Velocity derivative statistics in shearless turbulence

Euromech colloquium 501

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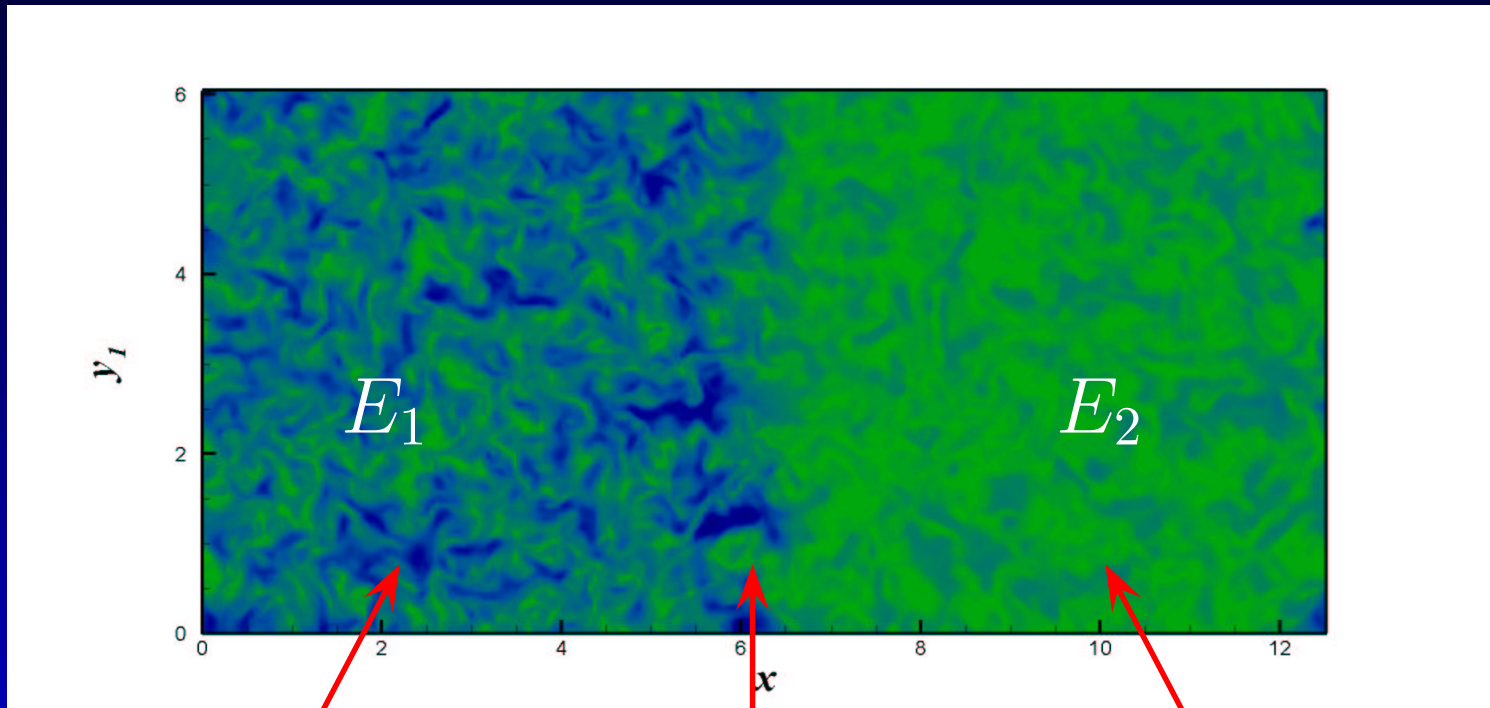*Dipartimento di Ingegneria Aeronautica e Spaziale*

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Turbulent shearless mixing

Ref: *J. Fluid Mech.* **549**, 441-451, (2006).

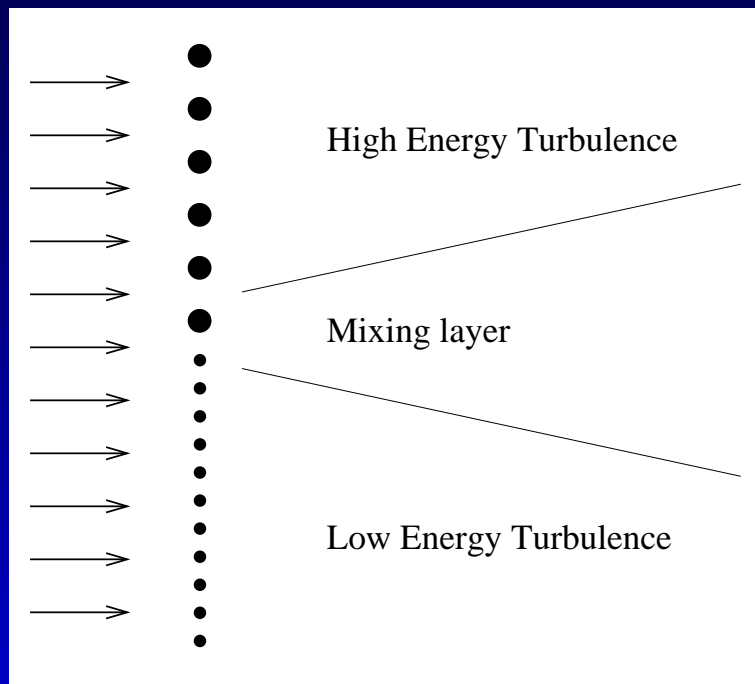
Run Movie 1-2

1-High energy turbulence   2-Low energy turbulence
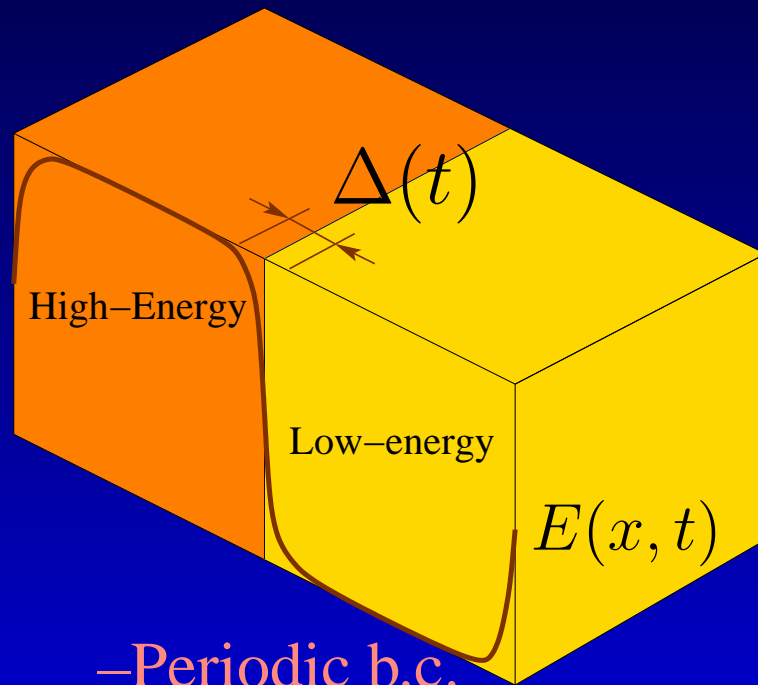
Mixing layer

State of the art

● Grid turbulence experiments:
  ► Gilbert *JFM* 1980
  ► Veeravalli-Warhaft *JFM* 1989

High Energy Turbulence

Mixing layer

Low Energy Turbulence

State of the art

High–Energy

$\Delta(t)$

Low–energy

$E(x,t)$

–Periodic b.c.

–Temporal decay

- Grid turbulence experiments:
  - ▶ Gilbert *JFM* 1980
  - ▶ Veeravalli-Warhaft *JFM* 1989

- Numerical experiments:
  - ▶ Briggs *et al.* *JFM* 1996
  - ▶ Knaepen *et al.* *JFM* 2004
  - ▶ Tordella-Iovieno *JFM* 2006
  - ▶ Iovieno-Tordella-Bailey *PRE* 2008)

Main features

- Self-similar stage of decay

- $\overline{u_1^2} \approx \overline{u_2^2} \approx \overline{u_3^2}$

- High intermittency, function of:

  ▶ gradient of turbulent kinetic energy $\Rightarrow \mathcal{E} = E_1/E_2$

  ▶ gradient of integral scale $\Rightarrow \mathcal{L} = \ell_1/\ell_2$

Method

- We consider two kinds of shearless mixings:
  – with a *uniform scale*: the mixing is generated by the gradient of energy only $\Rightarrow \mathcal{E}$ only parameter ($\mathcal{L} = 1$, $\mathcal{E} \neq 1$)
  – with a *scale* and *energy* gradient: $\Rightarrow \mathcal{E} \neq 1$ and $\mathcal{L} \neq 1$

- The behaviour of the mixing in the limit of very high energy ratio $\mathcal{E}$ is investigated.

- Method: DNS, parallelepiped domain ($2\pi \times 2\pi \times 2n\pi$, $n = 2$ and $n = 4$), $Re_\lambda = 45$ and 71, LES, same domain and $Re_\lambda = 44 - 450$

Velocity statistics

- No shear $\Rightarrow$ no kinetic energy production,

- The mixing is intermittent even if no gradient of scale is present (*PRE* 2008)

- Intermittency is (*JFM* 2006)

  - ▶ *ENHANCED* if the energy gradient is concurrent with the integral scale gradient ($\mathcal{L} > 1$)

  - ▶ *REDUCED* if the energy gradient is opposite to the integral scale gradient ($\mathcal{L} < 1$)

- Self-similar stage of evolution

Intermittency - $\mathcal{E} = 6.7$

$$S = \overline{u^3}/\overline{u^2}^{3/2} \qquad\qquad K = \overline{u^4}/\overline{u^2}^2$$
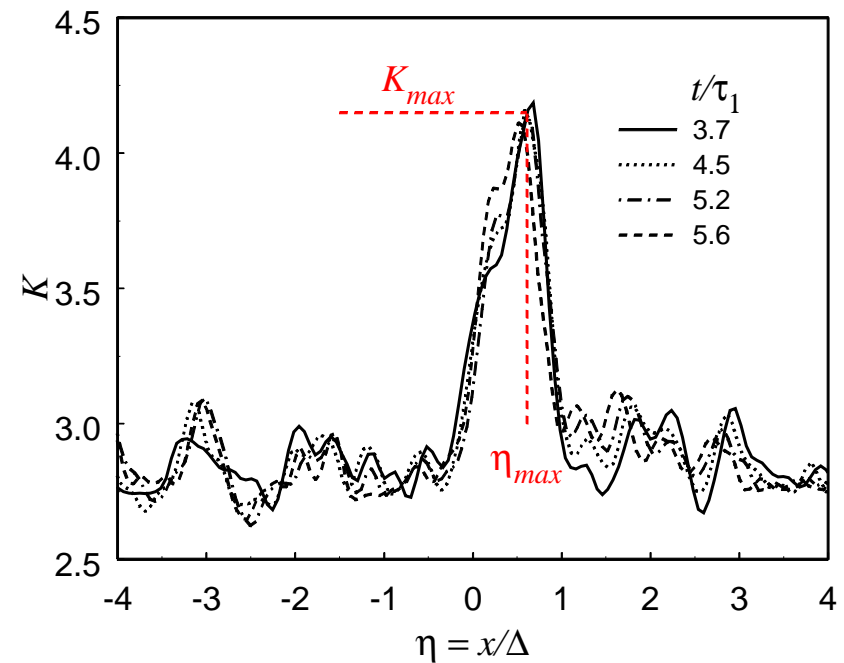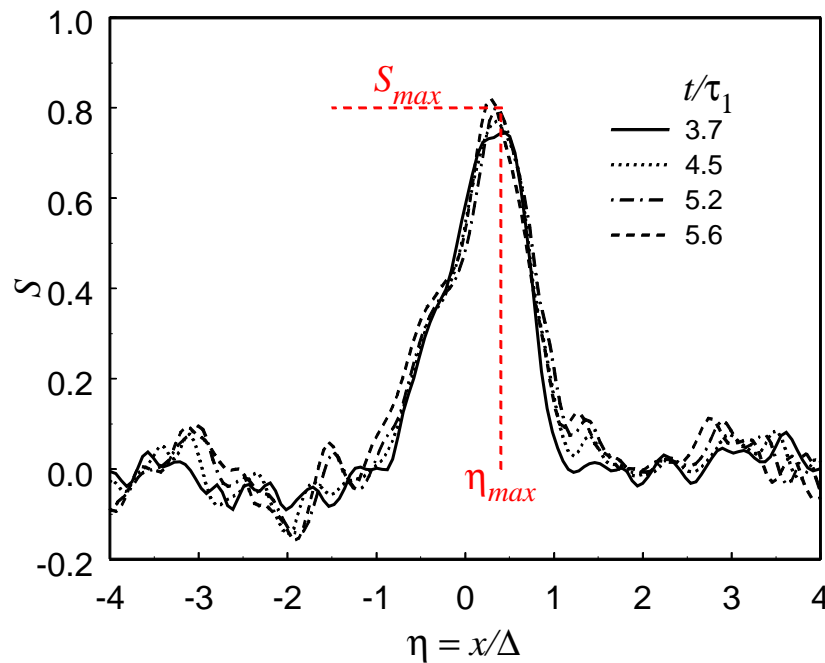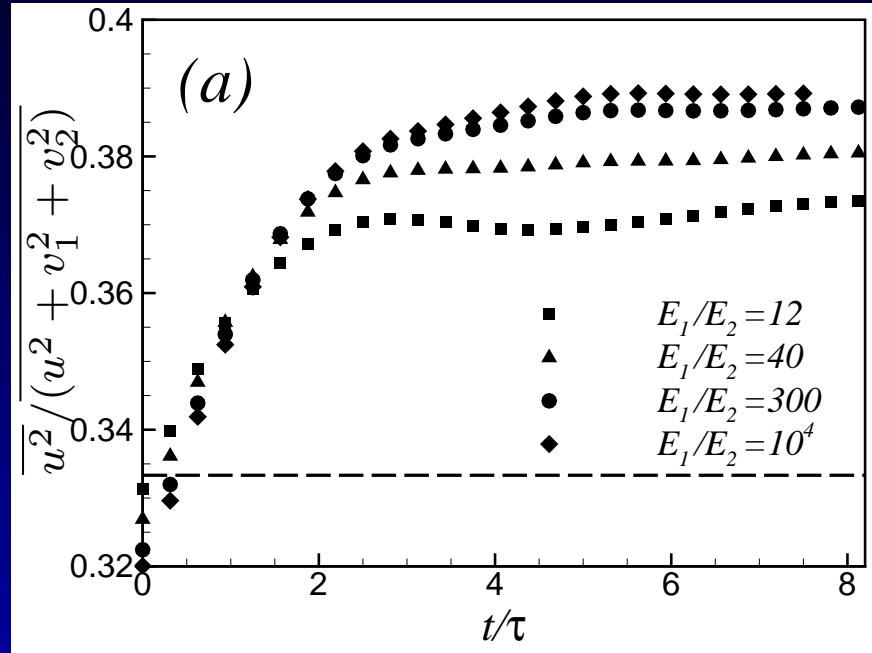

$S_{max}, \quad K_{max}$ = maximum of Skewness and Kurtosis in the mixing layer

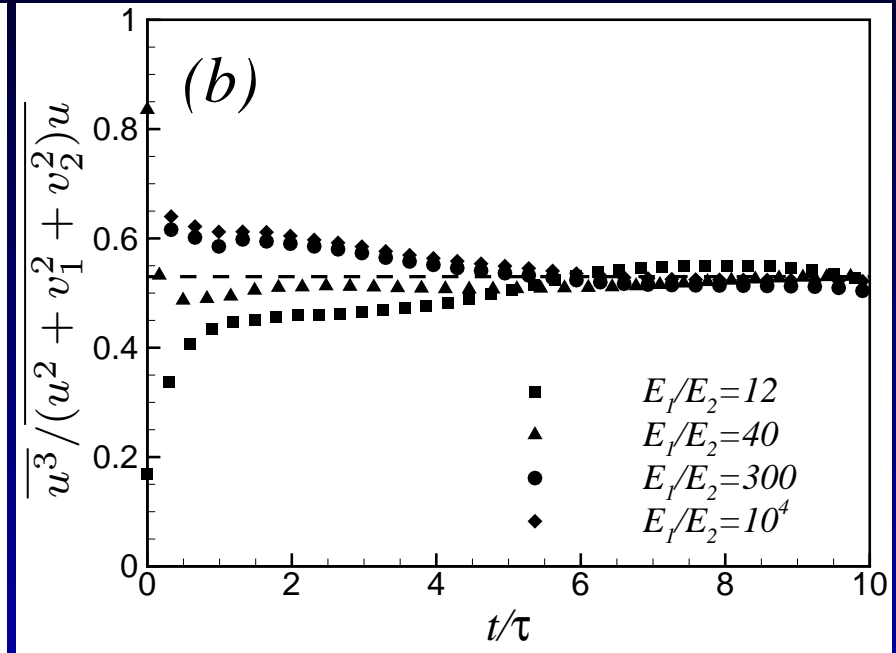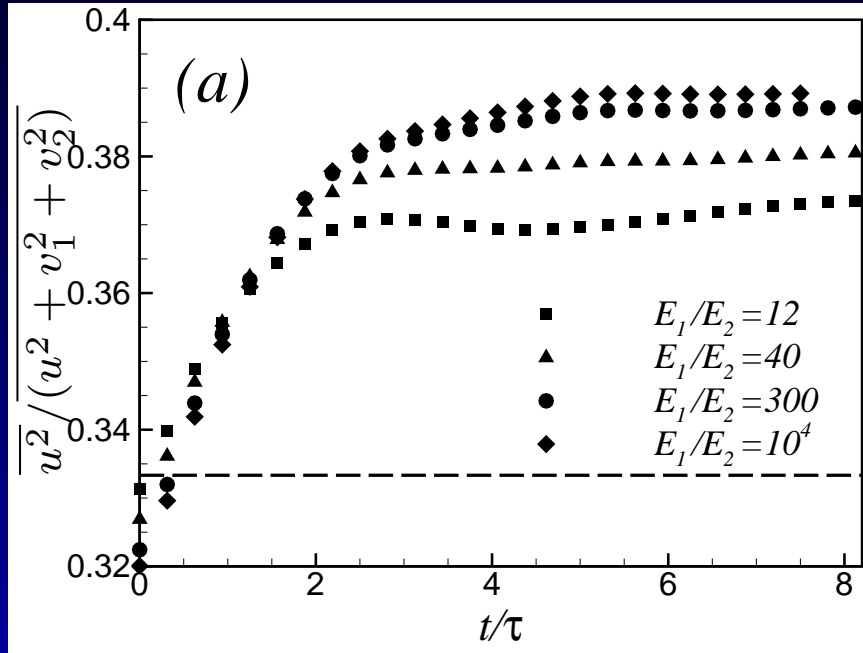η_{max} = position of the maximum in the mixing layer

Anisotropy

Left: second order velocity moment anisotropy
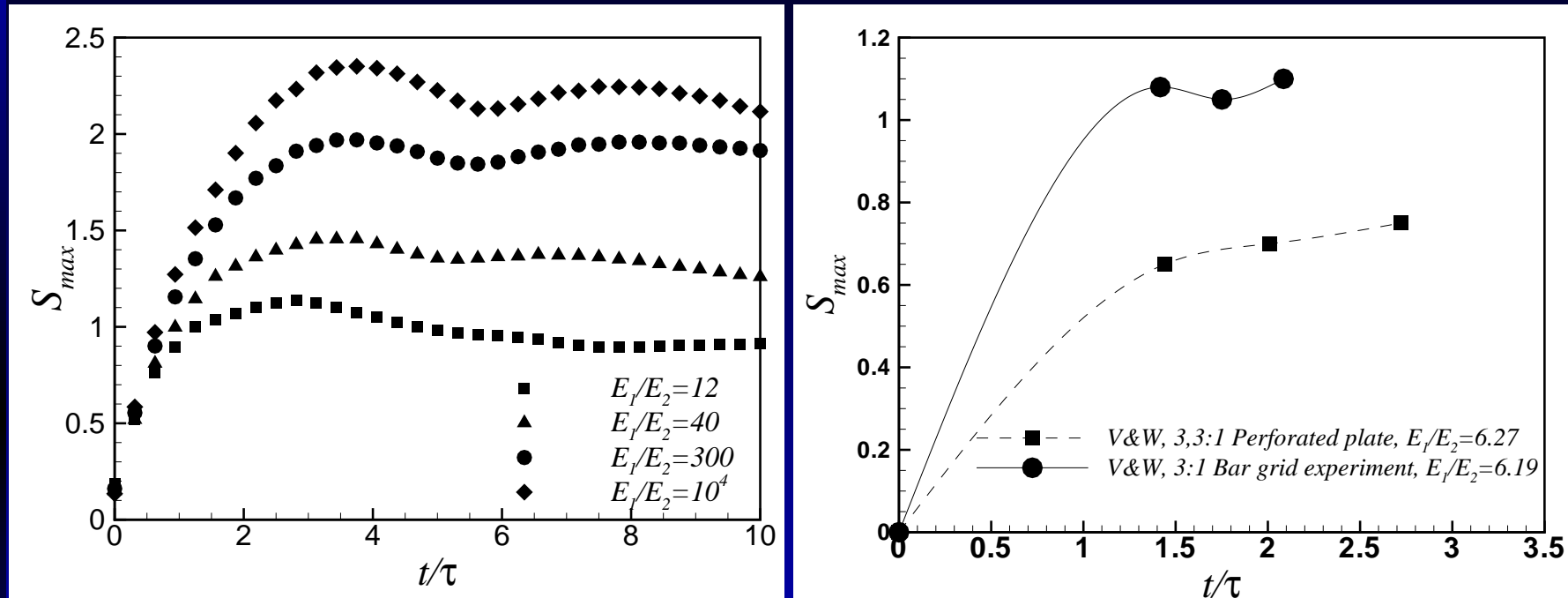
Anisotropy

Left: second order velocity moment anisotropy
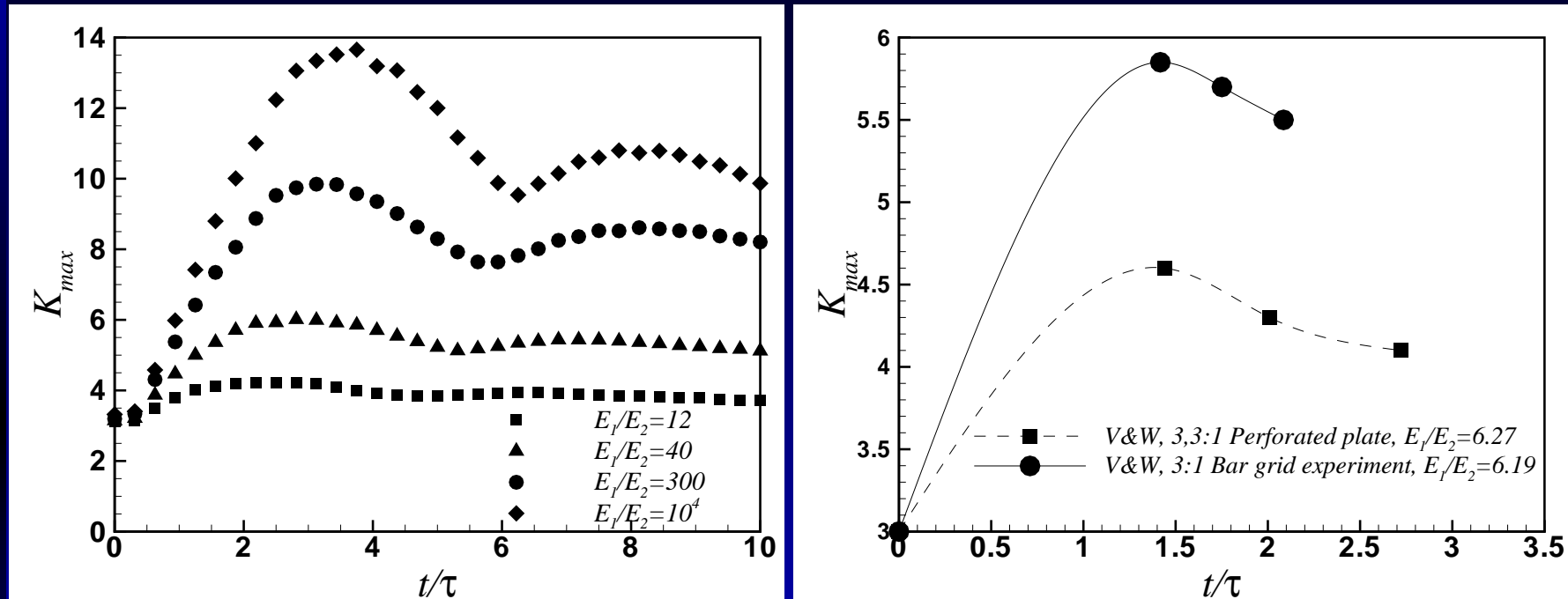Right: third order velocity moment anisotropy

Inhomogeneity: Skewness evolution

Time evolution of the maximum of skewness within the mixing.

Inhomogeneity: Kurtosis evolution

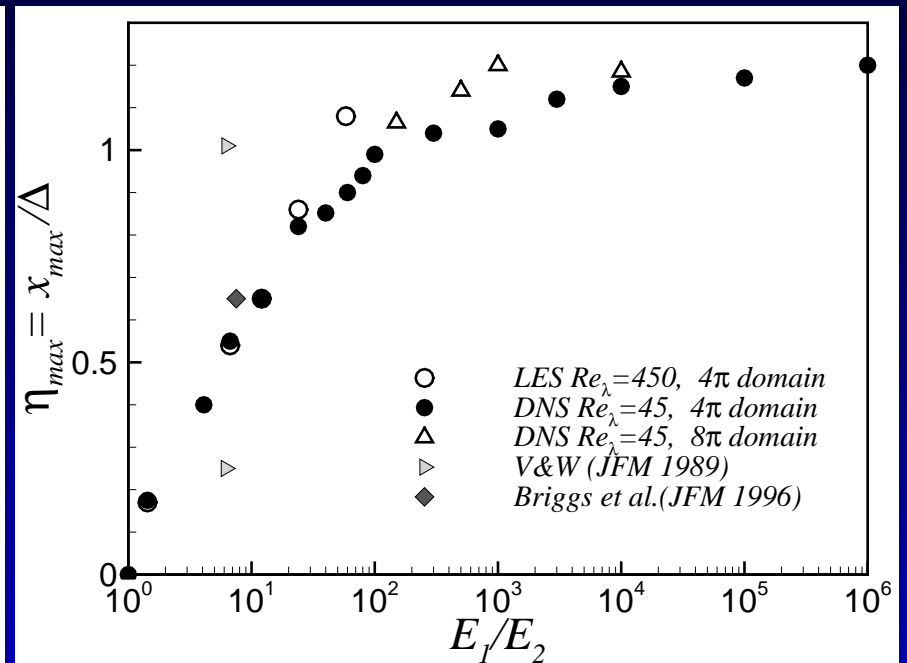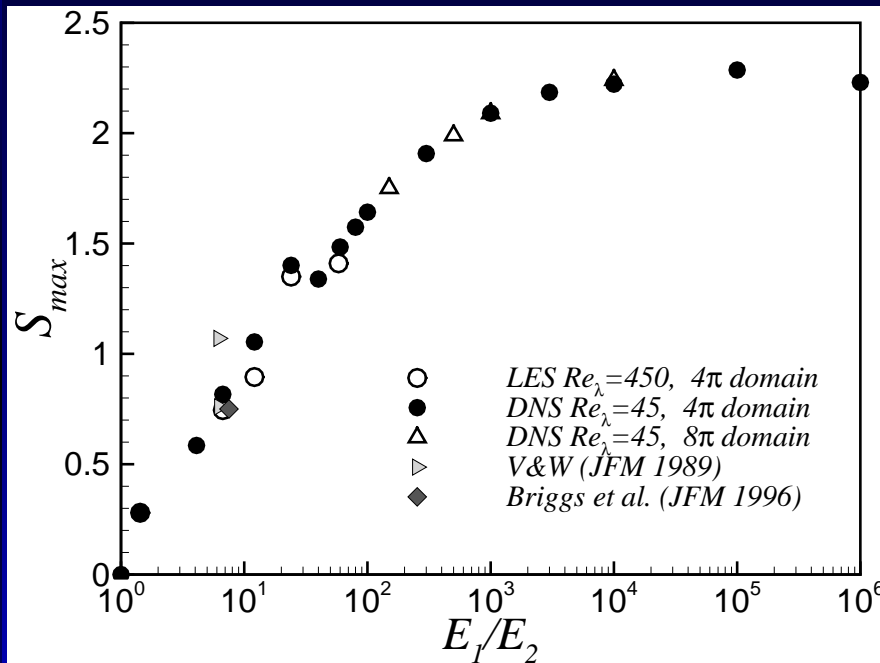

Time evolution of the maximum of kurtosis within the mixing.

Asymptote for $E_1/E_2 \to \infty$

Skewness and penetration

Asymptote for $E_1/E_2 \rightarrow \infty$

Kurtosis
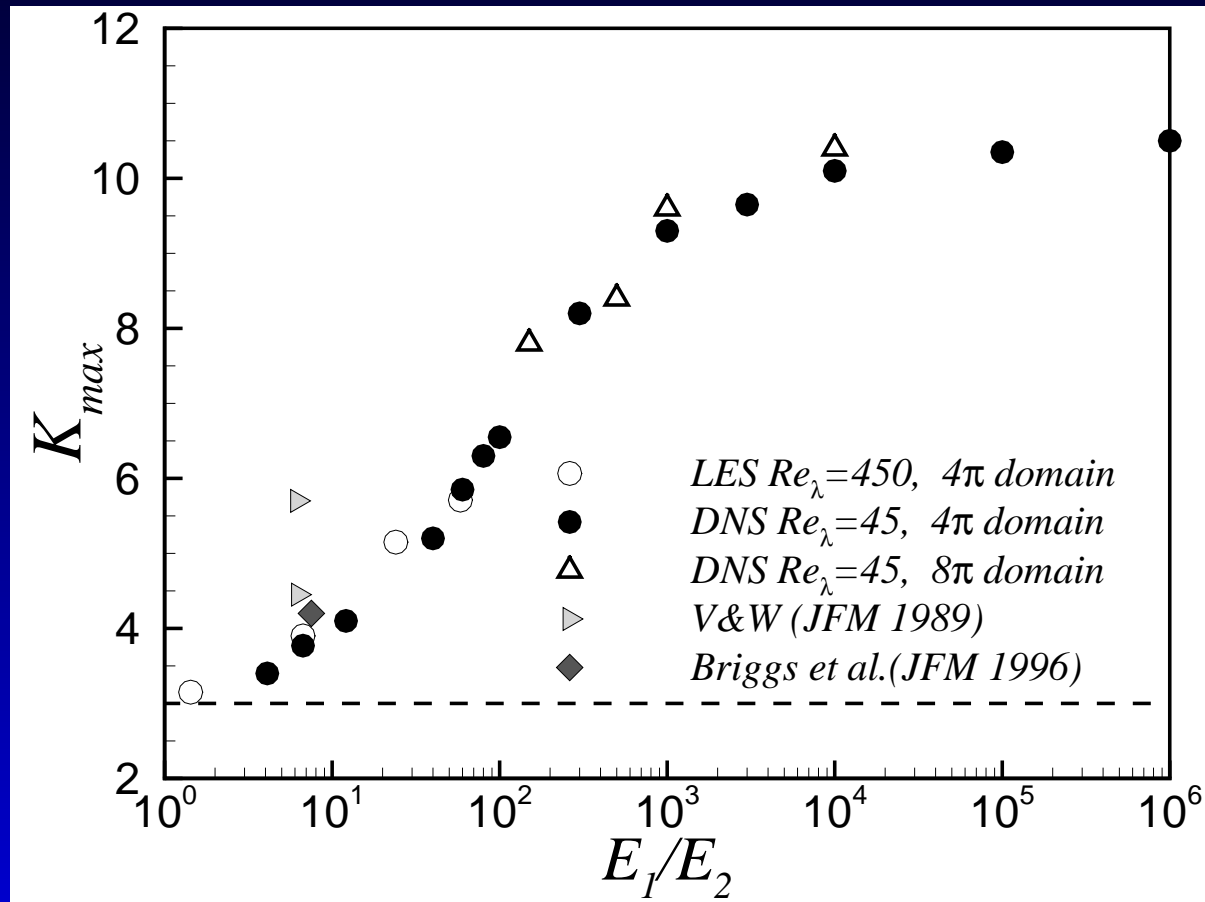
Longitudinal derivative moments distribution

$E_1/E_2 = 6.7$, $\ell_1/\ell_2 = 1$
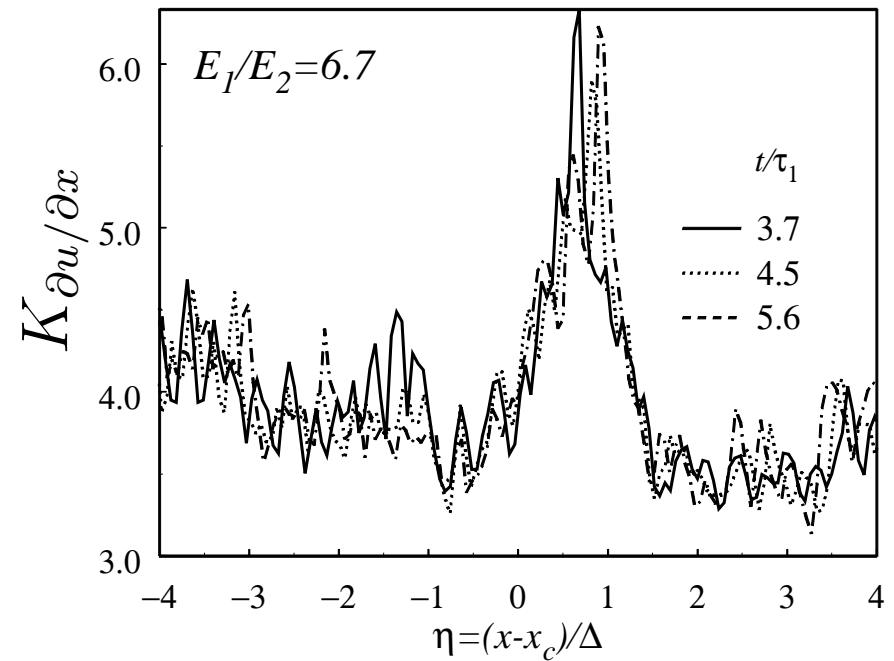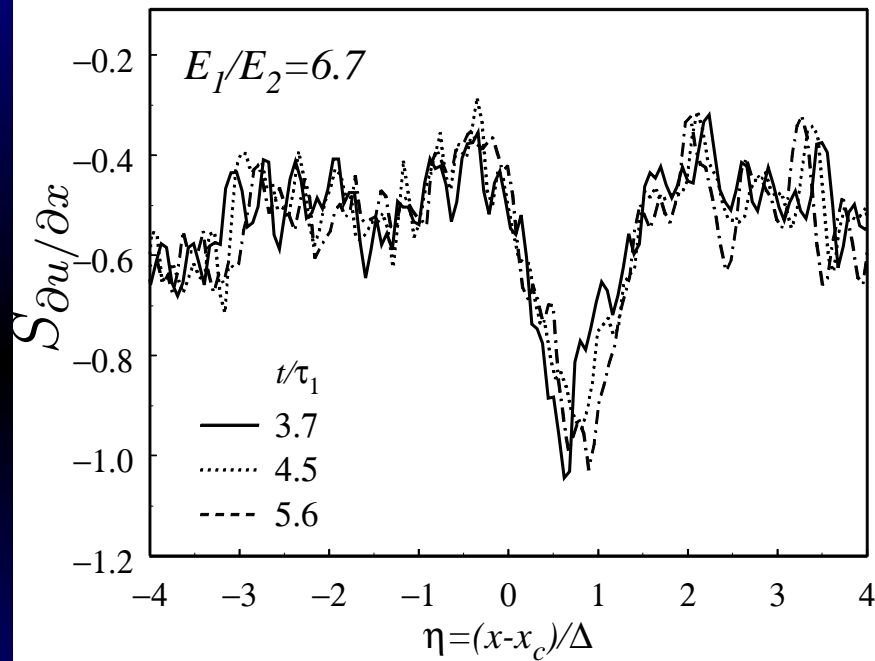
η is the dimensionless coordinate along the mixing
Δ is the mixing half-width

Mixing: Longitudinal derivatives

Time evolution of the peak of the derivative statistics
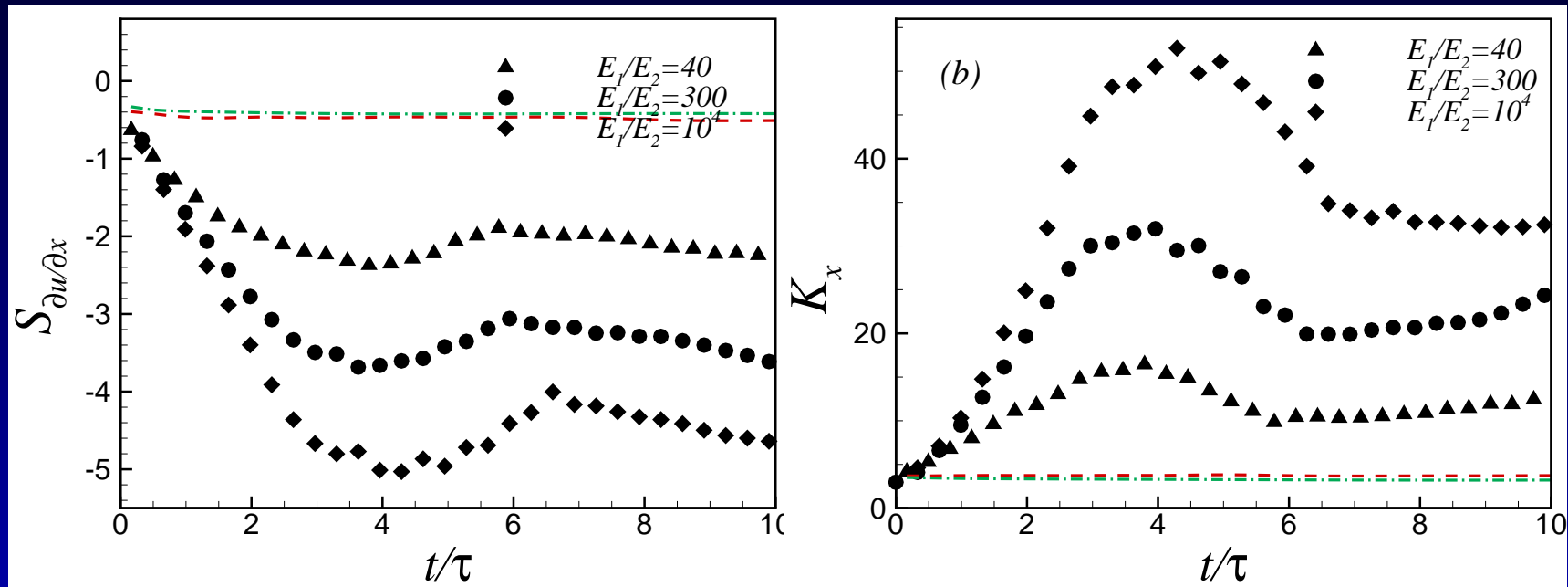
u = velocity component in the mixing direction
x = direction along the mixing
τ = initial eddy turnover time
$----$ high energy homogeneous region
$-\cdot-\cdot-$ low energy homogeneous region
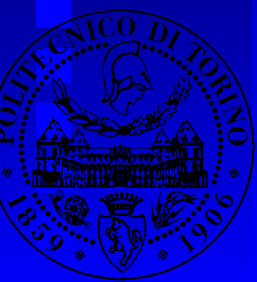
Asymptote for $E_1/E_2 \to +\infty$

Velocity Skewness
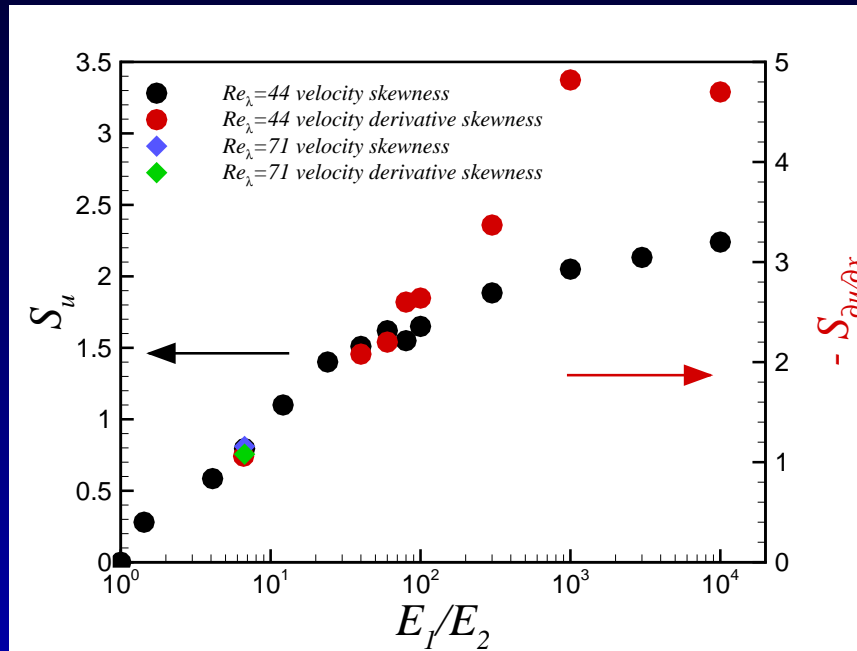
Velocity Kurtosis
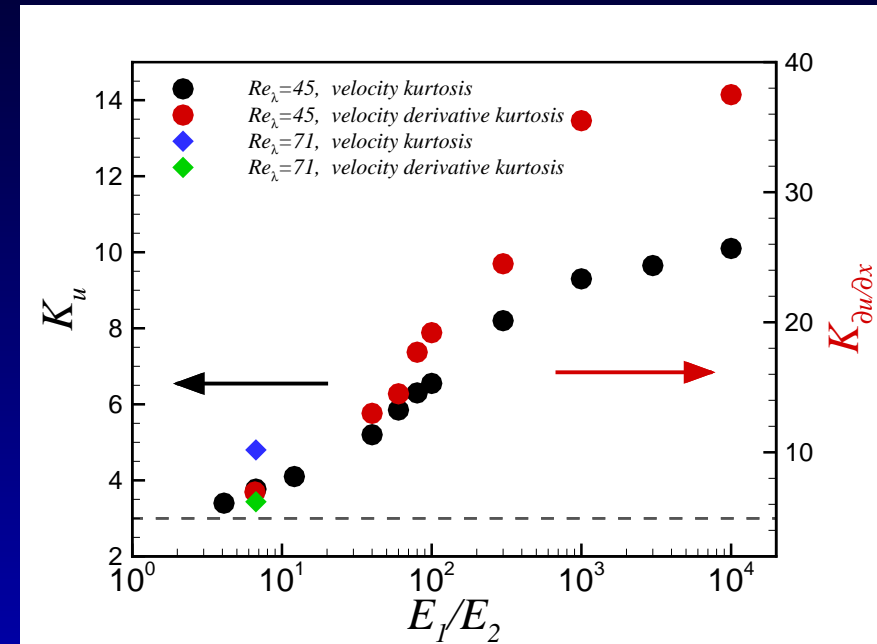
Longitudinal derivative Skewness and Kurtosis of the velocity component along the mixing direction

Comparison with homogeneous turbulence

Comparison of longitudinal moments inside the mixing with logitudinal moments in homogeneous and isotropic turbulence (data from Sreenivasan and Antonia, *Ann.Rev.Fluid Mech* 1997)
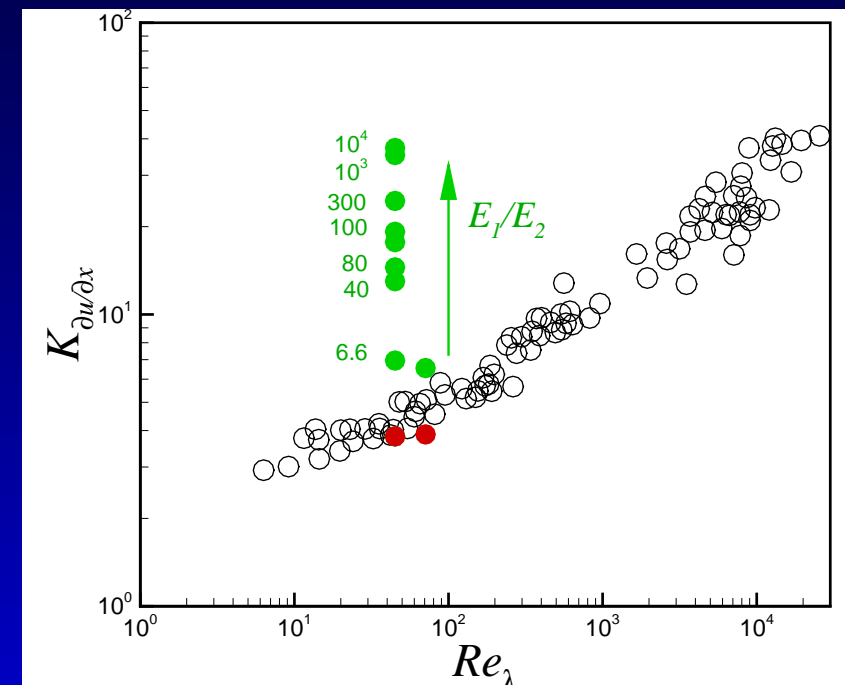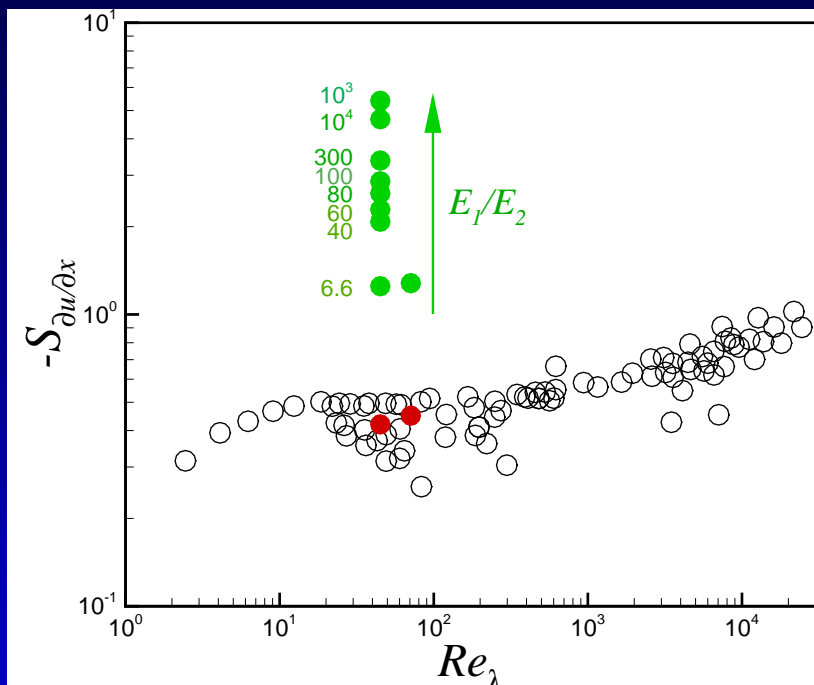
- ● *HIT, present simulations*
- ● *Shearless mixings, present simulations*
- ○ *Homogeneous turbulence*

Comparison with homogeneous turbulence (II)
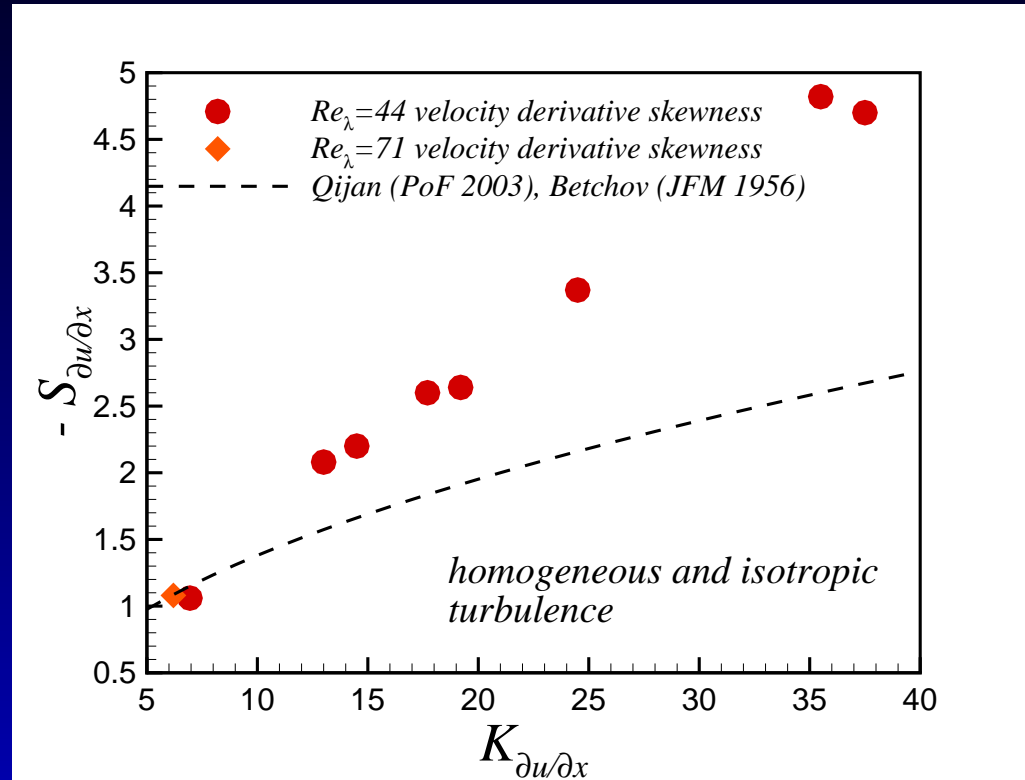
Comparison with the upper bound

$$-S_{\partial u/\partial x} \leq 2\left(\frac{K_{\partial u/\partial x}}{21}\right)^{\frac{1}{2}}$$

of the longitudinal skewness in homogeneous turbulence

Mixing: Transversal derivatives

Time evolution of the peak of the derivative statistics
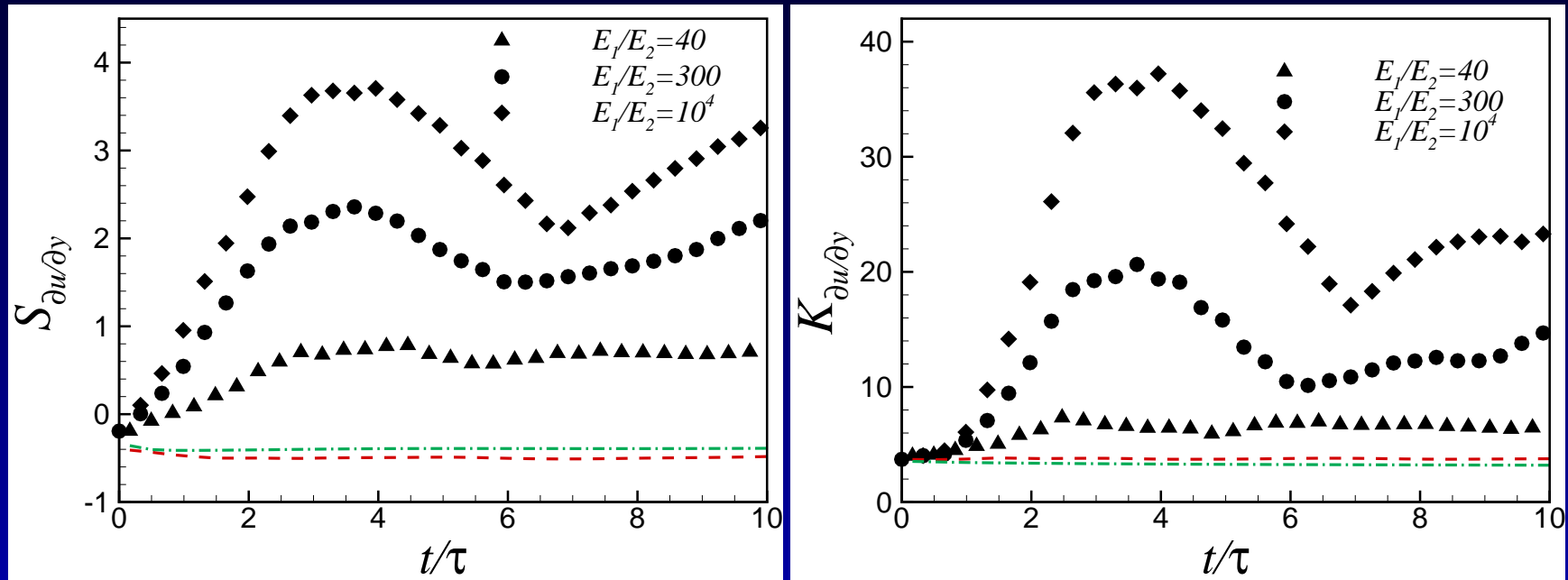
u = velocity component in the mixing direction
y = direction normal to the mixing
τ = initial eddy turnover time
$- - - -$ high energy homogeneous region
$- \cdot - \cdot -$ low energy homogeneous region

Small scale anisotropy: uniform integral scale
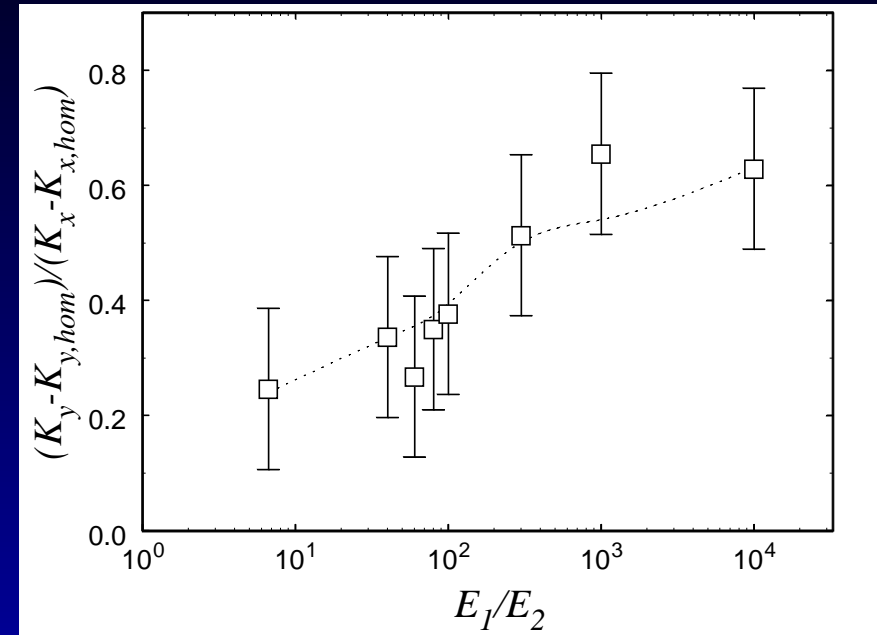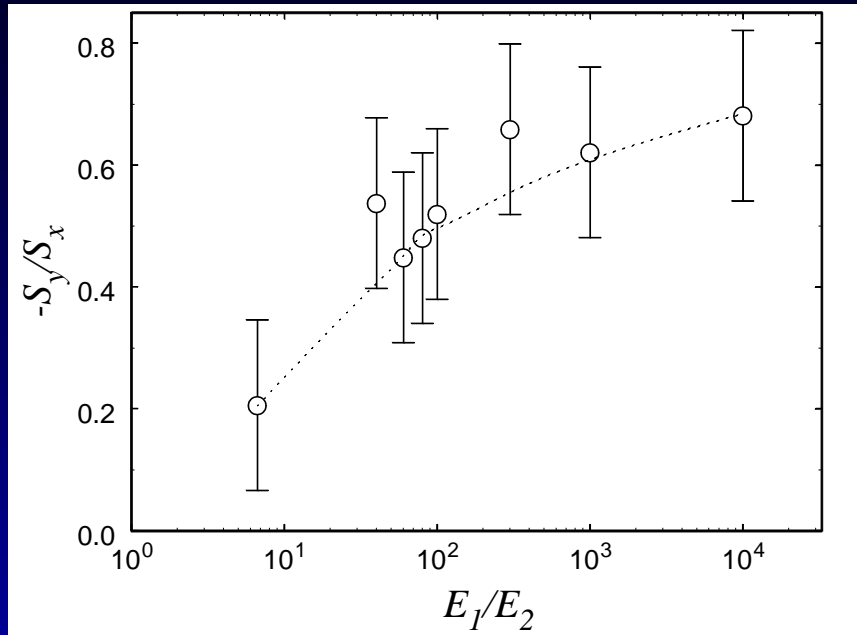
(a) isotropy of the derivative skewness;

(b) isotropy of the derivative kurtosis.
The bars represent the amplitude of the fluctuations of these ratios.

Comparison with homogenous shear turbulence
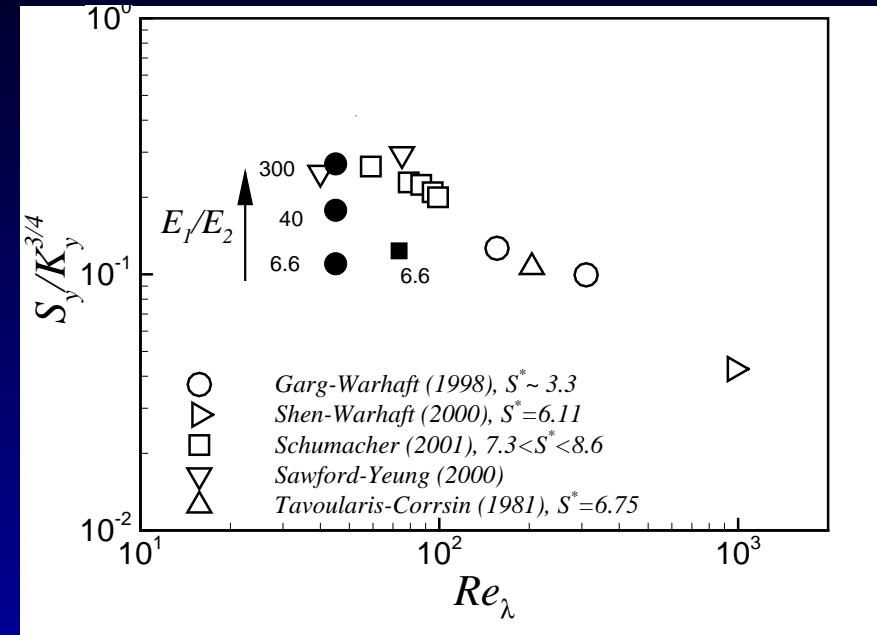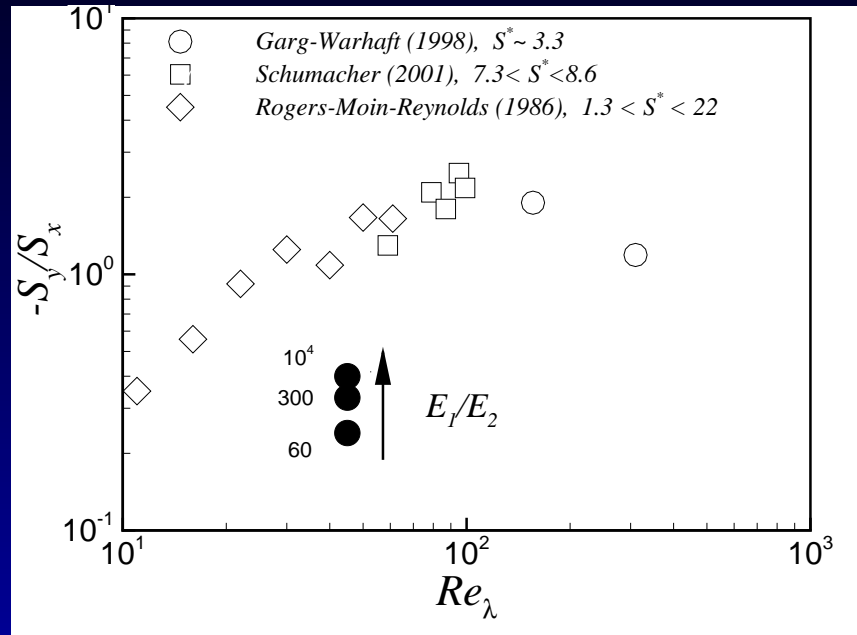
Comparison of anisotropy and intermittency:

– shearless mixing data, $\bullet\ Re_\lambda = 45,\ \square\ Re_\lambda = 71$

– empty symbols: homogeneous shear flows data.

Derivative statistics: non uniform integral scale
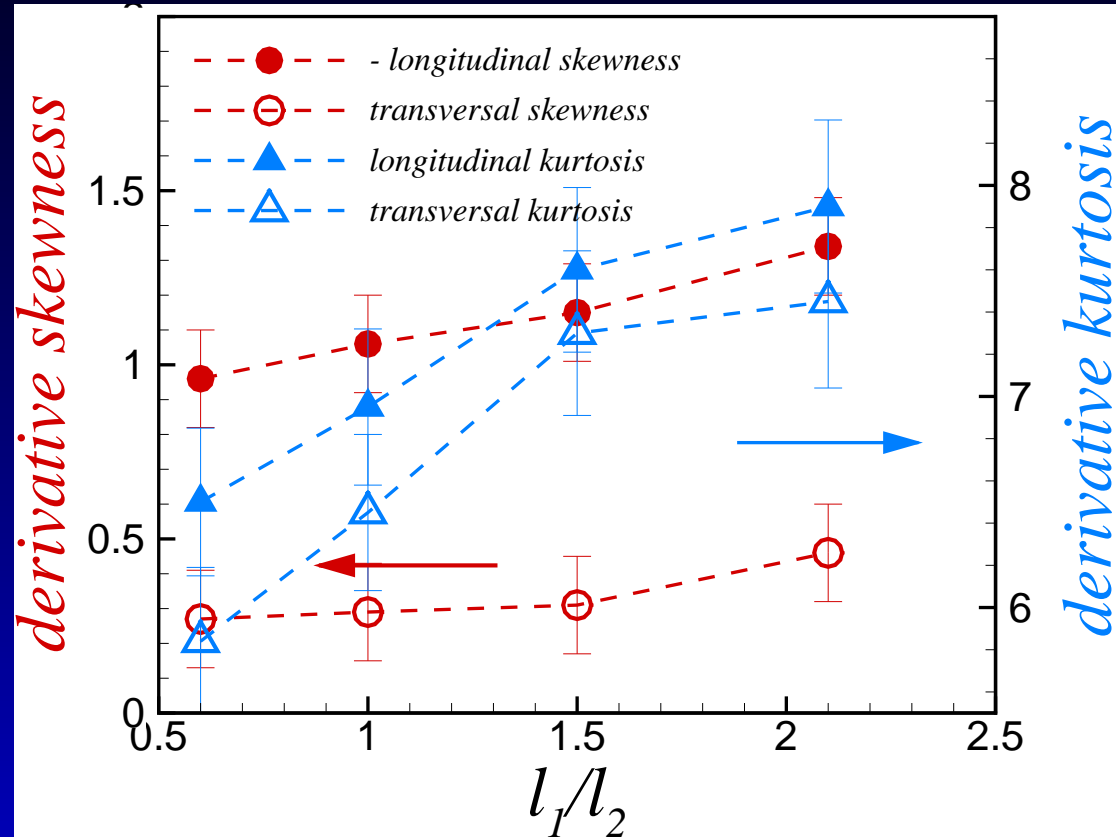
Conclusions

Direct numerical simulations carried over a large range of turbulent energy ratios ($6 \leq \mathcal{E} \leq 10^4$) show:

- a high level of intermittency, comparable with that of shear flows, on longitudinal and transversal derivative moments

- anisotropy of third and fourth order derivative moments increasing with the energy ratio E_1/E_2.

- a minor influence of the gradient of integral scale on the derivative statistics (small scales) than on the velocity statistics (large and inertial scales)
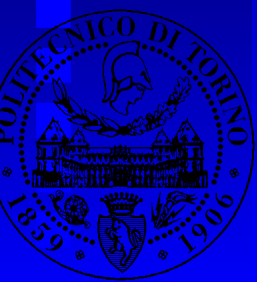
Conclusions

Direct numerical simulations carried over a large range of turbulent energy ratios ($6 \leq \mathcal{E} \leq 10^4$) show:

- a high level of intermittency, comparable with that of shear flows, on longitudinal and transversal derivative moments

- anisotropy of third and fourth order derivative moments increasing with the energy ratio E_1/E_2.

- *Future work*

- higher Reynolds number

- study of the mixing where the initial nonhomogeneity is associated to the integral scale variation only

Appendix: Shearless mixing statistics

	$\overline{u_i u_i u_3}/\overline{u_3^2}^{3/2}$			Velocity Kurtosis			Long. derivative skewness			Long. derivative kurtosis			Trans. Moments	
	i=1	i=2	i=3	K_{u_1}	K_{u_2}	K_{u_3}	$S_{\partial_1 u_1}$	$S_{\partial_2 u_2}$	$S_{\partial_3 u_3}$	$K_{\partial_1 u_1}$	$K_{\partial_2 u_2}$	$K_{\partial_3 u_3}$	$S_{\partial_1 u_3}$	$K_{\partial_1 u_3}$
E_1/E_2 :	Mixings with $\ell_1/\ell_2 = 1$, $R_\lambda = 45$													
6.6	0.34	0.36	0.82	3.6	3.4	4.07	-0.11	-0.10	-1.06	5.0	4.85	6.95	0.29	6.55
40	0.54	0.59	1.34	5.6	6.0	5.56	0.55	0.80	-2.08	7.1	6.77	13.0	0.50	8.50
80	0.63	0.69	1.57	6.4	6.7	6.67	0.95	1.1	-2.60	8.5	8.6	17.7	0.60	13.0
300	0.78	0.87	1.91	7.5	8.2	8.93	1.5	2.0	-3.37	16	14	24.5	1.0	13.6
10e4	0.92	0.95	2.20	7.8	8.2	11.6	3.4	3.2	-4.70	20	26	37.3	1.05	23.1
ℓ_1/ℓ_2 :	Mixings with $E_1/E_2 \approx 6.6$, $R_\lambda = 45$													
0.6	0.32	0.33	0.77	3.45	3.4	3.85	-0.14	-0.10	-0.94	5.0	4.8	6.50	0.27	5.90
6.6	0.34	0.36	0.82	3.6	3.4	4.07	-0.11	-0.10	-1.06	5.0	4.85	6.95	0.29	6.55
1.5	0.49	0.52	1.20	4.0	3.6	5.5	-0.12	-0.08	-1.14	5.2	4.9	7.60	0.31	7.70
2.1	0.55	0.58	1.40	4.9	4.4	7.5	-0.05	-0.02	-1.34	5.2	5.0	7.80	0.46	8.22
	Mixings with $E_1/E_2 = 6.7$ $\ell_1/\ell_2 = 1$, $R_\lambda = 71$													
	0.42	0.37	0.81	3.65	3.55	4.8	-0.15	-0.19	-1.08	4.45	4.65	6.20	0.30	5.95
	Veeravalli and Warhaft(1989), $E_1/E_2 \approx 7$, $\ell_1/\ell_2 \approx 1.5 \div 1.7$													
bars	==	==	1.06	4.36	4.23	5.53	==	==	==	==	==	==	=	=
plate	==	==	0.63	3.47	3.49	4.07	==	==	==	==	==	==	=	=

Legend:

3 = inhomogeneous direction

1,2 = homogeneous directions

S_{u_i}, K_{u_i} = skewness and kurtosis of u_i

$S_{\partial_j u_i}$, $K_{\partial_j u_i}$ = skewness and kurtosis of $\partial u_i/\partial x_j$

Appendix: homogeneous shear turbulence

Homogeneous shear turbulence						Shearless mixings						
Re_λ	S^*	Sd-lg	Kd-lg	Sd-tr	Kd-tr	Re_λ	E_1/E_2	ℓ_1/ℓ_2	Sd-lg	Kd-lg	Sd-tr	Kd-tr
Garg-Warhaft 1998												
156	3.39	-0.42	7.1	0.8	11.7							
310	3.31	-0.42	6.9	0.5	8.6							
Shen-Warhaft 2000												
974	6.11	=	=	0.24	*10							
Gylfason-Warhaft 2004												
452	=	-0.546	9.94	=	=							
877	=	-0.586	13.1	=	=							
Schumacher 2001												
59	7.32	-0.74	6.3	0.96	5.6	45	6.6	1.0	-1.06	6.95	0.29	6.55
79	8.18	-0.44	4.8	0.92	6.4	71	6.6	1.0	-1.08	6.25	0.30	5.95
87	8.58	-0.50	5.3	0.90	6.4	45	40	1.0	-2.08	13.0	0.50	8.50
95	8.22	-0.37	4.3	0.92	7.2	45	300	1.0	-2.60	17.7	0.60	13.0
99	8.37	-0.40	5.0	0.87	7.1	45	10^4	1.0	-3.37	24.5	1.0	13.6
Rogers-Moin-Reynolds 1986												
11	1.3	-1.2	=	0.42	=	45	6.5	0.6	-0.94	6.50	0.27	5.90
16	3.3	-0.95	=	0.53	=	45	6.7	1.5	-1.24	7.60	0.31	7.70
22	6.2	-0.61	=	0.56	=	45	6.7	2.1	-1.34	7.80	0.46	8.22
30	10	-0.48	=	0.60	=							
40	14	-0.58	=	0.63	=							
50	19	-0.42	=	0.70	=							
61	22	-0.43	=	0.71	=							
Sawford-Yeung 2000												
40	=	=	=	0.92	*5.7							
75	=	=	=	1.20	*6.5							
Tavoularis-Corrsin 1981												
204	6.75	=	=	*0.6	*10							

Legend:

$$S^* = S \left\langle u^2 \right\rangle / \epsilon$$

dimensionless shear parameter

Sd = derivative skewness,

Kd = derivative kurtosis,

tr = transversal,

lg = longitudinal