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Size-independent Cylindrical Resonator Half-filled with DNG Metamaterial

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Abstract—**A circular cylindrical metallic resonator half filled with DPS material and half with DNG metamaterial is analyzed, in the frequency domain. The two materials are linear, lossless, homogeneous, and anti-isorefractive to each other. The electric field is assumed to be parallel to the cylinder axis. It is shown that the resonator performs independently of diameter size. Numerical results are presented and discussed for a resonator excited by a line source parallel to the axis.**

I. Introduction

A metallic cylindrical resonator half-filled with anti-isorefractive lossless DNG metamaterial is analyzed in the frequency domain, with time-dependence factor $exp(+j\omega t)$ omitted throughout. The geometrical configuration of the resonator is independent of the direction along the axis z of the cylinder, that is, the cross section of the structure in a plane $z = constant$ is independent of z. Half of the resonator is filled with a double-positive (DPS) medium characterized by a real positive electric permittivity ε and a real positive magnetic permeability μ, while the other half is filled with a double-negative (DNG) metamaterial characterized by a real negative permittivity $-\varepsilon$ and a real negative permeability $-\mu$. Consequently, the DNG medium has a real negative refractive index that is the opposite of the refractive index of the DPS medium, while the two media have the same real positive intrinsic impedance. The interface separating the two media is a plane containing the cylinder axis, so that the resonator consists of two equal half cylinders, each filled with one of the two media. The electric field is assumed to be parallel to the cylinder axis, so that the boundary conditions at the ends $z = constant$ of the resonator are satisfied and the resonator length plays no role in the analysis.

The concept of utilizing DNG metamaterial inclusions to render a resonator size-independent was introduced by Engheta [1]-[2] for a one-dimensional structure, to show that it is possible to build resonators that functions independently of dimensions at those frequencies for which the metamaterial behaves as postulated. This concept was later extended to fully three dimensional cavity resonators by Couture et al. et al. [3]-[4] and by Uslenghi [5]-[7]. Recently, Daniele et al. [8]-[9] studied in detail a cylindrical resonator sectorally filled with metamaterial; in particular, their analysis showed that phase compensation leading to size independence is possible only if the cylinder is half-filled with metamaterial. This is the case examined in the present work, in which the excitation is provided by an electric line source located anywhere inside the DPS half cylinder.

II. Geometry of the problem

With reference to rectangular coordinates (x, y, z), the metallic cylindrical resonator has the z axis as symmetry axis, the inner radius of its circumference in any plane $z = constant$ is a, and the length of the resonator in the z direction does not come into play because the resonator is assumed to be excited by an electric line source parallel to its axis, leading to an electric field that is everywhere parallel to z, so that the boundary conditions at the two circular bases of the cylinder are always satisfied. The half-cylinder occupying the volume $x < 0$ is filled with DPS material characterized by a wavenumber $k = \omega\sqrt{\varepsilon\mu}$ and an intrinsic impedance $Z = \sqrt{\frac{\mu}{\varepsilon}}$. The half-cylinder occupying the volume $x > 0$ is filled with a DNG material whose wavenumber $-k$ is the opposite of the wavenumber in the DPS region, and whose intrinsic impedance has the same value Z of the DPS region. The planar interface $x = 0$ separates the DPS and DNG regions.

With reference to circular cylindrical coordinates (ρ, φ, z), the DPS material fills the cross-sectional area ($0 \leq \rho \leq a$, $\pi/2 \leq \varphi \leq 3\pi/2$), whereas the DNG material fills the cross-sectional area ($0 \leq \rho \leq a$, $-\pi/2 \leq \varphi \leq \pi/2$). The electric line source $\mathbf{J_0}$ located inside the DPS region at $\boldsymbol{\rho_0}$ ($\rho = \rho_0$, $\varphi = \varphi_0$) is assumed to be:

$$\mathbf{J_0} = I\delta(\boldsymbol{\rho} - \boldsymbol{\rho_o})\hat{z} = I\frac{1}{\rho_o}\delta(\rho - \rho_o)\delta(\varphi - \varphi_o)\hat{z} \quad (1)$$

where I is the intensity (in A), $\hat{z}$ is a unit vector parallel to the z axis, and δ is the delta function.

A cross section of the resonator in a plane perpendicular to its axis is shown in Fig. 1.
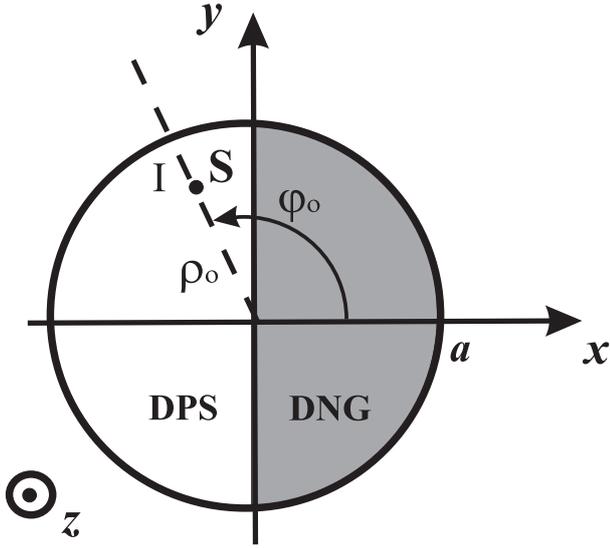
Fig. 1. Cross section of the resonator

III. DESCRIPTION OF THE ANALYSIS

The dispersion relation for any homogeneous penetrable wedge is obtained by expressing the solution to the boundary-value problem in terms of circular cylindrical coordinates and imposing the continuity of the tangential electric and magnetic fields across the two wedge faces. As shown by Osipov [10], such a relation always results in the product of trigonometric functions being equal to zero. In the particular case of a wedge of semi-aperture angle α, whose faces separate two regions of space that are anti-isorefractive to each other, the dispersion relation takes the simple form (see [9]):

$$\sin((\pi - 2\alpha)\nu) = 0 \qquad (2)$$

where ν is a constant arising in the solution of the wave equation by separation of variables. In the present work, $\alpha = \pi/2$, and therefore equation (2) is satisfied for any value of ν. It follows that the boundary condition on the PEC wall of the resonator

$$J_\nu(ka) = 0 \qquad (3)$$

where J_ν is the Bessel function, yields the allowed values of ν for any given frequency, that is, the cylinder resonates at all frequencies for which the DNG material behaves as postulated, and therefore the resonator can be miniaturized.

For a resonator partially filled by a DNG sectoral wedge whose semiaperture angle α is different from $\pi/2$, the solution for line source excitation has been obtained in [9] by employing a Green resolvent technique [11]. However, this technique fails when $\alpha = \pi/2$ because the characteristic Green function cannot be defined in such a case. Hence, a different approach is introduced in this work.

We introduce an image line source $\mathbf{J_1}$ having the same magnitude but opposite sign of the line source (1), and located at the image line $\boldsymbol{\rho_1}$ ($\rho = \rho_0$, $\varphi = \pi - \varphi_0$) with respect to the interface $x = 0$ separating the DPS and DNG regions:

$$\mathbf{J_1} = -I\delta(\boldsymbol{\rho} - \boldsymbol{\rho_1})\hat{z} \qquad (4)$$

Let us designate with $\mathbf{E_0} = \mathbf{E_0}\hat{z}$ the electric field produced by the line source $\mathbf{J_0}$ when the resonator is completely filled with DPS material, and with $\mathbf{E_1} = \mathbf{E_1}\hat{z}$ the electric field produced by the image line source $\mathbf{J_1}$ when the resonator is completely filled with DNG metamaterial.

It can be proven that the electric field in the half-filled resonator considered in this work is everywhere given by

$$\mathbf{E} = E_z\hat{z} = \mathbf{E_0}p_0 + \mathbf{E_1}p_1\hat{z} \qquad (5)$$

where p_0 and p_1 are window functions vanishing in the DNG half-cylinder and in the DPS half-cylinder, respectively.

Slightly modifying the procedure proposed in [12], the fields E_n ($n = 0$ or 1) are given by

$$E_n(\rho, \varphi) = jkZIg_n(\boldsymbol{\rho}, \boldsymbol{\rho}_o) \qquad (6)$$

where g_0 and g_1 are:

$$g_o(\rho, \rho_o) = \frac{1}{2\pi}\sum_{m=0}^{\infty}\varepsilon_m\hat{g}_{om}(\rho, \rho_o)\cos[m(\varphi - \varphi_o)] \qquad (7)$$

$$g_1(\rho, \rho_1) = \frac{1}{2\pi}\sum_{m=0}^{\infty}\varepsilon_m\hat{g}_{1m}(\rho, \rho_1)\cos[m(\varphi - \varphi_1)] \qquad (8)$$

where $\varepsilon_0 = 1$, $\varepsilon_n = 2$ for $n > 0$, and

$$\hat{g}_{om}(\rho, \rho_o) = \frac{\pi J_m(k_o\rho <)D_m(k_o\rho >)}{2J_m(k_oa)} \qquad (9)$$

$$\hat{g}_{1m}(\rho, \rho_1) = \frac{\pi J_m(-k_o\rho <)D_m(-k_o\rho >)}{2J_m(-k_oa)} \qquad (10)$$

with

$$D_m(k\rho) = J_m(k\rho_>)Y_m(k\rho_>) - J_m(k\rho_<)Y_m(k\rho_<) \qquad (11)$$

and $\rho_<$ ($\rho_>$) is the smaller (larger) between ρ and ρ_0. In (9)-(11) J_m and Y_m are respectively the Bessel functions of the first kind and second kind.

The magnetic field components are obtained from the electric field via Maxwell's equations. It can be verified that the above solution satisfies all the boundary conditions.

Numerical validations and results will be presented at the conference and they will be published in [13].

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