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(Article begins on next page)

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Modelling of junctions with edges in surface integral equations

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***Abstract*— When the surface integral equation method is applied to metallic structures with junctions, a special procedure to handle the basis functions and the unknowns is needed. In this summary paper we present the possibility to combine the capability of modeling edges/wedges with patch junctions. Numerical simulations will illustrate the validity of the proposed method as well as some applications will be presented in terms of evaluation of current density and RCS in PEC structures.**

I. INTRODUCTION

The Method of Moments (MoM) has been successfully validated for modeling 3D sharp-wedge structures using Surface Integral Equations (SIE) [1].

From the other side, the procedure for the handling junctions in SIE are successfully implemented in several papers [] in structures composed by piecewise inhomogeneous geometries, in particular the use of Rao-Wilton-Glisson (RWG) basis functions has been dealt in junctions with PEC and dielectric regions.

In this summary we consider the modeling capability of singular divergence-conforming vector bases with the procedure for handling surface junctions of PEC structures.

Numerical simulations will illustrate the validity of the proposed method as well as some applications will be presented in terms of evaluation of current density and RCS in PEC structures.

II. SINGULAR DIVERGENCE-CONFORMING VECTOR BASES

As proposed in [] we have defined singular divergence-conforming vector based that models 3D sharp-wedge structures. These functions are able to model singularities in curvilinear patches of triangular and quadrilateral shapes. Moreover the bases are complete to arbitrarily high order and they constitute and additive subset to regular high order bases that are commonly used in MoM applications[].The singular subset are located at the edge profile as shown in Fig. 1 and in the case of triangular discretization of the geometry they are of two kinds: edge singular elements and vertex singular elements.

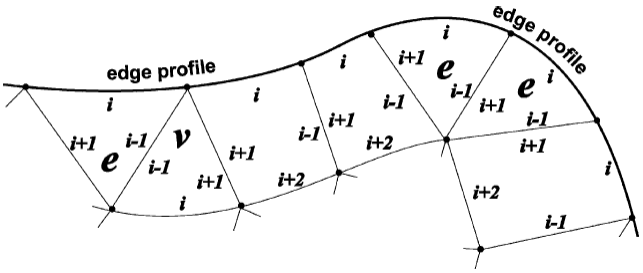

Figure 1. Edge singularity quadrilaterals and edge (e) and vertex (v) singularity triangles with local edge numbering scheme.

III. JUNCTIONS IN JOINED ELECTRICALLY CONDUCTING PLATES

Without loss of generality, we focus the attention on joined electrically conducting plates, see Fig. 2 the case of tri-plate.

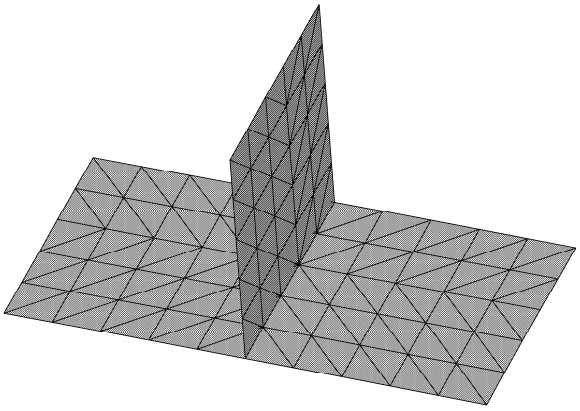

Figure 2. Edge singularity quadrilaterals and edge (e) and vertex (v) singularity triangles with local edge numbering scheme

the a perfect electric conductor (PEC) wedge lying on a dielectric half-space, see Fig. 1, *i.e.* a PEC wedge with a face

tangent to the dielectric half-space. The PEC wedge is with aperture angle $\pi - \Phi_e$ with $\Phi_e > \pi/2$.

Without loss of generality, the structure is illuminated by an E-polarized plane wave with direction φ_o ($0 \le \varphi_o \le \Phi_e$) at normal incidence (1) with $k_o = \omega\sqrt{\mu_o \varepsilon_o}$, located in the upper half-space constituted by free-space (permittivity ε_o and permeability μ_o). The lower half-plane is constituted by dielectric region with permittivity $\varepsilon = \varepsilon_r \varepsilon_o$ and permeability $\mu = \mu_r \mu_o$.

$$E_z^i = E_o e^{jk_o \rho \cos(\varphi - \varphi_o)} \tag{1}$$

The skew incidence case will not introduce conceptual difficulties but it will double the number of equations to be solved.

The literature on this problem is apparently very poor. Particular cases of a wedge immersed in a stratified medium were studied by Bertoncini and al. [1-2] by using the Uniform Theory of Diffraction (UTD). However the application of this method is limited to edges not close to the stratified regions.

Our approach is based on a formulation that uses the generalized Wiener-Hopf equations (GWHE) developed in [3-12] for different kind of wedge problems, *i.e.* scattering and diffraction by impenetrable wedge and penetrable wedge illuminated by plane waves.

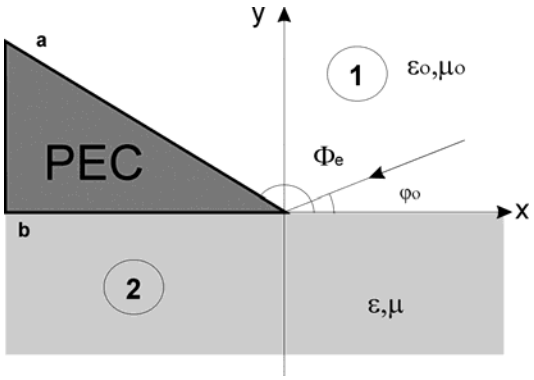

Figure 1. Perfect electric conductor (PEC) wedge lying on a dielectric half-space.

This technique has been recently extended to investigate wedge over a stratified medium [13].

Since closed form solutions of classical/generalized Wiener-Hopf equations are possible only for a small class of problems, we have introduced a semi-analytical method based on the reduction of GWHE to Fredholm integral equations (FIE) [13-17] to obtain solutions for more general problems.

The aim of this summary paper is to successfully introduce the extension of the application of Wiener-Hopf techniques to the present problem.

IV. FORMULATION

Following the procedure presented in [13] we formulate the problem in terms of radial transmission line [3,18,5] in the upper free-space region and in terms of longitudinal transmission line in the lower dielectric region.

With reference to Fig. 1, we only consider time harmonic electromagnetic fields with a time dependence specified by the factor $e^{j\omega t}$ which is omitted in the following.

We define the following Wiener-Hopf spectral unknowns in the η-plane in the upper region for $(y > 0, 0 \le \varphi \le \Phi_e)$

$$\begin{aligned} V_+(\eta,\varphi) &= \int_0^\infty E_z(\rho,\varphi) e^{j\eta\rho} d\rho \\ I_+(\eta,\varphi) &= \int_0^\infty H_\rho(\rho,\varphi) e^{j\eta\rho} d\rho \end{aligned} \tag{2}$$

and in the lower region for ($y<0$)

$$\begin{aligned} v_\eta(y) &= \int_{-\infty}^\infty E_z(x,y) e^{j\eta x} dx \\ i_\eta(y) &= \int_{-\infty}^\infty H_x(x,y) e^{j\eta x} dx \end{aligned} \tag{3}$$

By recalling the boundary conditions on the PEC and dielectric faces, the radial transmission line $(y > 0, 0 \le \varphi \le \Phi_e)$ is modeled by the following GWHE

$$Y_c V_+(\eta) - I_+(\eta) = -I_{a+}(-m) \tag{4}$$

where $Y_c = \dfrac{\xi}{\omega\mu_o}$, $\xi = \xi(\eta) = \sqrt{k_o^2 - \eta^2} = \tau_o$, $m = -\eta\cos\Phi_e + \xi\sin\Phi_e$.

The longitudinal transmission line $(y < 0)$ is modeled by the following classical Wiener-Hopf equation

$$Y_d V_+(\eta) + I_+(\eta) = -I_-(\eta) \tag{5}$$

where $Y_d = \dfrac{\xi_d}{k_d Z_d}$, $\xi = \xi(\eta) = \sqrt{k_d^2 - \eta^2}$, $k_d = k_o\sqrt{\varepsilon_r}$.

III. STEPS FOR THE SOLUTION

Starting from the system of Wiener-Hopf equations (4) and (5) we apply a generalized version of the Fredholm factorization method [14-17] as reported in [13] yielding:

$$Y_c(\eta)V_+(\eta) - I_+(\eta) + \frac{1}{2\pi j}\left[\int_{-\infty}^{\infty} \frac{Y_c(\eta')}{\alpha(\eta') - \alpha(\eta)} \alpha'(\eta') - \frac{Y_c(\eta)}{\eta' - \eta}\right] V_+(\eta')d\eta' = f(\eta) \tag{6}$$

where $f(\eta)$ is a source term extracted from the Wiener-Hopf unknowns as offending pole. This term is related to the Geometrical Optics field (GO).

The numerical discretization of (6) [19] yields the solution in terms of $V_+(\eta)$.

Using the inverse Fourier transform and asymptotic evaluation it is possible to evaluate the GTD diffraction coefficient and the total far-fields.

III. CONCLUSION

The method proposed in this summary paper shows the possibility of the GWHE technique to solve the problem of evaluating the electromagnetic field of a perfect electric conductor wedge lying on a dielectric half-space.

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