Fair Traffic Relaying for Two-Source-One-Destination Wireless Networks

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Alessandro Nordio, Carla-Fabiana Chiasserini, Tamer ElBatt

***Abstract*—We propose a communication strategy for a three-node wireless network, where the relay nodes generate their own data besides decoding and forwarding other nodes messages. Unlike previous work, we consider that the nodes are arbitrarily located on a 2D plane, are equipped with half-duplex radios and require a fair rate allocation. We quantify the performance in terms of achievable rate as the SNR conditions, the network geometry and the nodes traffic demand vary, and compare it to the cut-set bound that we derive for the network under study. Furthermore, we show that our strategy outperforms that proposed in [2].**

I. Introduction

Cooperative relaying has received a growing interest in the literature since it is frequently encountered in a variety of wireless systems, e.g., cellular, ad hoc and sensor networks.

The basic three-node relay channel, where one source node transmits to a destination via an intermediate node (relay), has been studied in [1], where different coding strategies are defined under the asumptions of full-duplex nodes. More recently, the work in [2] introduces two cooperative protocols that encompass the ones previously proposed. The case of multi-source, multi-destination, multi-relay networks has been addressed in [3]. Unlike our study, all these works hinge on the rather strong assumption of full-duplex radios.

A network with half-duplex nodes (i.e., unable to transmit and receive simultaneously) has been studied in [4], [5], in the case of a diamond-shaped network topology. The protocols proposed there achieve rates close to the cut-set upper bound derived in [6]. Note, however, that the scenarios addressed in [2], [4]–[6] are fundamentally different from ours, since the relay nodes do not generate their own data. The achievable rate in half-duplex networks with two sources, one destination and multiple relays is studied in [7], but assuming a noise-free channel and single-hop transmissions only.

The cases of a three- and a four-node network where nodes can be both sources and relays have bee investigated in [8], [9], respectively. However, unlike our work and similar to the aforementioned information-theoretic approach, the study in [8] assumes full-duplex nodes. The study in [9], instead, deals with a half-duplex network where two sources can reach their destination only using the other source as a relay. The symmetry of such a scenario allows the authors to exploit the so-called broadcast channel with cognitive receiver, which cannot be applied to our case.

Our objective in this work is to revisit the three-node, decode-and-forward (DF) relay channel with four key differences with respect to previous work: i) each node can act as both a source and a relay, ii) nodes are half duplex, iii) they use the same frequency channel and the signal of both sources can reach the destination, possibly with an arbitrarily low power, iv) a fair rate allocation is required, i.e., nodes need to achieve different data rates according to their traffic demand. Considering this network scenario, we derive the cut-set upper bound on the achievable rates and propose a relaying strategy that closely approximates such a bound. We also show that our strategy outperforms the one presented in [2].

Fig. 1. The network under study (left) and the network cuts used for computing the bound (right).

II. System Model

We consider a network composed of three nodes, as depicted in Fig. 1 (left). We assume that the network lies on a plane where the positions of the nodes 1, 2, and D are given by $(-1/2, 0)$, $(+1/2, 0)$ and (z_1, z_2), respectively. Let d_{ij} be the distance between node i and node j, with $i, j \in \{1, 2, D\}$. We set $d_{12} = 1$, which yields: $d_{1D} = \sqrt{(z_1 + 1/2)^2 + z_2^2}$ and $d_{2D} = \sqrt{(z_1 - 1/2)^2 + z_2^2}$.

Nodes 1 and 2 are sources of data traffic to be delivered to the destination D. These nodes may also cooperate by relaying each other traffic toward D; in this case they adopt the DF relaying technique [1].

The nodes operate on the same frequency channel and transmit at the same power level. We assume free space propagation and that the received signal is corrupted by additive white Gaussian noise with the same variance at every receiver. For simplicity of notation, we define γ as the signal-to-noise (SNR) ratio observed at a receiving node located at distance $d = 1$ from the transmitter. It follows that the SNRs observed at D when nodes 1 and 2 transmit are given, respectively, by $\gamma_1 = \gamma/d_{1D}^2$ and $\gamma_2 = \gamma/d_{2D}^2$. In the following, we focus on the case where $\gamma_1 < \gamma_2$, i.e., $z_1 > 0$; the extension to the opposite case is however straightforward. Also, note that, when the distance d_{2D} is close to zero, γ_2 may become very large and the free space propagation model might not hold any longer.

Since nodes 1 and 2 operate in half-duplex mode, each of them has two operational states: transmit ($\mathfrak{t}$) and receive ($\mathfrak{r}$). D instead is always in receiving mode. We define the operational state of the network, $\boldsymbol{\sigma}$, as the vector of the states of nodes $1, 2, D$, respectively. Since we are interested in studying the nodes achievable rate, we only consider the following states:

$$\boldsymbol{\sigma}_1 = [\mathfrak{t}, \mathfrak{r}, \mathfrak{r}], \quad \boldsymbol{\sigma}_2 = [\mathfrak{r}, \mathfrak{t}, \mathfrak{r}], \quad \boldsymbol{\sigma}_3 = [\mathfrak{t}, \mathfrak{t}, \mathfrak{r}],$$

i.e., we neglect the state where all nodes are receiving as this would imply that no data transfer occurs in the network.

III. COOPERATIVE RELAYING STRATEGY

We propose a communication strategy for the network described in Sec. II. We describe the scheme by taking node 2 to act as a relay for node 1; the symmetric case where node 1 acts as a relay for node 2 can be easily derived from there. Instead, the case where nodes 1 and 2 relay each other's traffic would lead to a totally different analysis and is out of the scope of this work.

Since nodes operate in half-duplex mode, we consider a time-division approach where transmissions occur over a two-slot frame, with slots of equal duration. We assume that in Slot 1 the network is in state $\boldsymbol{\sigma}_1$, while in Slot 2 the network is in state $\boldsymbol{\sigma}_3$. In other words, according to the proposed scheme node 1 always transmits while node 2 receives in Slot 1 and transmits in Slot 2.

More precisely, assume that node 1 has two independent messages to send to D, denoted by W_{11} and W_{12}, respectively, while node 2 has a single message, W_2, (independent of W_{11} and W_{12}) to be delivered to D. The messages W_{11}, W_{12}, and W_2 are encoded into the complex signals[1] x_{11}, x_{12}, and x_2, by using codebooks of rate $\mathcal{R}_{11}$, $\mathcal{R}_{12}$, and $\mathcal{R}_2$, respectively. We assume that these signals have zero mean and unit variance. Then, our relaying strategy works as follows.

Slot 1. The network is in state $\boldsymbol{\sigma}_1$ and node 1 transmits x_{11}. Denoting the signal and the noise at the receiver $k \in \{2, D\}$ in slot $j \in \{1, 2\}$ by $y_k^{(j)}$ and $n_k^{(j)}$, respectively, we can write the signals received at node 2 and D as, $y_2^{(1)} = \sqrt{\gamma} x_{11} + n_2^{(1)}$ and $y_D^{(1)} = \sqrt{\gamma_1} x_{11} + n_D^{(1)}$. By processing $y_2^{(1)}$, node 2 can successfully decode the signal x_{11} and retrieve the message W_{11} if

$$\mathcal{R}_{11} \le \mathcal{C}(\gamma) \tag{1}$$

where the function $\mathcal{C}(\cdot)$ is defined as $\mathcal{C}(x) = \log_2(1 + x)$ and γ is the SNR associated to $y_2^{(1)}$.

Slot 2. The network is in state $\boldsymbol{\sigma}_3$: node 1 transmits a linear combination of the signals x_{11} and x_{12}, while node 2 transmits a linear combination of the signals x_{11} and x_2. Note that x_{11} is available at node 2 if the constraint in (1) is satisfied. Thus, the signal received at D is given by $y_D^{(2)} = \sqrt{\alpha\gamma_1} x_{11} + \sqrt{(1-\alpha)\gamma_1} x_{12} + \sqrt{\beta\gamma_2} x_{11} + \sqrt{(1-\beta)\gamma_2} x_2 + n_D^{(2)}$ where the parameters $\alpha, \beta \in [0, 1]$ represent the transmit power share that nodes 1 and 2, respectively, devote to the transmission of x_{11}.

The signals $y_D^{(1)}$ and $y_D^{(2)}$ can be rewritten as[2] $\mathbf{y}_D = \mathbf{H}\mathbf{x} + \mathbf{n}_D$, where $\mathbf{y}_D = [y_D^{(1)}, y_D^{(2)}]^\mathsf{T}$, $\mathbf{n}_D = [n_D^{(1)}, n_D^{(2)}]^\mathsf{T}$, $\mathbf{x} = [x_{11}, x_{12}, x_2]^\mathsf{T}$, and

$$\mathbf{H} = \begin{bmatrix} \sqrt{\gamma_1} & 0 & 0 \\ \sqrt{\beta\gamma_2} + \sqrt{\alpha\gamma_1} & \sqrt{(1-\alpha)\gamma_1} & \sqrt{(1-\beta)\gamma_2} \end{bmatrix}. \tag{2}$$

Hence, given z_1, z_2, α, and β, the instantaneous rates achievable by the source nodes are limited by the following constraints [1]:

$$\begin{aligned} \mathcal{R}_{11} &\le \mathcal{C}(\|\mathbf{h}_1\|^2) & \text{(3a)} \\ \mathcal{R}_{12} &\le \mathcal{C}(\|\mathbf{h}_2\|^2) & \text{(3b)} \\ \mathcal{R}_{2} &\le \mathcal{C}(\|\mathbf{h}_3\|^2) & \text{(3c)} \\ \mathcal{R}_{11} + \mathcal{R}_{12} &\le \mathcal{C}\left(\mathbf{H}_3\mathbf{H}_3{}^\mathsf{H}\right) & \text{(3d)} \\ \mathcal{R}_{11} + \mathcal{R}_{2} &\le \mathcal{C}\left(\mathbf{H}_2\mathbf{H}_2{}^\mathsf{H}\right) & \text{(3e)} \\ \mathcal{R}_{12} + \mathcal{R}_{2} &\le \mathcal{C}\left(\mathbf{H}_1\mathbf{H}_1{}^\mathsf{H}\right) & \text{(3f)} \\ \mathcal{R}_{11} + \mathcal{R}_{12} + \mathcal{R}_{2} &\le \mathcal{C}\left(\mathbf{H}\mathbf{H}^\mathsf{H}\right) & \text{(3g)} \end{aligned}$$

where $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3]$ and $\mathbf{H}_k$ is obtained from $\mathbf{H}$ by removing its k-th column, $k = 1, 2, 3$. Also, for a generic matrix $\mathbf{X}$, we defined $\mathcal{C}(\mathbf{X}) = \log_2 \det(\mathbf{I} + \mathbf{X})$.

Next, we denote by $\mathcal{R}_1$ the overall instantaneous rate of node 1, i.e., $\mathcal{R}_1 = \mathcal{R}_{11} + \mathcal{R}_{12}$. Prior work on cooperative relaying has been focused mainly on maximizing the sum rate of the source nodes, i.e., $\mathcal{R}_1 + \mathcal{R}_2$. However, such a maximization does not guarantee fairness between the traffic requirements of the source nodes. In order to solve this problem, we impose the additional fairness constraints:

$$\mathcal{R}_1 = \mathcal{R}; \quad \mathcal{R}_2 = \rho\mathcal{R} \tag{4}$$

and aim at maximizing $\mathcal{R}$. In (4), $\rho \in [0, +\infty)$ is the ratio between the nodes traffic requirements and is assumed to be known. We also stress that for $\rho = 0$ (i.e., $\mathcal{R}_2 = 0$) the problem reduces to maximizing the rate of the pure relay channel, as done for example in [2].

By using the expression of $\mathcal{R}_1$ and (4), we can rewrite the constrains (3a)–(3g) as functions of $\mathcal{R}$ and $\mathcal{R}_{11}$ only, i.e.,

$$\begin{aligned} \mathcal{R}_{11} &\le \mathcal{C}(\|\mathbf{h}_1\|^2) & \text{(5a)} \\ \mathcal{R}_{11} &\ge \mathcal{R} - \mathcal{C}(\|\mathbf{h}_2\|^2) & \text{(5b)} \\ \mathcal{R} &\le \mathcal{C}(\|\mathbf{h}_3\|^2)/\rho & \text{(5c)} \\ \mathcal{R} &\le \mathcal{C}\left(\mathbf{H}_3\mathbf{H}_3{}^\mathsf{H}\right) & \text{(5d)} \\ \mathcal{R}_{11} &\le -\rho\mathcal{R} + \mathcal{C}\left(\mathbf{H}_2\mathbf{H}_2{}^\mathsf{H}\right) & \text{(5e)} \\ \mathcal{R}_{11} &\ge (1+\rho)\mathcal{R} - \mathcal{C}\left(\mathbf{H}_1\mathbf{H}_1{}^\mathsf{H}\right) & \text{(5f)} \\ \mathcal{R} &\le \mathcal{C}\left(\mathbf{H}\mathbf{H}^\mathsf{H}\right)/(1+\rho) & \text{(5g)} \end{aligned}$$

where $\mathcal{R}_{11} \le \mathcal{R}$. Equations (1) and (5a) can be compactly rewritten as (6a), where $C_{11} = \min\{\mathcal{C}(\gamma), \mathcal{C}(\|\mathbf{h}_1\|^2)\}$. Also, (5c), (5d) and (5g) can be rewritten as (6b)

$$\begin{aligned} \mathcal{R}_{11} &\le C_{11} & \text{(6a)} \\ \mathcal{R} &\le C_0 & \text{(6b)} \end{aligned}$$

were $C_0 = \min\{\mathcal{C}(\|\mathbf{h}_3\|^2)/\rho, \mathcal{C}\left(\mathbf{H}_3\mathbf{H}_3{}^\mathsf{H}\right), \mathcal{C}\left(\mathbf{H}\mathbf{H}^\mathsf{H}\right)/(1+\rho)\}$.

Next, we are interested in finding the maximum rate $\mathcal{R}$ for which there exists at least a solution to the system of inequalities given by (5b), (5e), (5f), (6a), and (6b). By looking at (6a) and (5b), it is straightforward to see that a solution for $\mathcal{R}_{11}$ exists if the term on the right hand side of (5b) is lower than the term on the right hand side of (6a), i.e., $\mathcal{R} - \mathcal{C}(\|\mathbf{h}_2\|^2) \le C_{11}$. Then, by solving with respect to $\mathcal{R}$, we obtain that the rate is limited by

$$\mathcal{R} \le C_1 = C_{11} + \mathcal{C}(\|\mathbf{h}_2\|^2). \tag{7}$$

[1] Although a signal can be represented by a sequence of N random symbols, $x[n]$, $n = 1, \ldots, N$, where n is the symbol time index, for simplicity, we consider a symbol-by-symbol transmission and drop the index n.

[2] Bold lowercase and uppercase letters denote vectors and matrices, respectively. Vectors are column vectors, the conjugate transpose operator is denoted by $(\cdot)^\mathsf{H}$, and the identity matrix is denoted by $\mathbf{I}$.

Similarly, considering the pairs of equations: (6a) and (5f), (5e) and (5b), (5e) and (5f), $\mathcal{R}_{11} \le \mathcal{R}$ and (5f), we obtain:

$$\mathcal{R} \le C_2 = (C_{11} + \mathcal{C}(\|\mathbf{H}_1\|^2))/(1+\rho) \quad (8)$$
$$\mathcal{R} \le C_3 = (\mathcal{C}(\|\mathbf{h}_2\|^2) + \mathcal{C}(\|\mathbf{H}_2\|^2)/(1+\rho) \quad (9)$$
$$\mathcal{R} \le C_4 = (\mathcal{C}(\|\mathbf{H}_1\|^2) + \mathcal{C}(\|\mathbf{H}_2\|^2)/(1+2\rho) \quad (10)$$
$$\mathcal{R} \le C_5 = \mathcal{C}(\|\mathbf{H}_1\|^2)/\rho\,. \quad (11)$$

Note that we do not compare equations $\mathcal{R}_{11} \le \mathcal{R}$ and (5b) because the solution turns out to be independent of $\mathcal{R}$ and $\mathcal{C}(\|\mathbf{h}_2\|^2) \ge 0$. In conclusion, given the parameters γ, z_1, z_2, and ρ, the rate $\mathcal{R}$ is limited by

$$\mathcal{R} \le C^* = \max_{\alpha,\beta\in[0,1]} \min_{i=0,\dots,5} \{C_i\} \quad (12)$$

where the maximization is over the power share parameters α and β. From (12), it also follows that the achievable rate (averaged over the two-slot frame) is limited by

$$R \le C^*/2 \quad (13)$$

where the factor $1/2$ takes into account that the transmission is organized over two time slots of the same duration.

IV. Cut-set upper bound

In order to assess the performance of the proposed strategy, we compare it to the cut-set upper bound for the network under study. We derive the bound using the notation introduced in [10, chapter 10.2] and by following the approach in [6].

We denote by $\mathcal{T} = \{1, 2, D\}$ the set of nodes and assume that nodes 1 and 2 generate two independent messages, W_1 and W_2 with rates $\mathcal{R}_1 = \mathcal{R}$ and $\mathcal{R}_2 = \rho\mathcal{R}$, respectively, as defined in (4). Estimates of these messages are obtained at node D. X_1 and X_2 represent the signals transmitted by nodes 1 and 2, respectively, while Y_1, Y_2, and Y_D the signals received at nodes 1, 2, and D, respectively. The signals X_1 and X_2 are assumed to have zero mean and unit variance and have joint distribution p_{X_1,X_2}. We also denote by $N_1, N_2, N_D \sim \mathcal{N}_{\mathbb{C}}(0, 1)$ the independent noise terms at the receivers. Finally, let $\widehat{W}_1$ and $\widehat{W}_2$ be the estimates of the messages W_1 and W_2 obtained at node D.

We then consider the cuts of the network, $\mathcal{S}_i$, $i = 1, 2, 3$ (see Fig. 1 (right)) and their complement, $\mathcal{S}_i^c = \mathcal{T} \setminus \mathcal{S}_i$, which separate some of the messages from their corresponding estimates. Following [10, chapter 10.2] the cut-set upper bound to the achievable rate $\mathcal{R}$ is given by

$$C \le \max_{\substack{\zeta\in[0,1] \\ t_1,t_2,t_3\ge 0 \\ t_1+t_2+t_3=1}} \min\left\{\sum_{j=1}^{3} t_j I_{1j}, \sum_{j=1}^{3} t_j \frac{I_{2j}}{\rho}, \sum_{j=1}^{3} t_j \frac{I_{3j}}{1+\rho}\right\} \quad (14)$$

where $I_{ij} = I(X_{\mathcal{S}_i}; Y_{\mathcal{S}_i^c}|X_{\mathcal{S}_i^c}, \boldsymbol{\sigma}_j)$, $X_{\mathcal{S}_i} = \{X_k|k \in \mathcal{S}_i\}$ is the set of outputs from the nodes in $\mathcal{S}_i$, $X_{\mathcal{S}_i^c} = \{X_k|k \in \mathcal{S}_i^c\}$ is the set of outputs from the nodes in $\mathcal{S}_i^c$, and $Y_{\mathcal{S}_i^c} = \{Y_k|k \in \mathcal{S}_i^c\}$ is the set of inputs to the nodes in $\mathcal{S}_i^c$. The variable t_j, $j = 1, 2, 3$ in (14) represents the time fraction the network operates in state $\boldsymbol{\sigma}_j$, with $t_1 + t_2 + t_3 = 1$. The maximization in (14) is also performed over $\zeta = |\mathbb{E}[X_1X_2^*]|$, i.e., the correlation between the (assumed Gaussian) inputs X_1 amd X_2 (see [10, chapter 10.2] for details).

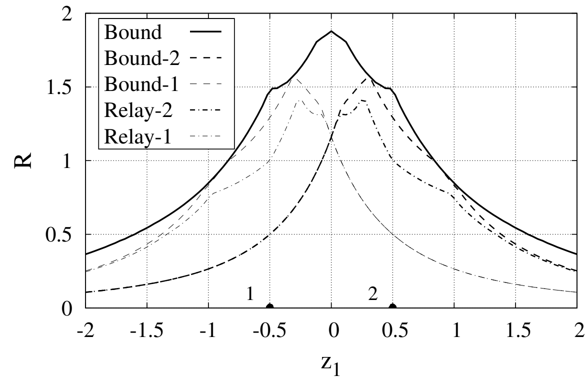

Fig. 2. Comparison between the average rates achievable by our relay strategy and the upper bounds as z_1 varies, for z_2=0, γ=0 dB and ρ=1.

As the last step, we compute the mutual information I_{ij}'s that appear in the expression of the cut-set bound in (14). To do so, we analyze each network state, separately.

1) In state $\boldsymbol{\sigma}_1$ (only node 1 transmits), the signals received at nodes 2 and D are given by $Y_2 = \sqrt{\gamma}X_1 + N_2$ and $Y_D = \sqrt{\gamma_1}X_1 + N_D$, respectively. Thus, $I_{11} = \mathcal{C}(\gamma + \gamma_1)$ and $I_{31} = \mathcal{C}(\gamma_1)$. Also, $I_{21} = 0$ since node 2 does not transmit.

2) In state $\boldsymbol{\sigma}_2$ (only node 2 transmits), the received signals are $Y_1 = \sqrt{\gamma}X_2 + N_1$ and $Y_D = \sqrt{\gamma_2}X_2 + N_D$, and we can write the mutual informations as $I_{12} = 0$, $I_{22} = \mathcal{C}(\gamma + \gamma_2)$ and $I_{32} = \mathcal{C}(\gamma_2)$.

3) In state $\boldsymbol{\sigma}_3$ (both nodes 1 and 2 transmit), the signal received at D is given by $Y_D = \sqrt{\gamma_1}X_1 + \sqrt{\gamma_2}X_2 + N_D$, and, by considering that X_1 and X_2 are correlated with correlation coefficient ζ, the mutual informations are given by $I_{13} = \mathcal{C}(\gamma_1 - \gamma_1\zeta^2)$, $I_{23} = \mathcal{C}(\gamma_2 - \gamma_2\zeta^2)$, and $I_{33} = \mathcal{C}(\gamma_1 + \gamma_2 + 2\zeta\sqrt{\gamma_1\gamma_2})$.

V. Results

We now show the performance of our strategy in terms of achievable rate, and compare it to the bound derived in Sec. IV. In particular, we evaluate the rates obtained when:

- node 2 acts as a relay for node 1. This is the situation described in Sec. III, hereinafter referred to as "Relay-2". We recall that in this case the network operates in states $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_3$, with $t_1 = t_3 = 1/2$;
- node 1 acts as a relay for node 2 (i.e., nodes 1 and 2 swap their roles with respect to the Relay-2 case). This situation is referred to as "Relay-1", and corresponds to letting the network operate in states $\boldsymbol{\sigma}_2$ and $\boldsymbol{\sigma}_3$, with $t_2 = t_3 = 1/2$.

Fig. 2 shows the results as z_1 varies, for $z_2 = 0$, $\gamma = 0$ dB and $\rho = 1$. In the plot, the bullets labeled "1" and "2" on the x-axis represent the position of the nodes 1 and 2, respectively, while z_1 represents the position of the destination. The curve labeled by "Bound" is obtained by evaluating (14), while the curves labeled by "Bound-2" and "Bound-1" represent the bounds obtained from (14) where, instead of maximizing over t_1, t_2 and t_3, we set $(t_1, t_2, t_3) = (1/2, 0, 1/2)$ and

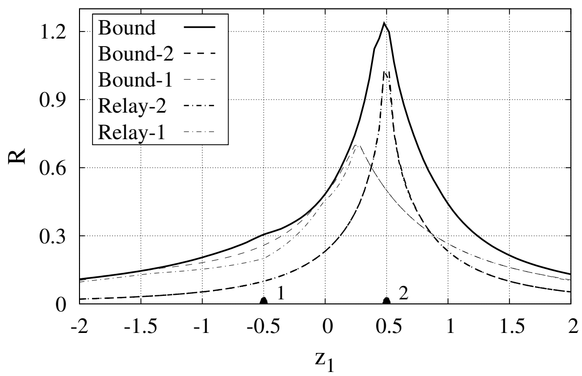

Fig. 3. Comparison between the average rates achievable by our relay strategy and the upper bounds as z_1 varies, for z_2=0, γ=0 dB, and ρ=5.

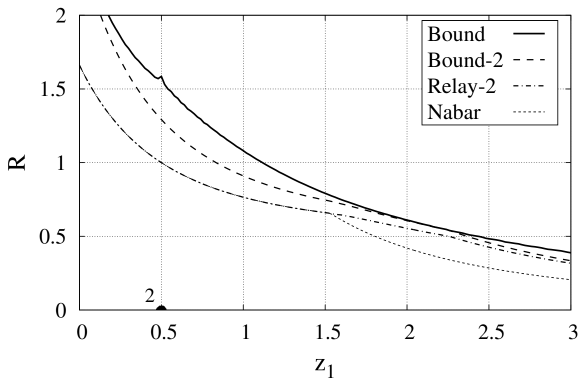

Fig. 4. Comparison between the average rates achievable by our relay strategy and the upper bounds as z_1 varies, for z_2=0, γ=0 dB and ρ=0.

$(t_1, t_2, t_3) = (0, 1/2, 1/2)$, respectively. These latter two are upper bounds to the rates achievable by communication strategies employing only two network states (out of three), and for the same amount of time.

As expected, the Relay-2 strategy outperforms the Relay-1 scheme. Indeed, $\rho = 1$ corresponds to an equal traffic demand at the two nodes and $z_1 > 0$ implies that D is closer to node 2 than to node 1. It is important to note, however, that the results obtained through Relay-2 are very close to the corresponding bound (Bound-2). This is true for any $z_1 > 0.75$; when instead the destination is very close to the nodes, a relaying strategy becomes less effective than the multiple access channel where both nodes simultaneously transmit toward D.

In Fig. 3, the average achievable rates and the bounds are plotted for $z_2 = 0$, $\gamma = 0$ dB and $\rho = 5$ (i.e., $\mathcal{R}_2 = 5\mathcal{R}_1$). Interestingly, although D is closer to node 2, Relay-1 (which uses node 1 as a relay) outperforms Relay-2 for any $z_1 > 0.8$. This is due to the fact that, since now node 2 requires a much higher rate than node 1, its traffic load is already very high and using node 2 as a relay (thus further increasing its load) is not beneficial. Finally, we observe that the curve corresponding to Relay-1 (resp. Relay-2) overlaps with the curve representing Bound-1 (resp. Bound-2), for almost any value of z_1.

Fig. 4 refers to the case $z_2 = 0$, $\gamma = 0$ dB and $\rho = 0$. Here node 2 is not a source and can only act as a relay for node 1. Thus, in the plot we only show the rates achieved by Relay-2 and compare them to Bound and Bound-2. Additionally, the curve labeled by “Nabar” shows the performance of the strategy named “Protocol I” by Nabar et al. in [2]. This strategy corresponds to setting $\alpha = 0$ and $\beta = 1$ in (2), and it is outperformed by our Relay-2 scheme when $z_1 > 1.5$.

VI. Conclusion and Future Work

We studied cooperative relaying in a three-node wireless network, where nodes use the same frequency channel, are half duplex and can act as both sources and relays. We proposed a transmission strategy and characterized the corresponding achievable rate when a fair rate allocation is provided. We then derived a cut-set upper bound in our network scenario, and showed that the rate achievable by the proposed strategy closely approximates such a bound. The results also showed that our strategy outperforms that presented in [2].

This work can be extended along several directions: i) given an achievable rate, we can minimize the node energy consumption through an efficient power allocation policy, ii) the study can be extended to fading channels to assess the diversity gains, and iii) the assumption on equally-sized slots can be relaxed in order to maximize the achievable rate with respect to the slot duration.

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