Sensitivity analysis of damaged monumental structures: the example of S. Maria del Suffragio in L'Aquila

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SUMMARY. In Italy, which accounts for an impressive number of architectural heritage sites, a large part of the territory is subject to seismic risk. Nonetheless, also the two recent examples of the 2009 L'Aquila earthquake and 2012 Emilia earthquake confirmed and highlighted the vulnerability of cultural heritage structures to these types of events.

In this paper the church of *Santa Maria del Suffragio* (*Anime Sante*) in L'Aquila is used as a benchmark for the experimental validation of a finite element model on the basis of the data gathered by the permanent structural health monitoring system installed on the building by IUAV in 2009. Structural health monitoring techniques have been largely applied to cultural heritage buildings in recent times, mostly because of their non-destructive nature, and they have proven to be a valid tool in assessing the damage evolution and in characterising the global dynamic behaviour of the structure.

In particular, a global sensitivity analysis technique has been applied to a finite element model. The model underwent a model updating procedure on the parameters chosen in the sensitivity analysis. The calibrated model is an invaluable tool in assessing the dynamic behaviour of the structure and may serve for several purposes.

1 INTRODUCTION

On April 6, 2009, at 03:32 an earthquake of magnitude 6.3 on the Richter scale struck central Italy. The epicentre of the quake was near the medieval city of L'Aquila, at 42.4228°N 13.3945°E. From the point of view of architectural heritage, in L'Aquila, neglecting the other cities of the seismic crater, the earthquake has made 355 out of 552 churches unusable, 112 out of 171 buildings, 13 towers out of 27. The extent of the damaged and the need to assess conservation, structural safety (ICOMOS 2003), the intrinsic vulnerability of the historic buildings and the safeguarding of human life require the definition of a methodology for monitoring, diagnosis and structural identification [1]. The observation and interpretation of full scale performance of civil infrastructures and historical buildings, through Structural Health Monitoring (SHM) and its subsets, has found favour and generated great interest in the academic research community.

SHM integrates sensing, data-communication and computing systems with non-destructive evaluation including geometric-physical surveys and vibration measurements. Dynamic monitoring is, to date, the only non-obtrusive methodology that allows global control and structural identification, through the model updating process [1]. SHM based on Vibration Based Monitoring (VBM) can be carried out by the statistical analysis of the data or by numerical model calibration [2]. In statistical analysis the data-process is based on the extraction of the data recorded during a long period of monitoring. This allows the identification of the variations in time and the determination of the current status of the system. Numerical model calibration, indeed, is based on a sensitivity analysis of numerical models of the structure. This analysis can be used to determine which parameters are the most important and most likely to affect system behaviour. Following a sensitivity analysis, values of critical parameters must be refined, while parameters that have little effect can be simplified or ignored [3].

These techniques involves use forced (Experimental Modal Analysis EMA) or ambient (Operational Modal Analysis OMA) response vibration data to identify modal characteristics. These modal parameters and their derivations reflect the structure mass, stiffness and damping properties which depending on the condition of the structure. Changes of the modal parameters identify changes in structure, with the possibility of detecting and/or quantifying the damage through the Vibration-Based Damage Detection (VBDD). For the non-obtrusive peculiarities the VBM and VBDD procedures are widely used in historic masonry structures. Structural identification of cultural heritage using ambient vibration data has been studied in [4, 5, 6].

The case dealt with in this paper shows the important role played by sensitivity analysis in the updating of a FE model using ambient vibration data of damage structure. Santa Maria del Suffragio church in L'Aquila was severely damaged by the 2009 earthquake. It is a complex example for the different variables interacting, such as the intrinsic characteristics of historic masonry, widespread damage, partial collapse, safety measure and degradations of global and local stiffness.

2 THE STRUCTURE

2.1 *The damage*

Santa Maria del Suffragio, also known as Anime Sante, is a 18th-century church in L'Aquila, Italy. Being this monument one of the most important churches of the city, it became one of the main symbols of the 2009 L'Aquila earthquake.

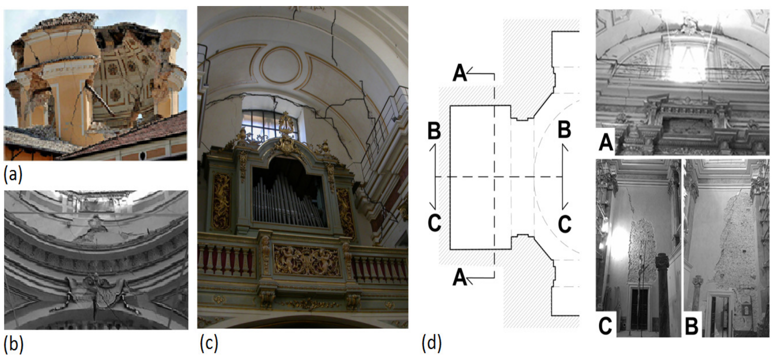

Figure 1: Damaged: (a) Dome, (b) Façade, (c) Transept, (d) Apse

The main shock caused damage mechanisms in the macro-elements of the monument (Figure 1). The macro-element cupola was affected by the failure of the belfry and the partial collapse of the cupola's and tambour's walls. The overspread crack patterns characterized by shear and compression cracks has arisen by the pendulum movement of this macro-element and by the simultaneous collapse of a great part of it. The out of plane bending of the façade, of the transept and of the apse walls have compromised the box behaviour of the structure. The noticeable shear cracking of all macro-elements causes the loss of the effectiveness of interconnections among the walls.

In general, all the arches and the apse of the church show cracks at the key zone and all the walls are affected by horizontal cracks at 1 meter from floor. These had been induced, probably, by the predominant vertical component of the earthquake.

2.2 *Safety measurement*

The strengthening activities are started after few weeks the main shock (6th April of 2009) and continued until November 2011 thanks to the structural control activity carried out during this period of static and dynamic monitoring (from November 2009 to March 2013).

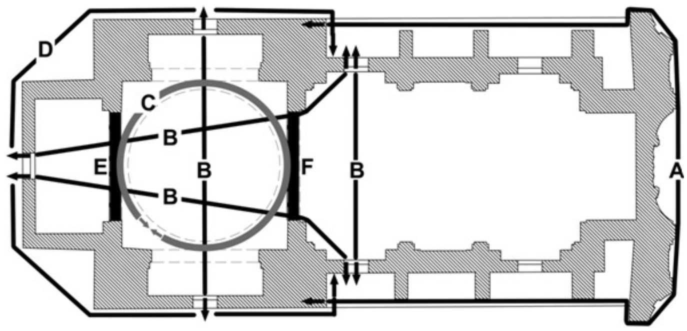

Figure 2: General scheme of safety measures: ties for façade (A) and for transept-nave-apse walls (B and D), scaffold and confinement for tambour (C), scaffolds for apse arch (E) and transept-nave arch (F).

The safety measures are illustrated in Figure 2. The ties labelled by capital letter A (Figure 2) avoid the overturning of the façade macro-element through the strength of longitudinal walls. The ties around the tambour ensure the confinement effect spreading the forces on whole perimetral surface through the temporary scaffold built up to fill the missing parts collapsed during the earthquake (Figure 2, letter C). The ties linking the bottom wall of the apse to the longitudinal walls and the ties between the bottom walls of the transept, labelled with letter B, work together with the external ties specified with letter D. The last safety measures regard the arches between the transept, the nave and the apse (Figure 2, letters E and F). The scaffolds were assembled at November 2011 to shore the arches weakened by the cracks in key.

3 DYNAMIC MONITORING

The monitoring activity began in November 2009 and finished in March 2013. The dynamic monitoring system (Figure 3) is based on 28 accelerometer and 30-channels. Each channel has its own threshold, calibrated on the signal. This is digitalised and pre-elaborated through high-pass digital filters, with a cut-off frequency of 0.31 Hz, to remove the sensor's offset. The dynamic sensors are 16 piezo-electric mono-directional accelerometers, with nominal sensitivity of 1000 mV/g, frequency interval (±5%) from 0.025 to 800 Hz, and 4 piezo-electric tri-directional accelerometers, with nominal sensitivity of 1000 mV/g, frequency interval (±5%) from 0.5 to 3000 Hz. The signals were sampled at a frequency of 500 Hz per channel.

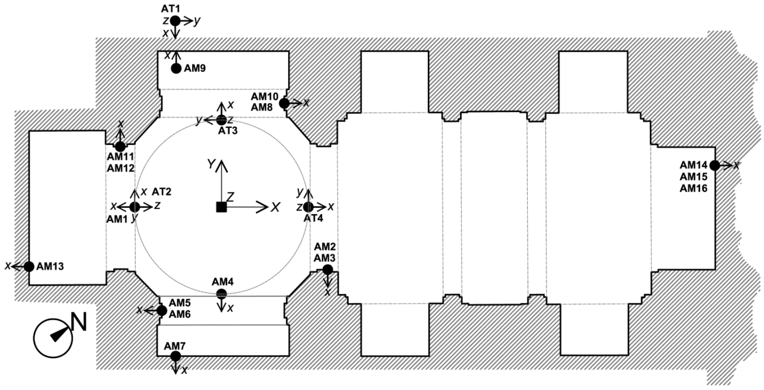

Figure 3: Plan scheme of accelerometric directions (AM mono-directional sensor, AT tri-directional sensor).

4 FINITE ELEMENT MODEL AND MODAL ANALYSIS

The church of Anime Sante shares the usual complex geometry and topology of cultural heritage buildings. Its structural dynamic behaviour is complicated of the severe damage caused by the 2009 Earthquake and by the successive seismic retrofitting interventions.

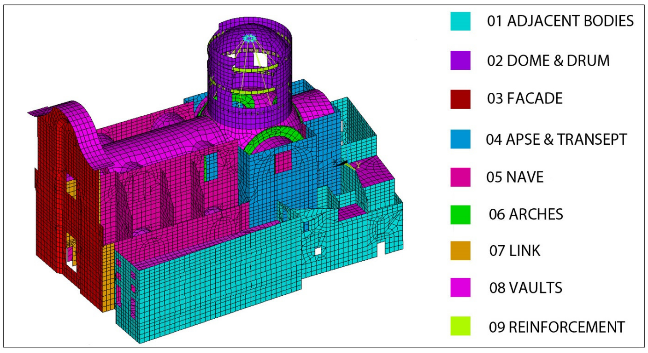

Figure 4: Macro-elements subdividing the damaged FE model.

The FE model had to consider not only the damage of the structure but also the seismic retrofit intervention that followed the collapse of the dome.

The diffuse cracks in the nave and in the transept have been modelled by modifying material properties while the extensive damage in the dome have been incorporated directly in the geometric model. The 3-levels reinforcement with metal tie rods and the 3D spatial frame, which carry the steel-glass roof, have been modelled respectively using shell element and beam elements. These have an equivalent stiffness and a mass density reduced in order to have the real behaviour.

The masonry has been modelled as a homogeneous material but different macro-zones are individuate. In each macro-zone the material could be different due to historic, structural or modelling reasons (Figure 4). A different material was also introduced in the interface region between the façade and the nave (Link in Figure 4) because of the extensive damage in that zone due to the earthquake.

4.1 *Structural dynamic identification*

Ambient vibration noise data were available for the structural dynamic identification of the building. The event of 14th November 2011 was used, having a signal length of 7200 s and sampled at frequency of 500 Hz. Standard signal conditioning techniques were used at first to clean the rough data: mean removal, polynomial de-trend (of order 1) and signal decimation (factor 5) together with a band pass filter of order 3 between 0.5 Hz and 20 Hz.

The structural dynamic identification, in terms of frequencies, damping ratios and modal shapes, has been carried out in the time domain using a stochastic subspace identification, in the algorithmic version proposed by Larimore, known as Canonical Variate Analysis (CVA) [1, 7].

The results of identification, carried out using all the channels except ones located below +1.5 m height, have shown good stability of the first 11 modes for different model order (Figure 5). It is possible to notice that stabilisation occurs for all the modes that are clearly seen in the frequency domain and also for the modes almost invisible in part of the structure. Even clusterisation in the frequency-damping plane has confirmed the good quality of the results, in fact in Figure 5 several cluster are clearly distinguishable with low dispersion of the esteem both in frequency and damping.

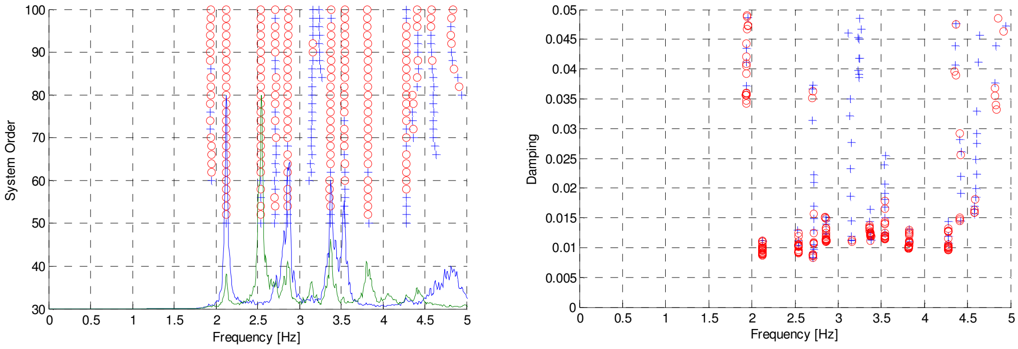

Figure 5: Stabilisation diagram: o stable modes + unstable modes —AT3X —AT3X
Stabilisation criteria: 1% Frequency, 5% Damping, 2% MAC.

Table 1: Identified frequencies, damping and modal shapes with SSI method.

Mode number	Frequency (Hz)	Damping (%)	Mode description
1	1.94	4.1	1st flexural façade
2	2.12	1	1st flexural dome Y
3	2.54	1	1st flexural dome X
4	2.71	1.1	2nd flexural dome Y
5	2.86	1.2	
6	3.15	1.1	
7	3.24	4.3	
8	3.37	1.4	
9	3.54	1.1	
10	3.82	4.1	
11	4.28	1.1	

The first mode, which is the 1st flexural mode of the façade, seems to have a relatively high dispersion index in terms of damping, whilst the majority of the other identified modes have a really good esteem of the damping (Table 1). The frequencies identified with the SSI method have been confirmed by an analysis in time-frequency domain of the most sensitive channels.

For what concerns the modal shapes, the location of the sensors was not optimal to distinguish higher modes, but the first 4 frequencies can be still classified (Table 1).

5 SENSITIVITY ANALYSIS AND MODAL UPDATING

The sensitivity analysis is a method to do modal updating in order to reduce the discrepancies, that can be significant, between the behaviour of a numerical model and the real system. These methods [7, 8] have seen larger widespread compared to direct methods because of their capability to calibrate the model taking the influence of the updating parameters of the different structural elements into account. They offer a wide range of parameters to update that have physical meaning and allow a degree of control over the optimisation process. All these parametric methods rely on the definition of a so-called penalty function which is usually computed as the quadratic norm of the differences between the measured and the numerical quantities.

5.1 *Optimisation procedure and sensitivity analysis*

In this paper the sensibility analysis has been used by minimising a penalty function. In order to define a robust penalty function the method proposed by Bakir, Reynders and De Roeck [8] has been pursued. One can define a residual vector as the weighted difference between the measured quantities $\boldsymbol{v_m}$ and calculated quantities $\boldsymbol{v(p)}$, as follows:

$$\mathbf{r}(\mathbf{p}) = \begin{bmatrix} \mathbf{r}_e(\mathbf{p}) \\ \mathbf{r}_s(\mathbf{p}) \end{bmatrix} = \mathbf{W}(\mathbf{v_m} - \mathbf{v}(\mathbf{p})) = \mathbf{W} \begin{bmatrix} \lambda_{1,EXP} - \lambda_{1,FEM} \\ \dots \\ \lambda_{m,EXP} - \lambda_{m,FEM} \\ \phi_{1,EXP} - \phi_{1,FEM} \\ \dots \\ \phi_{m,EXP} - \phi_{m,FEM} \end{bmatrix} \tag{1}$$

where $\lambda_{i,FEM}$ and $\lambda_{i,EXP}$ are the i-th analytical and experimental eigenvalues, respectively, whilst $\Phi_{i,FEM}$ and $\Phi_{i,EXP}$ are the i-th analytical and experimental eigenmodes. The other term $\boldsymbol{W}$ is a diagonal weighting matrix which normalise the residue of eigenvalues and eigenmodes because they can be of different order of magnitude. **W** matrix is thus defined as follows:

$$\mathbf{W} = \mathbf{diag}\left(1/\lambda_{1,EXP}, \dots, 1/\lambda_{m,EXP}, \dots, \omega_{\varphi 1}, \dots, \omega_{\varphi m}\right) \tag{2}$$

In order to compute the modal shape weights ω_i one must take in account that is common to have analytical modal shapes normalised with respect to the mass matrix, whilst experimental modal shapes are normalised to the unity or unscaled. Therefore, by defining the weighted eigenfrequency residual and mode shape residuals as:

$$r_e(\mathbf{p}) = \frac{\lambda_{i,FEM} - \lambda_{i,EXP}}{\lambda_{i,EXP}} \tag{3}$$

$$r_s(\mathbf{p}) = \frac{\varphi^l_{j,FEM}}{\varphi^r_{j,FEM}} - \frac{\varphi^l_{j,EXP}}{\varphi^r_{j,EXP}} = \omega^l_j \left(\varphi^l_{j,FEM} - \varphi^l_{j,EXP}\right) \tag{4}$$

the weighting matrix coefficients are obtained as:

$$\omega^k_i = \frac{\left(\varphi^k_{j,FEM} \cdot \varphi^r_{j,EXP} - \varphi^k_{j,EXP} \cdot \varphi^r_{j,FEM}\right)}{\varphi^r_{j,FEM} \cdot \varphi^r_{j,EXP}\left(\varphi^k_{j,FEM} - \varphi^k_{j,EXP}\right)} \tag{5}$$

Finally, one can update the FE model by minimising the residual:

$$min\left(0.5 \cdot \mathbf{r}(\mathbf{p})^T \, \mathbf{W} \;\; \mathbf{r}(\mathbf{p})\right) \tag{6}$$

This penalty function allows minimizing the discrepancies between frequencies and modal shapes modifying the values of previously chosen parameters in accordance to a previously performed sensitivity analysis. In order to perform the sensitivity analysis, a sensitivity index has been calculated, in the form defined by Saltelli et al. (2000):

$$S_i = \frac{ln(\varepsilon) - ln(\varepsilon_0)}{ln(x_i) - ln(x_{i0})} \tag{7}$$

where x_{i0} and $x_i = x_{i0}(1+\delta)$ are, respectively, the initial nominal value of the i-th updating parameter and its value incremented of a given percentage δ (here assumed equal to 5%), ε_0 is the error corresponding to the nominal model, and ε is the error corresponding to equalling all parameters to their nominal value while setting the i-th parameter to x_i.

The relative sensitivity index values are shown in Figure 6 with respect to the first 4 frequencies of the FE model.

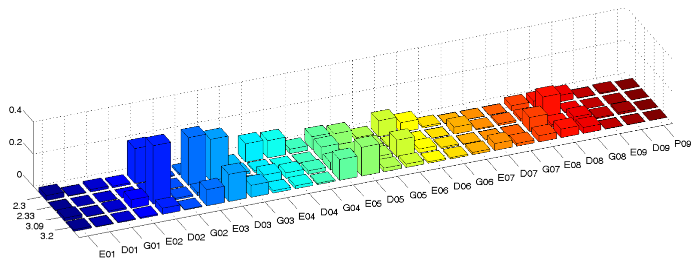

Figure 6: Frequency sensitivity index with respect to the parameters of the 9 materials chosen.

Based on the sensitivity analysis, only the most sensible parameters have been chosen as updating parameters. Among the macro-zones, in which the FE model had been divided (Figure 4), it has been chosen to tune the parameters of six elements: the apse, the façade, the nave, the interface between façade and nave, the arches and the metal tie-rod system.

The material's density has been taken constant to 1700 Kg·m^3. The full list of parameters is reported in Table 2, with their initial nominal values listed. Moreover, for brevity's sake, also the values obtained after the updating procedure are reported. The solution of Eq. (6) has been computed by using a pattern search algorithm.

Table 2: Updating parameters. Nominal and updated values.

Material	$E_{initial}$ (Gpa)	$G_{initial}$ (Gpa)	$E_{updated}$ (Gpa)	$G_{updated}$ (Gpa)
01 Adjacent bodies	1.50	0.80	2.60	0.44
02 Dome & Drum	1.50	0.80	1.50	0.98
03 Facade	1.50	0.80	0.80	0.37
04 Apse & Transept	1.50	0.80	2.00	0.68
05 Nave	1.50	0.80	1.00	0.65
06 Arches	1.50	0.80	1.00	0.99
07 Link	1.00	0.40	1.90	0.64
08 Vaults	1.50	0.80	1.00	0.43
09 Reinforcement	2.00E+03	0.30	1.50	0.30

5.1 *Updated model*

In Figure 7 a graphical representation of the comparison between numerical and experimental frequencies is shown for the updated FE model. Also the MAC between identified and analytic modal shapes is represented and reported in the graph by numerical values. The line indicates the ideal linear correlation that one should aim when updating a FE model.

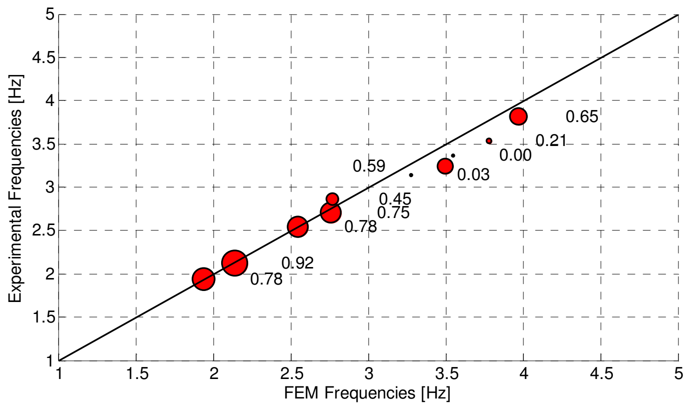

Figure 7: Frequency sensitivity index with respect to the parameters of the 9 materials chosen.

One may notice that the first four modal frequencies used for the updating show a good match with the experimental data, both in terms of frequencies and of modal shapes. Higher modes (in Figure 7 are shown up to the tenth) retain as well a strong correlation in terms of frequencies, but some of them are swapped, resulting in very low values of MAC.

6 CAPABILITY OF THE UNDATED MODEL FOR SEISMIC ASSESSMENT OF THE REPAIRED STRUCTURE

The predictive capabilities of the updated FE model, as resulting from the sensitivity analysis, have been verified by performing a time-history analysis.

Several low-intensity accelerograms have been recorded during the last three years by the dynamic monitoring system. Therefore, the propose FE updating procedure was assessed with respect to an intense seismic events recorded in the recent past. In particular, the event supplied as input of the updated FE model occurred on 02/17/2013 at 1:00:07 hours (UTC, Coordinated Universal Time), and was attributed a magnitude of 3.7ML (see Italian Seismic Instrumental and parametric Data-base ISIDE http://iside.rm.ingv.it/iside/standard/result.jsp).

The experimental displacements calculated by double numerical integration of the accelerometric signals have been compared with those obtained by the FE model in the monitored positions. This comparison shows a good agreement in terms of displacement amplitudes (Figure 8). The average error is of about 24% on the esteem of the maximum displacements.

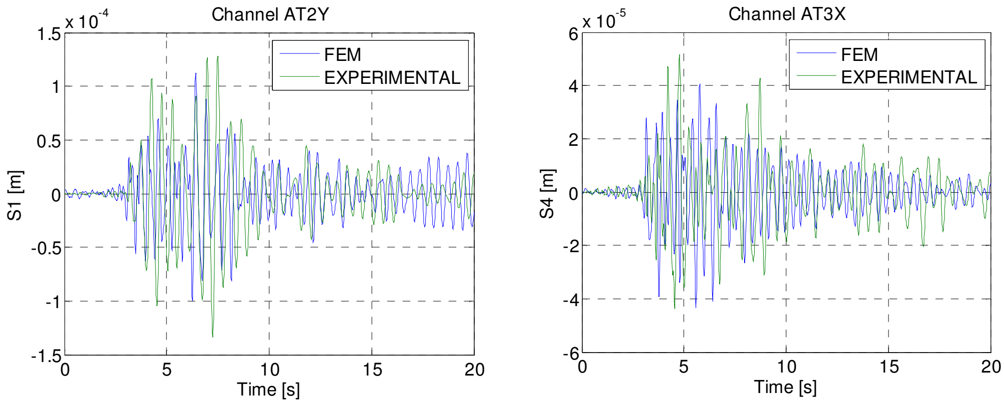

Figure 8 : Comparison of numerical and experimental time-history response for channel AT2Y and AT3X located at the bottom of the tambour.

7 CONCLUSION

This paper has presented a sensitivity analysis procedure for the calibration of a finite element model of the church S. Maria del Suffragio in L'Aquila. The updating process was based on data acquired by a permanent monitoring system installed on the building after the 2009 L'Aquila earthquake. This application was complicated by the uncertain dynamic behaviour of the monument, which is characterised by extensive and severe damage, and by the interaction with post-earthquake safety interventions. Sensitivity methods make it possible to take into account a reduced but meaningful set of parameters associated to different structural macro-elements. In this specific case, a good match has been achieved for the first four modes with a MAC value ≥0.75. The performance of the updated model was validated through a time-history analysis and an average error of 24% was found. This error is justified by the typical non-linear effects activated by the earthquake as well as with non-modelled dynamics (soil-structure interaction, uncertainty on mass matrix, etc.).

References

[1] Ceravolo, R. and Abbiati, G., “Time Domain identification of structures: a Comparative Analysis,” *ASCE Journal of Engineering Mechanics,* **139(4)**, 537-544 (2013).

[2] Farrar, C.R. and Worden, K., “An introduction to structural health monitoring,” *Phil. Trans. R. Soc. A.,* **15(265)**, 303-315 (2007).

[3] Santelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J. and al. *Global Sensitivity Analysis: The Primer*, Wiley, Chichester (2008).

[4] De Stefano, A. and Ceravolo, R., “Assessing the health state of ancient structures: the role of vibrational tests,” *Journal of Intelligent Material System and Structures.,* **18**, 793-807 (2007).

[5] Ramos, L.F., De Roeck, G., Lourenço, P.B., and Campos-Costa, A., “Damage identification on arched masonry structures using ambient and random impact vibrations,” *Engineering and Structures,* **32**, 146-162 (2010).

[6] Russo, S., “On the monitoring of historic Anime Sante church damaged by earthquake in L’Aquila,” *Structural Control and health Monitoring.,* 10.1002/stc.1531 (2012).

[7] Van Overschee, P. and De Moor, B., *Subspace identification for Linear Systems: theory and Implementation-Applications*, Kluwer Accademic Press , Dordrecht (1996).

[8] Mottershead, J.E., Link, M. and Friswell, M.I., “The sensitivity method in finite element model updating: A tutorial,” *Mechanical System and Signal processing,* **25**, 2275-2296 (2011).

[9] Barik, P.G., Reynders, E. and De Roeck, G., “Sensitivity-based finite element model updating using constrained optimization with a trust region algorithm,” *Journal of Sound and Vibration,* **305**, 211-225 (2007).