Supplemental material

Small scale anisotropy in the turbulent shearless mixing

Daniela Tordella, Michele Iovieno

email: daniela.tordella@polito.it

This PDF file includes the following material:

- Construction of the mixing layer: initial conditions and transient (Figures S.1 and S.2, Table S.1).
- Why a Reynolds stress and a mean flow are not generated inside the shearless mixing layer? (Figure S.3).
- Transversal velocity derivative skewness (Figure S.4).
- Movie description (shearless_mixing_energy.avi).

1 Construction of the mixing layer: initial conditions and transient

The shearless mixing layer is constructed by combining two cubic simulation domains that each contain isotropic turbulences with different energy levels. The computational domain is a parallelepiped with aspect ratio equal to two ($2L \times L \times L$, $L = 2\pi$ in dimensionless variables). Periodic boundary conditions are imposed in all directions. To satisfy this condition, the domain should host two shearless mixing layers as shown in figure S.1. In the initial condition, the two isotropic turbulent fields are matched over a layer as large as about one correlation length. The matched field is generated as a linear superposition of the two initial isotropic fields. The hypothesis is done that during the initial transient of the simulation (about 1-2 eddy turnover times) the superposition becomes a Navier-Stokes solution. Thus

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}_1(\mathbf{x})p(x) + \mathbf{u}_2(\mathbf{x})(1 - p(x)) \tag{1}$$

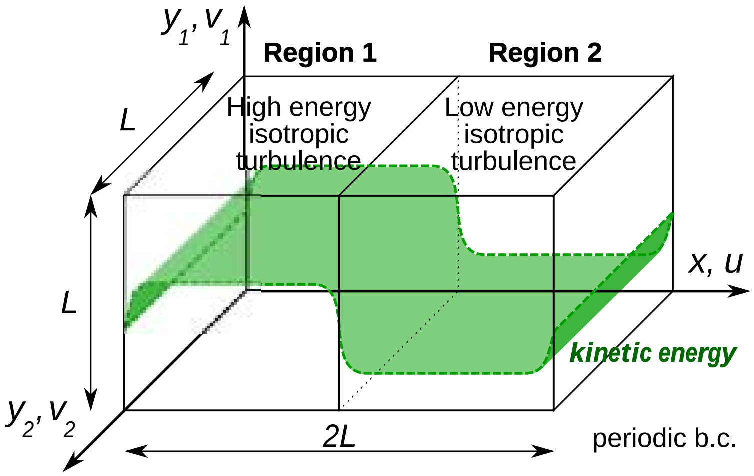

Figure S. 1: Scheme of the computational domain and initial conditions.

where the suffixes 1,2 indicate the high- and low-energy fields, respectively, x is the inhomogeneous direction. Function $p(x)$ is the weighting function:

$$p(x) = \frac{1}{2}\left[1 + \tanh\left(a\frac{x}{2L}\right)\tanh\left(a\frac{x-L}{2L}\right)\tanh\left(a\frac{x-2L}{2L}\right)\right]. \tag{2}$$

Constant a determines the initial mixing layer thickness Δ, conventionally defined as the distance between the points with normalized energy values 0.25 and 0.75 when the low-energy side is mapped to zero and the high-energy side to one. When $a = 50$, Δ is about 1/40 of the domain.

In the following, the reader can find a table where dimensional values of the main parameters for typical shearless mixings are listed. We have considered the flow studied in the laboratory by Veeravalli and Warhaft [2, 3] as a reference for the case at $Re_\lambda = 45$.

	$Re_\lambda = 45$	$Re_\lambda = 71$	$Re_\lambda = 150$
$2L$ [m]	0.41	0.82	4.1
E_1 [J/kg]	5.0×10^{-2}	7.2×10^{-2}	4.8×10^{-2}
u'_{rms} [m/s]	0.18	0.22	0.18
ℓ [cm]	1.03	2.16	11
λ [cm]	0.37	0.48	1.3
$\tau_1 = \ell/u'_1$ [s]	5.7×10^{-2}	9.6×10^{-2}	6.1×10^{-1}
ρ [kg/m^3]		1.2	
ν [m^2/s]		1.5×10^{-5}	
$(E_1 - E_2)/\Delta$ [J/(kg m)]	4.2	2.0	0.4
$\Delta t/\tau_1$	2.5	1.8	0.8

Table S. 1: Dimensional parameters of the initial conditions: $2L$ is the domain size, E_1 is the initial kinetic energy in the high energy isotropic regions, ℓ and λ are the initial integral scale and Taylor microscale, τ_1 is the initial eddy turnover time, Δt is the length of the initial transient. The data at $Re_\lambda = 45$ are chosen to match the 3:1 bar grid wind tunnel experiments by Veeravalli and Warhaft [2].

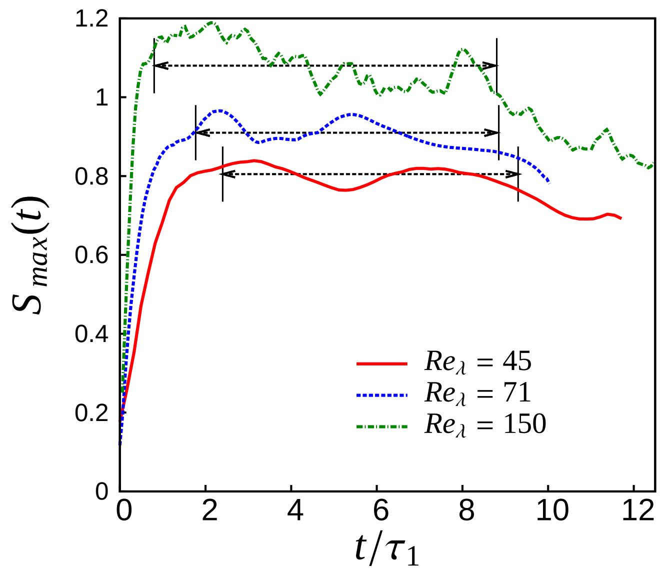

Figure S. 2: Maximum of the skewness of the velocity component u in the inhomogeneous direction inside the mixing layer. The arrows indicate the interval where we collected the results.

During the decay of the flow, we observe the velocity and velocity derivative statistics, which gradually depart from their isotropic values inside the interacting region. We observe an initial transient where the mixing is formed and the numerical solution becomes a physical Navier-Stokes solution. The mixing region gradually becomes intermittent, as can be seen from the appearance of significant peaks in the skewness and kurtosis distribution of the velocity component u (see figures 2-5 in [4], figure 9 in [2]).

The maximum of the skewness inside the interaction layer is reached after few eddy turnover times, then it slowly decreases, see figure S.2. The time at which the maximum skewness is obtained is used to define the end of the initial transient in which the shearless mixing emerges from the initial conditions. The length of the initial transient decreases as the Reynolds number is increased: it is about 2-3 initial eddy turnover times at $Re_\lambda = 45$ but less then one initial eddy turnover time at $Re_\lambda = 150$ in the present simulations with an imposed energy ratio equal to 6.7. This transient becomes longer with higher energy rations, see figures 6(a) and 7(a) in [1]. We see that, after 10-12 eddy turnover times, the mixing becomes spoiled by the growth of the correlation lengths and mixing layers thickness and by the decay of energy. The intervals we used to analyze the data of the present numerical experiment are shown in figure S.2.

2 Why a Reynolds stress and a mean flow are not generated inside the mixing layer?

The shearless turbulent mixing is a flow where the average momentum is zero since the initial condition and the boundary conditions are such as to not generate a mean flow. This can be also seen by considering the average momentum balance. In fact,

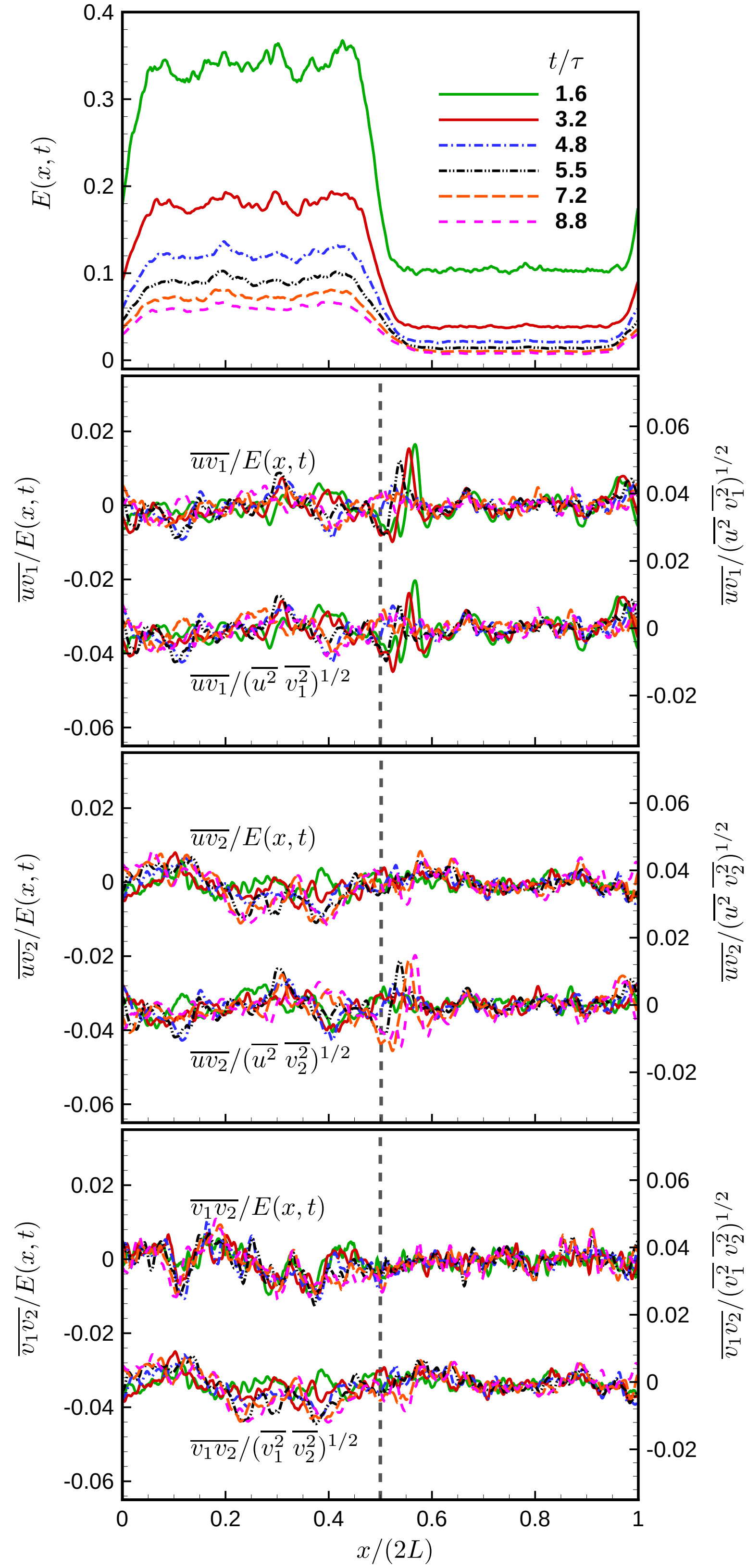

Figure S. 3: Spatial distributions of the turbulent kinetic energy $E(x,t)$ and of the deviatoric part of the Reynolds stress tensor, normalized by using the average kinetic energy $E(x,t)$, and correlation coefficients in the simulation at $Re_\lambda = 150$; x is the coordinate in the inhomogeneous direction and $2L$ is the domain size along x. The vertical dashed line indicates the centre of the mixing layer.

we can write

$$\partial_t U_i - \partial_j U_i U_j + (1/\rho)\partial_i P + \partial_j \overline{u_i u_j} - \nu \nabla^2 U_i = 0, \tag{3}$$

where the capital letters denote mean quantities, the small letters fluctuations, and the overline denotes the statistical average. From the initial conditions, we have $U_i = 0$ $\forall i$ at $t = 0$, so, the second and the last terms in equation (3) are everywhere equal to zero and the momentum equation in the x direction equations reduces to

$$\partial_t U = -(1/\rho)\partial_x P - \partial_x \overline{u^2}, \tag{4}$$

where U is the mean velocity in the mixing direction. Since the field is incompressible, the divergence of the mean velocity is zero. As consequence, we obtain the following equation for the mean pressure:

$$\nabla^2 P/\rho = -\partial_i \partial_j \overline{u_i u_j} - \partial_i \partial_j U_i U_j \tag{5}$$

which, by considering that the flow is uniform outside the mixing layer, yields

$$\partial^2_{xx} P/\rho = -\partial^2_{xx} \overline{u^2}, \;\; \partial_x P/\rho = -\partial_x \overline{u^2}. \tag{6}$$

Consequently, by inserting this mean pressure gradient into (4), the right hand side vanishes, and no mean acceleration, $\partial_t U$, is generated. The distributions of the resulting mean pressure and acceleration can be seen in [1], appendix A and figure 10. The order of magnitude of this pressure gradient is of a few Pascal/m. At the state of the art these small values are not observable in the laboratory.

Figure S.3 shows the spatial distributions of the components of the deviatoric part of the Reynolds stress tensor. They oscillate about a zero mean value. One can see that the oscillation never becomes larger than 2% of the turbulent kinetic energy.

3 Transversal velocity derivative skewness

In contrast with homogeneous shear flows, the transversal derivative skewness in the simulated flows is found to be very small and do not depart from the isotropic value. Thus, in general, the smallness of the transversal moments is not a sufficient condition for isotropy. Figure 4 shows the spatial distributions of the skewness of the six transversal derivatives. The skewness has a mean value close to zero and a standard deviation between 0.02 and 0.04.

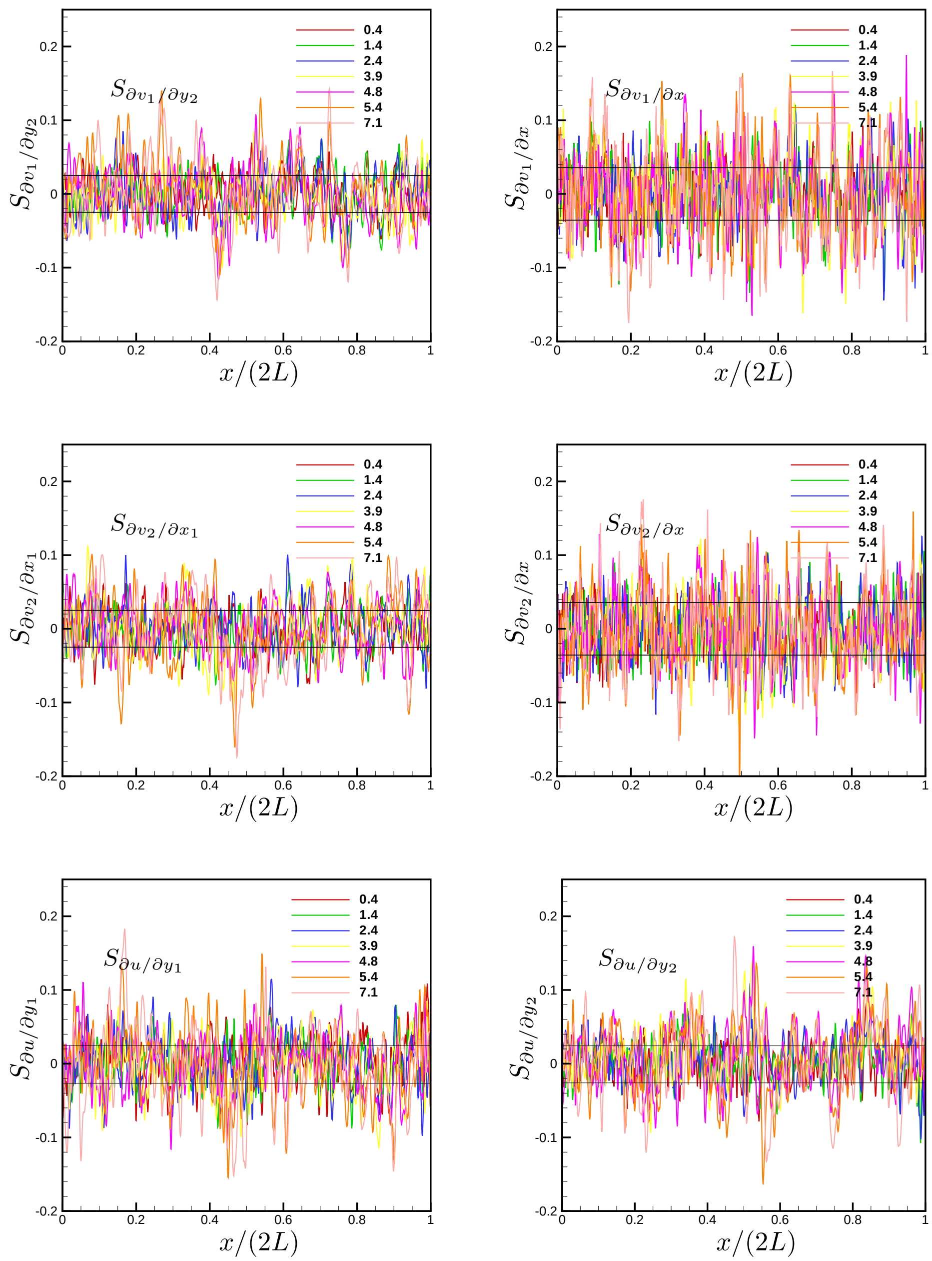

Figure S. 4: Spatial distribution of the skewness of the transversal derivatives in the simulation at $Re_\lambda = 150$. The horizontal lines indicate the computed standard deviation of the skewness distribution.

4 Movie description, `shearless_mixing_energy.avi`

The movie (`shearless_mixing_energy.avi`) shows the isolevels of the turbulent kinetic energy in the first 6 eddy turnover times in the simulation at $Re_\lambda = 150$ in a plane y_2 =const. The animation covers only a portion of the domain, roughly a square of 560×560 grid points, which is placed in the centre of the domain.

References

[1] Tordella D., Iovieno M., Bailey P.R. *Phys. Rev. E* **77**, 016309 (2008).

[2] Veeravalli S., Warhaft Z. *J. Fluid Mech.* **207**, 191–229, (1989).

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[4] Tordella D., Iovieno M. *J. Fluid Mech.*, **549**, 441-454, (2006).