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Number π from the Decorations of Ancient Artifacts

Amelia Carolina Sparavigna

Department of Applied Science and Technology, Politecnico di Torino, Italy
E-mail: amelia.sparavigna@polito.it

Abstract

The decorations of ancient objects can provide some information on the value of constant π as a rational number, known and used by the artists who made them. Number π is the dimensionless ratio of circumference to diameter, and then, by measuring the ancient decorations we can obtain its value and gain some hints on the human knowledge of mathematics and geometry in prehistoric times. Here we discuss two examples of this approach. The first is concerning some disks found in the tomb of Hemaka, the chancellor of a king of the First Dynasty of Egypt, about 3000 BC. The second is the decoration composed by several circles and spirals of the Langstrup belt disk, an artifact of the Bronze Age found in Denmark.

Keywords: History of Science, Mathematics of Bronze Age

Introduction

It is generally believed that the estimation of π constant, the dimensionless ratio of circumference to diameter, is quite old. According to [1], the value of π is involved in the proportions of the Great Pyramid of Giza, of the 26th century BC, in the form of 3+1/7 = 22/7. Some scholars consider this the result of a deliberate design proportion, others concluded that these ancient Egyptians had no concept of π, and that the observed pyramid design was based just on the choice of its slope [1-3].

In Egypt, the first written calculation of π can be found in the Rhind Mathematical Papyrus. An Egyptian scribe, Ahmes, wrote this text, which can be considered the oldest known treatise of mathematics. The Rhind Papyrus dates from the Egyptian Second Intermediate Period, but it seems that Ahmes stated that he copied a Middle Kingdom papyrus, therefore before 1650 BC [4]. In this papyrus we can find how we can approximate π. In the problem n.48 proposed by Ahmes, the area of a circle was computed by approximating the circle by an octagon. As we can see from Ref.5, the papyrus discusses the approximation of the area of circle, as shown in Fig.1. The procedure is the following. A square with the side length equal to the diameter of the circle is drawn. Then, the square (side length of 9 units) is subdivided in 9 squares, each of side length of 3 units. The area of seven such squares approximates the area of circle. Therefore π=4×7/9=28/9=3.11 (see Figure 1), having a difference from the true value of about 1%, a quite good value, if we consider the instruments the ancient Egyptian had to draw geometrical figures and subdivide them in equal parts [6].

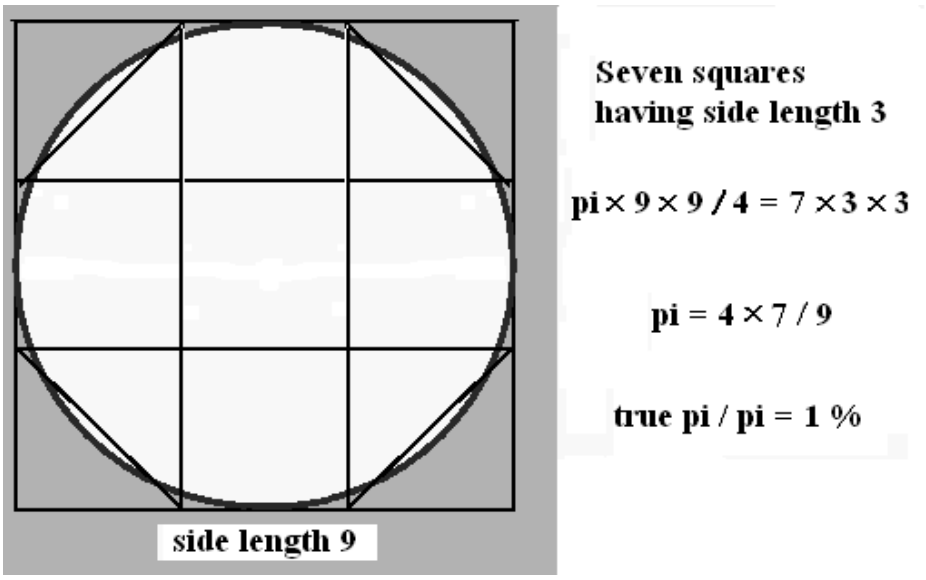

Figure 1. The figure shows how to approximate the area of the circle using squares, as proposed by scribe Ahmes in the Rhind Papyrus, Egyptian Second Intermediate Period.

Another written text is a clay tablet from Babylon, dated 1900–1600 BC, having geometrical statements that consider π as 25/8 [7]. However, besides these two, we have no other ancient written texts on π. Therefore, we could ask ourselves how to evaluate earlier knowledge of this constant. Here we suggest the analysis of ancient decorations. They can provide some information on the approximate values of constant π used by the artist who made them. Since π is the ratio of circumference to diameter, any analysis of circular decorations can give us this constant as a rational number. Here we discuss two examples of this approach. The first is concerning some disks found in the tomb of Hemaka, the chancellor of a king of the First Dynasty of Egypt, about 3000 BC. The second is the decoration composed by several circles and spirals of the Langstrup belt disk, an artifact of the Bronze Age found in Denmark. From the analysis of these two decorations, it is possible to argue the artists planned the decoration on their knowledge of π.

Hemaka's constant

As previously told, Egypt provides one of the first written text on π. But we can find an older estimation of π, based on the decorations of some items found in the tomb of chancellor Hemaka at Saqqara [8,9]. Hemaka was an important official during the reign of the First Dynasty Egyptian king Den. He was the royal seal-bearer, that is, the king's chancellor, second in power only to the king. His tomb, located in the northern part of Saqqara was enriched by many grave goods, including an inlaid gaming disc and the earliest surviving piece of papyrus [8, 9]. For what concerns the period, the reign of king Den lasted from 2975 BC to 2935 BC.

Information about the first dynasty is coming from a few monuments and other objects. At that time, the tombs of kings were built of wood and mud bricks, with some small insets of stone for walls and floors. Stone was largely used for producing ornaments, vessels and statues. It seems that it was during this period that Egyptians woodworkers invented the fixed mortise and tenon joint [10]. A variation of this joint became one of the most important features in Mediterranean shipbuilding. However, we have to tell that human sacrifices were practiced during funerary rituals of the kings. This practice ended with the dynasty, and small statuettes, the shabtis, took the place of persons to serve the kings in the afterlife [10].

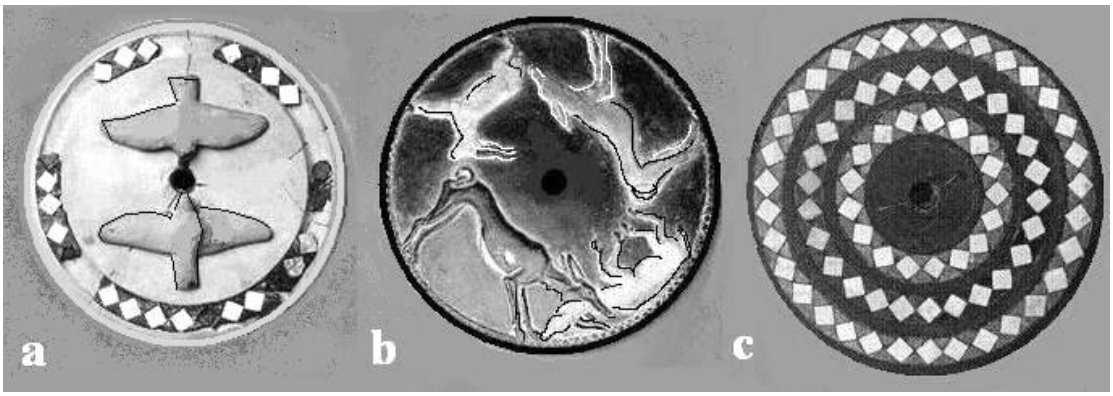

Figure 2. These sketches represent the Hemaka's gaming disks.

Hemaka is important for our research on π because he had in his tomb some decorated disks among the items for his afterlife. Sketches in Fig.2 show the disks. One disk (2a) is decorated in the central part with two birds, having a two-fold rotational symmetry. Disk (2b) has an interesting decoration composed of four animals, two gazelles and two jackals, in an anti-symmetric four-fold arrangement. The hunting is rendered by the rotational symmetry, as proposed in [11]. The third disk (2c) has a geometric decoration only. The reader can imagine such disks having diameters a little bit larger than ten centimetres.

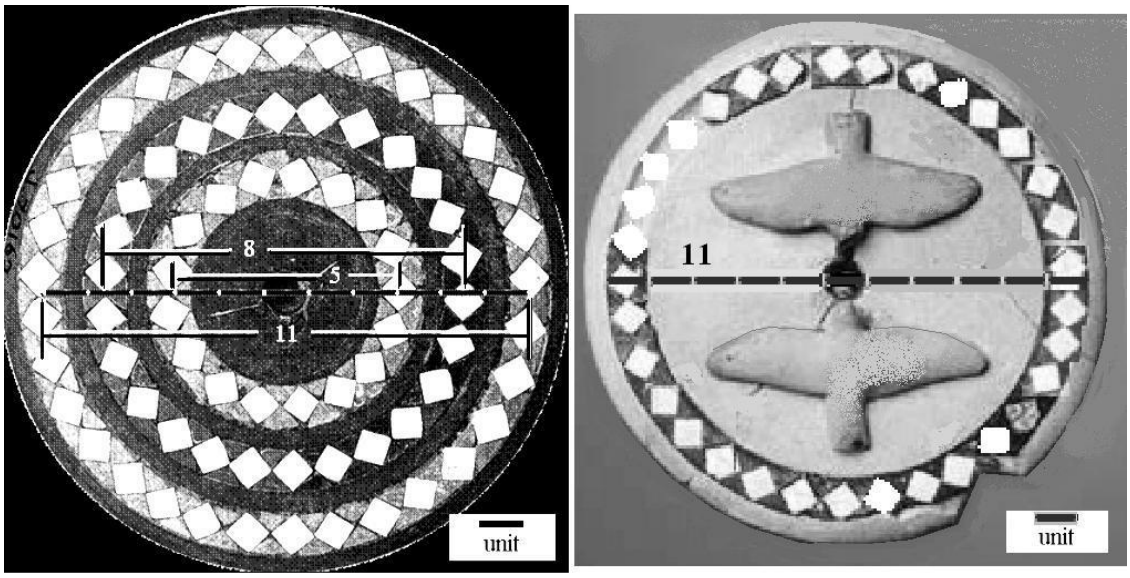

Figure 3. According to a given unit of measure (the diagonal *d* of the small white square) we can measure circumferences and diameters, having therefore the possibility to estimate π. The disk (2a) had been subjected to a digital restoration.

Small squares decorated disks (2a) and (2c). These decorations are really interesting for evaluating the Hemaka's knowledge of π. Let us start our discussion from (2c). Since the constant π is the dimensionless ratio of two measured quantities, circumference and diameter lengths, we have to measure these quantities, with an unit of measure. Let us choose this unit equal to the diagonal *d* of the small white squares of the decoration. For the measures on (2c), the reader can use Figure 3 on the left. We have therefore the data given in Table I, for the three annuli (annulus is the region lying between two concentric circles), containing the small white squares.

Table I.

Annulus	D, Diameter	C, Circumference	π=C/D
I	5 d	17 d	17/5=34/10=3.4
II	8 d	26 d	26/8=54/16=3.25
III	11 d	33 d	33/11=66/22=3.

In Table I we have fraction 26/8: we will see this value used in the Langstrup plate, which is the subject of the second example of this experimental approach to evaluate ancient π. Problems concerning the arrangement of small squares in I and II annuli arise from the fact that C is larger than the right one (there is one more square than necessary). Problems were solved by the artist with a small rotation about their centres of some squares. For the larger annulus, III, this problem does not exist. As a result of measurements, for this third annulus we have what is generally considered as the oldest approximation of π, that is, 3.

This same ratio 33/11 appears in the decoration of disk (2a). In fact, we need to digitally restore the decoration, as proposed in Figure 3 on the right. The best digital restoration we can have, is obtained by using 33 squares. If we choose a diameter of 10 times *d*, we have π=3.3; but, if we considered the diameter as (10+1/2)*d* instead of 11 or 10*d*, we had π evaluated as 33/(10+1/2)=66/21=22/7=3.1428. If we assume an uncertainty of about *d*/2 for the measures of diameters on (2c), we could arrange the following Table II.

Table II.

Annulus	D, Diameter	C, Circumference	π=C/D
I	(5+1/2) *d*	17 *d*	34/11=3.09
II	(8+1/2) *d*	26 *d*	54/17=3.18
III	(10+1/2) *d*	33 *d*	66/21=3.14

Quite probably Hemaka knew that the value of π was between two fractions, as, for instance, the best approximation 66/21=22/7 we found, which is involved in the proportion of the Great Pyramid, is between 66/22 and 66/20. Therefore, the decorations were probably created using some tables based on integers and their ratios, the fractions, developed for practical purposes. These persons, that accompanied their funeral rites with human sacrifices, had probably quite good practical knowledge of mathematics.

Langstrup belt disk

In 2006, the Danish National Bank proposed a new series of banknotes. Karin Birgitte Lund, the artist who developed the design of the series, decided to represent some objects of the Bronze Age found in Denmark. Among them there are the Trundholm sun chariot and the Langstrup belt plate (Fig.4). In Reference 12, I have discussed the Trundholm sun chariot, proposing a calendar of 360 days based on it. Other calendars based on this sun chariot and on the Langstrup belt are given in [13, 14]. In fact, this other ancient object, the Langstrup belt, can help us in the investigation on number π.

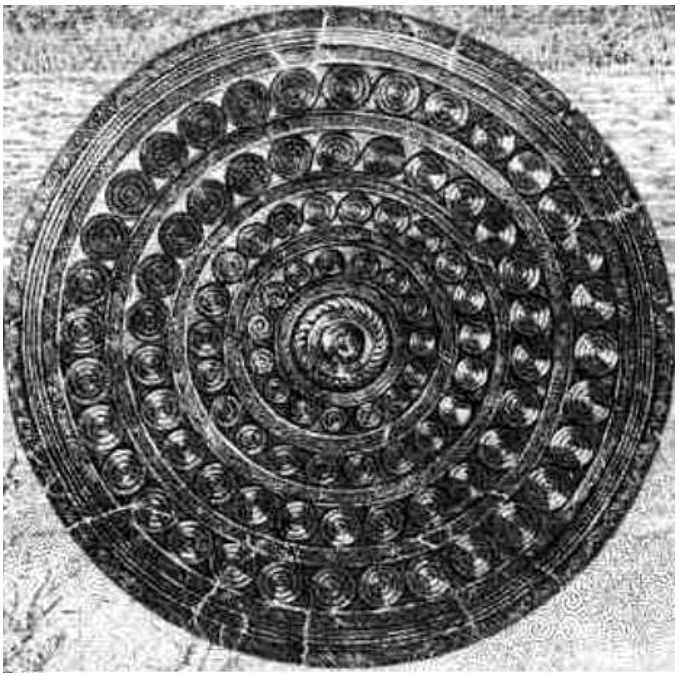

Figure 4. The Langstrup belt plate on the Danish banknotes. We see four annuli, decorated with spirals.

The Langstrup belt plate was found before 1880 together with a bronze knife and spiral bangles. It is coming from the early Bronze Age, approximately 1400 BC. The decoration is composed by circular grooves and spirals (see Fig. 5), stamped probably by means of some standard punches into wax model before casting. Belt plates were worn by women on the front of their belts, as shown by the mummy of the “Egtved girl” [15].

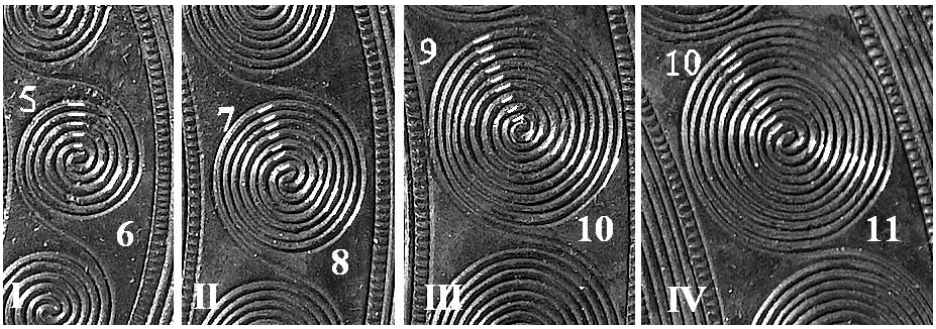

Figure 5. A detail of the decoration. The numbers are the turns of the spiral, counted at two different radial directions. The roman number gives the corresponding annulus.

In Ref.13, we find some interesting discussion on the figures emerging from the decoration. Let us report what this reference is telling about the Langstrup belt plate..."It has, apart from the point, four zones with 15+22+26+32 = 95 spirals in all. Still, a numerical pattern does not seem to emerge. However, if one … multiplies by the number of the factor of the zones, the sum of the spirals turns out to be 15×1+22×2+26×3+32×4 = 265, or exactly the number of days in 9 months of the Moon-year (265½), or, incidentally, also the length of the average human period of pregnancy. ... Going one step further, and again multiplying with the zonal factors, but now incorporating the point of the Langstrup belt-plate as Factor 1 (but with the value of 0), a sum of 0×1+15×2+22×3+26×4+32×5 = 360 appears."

Instead of using the numbers for a calendar, we can make some figures to investigate the knowledge of π. As we will see in the following discussion, a numerical pattern is clearly emerging, because the artist prepared the decoration on the wax using π approximated by rational numbers, that is, fractions having integers in numerators and denominators.

Table III is proposing some data on the spirals: the number of spiral in each annulus and the number of turns in each spiral. For what concerns the turns, their numbers depend on the manner we count them. In the Figure 5, I show two possible numbers, according with two different radial directions on the spiral.

Table III.
(see Figure 4 and 5)

Annulus	I	II	III	IV
N of spirals	15	22	26	32
N of spiral turns	5 or 6	7 or 8	9 or 10	10 or 11

We can calculate the length L of the circumference, on which there are the centres of spirals, to be two times the radius (R_i) of the spirals, multiplied by the number of spirals (see the second line of Table IV). The radius of the spirals could be estimated by the spiral turns. Let us note that for the first and the second annuli, we see that there is a certain distance between the spirals. We could assume this distance as two times the thickness of a spiral turn. Therefore, we could calculate the lengths, using a number of turns of 7 instead of 6, for the first annulus and 9 for the second.

Table IV. Length of the circumferences

Annulus	I	II	III	IV
L	$15\times2\times R_I$	$22\times2\times R_{II}$	$26\times2\times R_{III}$	$32\times2\times R_{IV}$
L	15×2×7	22×2×9	26×2×10	32×2×11

To have a possible evaluation of π, we need to give the radius of this circumference too. We can use the following approach to estimate it. Let us suppose the artist used a multiple value of the spiral radius, measured on the radial direction, to determine the radius of the circumference suitable for a decoration with the given spiral. Therefore, we can find it, as shown in the Figure 6.

The values of the radius are given in the following Table V.

Table V. Radius of the circumference of Table IV (with radii estimated as in Figure 6)

Annulus	I	II	III	IV
R	$5\times R_I$	$7\times R_{II}$	$8\times R_{III}$	$10\times R_{IV}$

Using Table IV and V, we can then determine the value of $\pi = L/(2R)$. Here the results in Table VI.

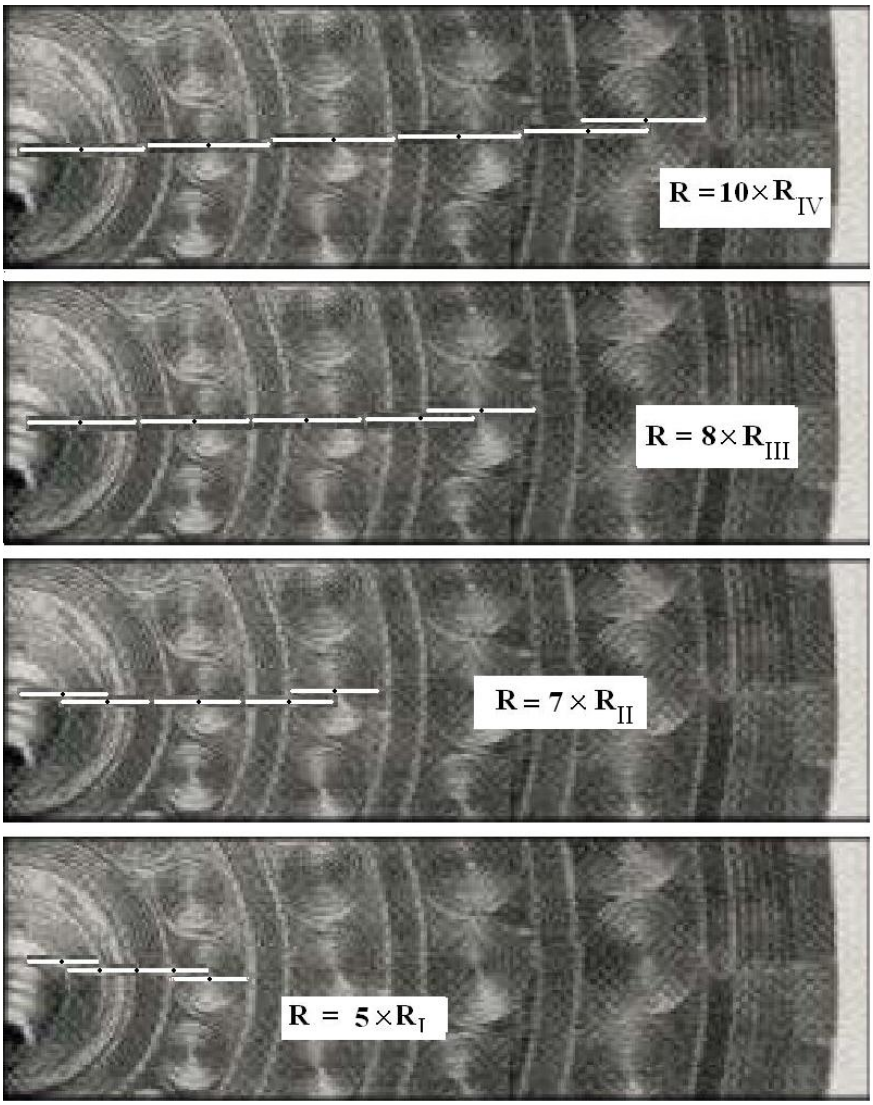

Figure 6. Probably, the artist used a multiple of the spiral radius, taken on the radial direction, to determine the radius of the circumference, on which there are the spiral centres, and suitable for the decoration. We can obtain the multiple for each annulus directly from the picture.

Table VI. π=L/(2R) as a rational number

Annulus	I	II	III	IV
π	15/5=3.0	22/7=3.1428	26/8=3.25	32/10=3.20

Supposing that the artist used for the radius a multiple of the spiral radius, Table VI shows that the artist knew some approximation of number π. Let us remember that the disk is coming from the early Bronze Age, approximately 1400 BC. We find again 22/7.

There is also another possibility to evaluate π, that of calculate the ratio between the number of spirals and the number of turns in each spiral. Choosing for this number the lowest value, we have therefore the following Table VII.

Table VII.

Annulus	I	II	III	IV
N of spirals	15	22	26	32
N of spiral turns	5	7	9	10
π	15/5=3.0	22/7=3.1428	26/9=2.89	32/10=3.20

Table VI, which is based on the measurements of radii and circumferences, seems the better and more realistic approach, showing clearly the origin of the number π as a dimensionless physical quantity. In conclusion, once again, we can tell that these artefacts are fundamental to understand the knowledge of mathematics of ancient people. Besides being amazing, the decoration of the Langstrup belt disk demonstrates that who made it knew the number π as a ratio of integers.

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