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Possible alterations of the gravitational field in a superconductor

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In this paper I calculate the possible alteration of the gravitational field in a superconductor by using the time-dependent Ginzburg-Landau equations (TDGL). I compare the behaviour of a high-T_c superconductor (HTCS) like $YBa_2Cu_3O_7$ (YBCO) with a classical low-T_c superconductor (LTCS) like Pb. Finally, I discuss what values of the parameters characterizing a superconductor can enhance the reduction of gravitational field.

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There is no doubt that the interplay between gravitational field and superconductivity is a very intriguing field of research, whose theoretical study has been involving many researchers for a long time[1, 2, 3, 4, 5]. Eight years ago, E. Podkletnov and R. Nieminem declared the achievement of an experimental evidence for a gravitational shielding due to a rotating high-T_c superconductor. After their announcement, other groups tried to repeat the experiment but they obtain controversial results [6, 7, 8], so that at the present moment the question is still open.

In 1996, G. Modanese interpreted the results by Podkletnov and Nieminem in the frame of quantum theory of General Relativity [9] but the complexity of the formalism he used makes it very difficult to extract quantitative predictions.

In a very recent paper, M. Agop [10] and collaborators wrote generalized Maxwell equations that simultaneously treat weak gravitational and electromagnetic fields. In the weak field approximation, the Einstein equations [11],

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = \frac{-8\pi G}{c^4}T^{\mu\nu}$$

lead to the following equations [12], formally similar to Maxwell's:

$$\nabla \cdot \mathbf{E}_g = -4\pi G\rho_g \tag{1}$$

$$\nabla \cdot \mathbf{B}_g = 0 \tag{2}$$

$$\nabla \times \mathbf{E}_g = -\frac{\partial \mathbf{B}_g}{\partial t} \tag{3}$$

$$\nabla \times \mathbf{B}_g = -\frac{4\pi G}{c^2}\mathbf{j}_g + \frac{1}{c^2}\frac{\partial \mathbf{E}_g}{\partial t} \tag{4}$$

where $\mathbf{E}_g$ and $\mathbf{B}_g$ are the gravitoelectric and gravitomagnetic field respectively, $\mathbf{j}_g$ is the mass current density vector such that $\mathbf{j}_g = \mathbf{v}\rho_g$, $\mathbf{v}$ is the velocity, ρ_g is the mass density. Obviously G is the Newton's constant and c is the speed of light in vacuum. As in the electromagnetic case, it is possible to define a gravitational permitivity $\varepsilon_g = 1/4\pi G$ and a gravitational permeability $\mu_g = 4\pi G/c^2$ of the vacuum. For example on the surface of the earth $\mathbf{E}_g$ is simply the Newtonian gravitational acceleration and $\mathbf{B}_g$ is related to angular momentum interactions [10, 12]. Then, they defined generalized electric field, magnetic field, scalar and vector potentials containing both an electromagnetic and a gravitational term, in the following way: $\mathbf{E} = \mathbf{E}_e + \frac{m}{e}\mathbf{E}_g$; $\mathbf{B} = \mathbf{B}_e + \frac{m}{e}\mathbf{B}_g$; $\phi = \phi_e + \frac{m}{e}\phi_g$ and $\mathbf{A} = \mathbf{A}_e + \frac{m}{e}\mathbf{A}_g$ where m and e are the electronic mass and charge and the subscripts e and g mean 'electromagnetic' and 'gravitational' respectively. The generalized Maxwell equations then become [10]:

$$\nabla \cdot \mathbf{E} = \left(-4\pi G + \frac{1}{\varepsilon_0}\right)\rho \tag{5}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{6}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{7}$$

$$\nabla \times \mathbf{B} = \left(\frac{-4\pi G}{c^2} + \mu_0\right)\mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \tag{8}$$

where the relations: $\rho_g = \frac{m}{e}\rho$ and $\mathbf{j}_g = \frac{m}{e}\mathbf{j}$ have been used and ϵ_0 and μ_0 are the electric permitivity and magnetic permeability in the vacuum. They also wrote the two generalized London equations [10]

$$\begin{aligned} \mathbf{E} &= (1/\rho)\,(\partial \mathbf{j}/\partial t) \\ \mathbf{B} &= (-1/\rho)\,\nabla \times \mathbf{j} \end{aligned} \tag{9}$$

and so they could define the generalized penetration depth

$$\lambda = \frac{\lambda_g \lambda_e}{\sqrt{\lambda_g^2 - \lambda_e^2}} \simeq \lambda_e \tag{10}$$

where $\lambda_e = \left[m/(\mu_0 e^2 n)\right]^{1/2}$, $\lambda_g = \left[c^2/(4\pi G m n)\right]^{1/2}$, n is the density of superelectrons and $\lambda_g/\lambda_e \simeq 10^{21}$.

For simplicity, I will study the case of an *isotropic* superconductor, in the gravitational field of the earth, in the absence of an electromagnetic field, thus taking $\mathbf{E_e} = \mathbf{0}$ and $\mathbf{B_e} = \mathbf{0}$. $\mathbf{B}_g$ in the solar system is very small [16] therefore, $\mathbf{E} = \frac{m}{e}\mathbf{E}_g$ and $\mathbf{B} = \mathbf{0}$. Moreover the gravitational effects of $\mathbf{j}_g$ are insignificant ($\mathbf{j}_g$ is related to gravitational effect of the superconductor) and so I assume $\mathbf{j}_g = 0$. I have also $\phi = \frac{m}{e}\phi_g$ and $\mathbf{A} = \frac{m}{e}\mathbf{A}_g$. This situation isn't analogous of the Meissner effect but, rather, to the case of a superconductor in a electric field. Since the gravitoelectric fied is formally analogous to electric field I will use the time-dependent Ginzburg-Landau equations (TDGL) [13, 14, 15], which, in the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$, are written in the form:

$$\frac{\hbar^2}{2mD}\left(\frac{\partial}{\partial t} + \frac{2ie}{\hbar}\phi\right)\psi - a\psi + b\left|\psi\right|^2\psi + \frac{1}{2m}(i\hbar\nabla + \frac{2e}{c}\mathbf{A})^2\psi = 0 \tag{11}$$

$$\nabla \times \nabla \times \mathbf{A} - \nabla \times \mathbf{H} = \frac{-4\pi\sigma}{c}(\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} + \nabla\phi) + \frac{4\pi}{c}\left[\frac{e\hbar}{mi}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{4e^2}{mc}\left|\psi\right|^2\mathbf{A}\right] \tag{12}$$

where D is the diffusion coefficient, σ is the conductivity in the normal phase, $\mathbf{H}$ is the applied field, $a(T) = a_0(T - T_c)$ and $b(T) \equiv b(T_c)$ where a_0 and b are the positive constants and T_c is the critical temperature of the superconductor. The boundary and initial conditions are:

$$\left.\begin{array}{c} (i\hbar\nabla\psi + (2e/c)\mathbf{A}\psi)\cdot\mathbf{n} = 0 \\ \nabla \times \mathbf{A}\cdot\mathbf{n} = \mathbf{H}\cdot\mathbf{n} \\ \mathbf{A}\cdot\mathbf{n} = 0 \end{array}\right\} \text{ on } \partial\Omega \times (0,t) \qquad \left.\begin{array}{c} \psi(x,0) = \psi_0(x) \\ \mathbf{A}(x,0) = \mathbf{A}_0(x) \end{array}\right\} \text{ on } \Omega \tag{13}$$

where $\partial\Omega$ is the boundary of a smooth and simply connected domain Ω in $\mathrm{R^n}$. In order to write equations 11,12 in a dimensionless form, the following quantities can be defined:

$$\Psi^2(T) = \frac{|a(T)|}{b};\ H_c(T) = \sqrt{\frac{4\pi\mu_0\left|a(T)\right|^2}{b}} = \frac{h/2e}{2\sqrt{2}\pi\lambda(T)\xi(T)} \tag{14}$$

$$\xi(T) = \frac{h}{\sqrt{2m\left|a(T)\right|}};\ \lambda(T) = \sqrt{\frac{bmc^2}{4\pi\left|a(T)\right|e^2}} \tag{15}$$

$$\kappa = \lambda(T)/\xi(T),\ \tau(T) = \lambda^2(T)/D,\ \eta = 4\pi\sigma D/(\varepsilon_0 c^2) \tag{16}$$

where $\lambda(T)$, $\xi(T)$ and $H_c(T)$ are the penetration depth, the coherence length and the thermodynamic field.

The dimensionless quantities are then:

$$x' = x/\lambda,\ t' = t/\tau,\ \psi' = \psi/\Psi \tag{17}$$

$$\mathbf{A}' = \mathbf{A}\kappa/(\sqrt{2}H_c\lambda),\ \phi' = \phi\kappa/(\sqrt{2}H_c D),\ H' = H\kappa/(\sqrt{2}H_c). \tag{18}$$

Inserting the eq. 17,18 in eqs. 11,12 and dropping the prime gives the dimensionless TDGL equations [13] in a bounded, smooth and simply connected domain Ω in R^n:

$$\frac{\partial\psi}{\partial t} + i\phi\psi + \kappa^2(|\psi|^2 - 1)\psi + (i\nabla + \mathbf{A})^2\psi = 0 \tag{19}$$

$$\eta(\frac{\partial\mathbf{A}}{\partial t} + \nabla\phi) + \frac{1}{2}i(\psi^*\nabla\psi - \psi\nabla\psi^*) + |\psi|^2\,\mathbf{A} + \nabla\times\nabla\times\mathbf{A} - \nabla\times\mathbf{H} = 0 \tag{20}$$

The boundary and initial conditions (13) become, in the dimensionless form:

$$\left.\begin{array}{c}(i\nabla\psi + \mathbf{A}\psi)\cdot\mathbf{n} = 0\\ \nabla\times\mathbf{A}\cdot\mathbf{n} = \mathbf{H}\cdot\mathbf{n}\\ \mathbf{A}\cdot\mathbf{n} = 0\end{array}\right\}\ \text{on } \partial\Omega\times(0,t) \qquad \left.\begin{array}{c}\psi(x,0) = \psi_0(x)\\ \mathbf{A}(x,0) = \mathbf{A}_0(x)\end{array}\right\}\ \text{on } \Omega \tag{21}$$

Our superconductor is immersed in the gravitational field of the earth which is very weak and approximately constant. So $\phi = -g^*x$ where $g^* = \lambda(T)\kappa m g/(\sqrt{2}eH_c(T)D) \ll 1$ and g is the gravity acceleration. The corrections to ϕ in the superconductor are of the second order in g^* and therefore they aren't considered here.

Now I search for a solution of the form:

$$\begin{aligned}\psi(x,t) &= \psi_0(x,t) + g^*\gamma(x,t)\\ A(x,t) &= 0 + g^*\beta(x,t)\\ \phi(x) &= -g^*x\end{aligned} \tag{22}$$

At order zero in g^*, eq.(19) gives:

$$\frac{\partial\psi_0(x,t)}{\partial t} + \kappa^2(|\psi_0(x,t)|^2 - 1)\psi(x,t) - \frac{\partial^2\psi_0(x,t)}{\partial x^2} = 0 \tag{23}$$

with the conditions:

$$\begin{aligned}\psi_0(x,t=0) &= 0\\ \psi_0(x=0,t) &= 0\\ \psi_0(x=L,t) &= 0\end{aligned} \tag{24}$$

where L is the length of the superconductor and $t = 0$ is the instant when the material undergoes the transition to the superconducting state.

The static classical solution of eq. 23 is:

$$\psi_0(x,t) \equiv \psi_0(x) = \left\{\tanh\left[\kappa x/\sqrt{2}\right] - \tanh\left[\kappa(x-L)/\sqrt{2}\right] - \tanh\left[\kappa L/\sqrt{2}\right]\right\}/\tanh\left[\kappa L/\sqrt{2}\right]. \tag{25}$$

At the first order in g^* one obtains from equation (19):

$$\frac{\partial\gamma(x,t)}{\partial t} - \frac{\partial^2\gamma(x,t)}{\partial x^2} + \kappa^2(3\,|\psi_0(x)|^2 - 1)\gamma(x,t) = ix\psi_0(x) \tag{26}$$

with the conditions:

$$\begin{aligned}\gamma(x,t=0) &= 0\\ \gamma(x=0,t) &= 0\\ \gamma(x=L,t) &= 0\end{aligned} \tag{27}$$

The equation at order one for the vector potential is

$$\eta\frac{\partial\beta(x,t)}{\partial t} + |\psi_0(x)|^2\,\beta(x,t) + J(x,t) - \eta = 0 \tag{28}$$

with the constraint

$$\beta(x,t=0) = 0 \tag{29}$$

Note that the second-order spatial derivative of β does not appear in eq.(28). This is due to the fact that, in one dimension, $\nabla^2 A = \frac{\partial}{\partial x}\nabla \cdot \mathbf{A}$ and therefore, in the Coulomb gauge, $\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 A = 0$. The quantity $J(x,t)$ which appears in eq. 28 is given by:

$$J(x,t) = \frac{1}{2}\left[\psi_0(x)\frac{\partial}{\partial x}\mathrm{Im}\gamma(x,t) - \mathrm{Im}\gamma(x,t)\frac{\partial}{\partial x}\psi_0(x)\right] \tag{30}$$

The solution of eq. 28 is

$$\beta(x,t) = \frac{\eta}{|\psi_0(x)|^2}\left[1 - \exp(-|\psi_0(x)|^2 t/\eta)\right] - \frac{\exp(-|\psi_0(x)|^2 t/\eta)}{\eta}\int_0^t dt J(x,t)\exp(|\psi_0(x)|^2 t/\eta) \tag{31}$$

Now, we have an expression for $\psi_0(x,t)$ (eq. 25), and an expression for $\beta(x,t)$ as a function of $\gamma(x,t)$ (eq. 31). Therefore, we can insert these expressions in eq. 22 and obtain both $\psi(x,t)$ and $\mathbf{A}(x,t)$ as functions of $\gamma(x,t)$. Using the relation:

$$\mathbf{E}_g = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} \tag{32}$$

we finally find the gravitoelectric field $\mathbf{E}_g$ in the superconductor:

$$\frac{\mathbf{E}_g(x,t)}{g^*} = \left[1 - \exp(-|\psi_0(x)|^2 t/\eta) - \frac{\partial}{\partial t}\left(\frac{\exp(-|\psi_0(x)|^2 t/\eta)}{\eta}\int_0^t dt J(x,t)\exp(|\psi_0(x)|^2 t/\eta)\right)\right] \tag{33}$$

From this formula we can see that for maximizing the effect of the reduction of the gravitational field in a superconductor it is necessary to reduce η and to have large spatial derivatives of $\psi_0(x)$ and $\gamma(x,t)$. The condition for having a small value of η is that the superconductor has a large normal-state resistivity and a small diffusion coefficient $D \sim v_F l/3$ (where v_F is the Fermi velocity, which is small in HTCS, and l is the mean free path). Therefore, the effect is enhanced in 'bad' samples with impurities, not in single crystals.

From the experimental viewpoint, the greater are the length and time scales over which there is a variation of $\mathbf{E}_g$, the easier is the observation of this effect. Actually, we started from non-dimensional equations and therefore the length and time scales are determined by $\lambda(T)$ and $\tau = \lambda^2(T)/D$, which should therefore be as large as possible. In this sense, some new materials with very large $\lambda(T)$ [18] could be interesting for the study of this effect. Moreover, as clearly seen in eq. 33, the relaxation time is inversely proportional to $|\psi_0(x)|^2$. As a result, $\psi_0(x)$ must be as small as possible, and this implies that κ is small (see eq.25). Then, $\lambda(T)$ and $\xi(T)$ must be both large.

Up to now we have dealt with the expression of $\beta(x,t)$ as a function of $\gamma(x,t)$. Actually, to obtain an explicit expression for $\mathbf{E}_g$ we have to solve the equation for $\gamma(x,t)$ (eq. 26). This is a difficult task which can be undertaken only in a numerical way. Nevertheless, if one puts $\psi_0(x,t) \simeq 1$ (good approximation in the case of $YBa_2Cu_3O_7$ (YBCO), in which κ=94.4), one can find the simple approximate solution:

$$\gamma(x,t) = i\gamma_0(x) + i\sum_{m=1}^{+\infty} A_m \sin(n\pi x/L)\exp\left[-(m^2\pi^2/L^2 + 2\kappa^2)t\right] \tag{34}$$

where

$$\gamma_0(x) = x/(2\kappa^2)\cdot\left\{1 - \left[\sinh(\sqrt{2}\kappa(L-x)) + \sinh(\sqrt{2}\kappa x)\right]/\sinh(\sqrt{2}\kappa L)\right\} \tag{35}$$

$$A_m = \frac{1}{L}\int_0^L \gamma_0(x)\sin(m\pi x/L)\mathrm{d}x = \frac{L}{2m\pi\kappa^2}\left\{(-1)^m - \frac{[A_{1m} + A_{2m}]}{m\pi\left[\left(\sqrt{2}\kappa L/(m\pi)\right)^2 + 1\right]}\right\} \tag{36}$$

and

$$A_{1m} = m\pi\left[1 - (-1)^m\cosh(\sqrt{2}\kappa L)\right] + 2\frac{(-1)^m\sqrt{2}\kappa L\sinh(\sqrt{2}\kappa L)}{\left[\left(\sqrt{2}\kappa L/(m\pi)\right)^2 + 1\right]} \tag{37}$$

$$A_{2m} = \left[\frac{1-\cosh(\sqrt{2}\kappa L)}{\sinh(\sqrt{2}\kappa L)}\right]\left\{\frac{2\sqrt{2}\kappa L\left[(-1)^m\cosh(\sqrt{2}\kappa L)-1\right]}{(m\pi)\left[\left(\sqrt{2}\kappa L/(m\pi)\right)^2+1\right]} - m\pi(-1)^m\sinh(\sqrt{2}\kappa L)\right\} \tag{38}$$

By inserting eq. 35 in eqs. 30,31 and taking into account eq. 25, the gravitoelectric field $\mathbf{E}_g$ becomes:

$$\mathrm{E}_g(x,t) = g^*\left[1-\exp(-\left|\psi_0(x)\right|^2 t/\eta)\left(1-\frac{J_0(x)}{\eta}\right)+\sum_{m=1}^{+\infty}\frac{A_m B_m(x) C_m(x,t)}{\eta}\right] \tag{39}$$

where

$$J_0(x) = \frac{1}{2\kappa^2}\left[\psi_0(x)\frac{\partial}{\partial x}\gamma_0(x)-\gamma_0(x)\frac{\partial}{\partial x}\psi_0(x)\right], \tag{40}$$

$$B_m(x) = (m\pi/L)\psi_0(x)\cos(m\pi x/L)-\sin(m\pi x/L)\frac{\partial}{\partial x}\psi_0(x), \tag{41}$$

$$C_m(x,t) = \frac{(m^2\pi^2/L^2+2\kappa^2)\exp\left[-(m^2\pi^2/L^2+2\kappa^2)t\right]-\left(\left|\psi_0(x)\right|^2/\eta\right)\exp(-\left|\psi_0(x)\right|^2 t/\eta)}{\left(\left|\psi_0(x)\right|^2/\eta\right)-(m^2\pi^2/L^2+2\kappa^2)}. \tag{42}$$

Note that making the very drastic approximation

$$\gamma(x) \simeq ix/(2\kappa^2) \tag{43}$$

leads to the apparently draft result

$$\mathrm{E}_g(x,t) = g^*\left[1-\exp(-\left|\psi_0(x)\right|^2 t/\eta)\left(1-\frac{J_{00}(x)}{\eta}\right)\right] \tag{44}$$

where

$$J_{00}(x) = \frac{1}{2\kappa^2}\left[\psi_0(x)-x\frac{\partial\psi_0(x)}{\partial x}\right] \tag{45}$$

In spite of its crudeness, this approximate solution (eq. 44) in the case of YBCO gives the same results of the exact solution (eq. 39). Moreover, nothing changes significantly if one neglects the finite size of the superconductor and uses $\psi_0(x) = \tanh\left(\kappa x/\sqrt{2}\right)$ instead of eq. 25. In the case of YBCO the variation of the gravitoelectric field E_g in time and space is shown in Figure 1a and 1b. It is easily seen that this effect is almost independent on the spatial coordinate.

The results in the case of Pb are reported in Figure 2a and 2b, which clearly show that, due to the very small value of κ, the reduction is greater near the surface. In this case, moreover, some approximations made in the case of YBCO are no longer allowed. For example, for small values of L the simplified relation (43) is not valid. When κ is small, in fact, the length L plays an important role. In particular, if L is small the effect is remarkably enhanced, as shown in Figure 3. In the same condition, a maximum of the effect (and therefore a minimum of E_g) can occur at $t \neq 0$, as can be seen in the same Figure.

In conclusion, for YBCO the shielding effect decays with a relaxation time $t_{\mathrm{surf}} \simeq 0.25\ \tau = 8.5\ 10^{-11}$ s near the surface ($x = 0.01\lambda = 3.3\cdot 10^{-9}$ m) and $t_{\mathrm{int}} \simeq 0.1\tau = 3.4\cdot 10^{-11}$s in the interior of the sample ($x = \lambda = 3.3\cdot 10^{-7}$m).

In the case of Pb, the same quantities take the values $t_{\mathrm{surf}} \simeq 3\cdot 10^7\tau = 1.8\cdot 10^{-8}$s ($x = \lambda = 7.8\cdot 10^{-8}$m) and $t_{\mathrm{int}} \simeq 5\cdot 10^5\tau = 3.1\cdot 10^{-9}$s ($x = 100\lambda = 7.8\cdot 10^{-6}$m) with $L \gg \lambda$.

Table 1 reports the values of the parameters of YBCO and Pb, calculated at a temperature T such that $(T-T_c)/T_c$ is the same in the two materials.

Table 1

	YBCO	Pb
T_c	89 K	7.2 K
T	77 K	6.3 K
$\lambda(T)$	$3.3{\cdot}10^{-7}$ m	$7.8{\cdot}10^{-8}$ m
$\xi(T)$	$3.6{\cdot}10^{-9}$ m	$1.7{\cdot}10^{-7}$ m
σ^{-1}	$4{\cdot}10^{-7}$ Ωm$(T = 90\text{K})$	$2.5{\cdot}10^{-9}$ Ωm$(T = 15\text{K})$
$H_c(T)$	0.2 T	0.018 T
$\tau(T)$	$3.4{\cdot}10^{-10}$ s	$6.1{\cdot}10^{-15}$ s
D	$3.2{\cdot}10^{-4}$ m^2/s	1 m^2/s
l	$6{\cdot}10^{-9}$ m	$1.7{\cdot}10^{-6}$m
v_F	$1.6{\cdot}10^5$ m/s	$1.83{\cdot}10^6$ m/s
κ	94.4	$4.8{\cdot}10^{-1}$
η	$1.27{\cdot}10^{-2}$	$6.6{\cdot}10^3$

Tables 2.a and 2.b summarize the variation of the fundamental quantities with the temperature [17]:

Table 2.a

YBCO	λ	τ	g^*
$T = 0$K	$1.7{\cdot}10^{-7}$ m	$9.03{\cdot}10^{-11}$s	$2.6{\cdot}10^{-12}$
$T = 70$K	$2.6{\cdot}10^{-7}$ m	$2.1{\cdot}10^{-10}$s	$9.8{\cdot}10^{-12}$
$T = 77$K	$3.3{\cdot}10^{-7}$ m	$3.4{\cdot}10^{-10}$s	$2{\cdot}10^{-11}$
$T = 87$K	$8{\cdot}10^{-7}$ m	$2{\cdot}10^{-9}$s	$2.8{\cdot}10^{-7}$

Table 2.b

Pb	λ	τ	g^*
$T = 0$K	$3.90{\cdot}10^{-8}$ m	$1.5{\cdot}10^{-15}$s	$1{\cdot}10^{-17}$
$T = 4.20$K	$4.3{\cdot}10^{-8}$ m	$1.8{\cdot}10^{-15}$s	$1.4{\cdot}10^{-17}$
$T = 6.26$K	$7.8{\cdot}10^{-8}$ m	$6.1{\cdot}10^{-15}$s	$8.2{\cdot}10^{-17}$
$T = 7.10$K	$2.3{\cdot}10^{-7}$ m	$5.3{\cdot}10^{-14}$s	$2.2{\cdot}10^{-15}$

It is clearly seen that λ and τ grow with the temperature, so that one could think that the effect is maximum when the temperature is very close to T_c. However, this is true only for low-T_c superconductors because in high-T_c superconductors (HTSC) fluctuations are of primary importance for some Kelvin around T_c. The presence of these opposite contributions makes it possible that a temperature $T_{\max} < T_c$ exist, at which the effect is maximum.

In all cases, the time constant T_{int} is very small, and this makes the experimental observation rather difficult. Here I suggest to use pulsed magnetic fields to destroy and restore the superconductivity within a time interval of the order of T_{int}.

The main conclusion of this work is that the reduction of the gravitational field in a superconductor, if it exists, is a transient phenomenon and depends strongly on the parameters that characterize the superconductor.

Note that in this paper I have used a very simplified model. For a more realistic description, one should take into account some features of real superconductors, for example:

1. The symmetry of the order parameter, which in HTCS can be different from a pure s-wave [19];
2. The fact that the relaxation constant η can be complex [20];
3. The high anisotropy and layered structure of HTCS [21];
4. The effect of superconducting fluctuations, which is very large in HTCS [22].

Finally, since in the general solution (eq. 33) time and space derivative of the order parameter are present, I suggest that this effect could be enhanced:

1. by the presence of impurities [23];
2. by using quickly variable (pulsed) magnetic fields [24];
3. by making the superconductor quickly rotate [25];
4. by using constant or time-dependent electric fields [26];
5. by manufacturing a superconductor made of layers of different materials, or different phases (with different T_c) of the same material [27].

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FIGURE CAPTIONS

Fig. 1 **(a)** The gravitational field E_g/g^* as a function of the normalized time and space for YBCO at $T = 77$ K; **(b)** the gravitational field as a function of the normalized time for increasing values of x: $x = 5 \cdot 10^{-3}\lambda$, $x = 10^{-2}\lambda$ and $x = \lambda$.

Fig. 2 **(a)** The gravitational field E_g/g^* as a function of the normalized time and space for Pb at $T = 6.3$ K; **(b)** the gravitational field as a function of the normalized time for increasing values of x: $x = \lambda$, $x = 2\lambda$ and $x = 10\lambda$.

Fig. 3 The gravitational field E_g/g^* as a function of the normalized time in the case of Pb, with L=6, 8 and 10 and x=4. The maximum of the effect is evident.

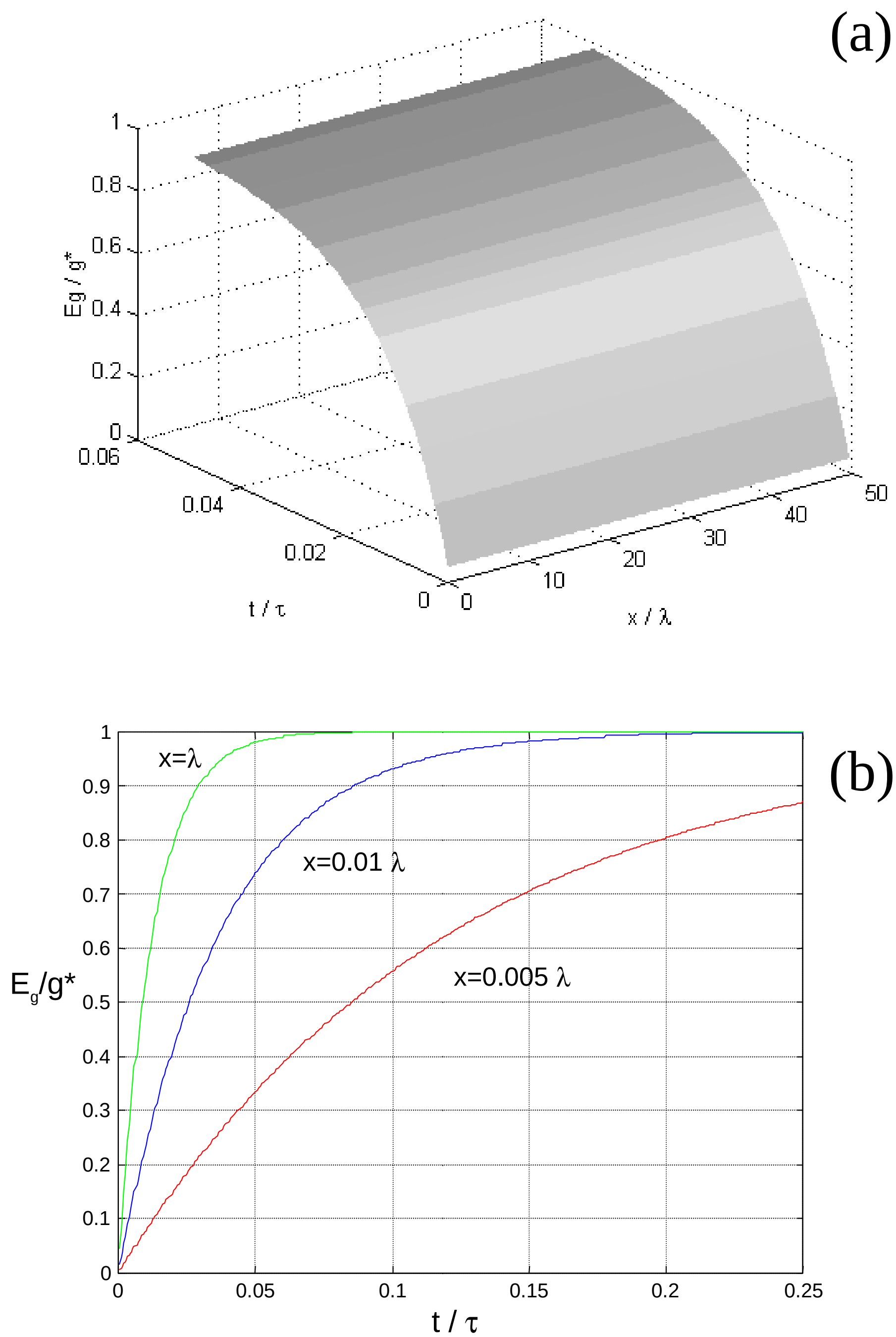

FIG. 1: G. A. Ummarino, *Possible alterations of the gravitational field in a superconductor*

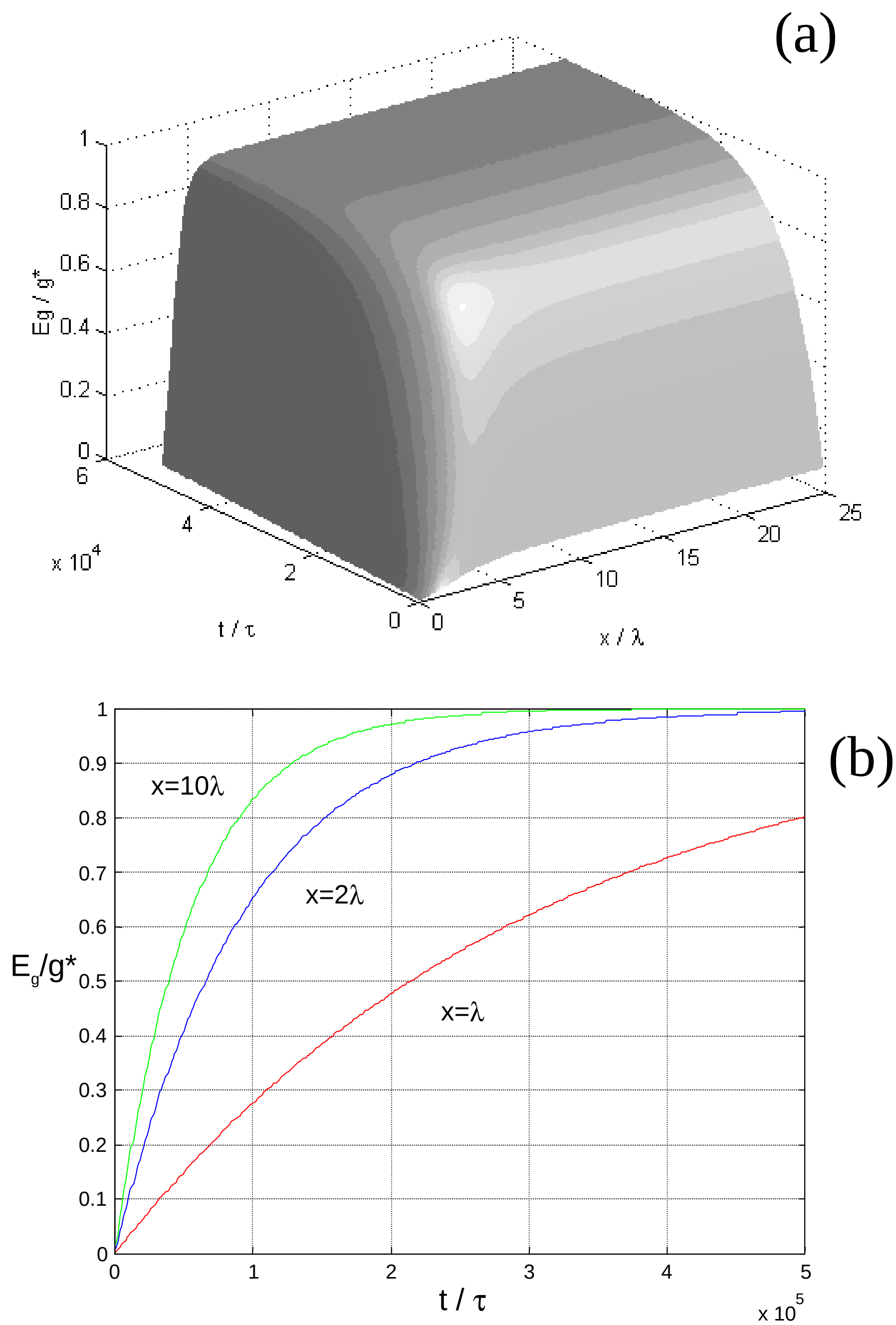

FIG. 2: G. A. Ummarino, *Possible alterations of the gravitational field in a superconductor*

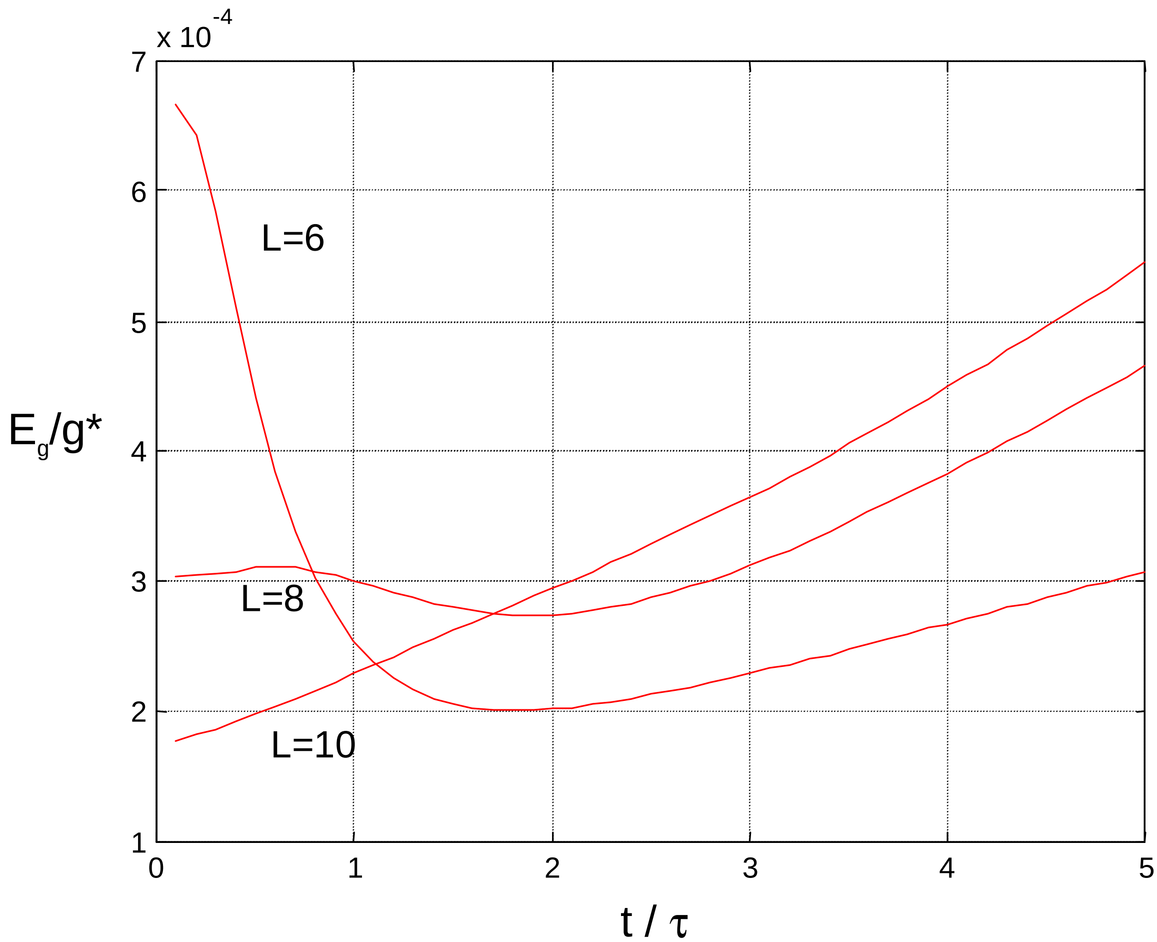

FIG. 3: G. A. Ummarino, *Possible alterations of the gravitational field in a superconductor*