Are carrier-to-noise algorithms equivalent in all situations?

GNSS Solutions:

Carrier-to-Noise Algorithms

"GNSS Solutions" is a regular column featuring questions and answers about technical aspects of GNSS. Readers are invited to send their questions to the columnist, **Dr. Mark Petovello**, Department of Geomatics Engineering, University of Calgary, who will find experts to answer them. His e-mail address can be found with his biography below.

Are C/N_0 Algorithms Equivalent in All Situations?

Fundamental to determining the status of GNSS tracking subsystems and controlling a GNSS receiver, the measure of C/N_0 (carrier-to-noise ratio) provides satellite signal health information in addition to the PVT (position, velocity, time) information. For example, tracking loops experience a rapid increase of tracking errors at low C/N_0, e.g., below 30 dBHz, until they completely lose lock.

Since high-performance GNSS software receivers have become a reality in navigation labs worldwide, software engineers must select algorithms capable of maximizing accuracy with minimal implementation complexity.

In this column, we present a comparison of C/N_0 algorithm performance. In digital receivers several methods may be used to estimate C/N_0, but all involve processing samples from the correlator output. Dr. Brad Badke discussed the most intuitive algorithm in the GNSS Solutions column in the September/October 2009 issue of *Inside GNSS* — the so-called *real signal-complex noise (RSCN)* method.

Texts on GPS architecture usually include another classic approach, the *narrowband-wideband power ratio* (NWPR) method. Additionally, literature on digital communication offers many other methods for estimating SNR (signal-to-noise ratio) — which, as discussed in Dr. Badke's contribution is not the same as C/N_0 — of M-PSK (*M*-phase shift keying) modulations in *additive white Gaussian noise* (AWGN). However, these SNR algorithms must be adapted before equating them to C/N_0.

Since high-performance GNSS software receivers have become a reality in navigation labs worldwide, software engineers must select algorithms capable of maximizing accuracy with minimal implementation complexity. For this reason, in our discussion we will compare the accuracy and implementation complexity of five different C/N_0 estimation algorithms.

Computational Complexity of Different Estimators

Here we will investigate the following SNR estimation algorithms (See Additional Resources section at the end of this column for more details):

- Real Signal-Complex Noise (RSCN)
- Beaulieu's method (BL): an "intuitively motivated" algorithm introduced in 2000 by N. C. Beaulieu et alia (see Additional Resources section at end of this article)
- Signal-to-Noise Variance (SNV): squared Signal-to-Noise Variance estimator, based on the first absolute moment and the second moment of the signal samples
- Moment Method (MM): employs the second- and fourth-order moments for the separate estimation of carrier strength and noise strength
- Narrowband-Wideband Power Ratio (NWPR) Method, reported in several books on GPS receivers. Note that this is the only algorithm that estimates C/N_0 directly; the others estimate SNR, which can be converted to C/N_0 as discussed later. Because we sample the "observable"

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signal stream from the prompt correlator output to compute C/N_0, we write the sample stream as a function of the discrete time, n,

$$r_C[n] = \sqrt{P_d}\,D[n] + \sqrt{P_n}\,\eta[n] \quad (1)$$

where $D[n] = \pm e^{j\theta_n}$ are the navigation bit samples; θ_n is the residual carrier phase error due to the carrier tracking loop; P_d and P_n are the powers associated to data and noise, respectively; and $\eta[n] = \eta_{Re}[n] + j\eta_{Im}[n]$ expresses the complex noise samples.

Thus the SNR related to the signal $r_C[n]$ is defined as $\lambda_C = P_d/P_n$, so that

$$\frac{C}{N_0} = \lambda_C B_{eqn} \quad (2)$$

where B_{eqn} represents the normalized *equivalent noise bandwidth* of the system.

Table 1 summarizes the algorithms defined by each of the foregoing methods, where we indicate their computational complexity in terms of the number of real (versus complex) sums, real multiplications, and square roots.

In Table 1, N is the number of observed samples used to produce one SNR estimate. We assume that we can select a high enough N — typically on the order of a few hundreds of samples — to prevent any additional estimation bias due to an insufficient number of observations.

The parameter M introduced in the NWPR estimator is the ratio between the bandwidth associated to the wideband power measurement WBP_k, and the bandwidth associated with the narrowband measurement NBP_k. An indicative evaluation of the computational complexity is also reported in **Figure 1**.

Method	Algorithm	Computational Complexity		
		sums	multiplications	square roots
RSCN	$\hat{P}_n = \frac{2}{N}\sum_{v=1}^{N} \lvert r_{C,\mathrm{Im}}[v]\rvert^2$ $\hat{P}_{tot} = \frac{1}{N}\sum_{v=1}^{N} \lvert r_C[v]\rvert^2$ $\hat{\lambda}_C = \frac{\hat{P}_{tot} - \hat{P}_n}{\hat{P}_n}.$	$3N+1$	$3N+5$	0
BL	$\hat{P}_{n,v} = \left(\lvert r_{C,\mathrm{Re}}[v]\rvert - \lvert r_{C,\mathrm{Re}}[v-1]\rvert\right)^2$ $\hat{P}_{d,v} = \frac{1}{2}\left(r_{C,\mathrm{Re}}[v]^2 + r_{C,\mathrm{Re}}[v-1]^2\right)$ $\hat{\lambda}_C = \left[\frac{1}{N}\sum_{v=1}^{N}\frac{\hat{P}_{n,v}}{\hat{P}_{d,v}}\right]^{-1}$	$3N+1$	$5N+5$	0
SNV	$\hat{P}_d = \left[\frac{1}{N}\sum_{v=1}^{N} \lvert r_{C,\mathrm{Re}}[v]\rvert\right]^2$ $\hat{P}_{tot} = \frac{1}{N}\sum_{v=1}^{N} \lvert r_C[v]\rvert^2$ $\hat{\lambda}_C = \frac{\hat{P}_d}{\hat{P}_{tot} - \hat{P}_d}$	$3N-1$	$2N+6$	0
MM	$\hat{M}_2 = \frac{1}{N}\sum_{v=1}^{N} \lvert r_C[v]\rvert^2$ $\hat{M}_4 = \frac{1}{N}\sum_{v=1}^{N} \lvert r_C[v]\rvert^4$ $\hat{P}_d = \sqrt{2\hat{M}_2^2 - \hat{M}_4}$ $\hat{P}_n = \hat{M}_2 - \hat{P}_d$ $\hat{\lambda}_C = \frac{\hat{P}_d}{\hat{P}_n}$	$4N$	$5N+7$	1
NWPR	$\mathrm{WBP}_k = \sum_{m=1}^{M} \lvert r_C[kM+m]\rvert^2, \quad k = 0,1,\ldots\left(\frac{N}{M}-1\right)$ $\mathrm{NBP}_k = \left(\sum_{m=1}^{M} \lvert r_{C,\mathrm{Re}}[kM+m]\right)^2 + \left(\sum_{m=1}^{M} \lvert r_{C,\mathrm{Im}}[kM+m]\right)^2$ $\hat{\mu}_{NP} = \frac{M}{N}\sum_{k=0}^{N/M-1} \frac{\mathrm{NBP}_k}{\mathrm{WBP}_k}$ $\gamma = \frac{C}{N_0} = \frac{1}{T_{int}}\frac{\hat{\mu}_{NP}-1}{M-\hat{\mu}_{NP}}$	$4N-K+1$ $(K = N/M)$	$2N+4K+4$	0

TABLE 1. SNR estimation algorithms. A "hat" (^) over a variable indicates an estimate from measured quantities (observables)

The Effect of Phase Noise

In the presence of a residual phase error (θ_n) in the carrier tracking loop, the above algorithms experience asymptotic biases for high C/N_0 values. Such biases can be theoretically demonstrated by computing the asymptotic limit for $P_d \rightarrow \infty$ of the SNR estimators with non-zero θ_n. Herein, θ_n is assumed to be a zero-mean random variable with variance σ_θ^2 and uniform distribution between $[-\sqrt{3}\sigma_\theta, +\sqrt{3}\sigma_\theta]$.

When the noise power contribution is very small, some algorithms cannot discriminate additive noise from θ_n or other factors, while other algorithms are sensitive to the noise power only.

On the other hand, in low SNR conditions the principal limiting phenomenon is that the carrier tracking loop no longer keeps the incoming and the local carriers synchronized. In this case the SNR algorithms use correlator outputs inconsistent with the assumed distribution of θ_n, thereby invalidating the SNR estimate. However, as long as the receiver maintains signal lock and the software tracking loops are not broken, the SNR estimators perform equivalently in medium-to-low SNR conditions.

Table 2 gives expressions for each estimator's expected *asymptotic esti-*

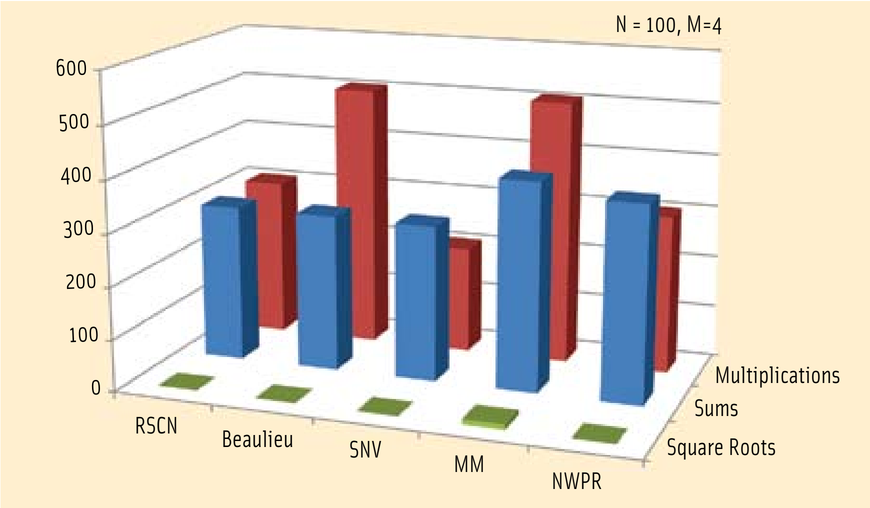

FIGURE 1 Relative computational complexity of SNR algorithms

Method	Asymptotic Value	Observations
RSCN	$\lambda_{RSCN,\infty}(\theta) = \lim_{P_d\to\infty} \mathbb{E}\{\hat{\lambda}_C(\theta)\} = \frac{\cos(\sqrt{3}\sigma_\theta)\sin(\sqrt{3}\sigma_\theta)}{\sqrt{3}\sigma_\theta - \cos(\sqrt{3}\sigma_\theta)\sin(\sqrt{3}\sigma_\theta)}$	Horizontal asymptote for increasing values of SNR, as a function of σ_θ^2
BL	$\lambda_{BL,\infty}(\theta) = \frac{P_d}{P_n}\left[\frac{1}{2} + \frac{\sin(2\sqrt{3}\sigma_\theta)}{4\sqrt{3}\sigma_\theta}\right]$	Asymptotically unbiased in absence of residual phase noise: $\lim_{\sigma_\theta\to 0}\lambda_{BL,\infty}(\theta) = \frac{P_d}{P_n}$
SNV	$\lambda_{SNV,\infty}(\theta) = \frac{\sin^2(\sqrt{3}\sigma_\theta)}{3\sigma_\theta^2 - \sin^2(\sqrt{3}\sigma_\theta)}$	Horizontal asymptote for increasing values of SNR, as a function of σ_θ^2
MM	$\lambda_{MM,\infty}(\theta) = \frac{P_d}{P_n}$	Unbiased with respect to the residual phase error
NWPR	$\lambda_{NWPR,\infty}(\theta) = \frac{\sin^2(\sqrt{3}\sigma_\theta)}{3\sigma_\theta^2 - \sin^2(\sqrt{3}\sigma_\theta)}$	Horizontal asymptote for increasing values of SNR, as a function of σ_θ^2

TABLE 2. Asymptotic behavior of the SNR estimation algorithms for $P_d \to \infty$

mation value, $\lambda_\infty(\theta)$, that is, the SNR estimate for signal power tending to infinity. Note that the limit for SNR as σ_θ^2 approaches zero is the *true SNR*, thus guaranteeing an unbiased asymptotic estimation capability of all the approaches in absence of residual phase noise.

Numerical simulations confirm the asymptotic behavior (for $P_d \to \infty$) foreseen by theoretical analysis. **Figure 2** shows the estimated C/N_0 versus the true C/N_0 obtained by simulating the signal using Equation 1, and for both cases where the residual phase noise variance $\sigma_\theta^2=(1°)^2$, as well as for the expected asymptotic limits of σ_θ^2 (computed using the equations given in Table 2).

Figure 3 shows a comparison of $\lambda_{(\cdot)\infty}(\theta)$ as a function of σ_θ for SNR values of 30, 40, and 50 decibels. The SNR for the RSCN, SNV, and NWPR estimators decreases asymptotically with increasing values of the residual phase noise. This inverse relationship indicates a bias in the estimated value.

Note that, as $\sigma_\theta \to 0$, the estimated SNR, $\lambda_{(\cdot)\infty}(\theta)$, for the RSCN, SNV and NWPR methods approaches the true

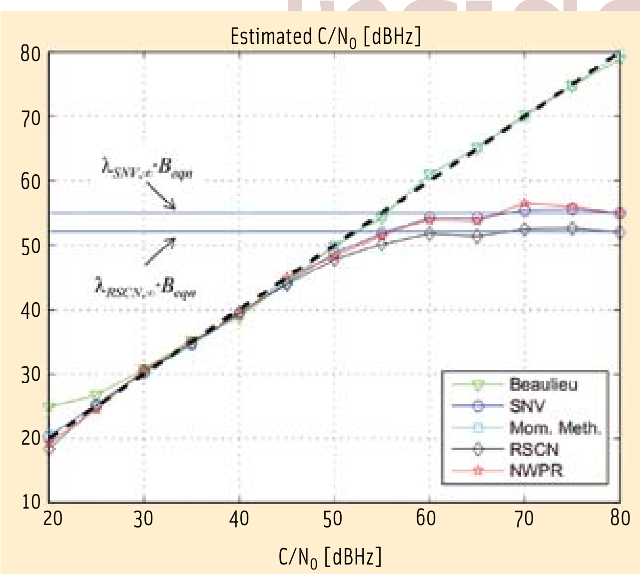

FIGURE 2 C/N_0 estimates for $\sigma_\theta = 1°$ The dark dashed line indicates the true C/N_0 values.

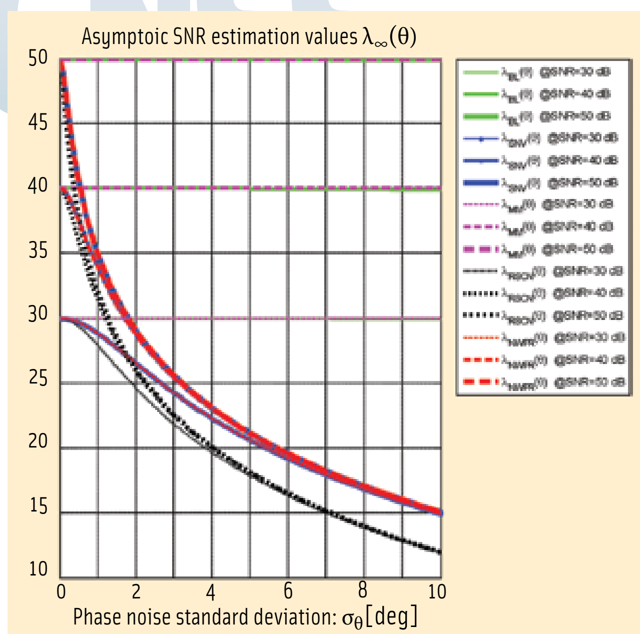

FIGURE 3 Asymptotic SNR estimates vs. the phase noise standard deviation.

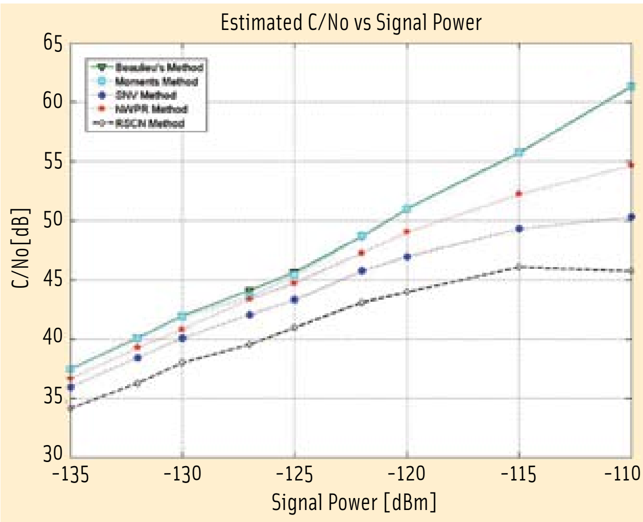

FIGURE 4 **Estimated C/N_0 from a real software receiver.**

SNR. These estimators exhibit fewer errors for decreasing SNR, that is, the bias from the SNR estimate at $\sigma_\theta \rightarrow 0$, is smaller for lower SNR values.

This simulation confirms the limits for the RSCN, SNV, and NWPR methods. For high values of C/N_0, the estimates of SNR provided by these algorithms is valid only for *extremely small* values of the residual phase noise, θ_n, while, vice-versa, for an increasing phase noise, their estimates are valid only in medium-to-low C/N_0 conditions.

As a final note, the bias of the BL estimator is practically negligible for the considered values of phase noise standard deviation. Also, the SNR estimated by the MM estimator, $\lambda_{MM,\infty}(\theta)$, coincides with the true SNR.

As a further verification of these theoretical considerations, we consider the performance of each estimator in a real time GPS software receiver with an RF signal supplied by a hardware GPS signal simulator tuned at different levels of power. **Figure 4** shows the estimated C/N_0 with respect to the GPS signal power.

As predicted, both the BL and MM methods show a linear trend as the signal power increases. These algorithms are not biased and have the same performance, indeed the difference between the estimated values is always less than 0.1 dB.

The RSCN, SNV, and NWPR algorithms perform less reliably. Their estimates vary linearly under normal power conditions, ranging between 35 and 50 dBHz, but as signal power increases to around -120 dBm, their error decreases asymptotically. Moreover these methods tend to underestimate C/N_0 in comparison with the BL and MM methods.

Additionally, we found that the RSCN algorithm underestimates the C/N_0 by more than three decibels at all signal power levels. We attribute this bias to a high sensitivity to residual carrier phase noise. As soon as the signal constellation is rotated by θ_n, a corresponding portion of signal power is interpreted as noise power, proportionally lowering the estimated C/N_0. Despite its simple algorithm and moderate computational burden, the RSCN method does not yield a suitable solution when accurate C/N_0 estimates are required.

Conclusions

Several algorithms developed for SNR estimation of biphase shift keying/quad phase shift keying (BPSK/QPSK) modulations in a noisy environment can be employed to compute the C/N_0 ratio in a GNSS digital receiver; however, they experience different estimation performance and implementation complexity. In particular they offer different sensitivities to the residual carrier phase noise, mainly for very high carrier-to-noise ratios. Receiver designers should select the algorithm that best suits their particular needs.

Additional Reading

Beaulieu, N. C., and A. S. Toms, and D. R. Pauluzzi, "Comparison of four SNR estimators for QPSK modulations," *IEEE Communications Letters,* vol. 4, no. 2, pp. 43–45, Feb. 2000.

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