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Macromodelling and its Applications to Signal and Power Integrity

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Macromodelling and its Applications to Signal and Power Integrity

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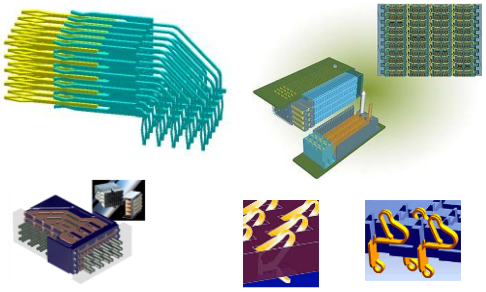
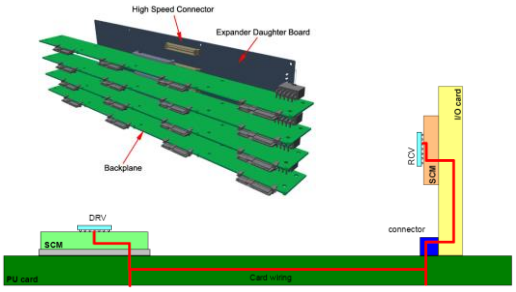
Outline

- Simulation of terminated interconnects
  - Frequency and time-domain analysis
- Transient analysis
  - Convolution-based approaches
  - Direct circuit simulation (when possible)
  - Black-box passive macromodeling
- Black-box passive macromodeling
  - Rational curve fitting
  - Passivity enforcement
- An application example
  - Coupled signal-power integrity analysis of a real board
- Conclusions

Interconnects: showcase

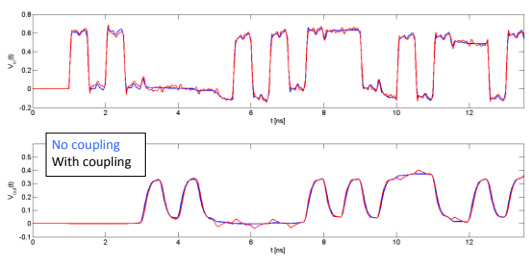


Interconnects: showcase

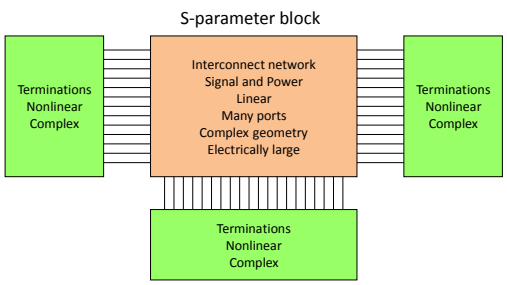


Courtesy D. Kaller, IBM Boeblingen, Germany

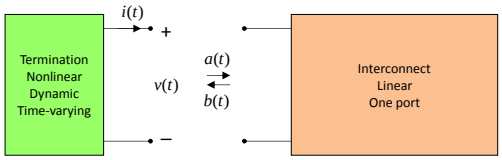
Signal Integrity



The objective



Nonlinear terminations



$$f\left(v,i;\frac{d}{dt};t\right)=0$$

$$B(j\omega_k)=S(j\omega_k)\,A(j\omega_k)$$

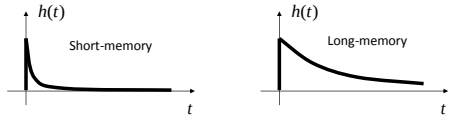
Inverse Fourier/Laplace transform

$$b(t)=h(t)*a(t)=\int_0^t h(t-\tau)a(\tau)d\tau$$

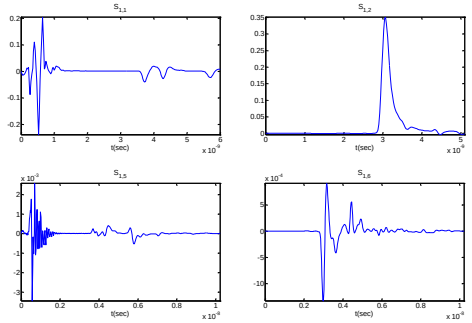
Discretizing convolution

$$b(t)=h(t)*a(t)=\int_0^t h(t-\tau)a(\tau)d\tau \qquad b(t_k)\approx\sum_{m=0}^{k-1}a(t_m)\Delta h_\Delta(t_k-t_m)$$

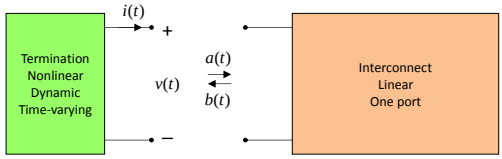
Memory: Number of non-vanishing time-samples in the impulse response



An example: CPU-I/O channel



Direct convolution



$$f\left(v,i;\frac{d}{dt};t\right)=0 \qquad b(t_k)\approx\sum_{m=0}^{k-1}a(t_m)\Delta h_\Delta(t_k-t_m)$$

$$\left.\frac{dv}{dt}\right|_{t=t_k}\approx\frac{v(t_k)-v(t_{k-1})}{\Delta}$$

(e.g., backward Euler)

Direct convolution

$$F_k\left(v(t_k),i(t_k),v(t_{k-1}),i(t_{k-1})\right)=0$$

Need nonlinear solver

$$b(t_k)\approx\sum_{m=0}^{k-1}a(t_m)\Delta h_\Delta(t_k-t_m)$$

Use many past samples

$$a(t_k)=\frac{1}{2}\left(Z_R^{-1/2}v(t_k)+Z_R^{1/2}i(t_k)\right)$$

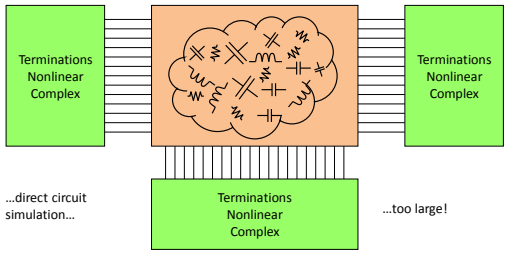
$$b(t_k)=\frac{1}{2}\left(Z_R^{-1/2}v(t_k)-Z_R^{1/2}i(t_k)\right)$$

May be very slow due to long memory in convolution

Very robust (when a good impulse response is available…)

Direct circuit simulation

If a circuit description of the interconnect is available…

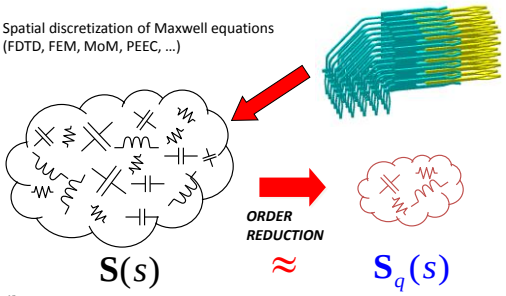


…direct circuit simulation…

…too large!

Model Order Reduction

Spatial discretization of Maxwell equations (FDTD, FEM, MoM, PEEC, …)



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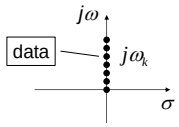
Black-Box Macromodeling

$$\mathbf{h}(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathbf{S}(j\omega)e^{j\omega t}d\omega$$



Parametric closed-form model fitting frequency samples

$$\mathbf{S}(s)\approx\sum_{n=1}^{N}\frac{\mathbf{R}_n}{s-p_n}+\mathbf{S}_\infty$$

Macromodeling via rational function fitting

$s=j\omega_k$

Analytic inversion of Laplace transform

$$\mathbf{h}(t)\approx\sum_{n=1}^{N}\mathbf{R}_n\exp(p_n t)u(t)+\mathbf{S}_\infty\delta(t)$$

May be used directly in SPICE via equivalent circuit extraction

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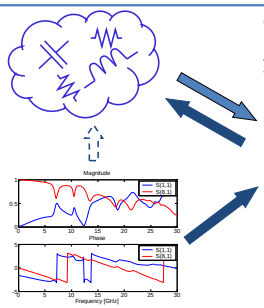
Rational function fitting: why?



Circuit solvers understand circuits

Any lumped circuit has rational frequency responses (poles-residues, poles-zeros, ratio of polynomials)

$$\mathbf{S}(s)\approx\sum_{n=1}^{N}\frac{\mathbf{R}_n}{s-p_n}+\mathbf{S}_\infty$$

Impedance, admittance, scattering

Extraction of an equivalent circuit is an inverse problem (two-step)

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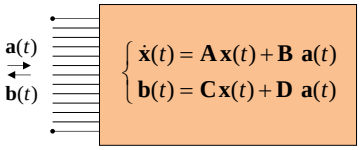
State-space realizations



$$\mathbf{S}(s)\approx\sum_{n=1}^{N}\frac{\mathbf{R}_n}{s-p_n}+\mathbf{S}_\infty=\mathbf{D}+\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}$$

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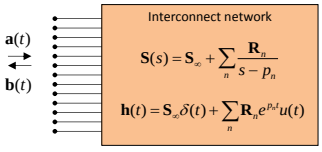
Recursive convolution



$$\mathbf{b}(t)=\mathbf{S}_\infty\mathbf{a}(t)+\sum_n\mathbf{R}_n\int_0^t e^{p_n(t-\tau)}\mathbf{a}(\tau)\mathrm{d}\tau=\mathbf{S}_\infty\mathbf{a}(t)+\sum_n\mathbf{R}_n\tilde{\mathbf{b}}_n(t)$$

Requires only one sample in the past!

$$\tilde{\mathbf{b}}(t_k)\approx e^{p\Delta}\tilde{\mathbf{b}}(t_{k-1})+\frac{1-e^{p\Delta}}{p}\mathbf{a}(t_k)\qquad t_k=t_{k-1}+\Delta$$

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Macromodel implementations

1. Synthesize an equivalent circuit in SPICE format
   - No access to SPICE kernel
   - Must use standard circuit elements
2. Direct SPICE implementation via recursive convolution
   - Laplace element, most efficient
3. Other languages for mixed-signal analyses
   - Verilog-AMS, VHDL-AMS, …
   - Equation-based

Example: board with 13 ports →

	CPU time
Standard convolution	389 seconds
Equivalent circuit	180 seconds
Recursive convolution	5.8 seconds

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Rational curve fitting

Model: $S(s)$

3 alternative rational forms

$$S(s) = \frac{\alpha_0 + \alpha_1 s + \alpha_2 s^2 + \cdots + \alpha_N s^N}{\beta_0 + \beta_1 s + \beta_2 s^2 + \cdots + \beta_N s^N}$$

$$S(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + S_\infty$$

$$S(s) = S_\infty \frac{(s - z_1)(s - z_2)\cdots(s - z_N)}{(s - p_1)(s - p_2)\cdots(s - p_N)}$$

Fitting: $S(j\omega_k) \approx \hat{S}(j\omega_k) = \hat{S}_k \quad k = 1, \cdots, K$ Input data

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Vector Fitting

$$\hat{S}(s) \approx S(s) = \frac{r_0 + \sum_{n=1}^{N} \frac{r_n}{s - q_n}}{c_0 + \sum_{n=1}^{N} \frac{c_n}{s - q_n}}$$

Input data

"starting poles" (arbitrary, as long as distinct)

Linearized (weighted) system: multiply by the denominator

$$\underbrace{\left[c_0 + \sum_{n=1}^{N} \frac{c_n}{s - q_n}\right]}_{\text{The VF "weight function"}} \hat{S}(s) \approx r_0 + \sum_{n=1}^{N} \frac{r_n}{s - q_n} \qquad s = j\omega_k, k = 1, \ldots, K$$

Linear Least Squares system!

$$w(s) = c_0 + \sum_{n=1}^{N} \frac{c_n}{s - q_n}$$

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Vector Fitting

$$w(s) = c_0 + \sum_{n=1}^{N} \frac{c_n}{s - q_n} = \frac{c_0 (s - q'_1)(s - q'_2)\cdots(s - q'_N)}{(s - q_1)(s - q_2)\cdots(s - q_N)}$$

"Pole relocation" process

$$\{q_n\} \rightarrow \{q'_n\} \rightarrow \cdots \rightarrow \{p_n\}$$ "true poles"

At convergence: $w(s) \rightarrow \text{constant}$

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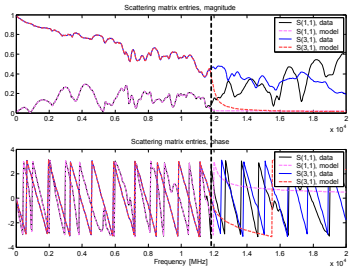
High-speed connector, measured



Modeled up to 12 GHz

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Advanced VF formulations

- Time-domain Vector Fitting
  - Processes time samples instead of frequency samples
- Orthonormal Vector Fitting
  - Further improvement in matrix conditioning using orthonormal rational functions
- Z-domain (orthonormal) Vector Fitting
  - Works on discrete-time/frequency systems
- Fast Vector Fitting
  - Uses smart QR decomposition (compressions) for systems with many ports
- Eigenvalue-based Vector Fitting
  - Possibly with relative error minimization, for improved robustness
- Multivariate/Parameterized Vector Fitting
  - Allows closed-form inclusion of geometry-material parameters in the macromodel equations
- Delayed Vector Fitting
  - Uses modified basis functions for representing propagation delays in closed form
- Parallel Vector Fitting
  - For multicore hardware architectures: close to ideal speedups, almost real-time modeling

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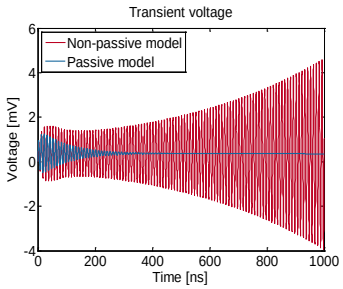
Passivity: why?



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Passivity conditions (scattering)

1. $\mathbf{S}(-j\omega) = \mathbf{S}^*(j\omega)$

   Guarantees real-valued impulse response.
   Always assumed by construction

2. $\|\mathbf{S}(j\omega)\| \le 1$ or $\max_i \sigma_i\{\mathbf{S}(j\omega)\} \le 1$

   Energy condition: structure must not amplify signals.
   Sometimes called simply "passivity" condition

3. $\mathbf{S}(j\omega)$ is causal

   No anticipatory behavior in time-domain.
   Note: causality is a prerequisite for passivity!
   Guaranteed if macromodel is stable.

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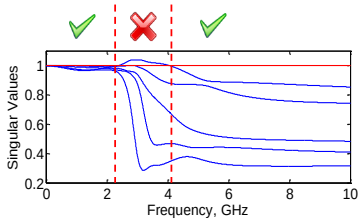
Passivity constraints (scattering)

$\mathbf{S}(s)$ is passive $\Leftrightarrow \{$singular values of $\mathbf{S}(j\omega)\} \le \mathbf{1}, \forall\omega$



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Passivity violations: why?

- Data from measurement
  - Improper calibration and de-embedding, human mistakes
  - Measurement noise
- Data from simulation
  - Poor meshing
  - Inaccurate solver
  - Bad models or assumptions on material properties
  - Poor data post-processing algorithms
  - Putting together results from two solvers
- Macromodel
  - Approximation errors in Vector Fitting
  - May be critical out-of-band, where no data sample is available

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Passivity enforcement

- Generate a new passive macromodel
- Apply small correction to preserve accuracy
  - original dataset should be passive
  - original macromodel should be accurate
  - (usually) preserve poles

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\,\mathbf{a} \\ \mathbf{b} = \mathbf{C}\mathbf{x} + \mathbf{D}\,\mathbf{a} \end{cases} \Rightarrow \begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\,\mathbf{a} \\ \mathbf{b} = (\mathbf{C} + \Delta\mathbf{C})\mathbf{x} + \mathbf{D}\,\mathbf{a} \end{cases}$$

$$\Delta\mathbf{S} = \Delta\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

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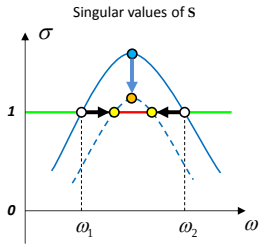
Model Perturbation



$$\begin{cases} \min \|\mathbf{w}\|_2 \\ \mathbf{Q}\mathbf{w} = \mathbf{y} \\ \mathbf{R}\mathbf{w} \le \mathbf{b} \end{cases}$$

accuracy

passivity

- Few and simple constraints
- Convex: robust solution

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A case study: coupled Signal/Power Integrity

This case study courtesy of

- Georgia Institute of Technology, Atlanta GA, USA
- E-System Design, Inc.
  - Provided field solver Sphinx
- Politecnico di Torino
- IdemWorks s.r.l.
  - Provided passive macromodeling tool IdEM

E-System Design — www.e-systemdesign.com
IDEMWORKS — www.idemworks.com

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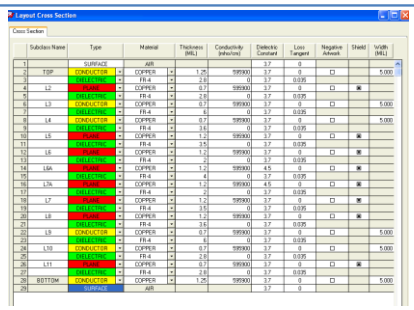
Board cross-section



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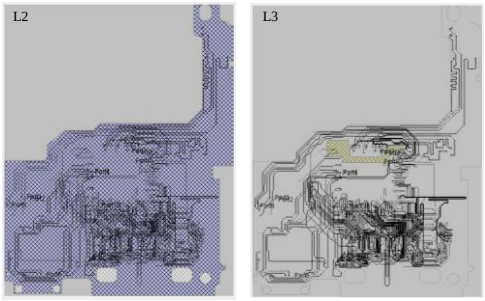
Layers L2 and L3



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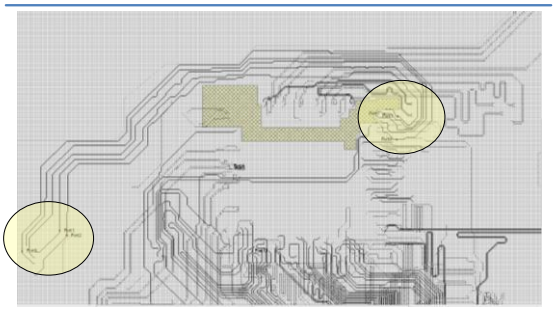
Port locations
L3 (Ref: L2) ports 1,7; 2,3; 8,9



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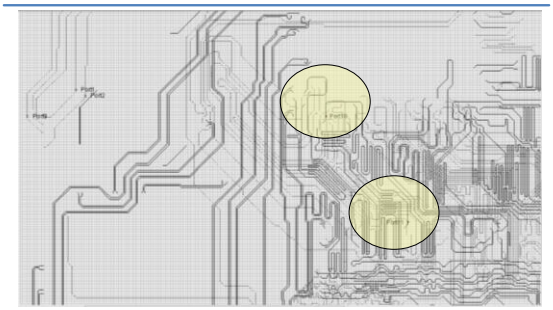
Port locations
L4 (Ref: L5) ports 10,11



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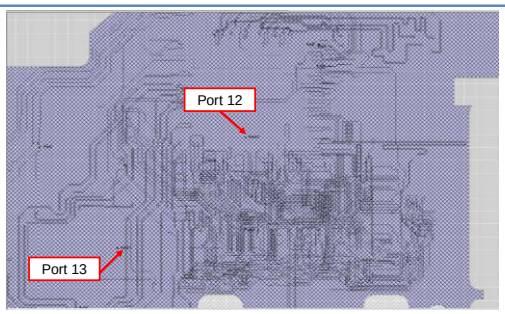
Power ports
L2 (Ref: L5) ports 12,13



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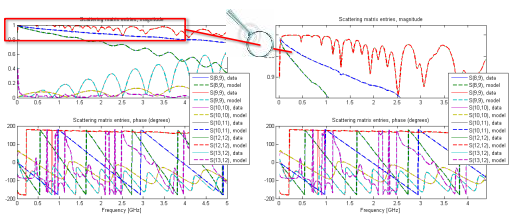
Macromodel vs S-parameters



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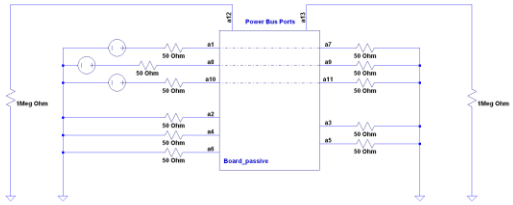
SPICE: excitation on signal lines



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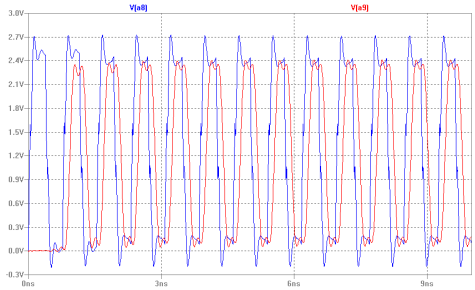
Response on a signal line, 1.3GHz



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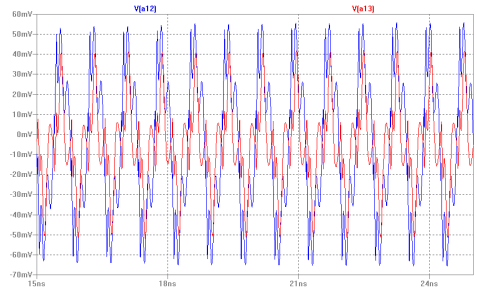
Coupling to power ports, 1.3GHz



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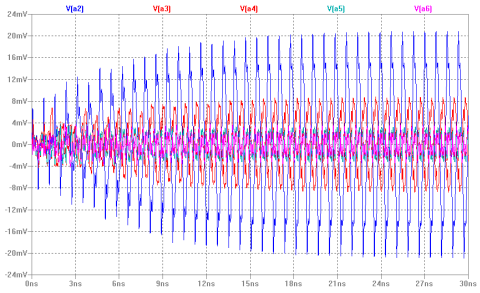
Xtalk and substrate coupling, 1.3GHz



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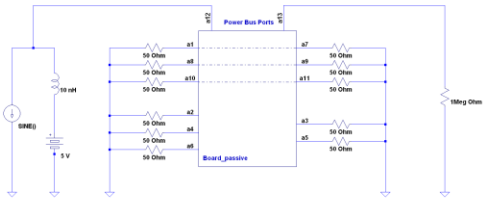
SPICE: excitation on PDN (core switching)



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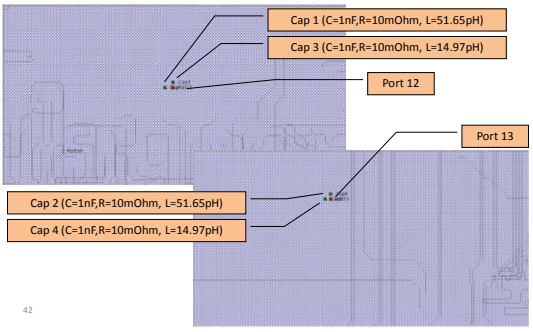
Decoupling capacitors



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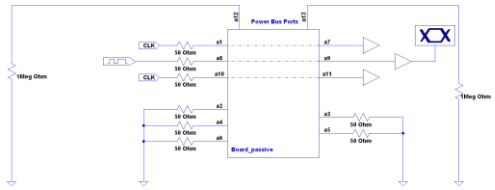
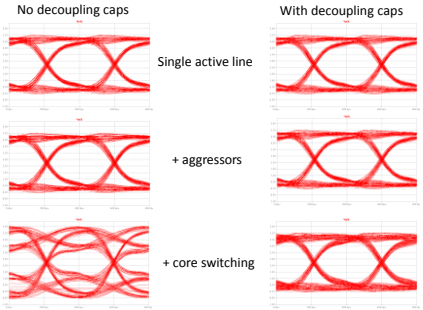
PDN response

Eye diagram simulation: setup



Eye diagram results, 2.6 Gb/s



„Signal Integrity Summary“

- **Application:**
  - **Fast numerical assessment of Signal and Power Integrity problems during early design stages**
- **Problems:**
  - **Mixing time-domain circuit-level models (NL) with frequency-domain description of interconnect networks, complexity, efficiency**
- **Solution:**
  - **Rational black-box macromodeling + smart implementation**
  - **Key enabling factors for fast system-level simulation, design optimization, what-if analyses**