\alpha = 32.5°$, $\phi_0 = -30°, -15°, 15°, 30°$, $\dot{\phi}_0 = 0°/s$.

A.5   Conclusions

This work describes the application of an $\mathcal{L}_1$ adaptive controller for the suppression of the wing rock phenomenon. Both an isolated highly swept wing and a complete configuration with fuselage and forebody are analyzed for different angles of attack and initial conditions. Disturbances and changes in the state parameters are included to validate the robustness of

(a) Free motion

(b) Controlled motion
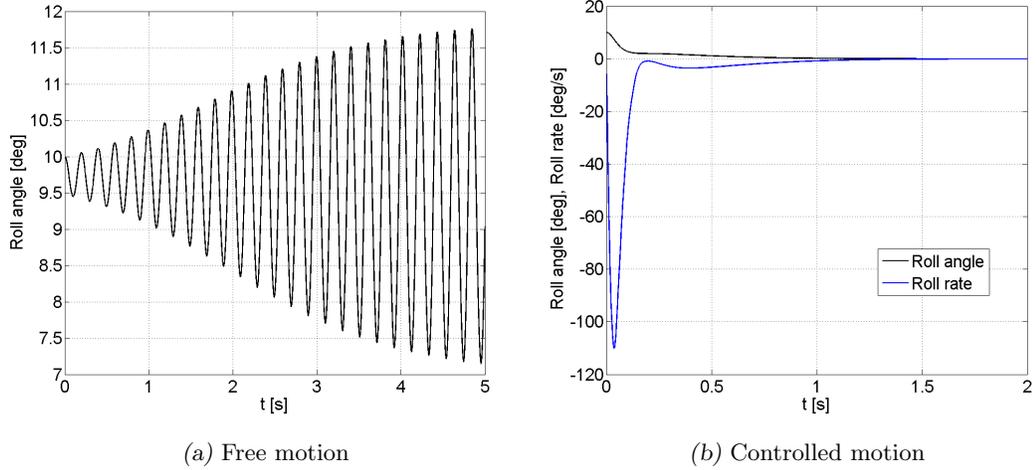
Fig. A.10: Model C free and controlled motion; $\alpha = 42.5°$, $\phi_0 = 10°$, $\dot{\phi}_0 = 0°/s$.

(a) Perturbation of a_0

(b) Perturbation of a_1
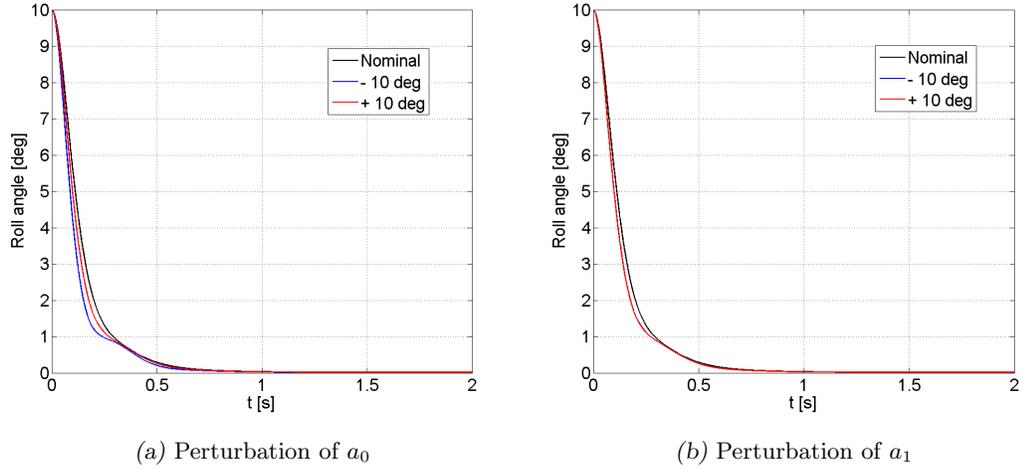
Fig. A.11: Model C perturbed controlled motion; $\alpha = 32.5°$, $\phi_0 = 10°$, $\dot{\phi}_0 = 0°/s$.

the controller to adapt to the level of uncertainty that characterizes wing-body rock modeling. In each of the simulated cases the controller effectively cancels oscillations in short time with a limited amount of energy input. No remarkable difference in performance is observable among the different scenarios suggesting that the design is able to handle the effect of aircraft configuration with adequate robustness.

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This test has helped to acquire confidence with the $\mathcal{L}_1$ technique and has demonstrated the excellent performance of this controller in dealing with robustness issues. Therefore, it is expected to guarantee satisfying results in the application to the control of fixed-wing UAVs, in particular when the model is affected by parametric uncertainties.

Appendix B

ULTRASTICK 25E ONBOARD CONNECTIONS SCHEMES

A schematic of the power setup and of the servos connections appears in Fig. B.1. Colors of lines represent different kind of connection: blue for signal, red for power and black for ground. On the lower bottom of Fig. B.1 the RX radio appears. The radio receives the PIC/CIC signal on the GEAR channel and the surfaces and throttle commands on the remaining channels. These, identified with the letters from C to G, are sent to the XMOS board where the logic described in Chapter 7 is implemented. The output of the board consists of the four commands, originated by the pilot or by the backstepping controller, letters J to M. A prototype board built with appropriate pins forwards the commands to the motor and to the servos.
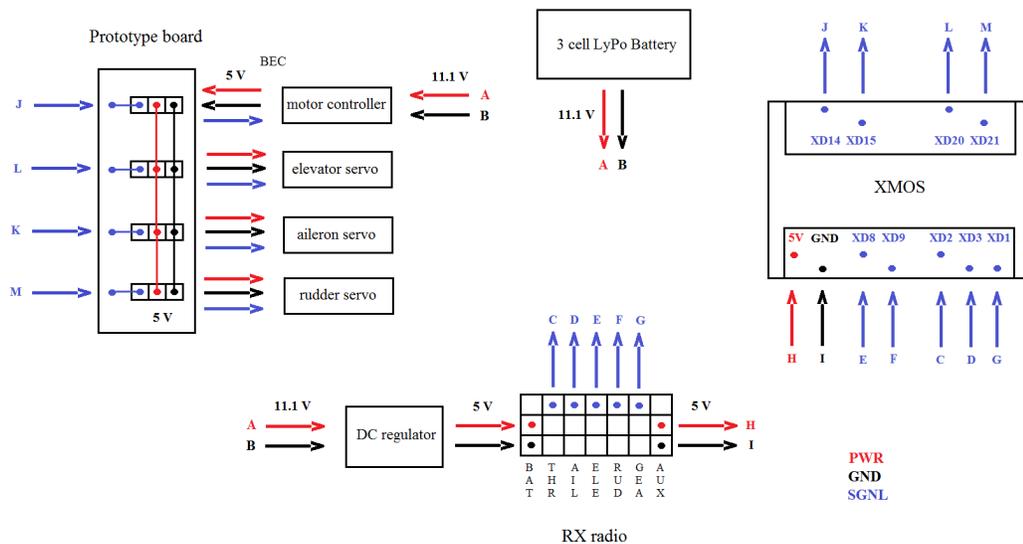
Fig. B.1: Ultrastick 25e power setup and servos integration scheme

Fig. B.2 illustrates the physical connections of the pitot sensor with the XMOS board,

including the Analog to Digital converter. Note that both the MPXV7002 and the LT1865 ADC are powered from XMOS, all devices are grounded together. Fig. B.3 illustrates the physical connections of the barometric pressure sensor with the XMOS board. Note that this sensor requires a voltage equal to 3.3V, the usual 5V could damage it. The connections for the IMU and the OpenLog data logger are more straightforwards. Both of them get the 5V and GND connection from the radio receiver as there are no more of these pins free on the XMOS board. Their physical connection is represented in Fig. B.4.
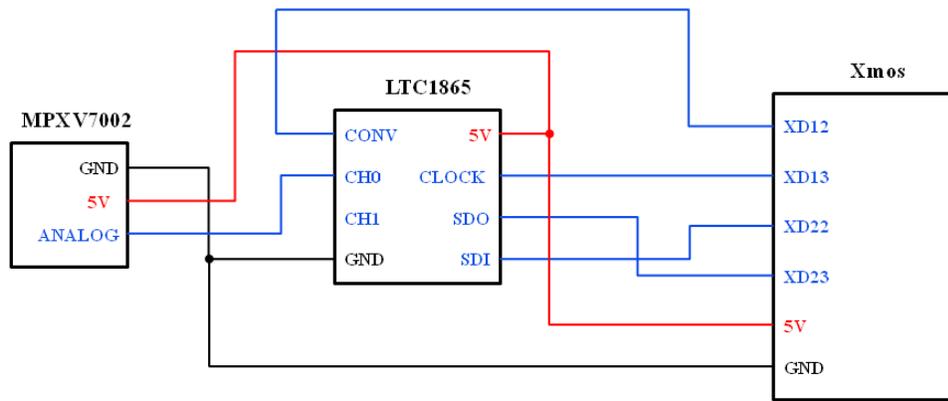

Fig. B.2: Ultrastick 25e pitot sensor integration scheme
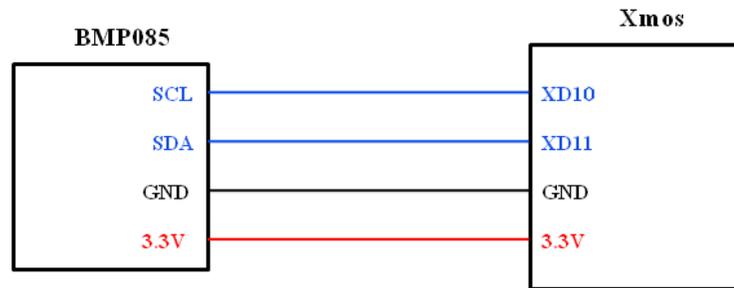
Fig. B.3: Ultrastick 25e barometric pressure sensor integration scheme
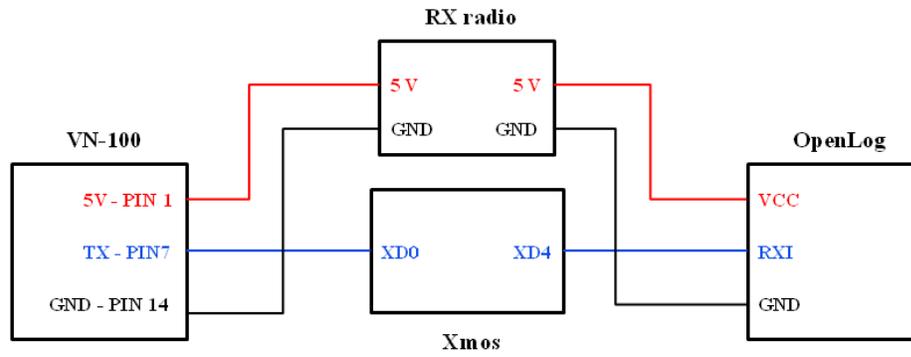
Fig. B.4: Ultrastick 25e IMU sensor and data logger integration scheme

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