Differential-difference equations for the transient simulation of lossy MTLs

Differential-Difference Equations For The Transient Simulation Of Lossy MTLs

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ABSTRACT

In this paper, we address the differential representation of the time-domain characteristics of lossy MTLs. This approach is of great interest for the efficient simulation of circuits with long interconnects and nonlinearities. The properties of this characterization method are discussed with particular emphasis on the bandwidth and on the order of the differential operators used. Our discussion is supported by a complete characterization example for a realistic wideband 3-conductor interconnect.

I. Introduction

As the number of circuits containing electrically long interconnects is steadily growing, the transient simulation of multiconductor transmission lines (MTLs), which can directly account also for the effects of nonlinear devices, is becoming increasingly important (*e.g.*, see [1]). The main problem of the transient simulation of MTLs (consisting of the numerical solution of a set of integral-differential equations for the variables at the line ends) is to achieve the required accuracy at an affordable computational cost, in view of the complexity of real applications and of the speed of available computers. Recently, in order to improve the numerical efficiency of the simulation schemes, the use of transient equations of differential-difference type has been widely considered: the system equations are obtained by replacing the linear operator describing the line effect in the original transient equations with a finite-order differential operator and with an ideal time delay. Although not immediately apparent, many simulation schemes (*e.g.*, the Asymptotic Waveform Evaluation (AWE) method [2]) share this common approach.

In this paper, we discuss the main features of the differential representation of the MTLs characteristics, namely the generation of the differential approximations, their order and bandwidth, and their use for the transient simulation.

II. Differential Line Characteristics

The line characteristic operators describing the transformations performed by a TL on the signals at its ends are usually expressed as convolution integrals with proper line impulse responses (IRs). The numerical representation of the line operators with convolution integrals, however, is computationally expensive, since the cost of any new time step is proportional to the number of previous time steps accounted by the convolution integrals. As a result, the cost of an n-step transient computation is proportional to n^2, while it becomes proportional to $n \times m$ with the use of a differential representation of order m for the line operators. In order to better assess this advantage, some further considerations may be useful. The most direct way to generate differential representations of the line operators is to compute rational approximations of the line frequency characteristics. It is intuitive that the order of the rational approximation grows indefinitely with the bandwidth over which a given accuracy is required. Even if the linear phase term responsible for the line delay is extracted before the approximation, no finite order rational function can globally approximate TL transfer functions, since, for large enough frequency, they have irrational behavior. The bandwidth of a simulation problem and the number of time steps in its solution, however, are strictly related, since the latter is determined by the time resolution needed and by the transient duration. This means that the order required for a rational representation of the line operators is a function (hopefully slowly growing) of n and that the cost of an n-step transient computed by differential characteristics is actually $n \times m(n)$. A proof of this general rule for a specific approximation scheme can be found in [3], where the increase of the method order is shown with respect to the transient duration.

The differential line characteristics have the important additional advantage to be integrable by a large number of standard numerical methods. Many available software packages can be directly applied to the solution of

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transient equations of differential-difference type, thereby leading to efficient MTL simulators at low developement cost. Furthermore, many standard circuit simulators accept arbitrary rational transfer functions, thus increasing even more the interest for the differential characterization of TLs. They can be easily extended to include custom MTLs, described by differential characteristics supplied by the user at the accuracy level he needs.

Differential line characteristics can be readily obtained in many different ways. Most approximation procedures for TL differential characterization rely on the AWE method to obtain rational frequency characteristics. Alternatively, rational approximations of network functions can be obtained by means of an optimization procedure, with the objective of achieving the required accuracy in the specified bandwidth. The variety of possible optimization strategies is quite large, but interesting results can be obtained already with MATLAB routine INVFREQS.

Differential representations of the line operators can be generated also in the time domain by approximating the IRs, or their integrals, with exponential sums (*e.g.*, see [4]). This approach does not extend the limits of the approximation, but apparently has two advantages, *i.e.*, the exponential approximations can be obtained by a real function fitting and the time error of the approximation is directly under control. On the other hand, the time-domain fitting requires the knowledge of the line IRs, which means additional computation for the characterization procedure. Finally, differential line characteristics can be obtained also from discrete models of the line, either based on chains of lumped equivalents or on the solution of the TL equation with a discrete along-the-line coordinate.

III. Differential Characteristics for a 3-conductor TL

In this Section, we develop a complete example of differential characterization for a MTL, with the aim of verifying in a real case the considerations developed in the previous Section. The method selected to generate the differential characteristics is the time-domain fitting of the MTL IRs.

A. Impulse responses of a MTL

The transient equations for a terminated 3-conductor line characterized by its matched scattering parameters are

$$\begin{aligned} \boldsymbol{b}_1 = \boldsymbol{h} * \boldsymbol{a}_2, \qquad \boldsymbol{b}_2 = \boldsymbol{h} * \boldsymbol{a}_1 \\ \boldsymbol{f}_p\Big(\boldsymbol{a}_p + \boldsymbol{b}_p - \boldsymbol{e}_p, \boldsymbol{y} * (\boldsymbol{a}_p - \boldsymbol{b}_p)\Big) = 0 \end{aligned} \tag{1}$$

where $*$ means time convolution, $\boldsymbol{a}_p$, $\boldsymbol{b}_p$ ($p = 1,2$) are the unknown voltage waves at the line ends, (related to voltages and currents by $\boldsymbol{a}_p = \frac{1}{2}(\boldsymbol{v}_p + \boldsymbol{z} * \boldsymbol{i}_p)$, $\boldsymbol{b}_p = \frac{1}{2}(\boldsymbol{v}_p - \boldsymbol{z} * \boldsymbol{i}_p)$, $\boldsymbol{v}_p = (v_{p1}, v_{p2})^T$, $\boldsymbol{i}_p = (i_{p1}, i_{p2})^T$, see Fig. 1), $\boldsymbol{h}$ and $\boldsymbol{y}$ (or $\boldsymbol{z}$) are the transient expressions of the matched transmission scattering parameter and the characteristic admittance (or impedance) matrices, respectively, $\boldsymbol{f}_p = 0$ are the vector equations describing the loads and $\boldsymbol{e}_p = (e_{p1}, e_{p2})^T$ are the driving voltage signals.

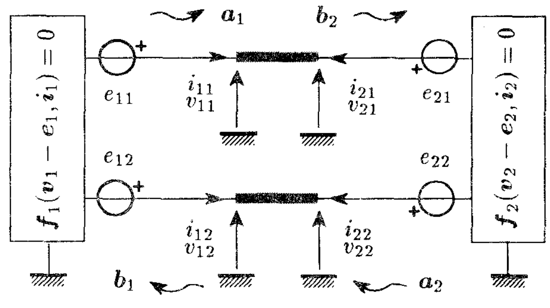

Figure 1: *Network representation of a loaded 3-conductor TL circuit with the variables used in this analysis.*

Although the impulse responses of (1), $\boldsymbol{h}$ and $\boldsymbol{y}$ are well-behaved functions, their accurate evaluation for wideband (*i.e.*, low loss) TL, as fast interconnects are, is not trivial. In the wideband case, these functions have a fast initial part and a slowly decreasing tail, which demand for nonuniform time sampling and prevent from a direct application of FFT inversion. This problem is discussed in [5], where an algorithm for Laplace Transform inversion (thereafter indicated by HILT) is effectively applied to the evaluation of the IRs of a 2-conductor TL. With the HILT algorithm, any sample of $\boldsymbol{h}(t)$ is obtained as a weigthed sum of $\boldsymbol{H}(s)$ values at selected complex s points. The numerical evaluation of $\boldsymbol{H}$ (*i.e.*, the solution of the MTL eigenvalue problem) is facilitated by the use of the following normalized time and complex frequency quantities: $T = \nu t$, $S = \Sigma + j\Omega = s/\nu$, where $\nu = R_m/L_m$, R_m and L_m being the largest elements of the per-unit-length DC resistance and inductance matrices, respectively. Besides, the linear phase terms of the propagation factors inside $\boldsymbol{H}$ must be extracted before the application of the HILT algorithm. To this end, the normalized transmission matrix is written in the form

$$\begin{aligned} \bar{\boldsymbol{H}}(S) = \bar{\boldsymbol{H}}_1 + \bar{\boldsymbol{H}}_2 e^{-S\Delta T} \\ \bar{\boldsymbol{H}}_1 = \bar{\boldsymbol{M}}_v \operatorname{diag}\{e^{\bar{\gamma}_1 \mathcal{L}}, 0\} \bar{\boldsymbol{M}}_v^{-1} \\ \bar{\boldsymbol{H}}_2 = \bar{\boldsymbol{M}}_v \operatorname{diag}\{0, e^{\bar{\gamma}_2 \mathcal{L}}\} \bar{\boldsymbol{M}}_v^{-1} \end{aligned} \tag{2}$$

where $\bar{\gamma}_k$ ($k = 1,2$) are the normalized modal propagation constants of the MTL in which the asymptotic terms $-ST_k$ are extracted, $\mathcal{L}$ is the line length, $\bar{\boldsymbol{M}}_v(S)$ is the matrix of the voltage modal profiles, and $\Delta T = T_1 - T_2$. The functions $\bar{\boldsymbol{H}}_k$ are inverse transformed by HILT and the IR matrix is obtained as a sum of two contributions, *i.e.*, $\boldsymbol{h}(t) = \nu\{\bar{\boldsymbol{h}}_1(\nu(t - T_1)) + \bar{\boldsymbol{h}}_2(\nu(t - T_2))\}$.

The structure of the example is a two-land PCB with length $\mathcal{L} = 25$ cm, dielectric constant $\epsilon_r = 4.7$, substrate height 200 μm, land thickness 30 μm, separation 90 μm, and widths 60 μm and 30 μm. This structure is a typical wideband (low-loss) interconnect, made strongly asymmetric to test the ability of the inversion method to handle frequency-dependent modal profile matrices. Losses in the ground plane are neglected, whereas the per-unit-length resistance of the lands is described by the simple Holt's resistance model, since in the considered time interval the behavior of the IRs is controlled by the skin losses [5]. When needed, more accurate resistance models can be freely used in this inversion procedure.

The normalized IR $\overline{h}_{11}(T)$ for this structure is shown in Fig. 2: logarithmic scales are used to include both the slow and the fast part of the waveform. Besides, the logarithmic time scale has a first origin in $T = 0$, at the arrival of the contribution of the faster mode (the odd one), and a second origin in ΔT, at the arrival of the contribution of the slower mode (the even one). Since the scale factor of this example is $\nu \approx 3.1 \times 10^6$, the time interval shown spans up to 30 ns, *i.e.*, about 30 time delays.

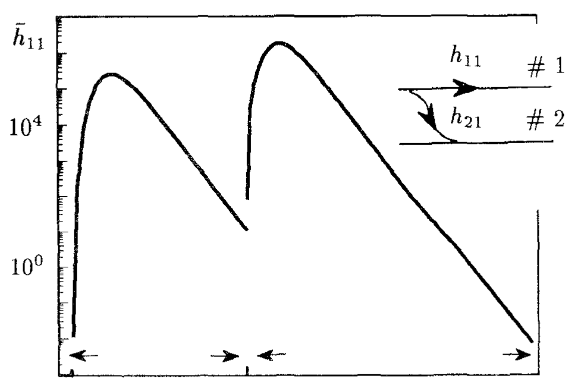

Figure 2: *Normalized IR $\overline{h}_{11}(T)$ of the RLC 3-conductor TL of the example. Special logarithmic scales, explained in the text, are used to highlight the features of the waveform. The insert explains the meaning of the $\bar{\mathbf{h}}$ elements.*

Fig. 3 shows, in conventional linear scales, all the four elements of the MTL transmission response matrix in the form of step responses, *i.e.*, $\bar{r}_{pq} = \int_0^T \bar{\mathbf{h}} dT'$. As it can be seen, the cross terms of this matrix consistently have the typical shape of the crosstalk signals caused by step input signals. For brevity, we skip the evaluation of the transient characteristic admittance matrix, which is obtained with the same procedure and is made of functions similar to the transient admittances of 2-conductor TLs.

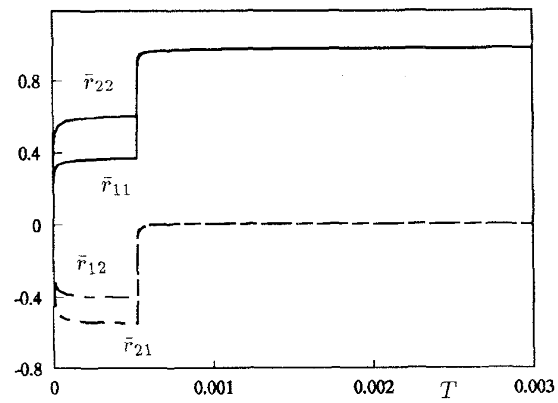

Figure 3: *Normalized step responses $\bar{r}_{pq}$ of the RLC 3-conductor TL of the example.*

B. Exponential fitting

In this Section, we generate a differential representation of the line operator $(\overline{h}_{11}(T)*)$ by approximating $\overline{h}_{11}(T)$ with a sum $\overline{h}_a$ of exponentials. We apply here, in a more general context, the idea of [4], and add some important remarks.

The most relevant point of a differential characterization obtained by time fitting is the selection of the time interval $[T_{min}, T_{max}]$ on which carrying out the approximation. The minimum time depends on the driving signals and on the bandwidth of the line loads, which is always finite owing to the parasitics present at the line ends. The maximum time coincides with the transient duration. Differential characteristics accurate outside this time interval are useless. It should be emphasized that approximations accurate for arbitrarily small time values are equivalent to infinite bandwith representations, and have no physical meaning. The additional condition $\int_0^{T_{min}} \overline{h}_a(T)dT = \int_0^{T_{min}} \overline{h}_{11}(T)dT$ must be imposed to ensure the consistency of the approximation, also when a significant part of the IR falls before T_{min}. This simple condition can account for the neglected part of the IR, because the signals transformed by the line operators are approximately constant over a time interval of duration T_{min}.

In order to automatically satisfy this condition and to deal with functions with a smaller dynamic range, it is preferable to fit the step response function $\bar{r}_{11}$. We seek an approximate exponential step response of the form $\bar{r}_a = \bar{r}_{a1} + \bar{r}_{a2}$, where $\bar{r}_{a1} = (c_1 - \sum_i c_{1i} e^{-\lambda_{1i} T}) u(T)$ is fitted to $\bar{r}_{11}$ in $[T_{min}, \Delta T]$,and $\bar{r}_{a2} = (c_2 - \sum_i c_{2i} e^{-\lambda_{2i}(T - \Delta T)}) u(T - \Delta T)$ is fitted to $\bar{r}_{11} - \bar{r}_{a1}$ in $[\Delta T + T_{min}, T_{max}]$. For our example, the time constants of the approximating function are assumed real and positive, and are determined by a standard least square fitting procedure over four decades, *i.e.*, in the interval $[T_{min} = 10^{-5}, T_{max} = 10^{-1}]$. Of course, the accuracy of the approximation or its interval of validity grow with the number of the time constants used. For our example, the influence of the approximation order can be seen in Fig. 4, where $\overline{h}_{11}(T)$ is compared with two approximations of order (2,3) and (2,5) ((m_1, m_2) indicates m_k time constants in $\bar{r}_{ak}$).

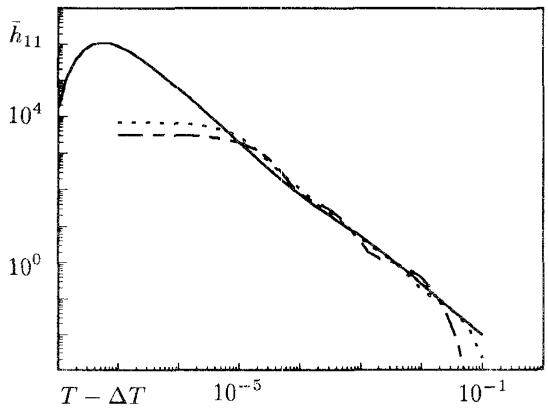

Figure 4: *Comparison of the second pulse of $\overline{h}_{11}(T)$ with two exponential approximations in the range $[\Delta T + 10^{-5}, 10^{-1}]$. The solid curve represents the exact IR; the dashed one refers to an approximation of order (2,3) , and the dotted one to an approximation of order (2,5). Note the use of doubly-logarithmic scales.*

C. *Line output from a differential characteristic*

The final differential-difference equations corresponding to our exponential approximation are obtained from

$$\begin{aligned} b_{21}(t) &= [h_{a1}(t - \tau_1) + h_{a2}(t - \tau_2)] * x(t) \\ b_{21}(t) &= h_{a1}(t) * x(t - \tau_1) + h_{a2}(t) * x(t - \tau_2), \end{aligned} \tag{3}$$

where $\tau_k = \nu T_k$ and h_{ak} are the IRs corresponding to $\bar{r}_{ak}$. To complete the example, we show in Fig. 5 a solution, computed by a Runge-Kutta routine, of the differential equation obtained from (3) for the (2,3) $\bar{h}_a$ approximation of Fig. 4. The input is a step voltage wave $\bar{a}_s$ with a normalized rise time $T_r = 2 \times 10^{-4}$ and the equation solved describe the transmission of this signal along the land # 1 when no signal is impinging on land # 2. For validation purposes, the exact $\bar{b}_{21}$, computed by HILT of $\bar{H}_{11}(S)\bar{A}_s(S)$, is also shown. Even if the rise time of the input signal is close to $T_{min} = 10^{-5}$, the agreement of the two curves is remarkably good.

IV. Conclusions

In this paper, we highlight the relationship between the bandwitdh of a MTL simulation problem and the order

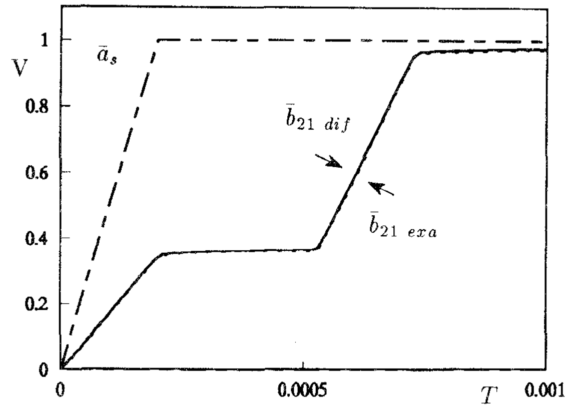

Figure 5: *Approximate (solid curve) and reference (dotted) output on land # 1, for a real step input signal (dashed). The approximation is computed by integration of a differential characteristic, and the reference is determined via HILT (see text). The output signals are shifted in $T = 0$.*

required for the differential characteristics of the MTL. The considerations are supported by a detailed example for a realistic asymmetric MTL, where a differential characteristic is obtained by fitting the accurate IRs of the structure. From this analysis, differential characteristics appear less efficient than commonly believed, yet easy to obtain and solve. Therefore, they constitute an important tool for the transient simulation of MTLs.

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