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Upper Bounds to the Performance of Cooperative Traffic Relaying in Wireless Linear Networks

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***Abstract*—Wireless networks with linear topology, where nodes generate their own traffic and relay other nodes' traffic, have attracted increasing attention. Indeed, they well represent sensor networks monitoring paths or streets, as well as multihop networks for videosurveillance of roads or vehicular traffic. We study the performance limits of such network systems when (i) the nodes' transmissions can reach receivers farther than one-hop distance from the sender, (ii) the transmitters cooperate in the data delivery, and (iii) interference due to concurrent transmissions is taken into account. By adopting an information-theoretic approach, we derive analytical bounds to the achievable data rate in both the cases where the nodes have full-duplex and half-duplex radios. The expressions we provide are mathematically tractable and allow the analysis of multihop networks with a large number of nodes.**

Our analysis highlights that increasing the number of cooperating transmitters beyond two leads to a very limited gain in the achievable data rate. Also, for half-duplex radios, it indicates the existence of dominant network states, which have a major influence on the bound. It follows that efficient, yet simple, communication strategies can be designed by considering at most two cooperating transmitters and by letting half-duplex nodes operate according to the aforementioned dominant states.

***Index Terms*—Wireless networks, traffic relaying, sensor networks.**

I. Introduction

Multi-hop communication systems are primarily implemented to extend the overall coverage of wireless networks, leading to a more efficient use of the available communication resources and to an increased network throughput.

As indicated by the information theory, the capacity of a wireless network increases when the nodes participate cooperatively in relaying the traffic toward their destinations. Thus, various cooperative schemes have been proposed in the literature for networks that include only full-duplex nodes (i.e., nodes that can simultaneously transmit and receive) [1], [2], only half-duplex nodes (i.e., nodes that at any time instant can either transmit or receive) [3], or a mix of full-duplex and half-duplex nodes [4].

In this paper, we consider a wireless network where n nodes have to deliver their traffic to a common destination node (e.g., a gateway node) through multi-hop data transfers. We focus on a network whose topology can be considered as linear, as, e.g., in the case of sensor networks for path and street monitoring, or multihop networks for videosurveillance of roads and vehicular traffic [5]. The nodes share the same radio resources and each of them may generate its own data at a different average rate. We assume that, if needed, the nodes cooperate to relay the traffic based on the decode-and-forward paradigm [6]. The nodes' transmission rates and powers correspond to optimal coding over a discrete-time additive white Gaussian noise (AWGN) channel, although more general channels and coding schemes could be considered as well. Furthermore, unlike previous work, we account for the fact that receivers may exploit signal transmissions from nodes farther than one-hop distance from the sender, and that nodes in radio visibility can cooperate to transmit toward one or more receiver nodes.

Under these conditions, we adopt an information-theoretic approach and we develop a method to obtain a fairly tight upper bound to the nodes' achievable rate, which also accounts for the interference due to simultaneous transmissions. Specifically, we study the cut-set upper bound [7], [8] of the network system, and obtain the timing and traffic links schedule of such a network under which the upper bound is satisfied. We carry out the analysis in presence of both full-duplex (FD) and half-duplex (HD) nodes; for the former, we study the general case where nodes may choose to operate either in FD or HD mode, as the second operational mode (i.e., HD) can be considered as a subcase of the first one (i.e., FD).

We stress that, since the nodes' operational states in FD mode are a superset of those under the HD mode, an upper bound for an FD network is an upper bound for the HD case too. However, such a bound would be loose for an HD network, where the data transfer towards the destination is expected to be significantly slower than in the FD case (recall that HD nodes cannot transmit and receive at the same time). We therefore carry out a different analysis for FD and HD networks, so as to obtain tight upper bounds under both operational modes.

We start our analysis by adopting the cut-set methodology as introduced in [7], [8]; this, however, would require us to consider all possible network cuts and operational states, which is unfeasible in our case due to their exceedingly high number (see Sec. IV for further details). We therefore limit the number of cuts to be considered and identify the dominant states in which the network can operate, and derive the upper bound to the nodes data rate accounting for such cuts and network states only. Also, in the case of an HD network, whose analysis becomes more complex due to the additional operational constraints, we are able to analyze a large-size network by resorting to an equivalent one, composed of five nodes only. To show the validity of our approach, we compute a lower bound to the traditional cut-set bound. By comparing our results to the aforementioned lower bound, we demonstrate that the upper bound we derive is tight. Finally, we use the bounds obtained for the FD and the HD case to investigate the

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system behavior as several parameters, like the signal-to-noise ratio (SNR), the dependence of the signal attenuation with the distance and the number of nodes, vary.

We remark that several works have appeared in the literature addressing a problem similar to the one we study, but for networks with only one node generating traffic and the others acting as relays [9], [10], or with multiple source nodes but operating in FD mode only [1], [2], or for networks with very few HD nodes [11]. The benefits of an integrated FD and HD relaying scheme have been studied in [4], for a network with a source-destination pair and an intermediate relay-only node. However, the solution in [4] holds only if the loop-back interference observed at the relay operating in FD mode is resolved. This imposes further hardware requirements, which limit the application of the strategy proposed there. A network scenario closer to ours has been analyzed in [12], [13], but with a different objective. There, the authors consider the problem of computing transmission powers, rates, and link scheduling for an energy-constrained wireless network and solve it by maximizing the network lifetime through a cross-layer design approach. Beside having different scope, our work differs from [12], [13] in that they consider the data rates of the source nodes as inputs to the problem of transmission scheduling, while we aim at deriving an upper bound to the nodes' achievable rate. Finally, in [14] Lutz et al. analyze relay cascades with HD constraints, in which adjacent node pairs are connected via error-free links. The information transfer is carried out by applying a coding scheme that allocates the transmission and reception time slots at the relays depending on the amount of information to be transferred. Through numerical results, the authors show that their strategy achieves the cut-set bound under certain conditions on the nodes' rates. Together with its rather complex coding scheme, the strategy in [14] requires the nodes to be synchronized at the symbol level. Unlike [14], in our paper we derive an upper bound to the rates achievable by the nodes, using an AWGN channel model and accounting for interference due to simultaneous transmissions.

In summary, to our knowledge, our work is the first one that provides an upper bound to the achievable data rates in a network where (i) the nodes may operate all in FD or HD mode, or some in FD and others in HD mode, (ii) a node's transmission can be exploited at a receiver located at more than one-hop distance from the sender, and (iii) interference is taken into account.

The rest of the paper is organized as follows. First, we describe the system model in Section II and provide some basic concepts on the cut-set bound in Section III. The upper bound to the nodes' achievable rate is investigated in Sections IV and V for, respectively, FD and HD networks. There, we also present some numerical results showing the impact of the system parameters on the performance. Finally, in Section VI we draw our conclusions and highlight directions for future research.

II. System model

We consider a wireless network with linear topology composed of n nodes and a destination node, as depicted in Fig. 1.

Fig. 1. Network topology.

Without loss of generality, we let node 1 be the node at the left end of the topology, while the destination is located at the right end and is denoted by $n+1$. For simplicity, we assume that the nodes are equally spaced along the path and denote by ℓ the inter-node distance, which we refer to as the one-hop distance. It follows that the network has length $L = n\ell$ meters, or, equivalently, it includes n-hops.

Node i ($i = 1, \dots, n$) generates messages at rate R_i, and it can decode and forward other nodes' messages. We consider an additive white Gaussian noise (AWGN) channel, and assume that all nodes transmit with power P while the noise power spectral density at each receiver is N_0. We then write the SNR measured at distance ℓ from a transmitting node as $\gamma = \frac{PG_tG_r}{WN_0}\left(\frac{\lambda}{4\pi\ell}\right)^a$ where G_t, G_r are, respectively, the transmit and receive antenna gains, λ is the carrier wavelength, and a is the path loss exponent.

We assume that each node i, $i = 1, \dots, n+1$, is equipped with directional antennas, so that it can receive signals only from upstream transmitting nodes and it can use its whole power to transmit towards downstream nodes. This is a reasonable assumption considering that our objective is to find an upper bound to the achievable data rates and that we deal with a linear network in which all nodes aim at delivering their data to the same destination located at one end of the topology.

Furthermore, since we are interested in finding bounds to practical cooperative communication strategies, for any receiver node, we define k_C as the maximum distance (with respect to the receiver itself) at which collaborating transmitters can be located; we refer to k_C as cooperation range. We define k_I ($k_I \geq k_C$) as the interference range of a node, i.e., the maximum distance at which a transmitted signal can cause interference at a receiver. Both the cooperation and the interference ranges are expressed in hops. From the above definitions, it follows that a node can receive *useful* signals from transmitters within distance k_C hops, while it receives interfering signals from nodes located at distance farther than k_C hops but within k_I hops. All signals arriving at the receiver from farther than k_I hops are assumed to have negligible power. Signals received from collaborating nodes are correlated, while interfering signals received from nodes farther than k_C hops are uncorrelated and independent of the useful signals. This is a fair assumption as, by definition of cooperation range, the signals from nodes farther than k_C hops are not exploited by a receiver, hence useful and interfering signals can be assumed to be uncorrelated. Also, under the system scenario outlined above, neglecting the correlation among interfering signals represents a best case (i.e., it never overestimates the effect of the interfering signals), thus it does not invalidate the derivation of the upper bound to the nodes' data rate.

Denoting by $\mathbf{y}$ the vector of signals received at the network nodes, we can write:

$$\mathbf{y} = \sqrt{\gamma}\mathbf{H}^{\mathsf{T}}\mathbf{x} + \sqrt{\gamma}\mathbf{W}^{\mathsf{T}}\mathbf{i} + \mathbf{z}\,. \tag{1}$$

In (1), $\mathbf{x} = [x_1, \ldots, x_n]^{\mathsf{T}}$ is the vector of signals transmitted by nodes $1, \ldots, n$; $\mathbf{i} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ is the vector of signals transmitted by interfering nodes (assumed to be uncorrelated and independent of $\mathbf{x}$); $\mathbf{H}$ is the matrix including the coefficients of the channels between the receiver nodes and the transmitters in their cooperation range; $\mathbf{W}$ is the matrix including the coefficients of the channels between the receivers and their corresponding interferers. Finally, $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ is the noise vector, independent of $\mathbf{x}$ and $\mathbf{i}$.

We assume that nodes employ Gaussian codebooks and that $\mathbf{x} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$, with $(\mathbf{\Sigma})_{ii} = 1$, $i = 1, \ldots, n$. The entries h_{ij} of the $n \times (n+1)$ channel matrix $\mathbf{H}$ are defined as

$$h_{ij} = \begin{cases} (i-j)^{-a/2} & \text{if } i - k_C \leq j < i \\ 0 & \text{else,} \end{cases} \tag{2}$$

while the elements w_{ij} of the $n \times (n+1)$ interference matrix $\mathbf{W}$ are given by

$$w_{ij} = \begin{cases} (i-j)^{-a/2} & \text{if } i - k_I \leq j < i - k_C \\ 0 & \text{else.} \end{cases} \tag{3}$$

Note that the elements h_{ij} and w_{ij} are assumed to be static in order to make the following analysis more readable; however, our derivations can be easily extended to the case of a time-varying channel model.

At last, we stress that, while most of previous work aims at maximizing the sum rates of source nodes, we consider that every node i may have a different amount of data to deliver to the destination in the unit time. Thus, our goal is to study the *maximum fair rate allocation* to all nodes, i.e., the average data rates that can be achieved by the nodes and that satisfy the desired proportion among the nodes' data generation rates. To do so, we should consider an n-dimensional problem, with the n variables representing the nodes' data rates. However, we can obtain a problem formulation that is mathematically tractable, by expressing the average[1] rate at which node i transfers its own data towards the destination node $n+1$ as

$$R_i = \rho_i R \qquad i = 1, \ldots, n\,. \tag{4}$$

In the above equation, the coefficients ρ_i's are (positive) input parameters representing the relationship among the nodes' data generation rates, hence the desired relationship among the nodes' traffic delivery rates. Such an expression allows us to consider only one system variable, R, which should be maximized.

Given the aforementioned scenario, we are interested in deriving a bound to the maximum achievable rate R, in both the FD and the HD case. FD nodes have the ability to transmit and receive simultaneously over the same frequency band; we denote the corresponding operational state by $\mathfrak{tr}$. HD nodes, instead, cannot do both tasks simultaneously, i.e., at a given time instant, they can either transmit ($\mathfrak{t}$) or receive ($\mathfrak{r}$). Under certain circumstances, an FD node may also operate in HD mode for a fraction of time, hence it may be in any of the states $\mathfrak{t}$, $\mathfrak{r}$ and $\mathfrak{tr}$. However, for FD nodes, state $\mathfrak{t}$ can be included in state $\mathfrak{tr}$ since reception does not increase the interference level at other nodes and it does not decrease the system capacity either. Note also that a sleep state could be considered, in which the nodes neither transmit nor receive but they just save energy. However, for the purposes of our analysis, a sleep state is equivalent to the receive state. In conclusion, we can limit our attention to states $\mathfrak{r}$ and $\mathfrak{tr}$ for FD nodes, and to $\mathfrak{r}$ and $\mathfrak{t}$ for HD nodes.

[1] Note that the average is computed over time, as the generic node i may take different operational states at different time instants (namely, transmit, reception and idle/sleep).

Since any network node can operate in two states, while the destination node $n+1$ always receives, the number of possible states the network can take is $J = 2^n$. We denote the j-th network state ($j = 1, \ldots, J$) by $\boldsymbol{\sigma}_j = [\sigma_{1j}, \ldots, \sigma_{nj}]$ where σ_{ij} is the state of node i when the network is in state $\boldsymbol{\sigma}_j$, that is, $\sigma_{ij} \in \{\mathfrak{r}, \mathfrak{tr}\}$ if k is an FD node, and $\sigma_{ij} \in \{\mathfrak{r}, \mathfrak{t}\}$ if k is an HD node. Also, we define the set of network states as $\mathcal{J} = \{\boldsymbol{\sigma}_j, j = 1, \ldots, J\}$, while the time fractions the network spends in the possible states are represented by the vector $\mathbf{t} = [t_1, \ldots, t_J]^{\mathsf{T}}$, with $0 \leq t_j \leq 1$ and such that $\sum_{j=1}^{J} t_j = 1$.

III. Background on the cut-set bound

The cut-set bound is an upper-bound to the achievable data rate of a wireless network of generic topology where nodes exchange messages among each other. As mentioned, in our case the network is composed of n wireless nodes and a destination node (see Fig. 1). We define the set of network nodes as $\mathcal{T} = \{1, \ldots, n+1\}$ and, as introduced in Section II, we assume that node i, $i = 1, \ldots, n$, generates a message W_i, of rate R_i, to be transferred to the destination. The messages W_i's are assumed to be mutually independent.

Following the notation introduced in [7, Chapter 10.2], we denote by x_i and y_i the random variables representing the signals, respectively, transmitted (channel inputs) and received (channel outputs), by node i, $i = 1, \ldots, n+1$. Moreover, since we assume that the destination node (i.e., node $n+1$) is always in receive state $\mathfrak{r}$, we set $x_{n+1} = 0$. The transmitted signals x_i's are assumed to have zero mean, unit variance and joint distribution $p_{x_1,\ldots,x_n}$. The destination node, on the base of the received signal y_{n+1}, derives estimates $\widehat{W}_i$ of the messages W_i, $i = 1, \ldots, n$.

In order to compute the cut-set bound, one should consider all possible partitions, hereinafter called *cuts*, of the network nodes $\mathcal{T}$ into two non overlapping sets, $\mathcal{S}$ and $\mathcal{S}_c = \mathcal{T} \setminus \mathcal{S}$. The former includes some of the nodes generating messages, while the latter contains the destinations of those messages (for which they compute an estimate). Note that, beside the sources and destinations of a set of tagged messages, $\mathcal{S}$ and $\mathcal{S}_c$ can include other nodes as well. In our network scenario, message estimates are derived only at the destination node, thus a valid cut is such that $\mathcal{S}_c$ contains at least node $n+1$. Let us now consider a generic cut $\mathcal{S}$. We denote by

- $\mathcal{M}(\mathcal{S})$ the set of messages transmitted by nodes in the cut $\mathcal{S}$,
- $R_{\mathcal{M}(\mathcal{S})}$ the sum of the rates of the messages in $\mathcal{M}(\mathcal{S})$,
- $\mathbf{x}_{\mathcal{S}} = \{x_k | k \in \mathcal{S}\}$ the set of channel inputs contained in $\mathcal{S}$,

- $\mathbf{x}_{\mathcal{S}_c} = \{x_k | k \in \mathcal{S}_c\}$ the set of channel inputs contained in $\mathcal{S}_c$, and by
- $\mathbf{y}_{\mathcal{S}_c} = \{y_k | k \in \mathcal{S}_c\}$ the set of channel outputs contained in $\mathcal{S}_c$.

By [7, Chapter 10.2], the rate $R_{\mathcal{M}(\mathcal{S})}$ can be written as $R_{\mathcal{M}(\mathcal{S})} = \sum_{i\in\mathcal{S}} R_i$, where R_i is the rate of message W_i. Then, the cut-set bound to the network capacity region is given by:

$$\mathcal{C} \subseteq \bigcup_{p_{x_1,\ldots,x_n}} \bigcap_{\mathcal{S}\in\Omega} \left\{ R_1, \ldots, R_n | \sum_{i\in\mathcal{S}} R_i \le I(\mathbf{x}_{\mathcal{S}}; \mathbf{y}_{\mathcal{S}_c} | \mathbf{x}_{\mathcal{S}_c}) \right\} \tag{5}$$

where $\Omega = \{\mathcal{S} | \mathcal{S} \subseteq \mathcal{T}, \mathcal{S} \neq \emptyset\}$ is the set of network cuts, whose cardinality is $|\Omega| = 2^n - 1$. The term $I(\mathbf{x}_{\mathcal{S}}; \mathbf{y}_{\mathcal{S}_c} | \mathbf{x}_{\mathcal{S}_c})$ denotes the mutual information[2] between the random variables $\mathbf{x}_{\mathcal{S}}$ and $\mathbf{y}_{\mathcal{S}_c}$, given $\mathbf{x}_{\mathcal{S}_c}$ and a joint distribution $p_{x_1,\ldots,x_n}$. We recall the the mutual information $I(X;Y|Z)$ between two random variables X and Y, given the random variable Z, can be written as $I(X,Y|Z) = h(Y|Z) - h(Y|X,Z)$, with $h(Y|Z)$ being the differential entropy[3] of the random variable Y given Z.

IV. Cut-set bounds: Full-duplex radios

We now derive an upper bound to the achievable data rate by applying the cut-set bound approach. We start by considering the expression in (5) and make some observations, as detailed next.

In our scenario, the network can operate in J possible states, i.e., $\boldsymbol{\sigma}_j$, $j = 1, \ldots, J$, characterized by the time fractions $\mathbf{t} = [t_1, \ldots, t_J]^\mathsf{T}$. The mutual information $I(\mathbf{x}_{\mathcal{S}}; \mathbf{y}_{\mathcal{S}_c} | \mathbf{x}_{\mathcal{S}_c})$ in (5) can therefore be expressed as $I(\mathbf{x}_{\mathcal{S}}; \mathbf{y}_{\mathcal{S}_c} | \mathbf{x}_{\mathcal{S}_c}) = \sum_{j=1}^J t_j I(\mathbf{x}_{\mathcal{S}}; \mathbf{y}_{\mathcal{S}_c} | \mathbf{x}_{\mathcal{S}_c}, \boldsymbol{\sigma}_j)$. The rate of the messages generated by the network nodes are such that $R_i = \rho_i R$, therefore we can write $\sum_{i\in\mathcal{S}} R_i = R\varrho_{\mathcal{S}}$ where $\varrho_{\mathcal{S}} = \sum_{i\in\mathcal{S}} \rho_i$. This allows us to reduce the n-dimensional problem in (5) to a formulation with one variable only, i.e., R, where the union and the intersection operators can be replaced with a max and a min operators, respectively. Additionally, the maximization must be performed also over all possible time fractions $\mathbf{t}$. Hence, using (5), we can write the cut-set upper bound to the data rate R as

$$B = \max_{p_{x_1,\ldots,x_n}, \mathbf{t}} \min_{\mathcal{S}\in\Omega} \frac{1}{\varrho_{\mathcal{S}}} \sum_{j=1}^J t_j I(\mathbf{x}_{\mathcal{S}}; \mathbf{y}_{\mathcal{S}_c} | \mathbf{x}_{\mathcal{S}_c}, \boldsymbol{\sigma}_j). \tag{6}$$

Then, under the assumption of a AWGN channel, FD nodes, a Gaussian codebook and the signal model in (1), we obtain the following expression:

$$B_{\mathrm{FD}} = \max_{\boldsymbol{\Sigma},\mathbf{t}} \min_{\mathcal{S}\in\Omega} \left\{ \frac{1}{\varrho_{\mathcal{S}}} \sum_{j=1}^J t_j I_{\mathcal{S},j} \right\} \tag{7}$$

where $I_{\mathcal{S},j} = I(\mathbf{x}_{\mathcal{S}}; \mathbf{y}_{\mathcal{S}_c} | \mathbf{x}_{\mathcal{S}_c}, \boldsymbol{\sigma}_j)$ and the joint density $p_{x_1,\ldots,x_n}$ is represented by the covariance matrix $\boldsymbol{\Sigma}$.

[2]The mutual information of two random variables measures the mutual dependence of the two variables [6].

[3]Similar to the information-theory concept of entropy, differential entropy is a measure of average surprisal but of a continuous random variable [6].

However, the computation of a tight cut-set bound, such as that in (7) would require us to consider any possible cut of the network, $\mathcal{S} \in \Omega$, separating some messages from their corresponding estimates, and its complement, $\mathcal{S}_c = \mathcal{T} \setminus \mathcal{S}$. Unfortunately, this is impractical for networks with a large number of nodes, since the number of cuts increases exponentially with n, i.e., as $2^n - 1$. Thus, in the following we derive an upper bound for B_{FD}, i.e., a looser upper bound to the achievable data rate. To demonstrate that our bound is still tight, we derive a lower bound for B_{FD} and show that our upper and lower bounds for B_{FD} are very close.

A. Upper bound to B_{FD}

A weaker, but mathematically tractable, upper bound to the rate R can be obtained by reducing the cuts to be considered in (7) to one cut only, which coincides with $\mathcal{T}$. Then, we have $B_{\mathrm{FD}} \le \frac{1}{\varrho_{\mathcal{T}}} \max_{\boldsymbol{\Sigma},\mathbf{t}} \sum_{j=1}^J t_j I_{\mathcal{T},j}$ where $I_{\mathcal{T},j} = I(x_1, \ldots, x_n; y_{n+1} | \boldsymbol{\sigma}_j) = I(x_{n-k_C+1}, \ldots, x_n; y_{n+1} | \boldsymbol{\sigma}_j)$. It follows that

$$\begin{aligned} B_{\mathrm{FD}} &\le \frac{1}{\varrho_{\mathcal{T}}} \max_{\boldsymbol{\Sigma}} \sum_{j=1}^J t_j I(x_{n-k_C+1}, \ldots, x_n; y_{n+1} | \boldsymbol{\sigma}_j) \\ &= \frac{1}{\varrho_{\mathcal{T}}} \max_{\boldsymbol{\Sigma}} I(x_{n-k_C+1}, \ldots, x_n; y_{n+1} | \boldsymbol{\sigma}^*) \\ &= \frac{1}{2\varrho_{\mathcal{T}}} \max_{\boldsymbol{\Sigma}} \log_2 \left(1 + \gamma \mathbf{h}_{n+1}{}^\mathsf{T} \boldsymbol{\Sigma} \mathbf{h}_{n+1}\right) . \end{aligned} \tag{8}$$

Since we aim at deriving an upper bound, in (8) we limited the possible network states to those in which nodes $n-k_C+1, \ldots, n$ are in state $\mathfrak{tr}$, nodes $n-k_I+1, \ldots, n-k_C$ are in state $\mathfrak{r}$ (so that they do not interfere with the nodes within distance k_C from the destination $n+1$), and the remaining ones can be either in $\mathfrak{tr}$ or $\mathfrak{r}$ ($\boldsymbol{\sigma}^*$ represents any of these network states). Note that the vector $\mathbf{h}_{n+1}$ is the $(n+1)$-th column of $\mathbf{H}$ and

$$\begin{aligned} \mathbf{h}_{n+1}{}^\mathsf{T} \boldsymbol{\Sigma} \mathbf{h}_{n+1} &\le \left| \sum_{i=1}^n \sum_{j=1}^n (\mathbf{h}_{n+1})_i (\mathbf{h}_{n+1})_j (\boldsymbol{\Sigma})_{ij} \right| \\ &\overset{(a)}{\le} \left(\sum_{\ell=1}^{k_C} h_\ell \right)^2 \end{aligned} \tag{9}$$

where the inequality in (a) is due to the fact that all elements of $\mathbf{h}_{n+1}$ are positive, $|(\boldsymbol{\Sigma})_{ij}| \le 1$, and only the nodes within distance k_C from the destination are transmitting. By substituting (9) in (8), we can write

$$B_{\mathrm{FD}} \le \frac{1}{2\sum_{i=1}^n \rho_i} \log_2 \left(1 + \gamma \left(\sum_{\ell=1}^{k_C} h_\ell \right)^2 \right) = B_{\mathrm{U-FD}} \tag{10}$$

B. Lower bound to B_{FD}

In order to assess how tight the bound $B_{\mathrm{U-FB}}$ is with respect to B_{FB}, we derive a lower bound for the latter, which we denote by $B_{\mathrm{L-FB}}$. The lower bound $B_{\mathrm{L-FB}}$ is obtained by assuming $\boldsymbol{\Sigma} = \mathbf{I}$ in (7), i.e., that the transmitted signals are uncorrelated. Under this condition, a node can decode some data by using one signal only out of the received ones, and it has to consider the latter as interference. Thus, by recalling (7) we have

$$B_{\rm FD} \geq \max_{\mathbf{t}} \min_{\mathcal{S}\in\Omega} \left\{ \frac{1}{\varrho_{\mathcal{S}}} \sum_{j=1}^{J} t_j \; I_{\mathcal{S},j}|_{\mathbf{\Sigma}=\mathbf{I}} \right\} , \qquad (11)$$

where $I_{\mathcal{S},j}|_{\mathbf{\Sigma}=\mathbf{I}}$ is the mutual information $I_{\mathcal{S},j}$ conditioned to $\mathbf{\Sigma} = \mathbf{I}$, i.e.,

$$I_{\mathcal{S},j}|_{\mathbf{\Sigma}=\mathbf{I}} = I(\mathbf{x}_{\mathcal{S}}; \mathbf{y}_{\mathcal{S}_c}|\mathbf{x}_{\mathcal{S}_c}, \mathbf{\Sigma} = \mathbf{I}, \boldsymbol{\sigma}_j).$$

Let us define the $1 \times n$ vector $\boldsymbol{\delta}_{\mathcal{S}}$, whose i-th element is $(\boldsymbol{\delta}_{\mathcal{S}})_i = 1$ if $i \in \mathcal{S}$ and 0 otherwise, and the diagonal matrices $\mathbf{\Delta}_{\mathcal{S}} = \text{diag}(\boldsymbol{\delta}_{\mathcal{S}})$ and $\bar{\mathbf{\Delta}}_{\mathcal{S}} = \text{diag}([\mathbf{1}-\boldsymbol{\delta}_{\mathcal{S}}, 1])$. Then, the mutual information $I_{\mathcal{S},j}|_{\mathbf{\Sigma}=\mathbf{I}}$ can be rewritten as

$$\begin{aligned} I_{\mathcal{S},j}|_{\mathbf{\Sigma}=\mathbf{I}} \quad = \quad & I\left(\mathbf{\Delta}_{\mathcal{S}}\mathbf{x};\, \sqrt{\gamma}\bar{\mathbf{\Delta}}_{\mathcal{S}}\mathbf{H}^{\mathsf{T}}\mathbf{x} + \sqrt{\gamma}\bar{\mathbf{\Delta}}_{\mathcal{S}}\mathbf{W}^{\mathsf{T}}\mathbf{i} + \right. \\ & \left. \mathbf{z}|(\mathbf{I}-\mathbf{\Delta}_{\mathcal{S}})\mathbf{x}, \mathbf{\Sigma} = \mathbf{I}, \boldsymbol{\sigma}_j\right) \end{aligned}$$

where the matrices $\mathbf{\Delta}_{\mathcal{S}}$ and $\bar{\mathbf{\Delta}}_{\mathcal{S}}$ select the nodes in the cut $\mathcal{S}$ and $\mathcal{S}_c$, respectively. Since $\mathbf{x} = \mathbf{\Delta}_{\mathcal{S}}\mathbf{x} + (\mathbf{I} - \mathbf{\Delta}_{\mathcal{S}})\mathbf{x}$ and we assume $\mathbf{\Sigma} = \mathbf{I}$, we have

$$\begin{aligned} I_{\mathcal{S},j}|_{\mathbf{\Sigma}=\mathbf{I}} = & \, I\left(\mathbf{\Delta}_{\mathcal{S}}\mathbf{x};\, \sqrt{\gamma}\bar{\mathbf{\Delta}}_{\mathcal{S}}(\mathbf{H}^{\mathsf{T}}\mathbf{\Delta}_{\mathcal{S}}\mathbf{x} + \mathbf{W}^{\mathsf{T}}\mathbf{i}) + \mathbf{z}|\mathbf{\Sigma} = \mathbf{I}, \boldsymbol{\sigma}_j\right) \\ = & \, h(\sqrt{\gamma}\bar{\mathbf{\Delta}}_{\mathcal{S}}(\mathbf{H}^{\mathsf{T}}\mathbf{\Delta}_{\mathcal{S}}\mathbf{x} + \mathbf{W}^{\mathsf{T}}\mathbf{i}) + \mathbf{z}|\mathbf{\Sigma} = \mathbf{I}, \boldsymbol{\sigma}_j) - \\ & \, h(\sqrt{\gamma}\bar{\mathbf{\Delta}}_{\mathcal{S}}\mathbf{W}^{\mathsf{T}}\mathbf{i} + \mathbf{z}|\boldsymbol{\sigma}_j) \end{aligned} \qquad (12)$$

where $h(\cdot)$ denotes the differential entropy. Now, let us define the vector $\mathbf{d}_j = [d_{1j}, \dots, d_{nj}]^{\mathsf{T}}$ whose entries, for $i = 1, \dots, n$, are such that $d_{ij} = 1$ if $\sigma_{ij} = \mathfrak{tr}$, and $d_{ij} = 0$ if $\sigma_{ij} = \mathfrak{r}$. From the above definitions, it follows that the vectors of signals $\mathbf{x}$ conditioned to the network state $\boldsymbol{\sigma}_j$ can be written as $\mathbf{x}|\boldsymbol{\sigma}_j = \mathbf{D}_j\mathbf{x}$, where $\mathbf{D}_j = \text{diag}(\mathbf{d}_j)$. Similarly, the interference vector conditioned to the network state $\boldsymbol{\sigma}_j$ is given by $\mathbf{i}|\boldsymbol{\sigma}_j = \mathbf{D}_j\mathbf{i}$. Then, from (12) we obtain

$$\begin{aligned} I_{\mathcal{S},j}|_{\mathbf{\Sigma}=\mathbf{I}} = & \, h(\sqrt{\gamma}\bar{\mathbf{\Delta}}_{\mathcal{S}}(\mathbf{H}^{\mathsf{T}}\mathbf{\Delta}_{\mathcal{S}}\mathbf{x} + \mathbf{W}^{\mathsf{T}}\mathbf{i}) + \mathbf{z}|\mathbf{\Sigma} = \mathbf{I}, \boldsymbol{\sigma}_j) - \\ & \, h(\sqrt{\gamma}\bar{\mathbf{\Delta}}_{\mathcal{S}}\mathbf{W}^{\mathsf{T}}\mathbf{i} + \mathbf{z}|\boldsymbol{\sigma}_j) \\ = & \, h(\sqrt{\gamma}\bar{\mathbf{\Delta}}_{\mathcal{S}}(\mathbf{H}^{\mathsf{T}}\mathbf{\Delta}_{\mathcal{S}}\mathbf{D}_j\mathbf{x} + \mathbf{W}^{\mathsf{T}}\mathbf{D}_j\mathbf{i}) + \mathbf{z}|\mathbf{\Sigma} = \mathbf{I}) - \\ & \, h(\sqrt{\gamma}\bar{\mathbf{\Delta}}_{\mathcal{S}}\mathbf{W}^{\mathsf{T}}\mathbf{D}_j\mathbf{i} + \mathbf{z}) \\ = & \, \frac{1}{2}\log_2 \frac{|\mathbf{I} + \gamma\bar{\mathbf{\Delta}}_{\mathcal{S}}(\mathbf{W}^{\mathsf{T}}\mathbf{D}_j\mathbf{W} + \mathbf{H}^{\mathsf{T}}\mathbf{\Delta}_{\mathcal{S}}\mathbf{D}_j\mathbf{H})\bar{\mathbf{\Delta}}_{\mathcal{S}}|}{|\mathbf{I} + \gamma\bar{\mathbf{\Delta}}_{\mathcal{S}}\mathbf{W}^{\mathsf{T}}\mathbf{D}_j\mathbf{W}\bar{\mathbf{\Delta}}_{\mathcal{S}}|} \\ = & \, a_{\mathcal{S},j} \end{aligned} \qquad (13)$$

where we used the fact that $\mathbf{x}$, $\mathbf{i}$ and $\mathbf{z}$ are mutually independent, $\mathbf{D}_j^2 = \mathbf{D}_j$, and $\mathbf{\Delta}_{\mathcal{S}}\mathbf{D}_j^2\mathbf{\Delta}_{\mathcal{S}} = \mathbf{\Delta}_{\mathcal{S}}\mathbf{D}_j$. Let $\mathbf{a} = [a_{\mathcal{S},1}, \dots, a_{\mathcal{S},J}]^{\mathsf{T}}$ (with $a_{\mathcal{S},j}$ as in (13)) and $\mathbf{t} = [t_1, \dots, t_J]^{\mathsf{T}}$. It follows that (11) can be rewritten as

$$B_{\rm FD} \geq \max_{\mathbf{t}} \min_{\mathcal{S}\in\Omega} \left\{ \frac{\mathbf{a}^{\mathsf{T}}\mathbf{t}}{\varrho_{\mathcal{S}}} \right\} . \qquad (14)$$

The max-min problem in (14) can be turned into the following linear programming (LP) problem, which can be easily solved:

$$\begin{aligned} & B_{\rm L-FD} = \max R \quad \text{s.t.} \\ & \frac{\mathbf{a}_{\mathcal{S}}{}^{\mathsf{T}}\mathbf{t}}{\varrho_{\mathcal{S}}} \geq R, \;\; \text{for any } \mathcal{S} \in \Omega \\ & \mathbf{1}^{\mathsf{T}}\mathbf{t} = 1 \\ & 0 \leq t_j \leq 1, \;\; \text{for any } j \in \mathcal{J} \, . \end{aligned}$$

As a last remark, note that, for the special case where $k_C = k_I = 1$, the expressions we derived for $B_{\rm L-FD}$, B_{FD} and $B_{\rm U-FD}$ coincide and take the value, $\frac{1}{2\varrho_{\mathcal{T}}}\log_2(1+\gamma)$.

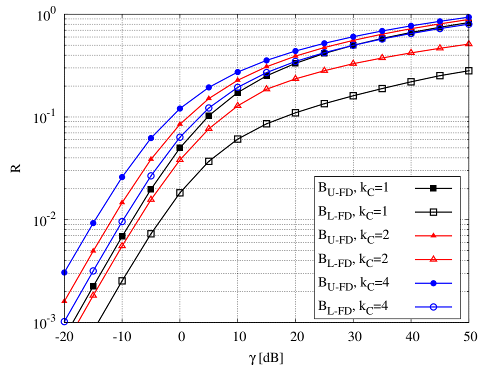

Fig. 2. Full-duplex radios: bounds for $n = 10$, $a = 2$, $\rho_i = 1\,\forall i$, $k_C = 1, 2, 4$ hops and $k_I = 5$.

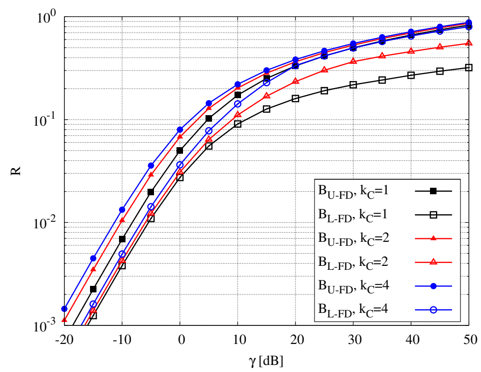

Fig. 3. Full-duplex radios: bounds for $n = 10$, $a = 4$, $\rho_i = 1\,\forall i$, $k_C = 1, 2, 4$ hops and $k_I = 5$.

C. Results

Let us consider a network composed of $n = 10$ nodes plus the destination. By using the above expressions and setting $\rho_i = 1$, $i = 1, \dots, n$, we compute the bounds to the achievable rate R in (4) as the value of SNR, γ, and the node cooperation and interference range vary. Recall that, by varying the latter two parameters, the values taken by the bounds in (10) and (14) vary as well.

Figs. 2 and 3 present the results obtained for a path loss exponent, a, equal to 2 and 4, respectively. The cooperation range k_C varies between 1 and 4, while the interference range is set to $k_I = 5$. As it can be seen by looking at the plots, in the medium-low SNR region the bounds, $B_{\rm U-FD}$ and $B_{\rm L-FD}$, are tight for any value of k_C, while in the high SNR region the gap is very limited for any $k_C \geq 2$. The figures also show that the distance between the two bounds decreases as the path loss exponent increases, especially in the high SNR region. The reason of this behavior is that the larger the k_C's and a's, the smaller the impact of the interference, as large k_C's imply that interferers are very far away from the receiver while large a's cause severe signal attenuation. Since the bounds $B_{\rm U-FD}$ and $B_{\rm L-FD}$ are always close (except for $a = 2$ and $k_C = 1$), we conclude that $B_{\rm U-FD}$ is a tight upper-bound of the cut-set bound $B_{\rm FD}$.

Furthermore, we observe that, by increasing k_C, the bounds also increase, as it can be exploited the cooperation among a larger number of transmitters. However, such a gain is evident only when k_C grows from 1 to 2, while a further increase of k_C to 4 provides only a limited increase in the data rate. Such a gain further reduces as the path loss exponent grows.

In conclusion, our results suggest that increasing the number of cooperating nodes beyond two provides a benefit which is little for medium-low SNR and negligible in the high SNR region. Also, such a gain in the achievable rate significantly decreases for $a > 2$. It follows that the complexity of designing and implementing a communication strategy that exploits cooperative transmissions from nodes, located farther than two hops away from a receiver, does not pay off in terms of performance.

V. Cut-set bounds: Half-duplex radios

We now consider that the n network nodes operate in HD mode, i.e., that at any time instant each node can be either in the transmit ($\mathfrak{t}$) or in the receive ($\mathfrak{r}$) state. As done in the case of FD radios, we denote by $I_{\mathcal{S},j} = I(\mathbf{x}_\mathcal{S}; \mathbf{y}_{\mathcal{S}_c}|\mathbf{x}_{\mathcal{S}_c}, \boldsymbol{\sigma}_j)$ the mutual information associated to cut $\mathcal{S}$ and conditioned to the network being in state $\boldsymbol{\sigma}_j = [\sigma_{1j}, \dots, \sigma_{nj}]$, where $\sigma_{ij} \in \{\mathfrak{r}, \mathfrak{t}\}$ is the state of node i when the network is in state $\boldsymbol{\sigma}_j$. It follows that the mutual information associated to the cut $\mathcal{S}$ can be written as $I_\mathcal{S} = I(\mathbf{x}_\mathcal{S}; \mathbf{y}_{\mathcal{S}_c}|\mathbf{x}_{\mathcal{S}_c}) = \sum_{j=1}^{J} t_j I_{\mathcal{S},j}$.

Following [8], the cut set bound to the rate that can be achieved in the HD scenario is:

$$B_{\text{HD}} = \max_{\mathbf{t},\boldsymbol{\Sigma}} \min_{\mathcal{S}\in\Omega} \left\{ \frac{I_\mathcal{S}}{\varrho_\mathcal{S}} \right\} . \quad (15)$$

where we recall that $\varrho_\mathcal{S} = \sum_{i\in\mathcal{S}} \rho_i$. The computation of the bound in (15) is again mathematically intractable for large networks, since it requires the maximization over the vector $\mathbf{t}$ and the matrix $\boldsymbol{\Sigma}$, and the minimization over $2^n - 1$ cuts. Thus, similarly to what done for the FD case, below we derive an upper and a lower-bound to B_{HD}.

A. Upper bound to B_{HD}

We first observe that the bound in (8) can be obtained again for the HD case by following the same approach as in Sec. IV-A, i.e., we can bound (15) by reducing the set of possible cuts, Ω. However, it is clear that a different derivation is needed in order to obtain a good bound for the HD case.

We now split the set of nodes $\mathcal{T}$ in two disjoint subsets: $\mathcal{T}_1$ containing the nodes $\{1, \dots, n-k-1\}$ and $\mathcal{T}_2$ including the nodes $\{n-k, \dots, n\}$, where $k \ge k_C$. We then upper-bound B_{HD} by considering only the set of network cuts, $\widetilde{\Omega}$, such that, for every $\mathcal{S} \in \widetilde{\Omega}$, the nodes in $\mathcal{T}_1$ are out of the cooperation range of all nodes in $\mathcal{S}$. Then, a first upper-bound to B_{HD} can be written as

$$B_{\text{HD}} \le \max_{\mathbf{t},\boldsymbol{\Sigma}} \min_{\mathcal{S}\in\widetilde{\Omega}} \left\{ \frac{I_\mathcal{S}}{\varrho_\mathcal{S}} \right\} . \quad (16)$$

Next, motivated by the results obtained for the FD radios (see Figs. 2 and 3 and related comments), let us limit our attention to the case where the cooperation range is equal to 2 hops, i.e., $k = k_C = 2$. The generalization to the case where $k_C > 2$, although more complicated, can be easily obtained. Under such an assumption, the right hand side of (16) can be rewritten as

$$\min_{\mathcal{S}\in\widetilde{\Omega}} \left\{ \frac{I_\mathcal{S}}{\varrho_\mathcal{S}} \right\} = \min_{1\le h\le 5} \min_{\mathcal{S}\in\widetilde{\Omega}_h} \left\{ \frac{I_\mathcal{S}}{\varrho_\mathcal{S}} \right\}$$

where the disjoint subsets of cuts, $\widetilde{\Omega}_h$'s, satisfy the condition $\widetilde{\Omega} = \bigcup_{h=1}^{5} \widetilde{\Omega}_h$ and are defined below.

1) $\widetilde{\Omega}_1 = \{\mathcal{S} = \{n-q, \dots, n-1\}, 2 \le q \le n-1\}$. In this case, we have $\mathcal{S}_c = \{1, \dots, n-q-1, n\}$ for $2 \le q < n-1$, and $\mathcal{S}_c = \{n, n+1\}$ for $q = n-1$. The corresponding mutual information can be written as

$$I_\mathcal{S} = I(\mathbf{x}_\mathcal{S}; \mathbf{y}_{\mathcal{S}_c}|\mathbf{x}_{\mathcal{S}_c}) = I(\mathbf{x}_\mathcal{S}; y_n, y_{n+1}|\mathbf{x}_{\mathcal{S}_c}) \quad (17)$$

where the last equality holds since the signals $\mathbf{y}_{\mathcal{S}_c}$, except for y_n, do not depend on $\mathbf{x}_\mathcal{S}$. We recall that the conditioned mutual information $I(X;Y|Z)$ can be written in terms of differential entropy as $I(X;Y|Z) = h(Y|Z) - h(Y|X,Z)$. In our case and for $2 \le q \le n-1$, we have

$$\begin{aligned} I(\mathbf{x}_\mathcal{S}; y_n, y_{n+1}|\mathbf{x}_{\mathcal{S}_c}) &= h(y_n, y_{n+1}|\mathbf{x}_{\mathcal{S}_c}) - \\ &\quad h(y_n, y_{n+1}|x_1, \dots, x_n) \\ &\le h(y_n, y_{n+1}|x_n) - \\ &\quad h(y_n, y_{n+1}|x_{n-2}, x_{n-1}, x_n) \\ &= I(x_{n-2}, x_{n-1}; y_n, y_{n+1}|x_n) \end{aligned} \quad (18)$$

since conditioning reduces the entropy and, under our assumptions, $h(y_n, y_{n+1}|x_1, \dots, x_n) = h(y_n, y_{n+1}|x_{n-2}, x_{n-1}, x_n)$. Recall that in (16) we need to minimize the ratio $I_\mathcal{S}/\rho_\mathcal{S}$ over all possible cuts. Therefore, by using the results in (17) and (18), we can write:

$$\begin{aligned} \min_{\mathcal{S}\in\widetilde{\Omega}_1} \frac{I_\mathcal{S}}{\rho_\mathcal{S}} &\le \min_{\mathcal{S}\in\widetilde{\Omega}_1} I(x_{n-2}, x_{n-1}; y_n, y_{n+1}|x_n) \left(\sum_{i\in\mathcal{S}} \rho_i \right)^{-1} \\ &= I(x_{n-2}, x_{n-1}; y_n, y_{n+1}|x_n) \\ &\quad \min_{2\le q\le n-1} \left(\sum_{i\in\{n-q,\dots,n-1\}} \rho_i \right)^{-1} \\ &= I(x_{n-2}, x_{n-1}; y_n, y_{n+1}|x_n) \left(\sum_{i=1}^{n-1} \rho_i \right)^{-1} \\ &= I_1 . \end{aligned} \quad (19)$$

2) $\widetilde{\Omega}_2 = \{\{n-1\}\}$. Following the same procedure as above, we obtain

$$\min_{\mathcal{S}\in\widetilde{\Omega}_2} \frac{I_\mathcal{S}}{\rho_\mathcal{S}} \le \frac{1}{\rho_{n-1}} I(x_{n-1}; y_n, y_{n+1}|x_{n-2}, x_n) = I_2 .$$

3) $\widetilde{\Omega}_3 = \{\mathcal{S} = \{n-q, \dots, n\}, 2 \le q \le n-1\}$. Then, we have $\mathcal{S}_c = \{1, \dots, n-q-1\}$ for $2 \le q \le n-1$, and $\mathcal{S}_c = \emptyset$ for $q = n-1$. Again, we obtain

$$\min_{\mathcal{S}\in\widetilde{\Omega}_3} \frac{I_\mathcal{S}}{\rho_\mathcal{S}} \le \frac{1}{\sum_{i=1}^{n} \rho_i} I(x_{n-1}, x_n; y_{n+1}) = I_3 .$$

4) $\widetilde{\Omega}_4 = \{\{n-1, n\}\}$, then $\min_{\mathcal{S}\in\widetilde{\Omega}_4} \frac{I_\mathcal{S}}{\rho_\mathcal{S}} \le \frac{1}{\rho_n + \rho_{n-1}} I(x_{n-1}, x_n; y_{n+1}|x_{n-2}) = I_4$.

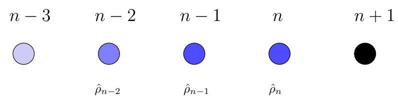

Fig. 4. Equivalent network for the computation of the bound in (20).

5) $\widetilde{\Omega}_5 = \{\{n\}\}$, then $\min_{\mathcal{S}\in\widetilde{\Omega}_5} \frac{I_{\mathcal{S}}}{\rho_{\mathcal{S}}} \leq \frac{1}{\rho_n} I(x_n; y_{n+1}|x_{n-2}, x_{n-1}) = I_5$.

Given that the interference range is larger than the cooperation range, i.e., $k_I > k_C$, it is clear that the terms I_h's also account for the interference. Since interfering signals are assumed to be uncorrelated, for simplicity in the bound computation, the terms of mutual information I_h's can be upper-bounded by considering $k_I = k_C + 1$. That is, we can account only for a single interfering node.

In conclusion, let $\widetilde{I}_h$ be the mutual information I_h conditioned to $k_I = k_C + 1$, $h = 1, \ldots, 5$. We can eventually write the upper-bound to B_{HD} as

$$B_{\text{U-HD}} = \max_{\substack{\mathbf{t},\boldsymbol{\Sigma} \\ \sum_{j=1}^{J} t_j = 1}} \min_{1\leq h\leq 5} \widetilde{I}_h \,. \tag{20}$$

Note that the bound in (20) refers to an equivalent network composed of 5 nodes (see Fig. 4), namely: (a) an interfering node $n-3$; (b) the node $n-2$ whose transmitted signal is considered as useful for nodes $n-1$ and n and as interference for node $n+1$; (c) the nodes $n-1$ and n whose transmitted signals are considered as useful to node $n+1$; and (d) the destination node $n+1$. Also, the terms $\widetilde{I}_1, \ldots, \widetilde{I}_5$ represent the mutual information associated to the cuts, respectively, $\{n-2, n-1\}, \{n-1\}, \{n-2, n-1, n\}, \{n-1, n\}, \{n\}$ of the equivalent network, where the equivalent traffic loads are described by the coefficients

$$\hat{\rho}_{n-2} = \sum_{i=1}^{n-2} \rho_i; \qquad \hat{\rho}_{n-1} = \rho_{n-1}; \qquad \hat{\rho}_n = \rho_n.$$

Since each node can operate in two states and node $n+1$ is always receiving, the above equivalent network has 2^4 states to be considered in the computation of (20).

As the last remark, we observe that the mutual information $\widetilde{I}_h$, $h = 1, \ldots, 5$, can be rewritten as $\widetilde{I}_h = \sum_j t_j \widetilde{I}_{h,j}$, where the terms $\widetilde{I}_{h,j}$ are the mutual information conditioned to network state $\boldsymbol{\sigma}_j$. Then, the bound in (20) can be written as

$$B_{\text{U-HD}} = \max_{\substack{\mathbf{t},\boldsymbol{\Sigma} \\ \sum_j t_j = 1}} \min_{1\leq h\leq 5} \sum_j t_j \widetilde{I}_{h,j} \,.$$

The above max-min problem can be efficiently solved as follows: for each covariance matrix $\boldsymbol{\Sigma}$, solve the LP problem

$$\begin{aligned} & B_{\text{U-HD}} = \max R \quad \text{s.t.} \\ & \textstyle\sum_j t_j \widetilde{I}_{h,j} \geq R, \;\; h = 1, \ldots, 5 \\ & \textstyle\sum_j t_j = 1 \\ & \sum_{j:\sigma_{j,n-3}=\mathfrak{t}} t_j \geq \frac{\rho_{n-3} R}{\frac{1}{2}\log_2(1+\gamma)} \end{aligned}$$

and choose the maximum over $\boldsymbol{\Sigma}$. Note that $\sum_{j:\sigma_{j,n-3}=\mathfrak{t}} t_j$ represents the time fraction during which the interfering node $n-3$ is transmitting (i.e., it is in state $\mathfrak{t}$). The constraint $\sum_{j:\sigma_{j,n-3}=\mathfrak{t}} t_j \geq \frac{\rho_{n-3}R}{\frac{1}{2}\log_2(1+\gamma)}$ bounds such a time fraction with the time required by node $n-3$ to transmit *at least* its own generated data ($\rho_{n-3}R$) through a single-hop channel with capacity $\log_2(1+\gamma)/2$.

B. Lower bound to B_{HD}

As done for the FD case, in order to verify that the upper bound $B_{\text{U-HD}}$ is tight enough, we derive a lower bound to B_{HD} and compare it to the $B_{\text{U-HD}}$. Again, we lower bound B_{HD} by assuming $\boldsymbol{\Sigma} = \mathbf{I}$. By doing so, we obtain:

$$B_{\text{HD}} \geq \max_{\mathbf{t}} \min_{\mathcal{S}} \left\{ \frac{\sum_{j=1}^{J} t_j I_{\mathcal{S},j}|_{\boldsymbol{\Sigma}=\mathbf{I}}}{\varrho_{\mathcal{S}}} \right\} . \tag{21}$$

The conditioned mutual information $I_{\mathcal{S},j}|_{\boldsymbol{\Sigma}=\mathbf{I}}$ can be expressed similarly to (13), as:

$$\begin{aligned} & \frac{1}{2}\log_2 \frac{\left|\mathbf{I} + \gamma \bar{\boldsymbol{\Delta}}_{\mathcal{S}} \bar{\mathbf{D}}_j (\mathbf{W}^\mathsf{T} \mathbf{D}_j \mathbf{W} + \mathbf{H}^\mathsf{T} \boldsymbol{\Delta}_{\mathcal{S}} \mathbf{D}_j \mathbf{H}) \bar{\mathbf{D}}_j \bar{\boldsymbol{\Delta}}_{\mathcal{S}}\right|}{\left|\mathbf{I} + \gamma \bar{\boldsymbol{\Delta}}_{\mathcal{S}} \bar{\mathbf{D}}_j \mathbf{W}^\mathsf{T} \mathbf{D}_j \mathbf{W} \bar{\mathbf{D}}_j \bar{\boldsymbol{\Delta}}_{\mathcal{S}}\right|} \\ = \;& b_{\mathcal{S},j} \end{aligned} \tag{22}$$

where the matrix $\bar{\mathbf{D}}_j = \text{diag}([\mathbf{1} - \mathbf{d}, 1])$ accounts for the fact that HD nodes cannot simultaneously transmit and receive, i.e., $\bar{\mathbf{D}}_j$ is used to force to 0 the signal received at the nodes that are transmitting. The right hand side of (21) can then be further bounded as:

$$\max_{\substack{\mathbf{t} \\ \mathbf{1}^\mathsf{T}\mathbf{t}=1}} \min_{\mathcal{S}} \left\{ \frac{\mathbf{b}_{\mathcal{S}}{}^\mathsf{T}\mathbf{t}}{\varrho_{\mathcal{S}}} \right\} \geq \max_{\substack{\hat{\mathbf{t}} \\ \mathbf{1}^\mathsf{T}\hat{\mathbf{t}}=1}} \min_{\mathcal{S}} \left\{ \frac{\hat{\mathbf{b}}_{\mathcal{S}}{}^\mathsf{T}\hat{\mathbf{t}}}{\varrho_{\mathcal{S}}} \right\} = B_{\text{L-HD}} \tag{23}$$

where the column vectors $\hat{\mathbf{b}}_{\mathcal{S}}$ and $\hat{\mathbf{t}}$ are defined as $\hat{\mathbf{b}}_{\mathcal{S}} = \{b_{\mathcal{S},j}\}$, $\hat{\mathbf{t}} = \{t_j\}$, with $j \in \hat{\mathcal{J}}$, and where we only considered a subset $\hat{\mathcal{J}}$ of the possible network states $\mathcal{J}$. The reduction of the number of considered states to $\hat{\mathcal{J}}$ allows a dramatic reduction of the computational complexity of (23) and is justified by the fact that, through numerical analysis, we have observed that most of the network states have little or no influence on the value of the bound. More specifically, the number of dominant states, i.e., those that provide significant contribution, increases just linearly with n. Also, for $k_I = k_C + 1$, the dominant network states are circular shifts of the vector $\boldsymbol{\sigma} = [\ldots \mathfrak{ttrttrttr} \ldots]$ where the pattern $\mathfrak{ttr}$ is repeated. This result was expected since, for the above value of k_I, the pattern $\mathfrak{ttr}$ both avoids interference and allows neighboring nodes to cooperate. This finding suggests that efficient communication strategies can be obtained by exploiting such network states. The max-min problem in (23) can then be turned into the following LP problem:

$$\begin{aligned} & B_{\text{L-HD}} = \max R \quad \text{s.t.} \\ & \tfrac{\hat{\mathbf{b}}_{\mathcal{S}}{}^\mathsf{T}\hat{\mathbf{t}}}{\varrho_{\mathcal{S}}} \geq R, \;\; \text{for any } \mathcal{S} \in \Omega \\ & \mathbf{1}^\mathsf{T}\hat{\mathbf{t}} = 1 \\ & 0 \leq t_j \leq 1, \;\; \text{for any } j \in \hat{\mathcal{J}} \,. \end{aligned}$$

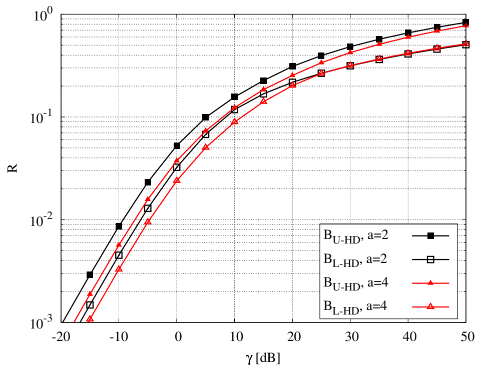

Fig. 5. Half-duplex radios: bounds for $n = 10$, $a = 2, 4$, $\rho_i = 1\,\forall i$, $k_C = 2$ hops and $k_I = k_C + 1$.

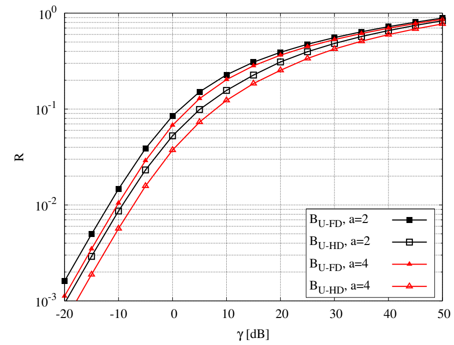

Fig. 7. Comparison between the cases of half-duplex and full-duplex radios. Bounds for $n = 10$, with $a = 2, 4$, $\rho_i = 1\,\forall i$, $k_C = 2$ hops and $k_I = k_C + 1$ hops.

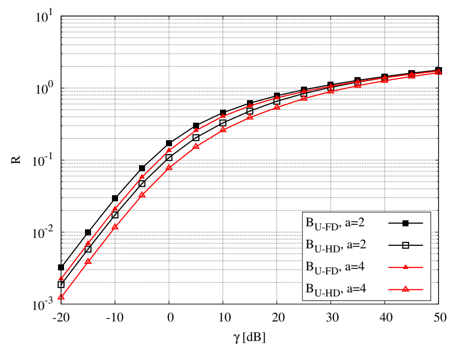

Fig. 6. Comparison between the cases of half-duplex and full-duplex radios. Bounds for $n = 5$, $a = 2, 4$, $\rho_i = 1\,\forall i$, $k_C = 2$ hops and $k_I = k_C + 1$ hops.

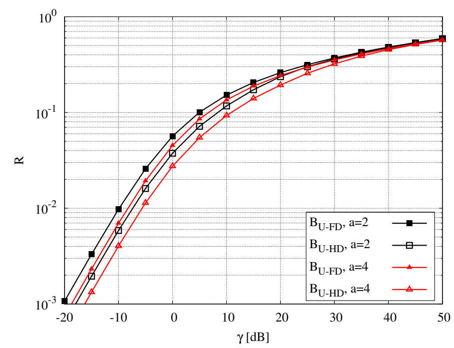

Fig. 8. Comparison between the cases of half-duplex and full-duplex radios. Bounds for $n = 5$, $a = 2, 4$, $k_C = 2$ hops, $k_I = k_C + 1$ hops, and different data generation rates at the nodes, namely, $\rho_i = i$, $i = 1, \ldots, n$.

C. Results

We now assume HD radios and compare the bounds in (20) and (23) to the achievable data rate as the SNR varies. We focus on the case where $\rho_i = 1$, $i = 1, \ldots, n$, path loss exponent $a = 2, 4$, cooperation range $k_C = 2$ and interference range $k_I = k_C + 1$. The results are shown in Fig. 5, for $n = 10$. The bounds we derived show to be very close for any value of γ and a, again proving that the upper-bound $B_{\mathrm{U-HD}}$ in (20) is a tight upper bound to the cut-set bound B_{HD}. Similar results have been obtained also varying the values of the system parameters over a larger range.

After assessing the tightness of the bounds in (10) and (20), we now investigate their behavior for different values of n. In particular, Figs. 6 and 7 compare the bounds $B_{\mathrm{U-FD}}$ and $B_{\mathrm{U-HD}}$ for $a = 2, 4$, $k_C = 2$, $k_I = k_C + 1$, and for $n = 5$ and $n = 10$, respectively. The bounds for the FD case are clearly higher than those obtained for HD radios, as the latter case constrains the nodes to operate in either transmit or receive mode while in the FD case the best operational mode for each node is selected.

We then analyze the case where the nodes have different traffic loads, i.e., they generate data traffic at different rates ρ_i. In particular, Figs. 8 and 9 show the case where $\rho_i = i$, $i = 1, \ldots, n$, i.e., the closer a node to the destination, the higher its load. The first plot presents the curves obtained for $n = 5$ and path loss exponent a equal to 2 and 4. In this case, we can observe a behavior very similar to the one exhibited by the bounds in Fig. 6, i.e., under a constant traffic load for all nodes. We stress that quantitevely the results shown in the two figures greatly differ, as they depict the parameter R. Indeed, when the nodes load increases as their distance from the destination decreases, the average rate of node i does not coincide with R but it is equal to $\rho_i R$, with $\rho_i = i$.

The plot in Fig. 9, instead, refers to the case where $a = 2$ and $n = 5, 10$. Comparing these results with those in Fig. 7, it is evident that the achievable value of R is greatly affected

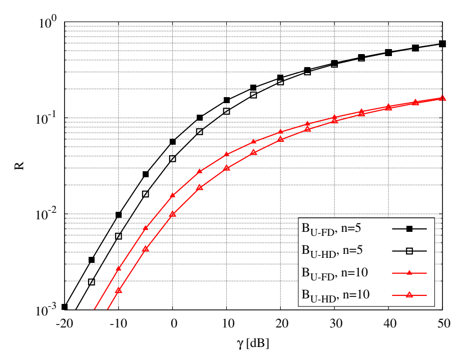

Fig. 9. Comparison between the cases of half-duplex and full-duplex radios. Bounds for $n = 5, 10$, $a = 2$, $k_C = 2$ hops, $k_I = k_C + 1$ hops, and different data generation rates at the nodes, namely, $\rho_i = i$, $i = 1, \ldots, n$.

by the number of nodes n when $\rho_i = i$, $i = 1, \ldots, n$. Again, it is worthwhile recalling that in this case the average data rate of node i is $R_i = iR$, $i = 1, \ldots, n$. Thus, although R decreases as n grows, the overall amount of traffic delivered to the destination in a time unit is still high.

VI. Conclusions

We studied the upper bound to the data rate that wireless nodes in a linear network can achieve. We carried out the analysis accounting for the interference due to simultaneous transmissions, and in presence of full as well as half-duplex nodes. Also, unlike previous work, we considered that nodes located at more than one-hop distance can cooperate to deliver the data traffic to the destination, and that nodes may have different requirements in terms of achievable data rate. The expressions we derived are mathematically tractable and allow the analysis of large-scale, multihop networks. Numerical results showed the impact on the performance of several system parameters, such as the SNR, the path loss exponent and the number of cooperating transmitters.

Our analysis suggests two important facts. First, in order to design efficient communication strategies, it is sufficient to use pairs of transmitters that cooperate to forward the data to the destination. Second, in half-duplex networks, there exist some dominant network states whose contribution determines the achievable data rate. Effective communication strategies can therefore be obtained by considering pairs of cooperating nodes and by letting the network operate in such states. Future work will focus on the definition of cooperative traffic relaying schemes that provide an achievable rate as close as possible to the upper bound provided in this study.

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