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A. Chinea
IdemWorks s.r.l.
C. Trento 13, 10129 Torino, Italy
e-mail a.chinea@idemworks.com

S. Grivet-Talocia, G. C. Calafiore
Politecnico di Torino
C. Duca degli Abruzzi 24, 10129 Torino, Italy
e-mail {grivet,calafiore}@polito.it

***Abstract*—This paper introduces a new algorithm for passivity enforcement of linear lumped macromodels in scattering form. As typical in most state of the art passivity enforcement methods, we start with an initial non-passive macromodel obtained by a Vector Fitting process, and we perturb its parameters to make it passive. The proposed scheme is based on a convex formulation of both passivity constraints and objective function for accuracy preservation, thus allowing a formal proof of convergence to the unique optimal passive macromodel. This is a distinctive feature that differentiates the new scheme with respect to most state of the art methods, which either do not guarantee convergence or are not able to provide the most accurate solution. The presented algorithm can thus be safely used for those cases for which existing techniques fail. We illustrate the advantages of proposed method on a few benchmarks.**

I. Introduction

Since the introduction of the Vector Fitting (VF) scheme [1], the generation of linear lumped macromodels for electrical interconnects has become a standard practice in Signal and Power Integrity analysis. Starting from tabulated frequency samples of the scattering matrix coming from measurement or full-wave analysis, the VF algorithm produces accurate and guaranteed stable rational approximations of the system transfer function, which can in turn be synthesized into equivalent circuits or equation-based state-space macromodels for fast system-level simulation and design optimization. Recent developments essentially resolved the complexity issues due to large dynamic order and/or port counts required by modern designs, through reformulation [2], parallelization [3] or compression [4].

Despite the reliability and efficiency of recent VF implementations [2]- [4], the obtained macromodels are generally not passive. Since passivity is a fundamental requirement that guarantees numerically stable system-level simulations [5], any macromodel obtained by VF should be subject to some postprocessing in order to enforce its passivity.

The passivity of scattering macromodels is implied by the bounded realness of the transfer matrix, which in general terms ensures that the energy gain of the system is less than unity. Various formulations of this constraint are available and have been used for passivity enforcement. If a state-space realization is available for the model, the Linear Matrix Inequality (LMI) provided by the Bounded Real Lemma (BRL) can be shown to be equivalent to bounded realness [6], [7], and various passivity enforcement schemes based on BRL constraints have been proposed, see e.g. [8]. Main advantage is the convexity of the corresponding formulation, which ensures convergence to the optimal solution. Unfortunately, the scalability of these schemes to medium and large-scale macromodels is quite limited due to excessive CPU and especially memory requirements.

For the above reasons, heuristic and sub-optimal but more efficient schemes have been proposed, based on iterative perturbation of Hamiltonian spectra [9], [10], of singular values of the transfer function at finite frequencies [10], [11], of residue matrices [12], and even of macromodel poles [13], [14]. All such formulations are not guaranteed to converge to a passive macromodel with acceptable accuracy, although in many applications they have been shown to work generally well. Some cases exist, however, where these methods fail [15].

This paper proposes a new passivity enforcement scheme that guarantees convergence to the passive macromodel with optimal accuracy, and that achieves its result by using limited computing resources. A first attempt in this direction was documented in [15], where a convex optimization scheme based on $\mathcal{H}_\infty$ norm minimization through a projected subgradient formulation was presented. The results in [15] demonstrated the excellent potential of this new approach, which however in the formulation of [15] required a large number of iterations to converge. In this paper, we continue along this track by proposing a new method based on a different and simpler alternate subgradient iteration, which is able to speedup convergence significantly. Several numerical examples illustrate the advantages of the new scheme.

II. Problem statement

Let us consider a strictly stable p-port macromodel in state-space form

$$\begin{array}{rcl}\dot{\boldsymbol{w}}(t) & = & \boldsymbol{A}\boldsymbol{w}(t)+\boldsymbol{B}\boldsymbol{u}(t)\\ \boldsymbol{y}(t) & = & \boldsymbol{C}\boldsymbol{w}(t)+\boldsymbol{D}\boldsymbol{u}(t)\,,\end{array} \tag{1}$$

where vector $\boldsymbol{w}\in\mathbb{R}^n$ collects the internal state variables and $\boldsymbol{A}\in\mathbb{R}^{n,n},\boldsymbol{B}\in\mathbb{R}^{n,p},\boldsymbol{C}\in\mathbb{R}^{p,n},\boldsymbol{D}\in\mathbb{R}^{p,p}$ are obtained as a result of a fitting process [1]- [4]. The input and output vectors $\boldsymbol{u}$ and $\boldsymbol{y}$ collect the p scattering incident and reflected waves at the device ports. Consequently,

$$\boldsymbol{H}(s)=\boldsymbol{C}(s\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}+\boldsymbol{D} \tag{2}$$

corresponds to the scattering matrix of the model evaluated at the complex frequency s.

We define the following *perturbed* macromodel through its transfer matrix as

$$\boldsymbol{H}(\boldsymbol{x}, s) = (\boldsymbol{C} + \boldsymbol{X})(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}\,, \tag{3}$$

where $\boldsymbol{X} \in \mathbb{R}^{p,n}$ and its vectorized form $\boldsymbol{x} = \mathrm{vec}(\boldsymbol{X}) \in \mathbb{R}^{np,1}$ parameterize a perturbation of the model, with the "vec" operator stacking the columns of its matrix argument in a single column. Clearly, when $\boldsymbol{x} = \boldsymbol{0}$ we have no perturbation and we recover the nominal macromodel (2). For any fixed value $\boldsymbol{x}$, we define the following scalar-valued function

$$h(\boldsymbol{x}) = \sup_{\omega \in \mathbb{R}} \sigma_1(\boldsymbol{H}(\boldsymbol{x}, \mathrm{j}\omega))\,, \tag{4}$$

where σ_1 denotes the maximum singular value of its matrix argument. The function $h(\boldsymbol{x})$ is usually denoted as the $\mathcal{H}_\infty$ norm of (3). The macromodel defined by (3) is passive if and only if $h(\boldsymbol{x}) \leq 1$. In the following, we will consider that the nominal macromodel is not passive, $h(\boldsymbol{0}) > 1$, and we state our passivity enforcement problem as finding some perturbation vector $\boldsymbol{x}_*$ such that $h(\boldsymbol{x}_*) \leq 1$.

The solution $\boldsymbol{x}_*$ is obviously not unique. In fact, each of the various passivity enforcement schemes available in the literature will provide a different answer. One can however measure the "goodness" of such solutions by defining some accuracy metric that is related to the amount of perturbation through a suitable norm $\|\boldsymbol{x}\|$. Good solutions are characterized by small perturbation amounts. The best solution is achieved when this perturbation is minimized.

For practical application, it is usually preferred to adopt the weighted norm [9]

$$f(\boldsymbol{x}) = \mathrm{tr}\{\boldsymbol{X}\boldsymbol{W}\boldsymbol{X}^T\}\,, \tag{5}$$

where "tr" denotes the matrix trace and $\boldsymbol{W} = \boldsymbol{W}^T > 0$ is the controllability gramian of the nominal macromodel, computed by solving the Lyapunov equation

$$\boldsymbol{A}\boldsymbol{W} + \boldsymbol{W}\boldsymbol{A}^T + \boldsymbol{B}\boldsymbol{B}^T = \boldsymbol{0}\,. \tag{6}$$

It can be shown that (5) equals the cumulative energy (squared $\mathcal{L}_2$ norm) of the induced perturbation in the model transfer matrix $\boldsymbol{\Delta}(\boldsymbol{x}, s) = \boldsymbol{H}(\boldsymbol{x}, s) - \boldsymbol{H}(\boldsymbol{0}, s)$,

$$f(\boldsymbol{x}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{tr}\left\{\boldsymbol{\Delta}(\boldsymbol{x}, \mathrm{j}\omega)\boldsymbol{\Delta}(\boldsymbol{x}, \mathrm{j}\omega)^H\right\} d\omega\,. \tag{7}$$

With the above notation defined, we formulate our optimal passivity enforcement scheme as the following optimization problem

$$\boldsymbol{x}_* = \arg\min_{\boldsymbol{x}} f(\boldsymbol{x}), \quad \text{s.t. } h(\boldsymbol{x}) \leq 1\,, \tag{8}$$

where the minimal perturbation condition is set as an objective function and the passivity condition appears as an inequality constraint.

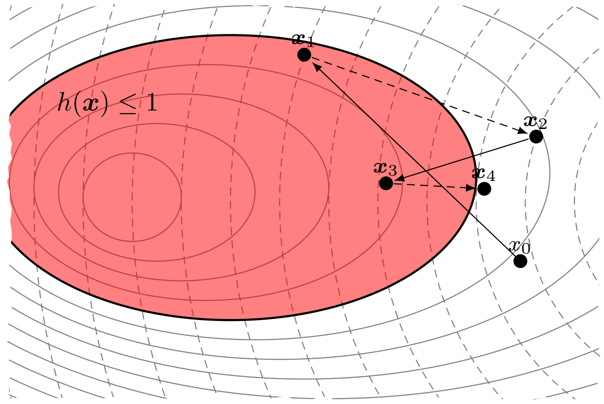

Fig. 1. Alternate subgradient algorithm. The level sets of $h(\boldsymbol{x})$ and $f(\boldsymbol{x})$ are depicted with solid and dashed gray lines, respectively. The region $h(\boldsymbol{x}) \leq 1$ is colored in red. The iteration steps in a direction in $\partial h(\boldsymbol{x})$ and $\partial f(\boldsymbol{x})$ are depicted with solid and dashed arrows, respectively.

III. Formulation

Both the objective $f(\boldsymbol{x})$ and the passivity constraint $h(\boldsymbol{x})$ are convex functions of the decision variables $\boldsymbol{x}$. This follows as a direct consequence of the fact that $f(\boldsymbol{x})$ and $h(\boldsymbol{x})$ are norms [17]. Therefore, the feasible set

$$\mathcal{X} = \{\boldsymbol{x} :\, h(\boldsymbol{x}) \leq 1\} \tag{9}$$

collecting all parameter configurations for which the macromodel is passive is a convex set. Problem (8) can thus be interpreted as the minimization of a convex function over a convex domain. It is well known that this problem admits a unique solution $\boldsymbol{x}_*$, which can be found numerically up to an arbitrary precision in a finite number of steps using the proper algorithm. This fact was not recognized in many works on passivity enforcement, where the same problem was restated or approximated in non convex forms for the sake of simplicity and/or numerical efficiency. It is however shown in [15] that these state of the art methods may fail in some cases, since their convergence cannot be assessed.

Our approach to solve (8) is actually very simple. Let us start by assuming that both $f(\boldsymbol{x})$ and $h(\boldsymbol{x})$ are smooth and differentiable functions of $\boldsymbol{x}$. We will see shortly that this is not true for $h(\boldsymbol{x})$, but we will release this assumption later. Figure 1 provides a graphical illustration of the main idea for $\boldsymbol{x} \in \mathbb{R}^2$. The solid lines represent the contour lines of $h(\boldsymbol{x})$, with the feasible set (9) highlighted in light red. The dashed lines are instead the contour lines of $f(\boldsymbol{x})$.

Suppose we start with an initial point $\boldsymbol{x}_0$ outside $\mathcal{X}$. We need to decrease the value of $h(\boldsymbol{x})$ to move the solution into the feasible set. To this end, we pick a descent direction $-\boldsymbol{g}^{(0)}$ and we compute a new point $\boldsymbol{x}_1 = \boldsymbol{x}_0 - \alpha_0\boldsymbol{g}^{(0)}$, where α_0 is a suitable step size. The direction $\boldsymbol{g}^{(0)}$ that guarantees the steepest descent (if $h(\boldsymbol{x})$ is differentiable) corresponds to the gradient $\nabla h(\boldsymbol{x})$.

In the second iteration, we assume to start from $\boldsymbol{x}_1 \in \mathcal{X}$. We need to stay in the feasible set, but we need to minimize $f(\boldsymbol{x})$. Therefore, pick a descent direction $-\boldsymbol{g}^{(1)}$, possibly coincident with the gradient $\boldsymbol{g}^{(1)} = \nabla f(\boldsymbol{x})$ to follow the steepest descent path, and we update the solution as $\boldsymbol{x}_2 = \boldsymbol{x}_1 - \alpha_1 \boldsymbol{g}^{(1)}$, where α_1 is again a suitable step size. Then, we iterate the algorithm, which can be stated at the k-th iteration as

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k - \alpha_k \boldsymbol{g}^{(k)} . \tag{10}$$

The direction of each step is defined according to

$$\boldsymbol{g}^{(k)} = \begin{cases} \boldsymbol{g}_f^{(k)} \in \partial f(\boldsymbol{x}^{(k)}) & \text{if } h(\boldsymbol{x}^{(k)}) \leq 1, \\ \boldsymbol{g}_h^{(k)} \in \partial h(\boldsymbol{x}^{(k)}) & \text{if } h(\boldsymbol{x}^{(k)}) > 1, \end{cases} \tag{11}$$

where operator ∂ denotes the differential which, in the smooth case, coincides with the unique gradient.

A. Technical considerations

There are two technical complications in the above formulation, which need to be analyzed with care. The first problem is related to the differentiability of $h(\boldsymbol{x})$. It turns out that $h(\boldsymbol{x})$ is convex but non-smooth, as shown in [15], [16]. In particular, any parameter configuration $\breve{\boldsymbol{x}}$ at which the supremum in (4) is attained at more than one frequency point $\{\bar{\omega}_i,\, 1 \leq i \leq q\}$ and/or by a maximum singular value of higher multiplicity ℓ_i, corresponds to a point where $h(\boldsymbol{x})$ is not differentiable. This makes the definition of the descent direction $\boldsymbol{g}^{(k)}$ ill-posed, since the local gradient cannot be defined. Fortunately, there exists a theoretical tool providing a generalization of the gradient to the (convex) non-smooth case (see [18], [19] for details). Denoting the singular value decomposition at frequency $\bar{\omega}_i$

$$\boldsymbol{H}(\boldsymbol{x}, \mathrm{j}\bar{\omega}_i) = \boldsymbol{U}^{(i)} \boldsymbol{\Sigma}^{(i)} [\boldsymbol{V}^{(i)}]^H \tag{12}$$

and collecting the first ℓ_i columns of $\boldsymbol{U}^{(i)}$ and $\boldsymbol{V}^{(i)}$ as $\boldsymbol{U}_1^{(i)}$ and $\boldsymbol{V}_1^{(i)}$, where ℓ_i is the multiplicity of the largest singular value $\sigma_1^{(i)}$, we have the following characterization of the so-called *subdifferential*

$$\partial h(\boldsymbol{x}) = \Big\{ \operatorname{vec} \Big(\sum_{i=1}^{q} \Re\{ \boldsymbol{\Psi}(\mathrm{j}\bar{\omega}_i) \boldsymbol{V}_1^{(i)} \boldsymbol{Y}_i \boldsymbol{U}_1^{(i)H} \}^{\top} \Big) \Big\}, \tag{13}$$

where $\boldsymbol{\Psi}(\mathrm{j}\omega) = (\mathrm{j}\omega \boldsymbol{I} - \boldsymbol{A})^{-1} \boldsymbol{B}$ and where the q matrices $\boldsymbol{Y}_i \in \mathbb{R}^{\ell_i, \ell_i}$ are such that $\boldsymbol{Y}_i = \boldsymbol{Y}_i^{\top} \geq 0$ and $\sum_{i=1}^{q} \operatorname{Tr} \boldsymbol{Y}_i = 1$. This subdifferential is a convex set with dimension $\sum_{i=1}^{q} \ell_i$. A generic element $\boldsymbol{g} \in \partial h(\boldsymbol{x})$ is denoted as *subgradient*, hence the denomination of proposed scheme as *alternate subgradient iteration*. A remarkable fact of the proposed formulation, is that we can pick any arbitrary subgradient as $\boldsymbol{g}_h^{(k)} \in \partial h(\boldsymbol{x}_k)$ in (11), and we are still able to prove [16] the convergence of the iterations to the optimal solution $\boldsymbol{x}_*$.

A second fundamental issue involves the choice of the step size α_k in (10). We follow here an adaptive step size selection based on [16]

$$\alpha_k = \frac{-G_k \zeta_{k-1} + \sqrt{G_k^2 \zeta_{k-1}^2 + R^2 + \xi_{k-1}}}{G_k} \tag{14}$$

where $G_k^2 = \|\boldsymbol{g}_f^{(k)}\|^2 + \|\boldsymbol{g}_h^{(k)}\|^2$ and

$$\xi_{k-1} = \sum_{i=1}^{k-1} \|\boldsymbol{g}^{(i)}\|^2 \alpha_i^2 , \quad \zeta_{k-1} = \sum_{i=1}^{k-1} \alpha_i , \tag{15}$$

with $\xi_0 = \zeta_0 = 0$, and where R is any constant such that $\|\boldsymbol{x}_0 - \boldsymbol{x}_*\| \leq R$. This constant R must be known a priori and provides an estimate of the amount of perturbation that will be required to reach a passive macromodel. In our particular case, a conservative choice is $R = \|\boldsymbol{C}\|_F$, where $\|\cdot\|_F$ denotes the Frobenius norm. With the above strategy, an explicit bound of the distance between the optimum at iteration k and the global optimum $f(\boldsymbol{x}_*)$ is available as

$$\min_{i=1,\ldots,k} f(\boldsymbol{x}_i) - f(\boldsymbol{x}_*) \leq \frac{R^2 + \sum_{i=1}^{k} G_i^2 \alpha_i^2}{2 \sum_{i=1}^{k} \alpha_i} . \tag{16}$$

TABLE I
NUMBER OF ITERATIONS REQUIRED BY DIFFERENT PASSIVITY ENFORCEMENT SCHEMES

Case	Ports	Order	Proposed	Proj [15]	Ham [9]
1	4	272	71	2202	6
2	2	60	132	2435	–
3	4	136	76	1856	5
4	4	88	84	1781	4

TABLE II
RELATIVE PERTURBATION ($\times 10^{-3}$) OBTAINED WITH DIFFERENT PASSIVITY ENFORCEMENT SCHEMES.

Case	Ports	Order	Proposed	Proj [15]	Ham [9]
1	4	272	0.123	0.131	0.216
2	2	60	2.502	2.703	–
3	4	136	0.028	0.027	0.050
4	4	88	0.401	0.413	0.730

IV. NUMERICAL EXAMPLES AND DISCUSSION

We illustrate the performance of proposed scheme on four simple test cases corresponding to various type of interconnects (cases 1,3,4) and a SAW filter (case 2). Given an initial non-passive macromodel obtained by VF, we applied three different passivity enforcement schemes based on Hamiltonian eigenvalue perturbation [9], convex optimization via projected subgradient iterations [15], and proposed alternate subgradient iterations. The number of required iterations and the corresponding perturbation amount (relative $\mathcal{L}_2$ norm) is reported in Table I and II, respectively.

We see that the proposed scheme outperforms the preliminary convex formulation of [15], which is characterized by similar theoretical properties. However, the number of required iterations is always about one order of magnitude larger than the Hamiltonian perturbation scheme [9]. The latter however fails to converge for case 2 and provides a worse accuracy than proposed technique for all cases. The runtime per iteration is approximately the same for each scheme, ranging from 0.4 to 1.0 seconds, depending on the case.

A few remarks are in order. With the proposed implementation, we are able to explicitly prove the following facts:

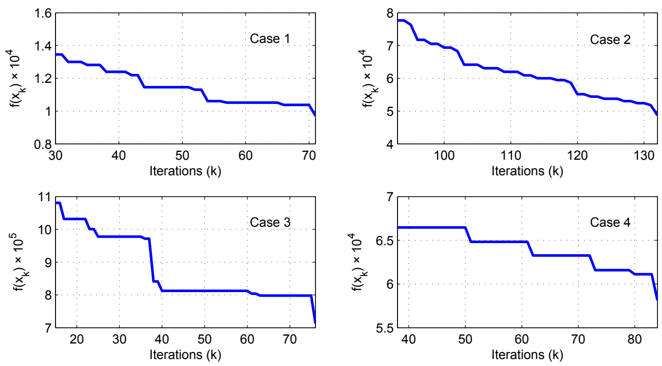

Fig. 2. Evolution of the objective function $f(\boldsymbol{x}^{(k)})$ through iterations.

- A unique passive macromodel $\mathcal{M}_*$ characterized by optimal accuracy exists.
- The macromodel $\mathcal{M}_k$ obtained at the k-th iteration deviates from the best macromodel $\mathcal{M}_*$ by some amount that is fully under control in the considered norm $f(\boldsymbol{x})$. More precisely, the lower bound (16) provides a quantitative measure of "how far" $\mathcal{M}_k$ is still from $\mathcal{M}_*$.
- The accuracy of $\mathcal{M}_k$ continuously improves as k increases through iterations, as depicted in Fig.2.

The above facts, which are supported by the presented numerical results, can be leveraged to combine the proposed method with any existing "standard" suboptimal passivity enforcement scheme based on Hamiltonian eigenvalue or direct singular value perturbation, according to the following guidelines.

1) Since the proposed scheme requires generally more iterations than "standard" techniques, the latter should be attempted in first place. If passivity enforcement succeeds and the accuracy is satisfactory, then there is no need to further proceed with convex optimization or refinement.
2) In case "standard" passivity enforcement fails, the proposed alternate subgradient scheme should be used to obtain a passive macromodel, stopping the iterations as soon as the accuracy is satisfactory.
3) In case "standard" passivity enforcement succeeds but the resulting passive macromodel is not sufficiently accurate, one can use the proposed alternate subgradient iteration to iteratively refine the passive macromodel until the desired accuracy is met.

We remark that in the above scenarios 2) and 3), the only actions before the availability of the proposed convex algorithm were either to give up with macromodeling, or to use the inaccurate macromodel with possibly unreliable results, or to regenerate the macromodel by tuning any available control parameter of the adopted algorithm. The key point is that the latter trial and error process has no guarantee of success when the underlying passivity enforcement approach is based on a non-convex formulation. Therefore, the proposed scheme provides at least some good solution, at the cost of a possibly larger number of iterations and runtime.

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