A strained space-time

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Abstract. The paper outlines a theory where spacetime is treated as a physical four-dimensional continuum with properties similar to the ones of ordinary three-dimensional elastic continua. Two manifolds are compared: the first is flat; the second is curved and is thought as being obtained from the first by deformation. The intrinsic "rigidity" os spacetime implies that the deformation correspond to a strained state, where the strain tensor is given by half the difference between the metric tensor of the deformed and the undeformed manifolds. As it would happen in three dimensions the strain is associated with an elastic potential energy. The theory adds this deformation potential energy term to the standard spacetime Lagrangian density of General Relativity. Using the new Lagrangian density the theory is able to reproduce the accelerated expansion of the universe and is also consistent with the data from Big Bang Nucleosynthesis, the acoustic horizon of CMB and the structure formation after the recombination era.

1. Introduction

Spacetime has a fundamental role in General Relativity (GR) and interacts with matter/energy, which means that it is not a mathematical artifact but possesses physical properties on its own. This point is not always fully considered when trying and describing the behaviour of gravitating systems, especially at the cosmic scale. On the other hand we know that there are a few observed behaviours in the universe which is not easy to explain in terms of plain GR. This is the case, for instance, of the accelerated expansion discovered in 1998 [6] as well as of the missing mass or dark matter problem in galaxies and galaxy clusters. In order to solve the open problems a host of solutions have been envisaged, but often the different theories explore the mathematical side without bothering about the physical meaning of the additional terms or changes they introduce into the Lagrangian density of spacetime.

Here I am sketching an approach that finally leads to an additional term into the Lagrangian of empty spacetime, but that term is molded on known physics, drawing onto the analogy that is found between spacetime and three-dimensional material continua. If spacetime is a deformable Riemannian manifold we expect the induced curvature to be associated to a strain in the manifold, then to a corresponding deformation energy. This is the reason why we introduce into the Lagrangian density a potential deformation energy term, depending on the strain of the curved manifold. The analogy further suggests to make use of a generalized linear elasticity theory to find the explicit form of the deformation energy.

Once the new Lagrangian density has been written, we are able, as we shall see, to work out of it the accelerated expansion of the universe as well as consistent values for a number of cosmological tests, like the primordial nucleosynthesis, the acoustic scale of the CMB and the large scale structure formation.

2. The strained manifold

The easiest way to describe the Strained State Theory (SST) is to have recurse to an embedding of the physical four-dimensional manifold into a higher dimensional flat Riemannian space. Actually we may imagine to have a four-dimensional flat manifold, called the *reference manifold* and the real curved manifold (the *natural manifold*), both embedded into a (4+1)-dimensional Riemannian space. Ideally we may think as having an "initial" state which coincides with the reference manifold and successively evolves into the natural manifold by deformation due to the presence of matter/energy or of texture defects in the original continuum. This logical sequence has nothing to do with an evolution in time since time is part of the manifolds. The Lorentzian signature may or may not be present from the very beginning. If it is there the reference manifold is Minkowskian, otherwise it is Euclidean. The situation is graphically represented in fig. 1.

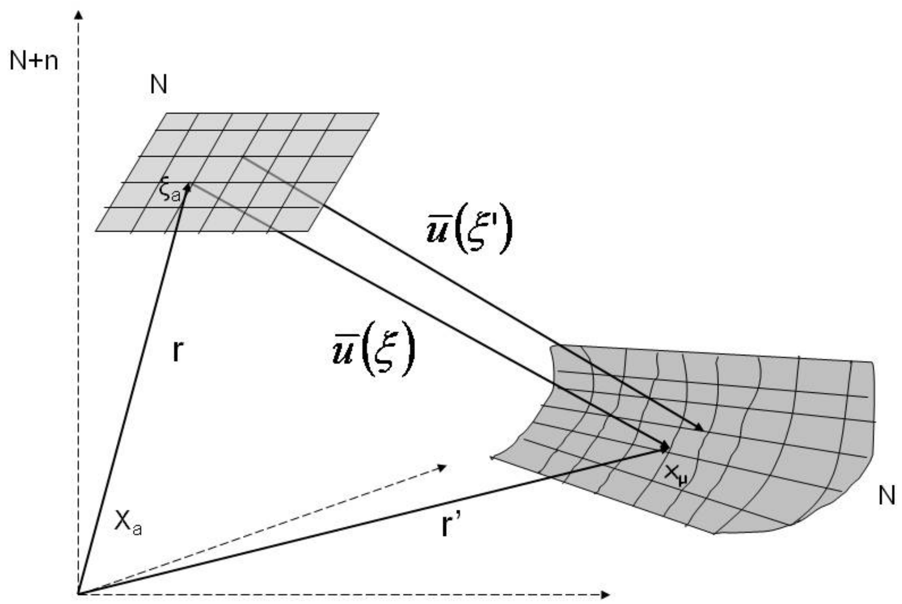

Figure 1. Schematic $(N+n)$-dimensional embedding (actually $4+1$) of a flat N-dimensional manifold and a curved N-dimensional one obtained by deformation from the first. The correspondence between line elements on the two manifolds (each one endowed with its own reference frame and coordinates) visualizes the strain induced in the curved one.

The (N+1)-dimensional vector field **u** visible in the figure establishes a one to one correspondence between events on the two N-dimensional submanifolds. In practice the **u**'s (the *displacement field*) define the distortion of the reference manifold into the natural one. On the basis of the established correspondence we may compare corresponding line elements on the two manifolds; each of them has its own 4-dimensional metric tensor: a general metric $g_{\mu\nu}$ for the natural manifold and a Minkowskian or Euclidean metric $\eta_{\mu\nu}$ for the reference manifold. The difference between the two tensors defines the strain tensor of the natural manifold according to:

$$\epsilon_{\mu\nu} = \frac{1}{2}(g_{\mu\nu} - \eta_{\mu\nu}) \tag{1}$$

The formal details of the theory may be found in [7].

2.1. The Lagrangian density

According to the model outlined in the previous section we can write the action integral of spacetime in the form:

$$S = \int [R + \frac{1}{2}(\lambda\epsilon^2 + 2\mu\epsilon_{\alpha\beta}\epsilon^{\alpha\beta}) + \kappa L_{matter}]\sqrt{-g}d^4x \tag{2}$$

The first term in the square brackets is of course the scalar curvature that plays the role of a "kinetic" term since it contains the derivatives of the dynamic coordinates, i.e. of the elements of the metric tensor; the second term is the potential deformation energy term; the third term is the Lagrangian density of ordinary matter/energy, coupled to geometry through $\kappa = 16\pi G/c^2$, as usual. The two paramters λ and μ are the Lamé coefficients of spacetime. The Lamé coefficients are the typical quantities that contain the elastic properties of a three-dimensional continuous material. Their number depends on the linearization of the theory and on the hypothesis of local isotropy of the manifold, but not on the number of dimensions, so we may use them also in our four-dimensional case.

Starting from eq. (2) and applying the variational principle we can solve various problems, finding the strained configuration of spacetime that minimizes the action integral consistently with the specific symmetry and constraints.

3. The Robertson-Walker symmetry

Studying an universe endowed with the typical Robertson-Walker symmetry we see that the Lagrangian density of empty spacetime becomes:

$$L = -6a(a\ddot{a}+\dot{a}^2)+\frac{1}{4}(\lambda+2\mu)a^3(1-b^2)^2+\frac{3}{8}(3\lambda+2\mu)\frac{(1-a^2)^2}{a}+\frac{3}{4}[\mu\frac{1-a^2}{a^2}-\lambda(1-b^2)]a(1-a^2) \tag{3}$$

The functions a and b depend on cosmic time τ only; a is the scale factor of the universe, b is a gauge function depending on the fact that we use the coordinates of the natural manifold to express the Minkowski metric tensor of the reference manifold and there are in principle an infinity of different ways to establish the correspondence between the two manifolds.

From the Lagrangian (3), after adding matter, we can obtain two equations, for a and b. Solving, we end up with the explicit form of the scale factor $a(\tau)$. Suppose the universe is filled up with dust and radiation, the result in terms of the Hubble function is:

$$H = \frac{\dot{a}}{a} = c\sqrt{\frac{B}{16}}\left\{3\left(1-\frac{(1+z)^2}{a_0^2}\right)^2 + \frac{8\kappa}{3B}(1+z)^3\left[\rho_{\mathrm{m}0} + \rho_{\mathrm{r}0}(1+z)\right]\right\}^{1/2}. \tag{4}$$

Here $B = \frac{\mu}{4}\frac{2\lambda+\mu}{\lambda+2\mu}$ is the bulk modulus of spacetime; $\rho_{\mathrm{m}0}$ and $\rho_{\mathrm{r}0}$ are the present matter and radiation energy densities; a_0 is the present value of the scale factor; z is the redshift parameter.

4. Cosmological tests

Eq. (4) contains four parameters. Two of them, $\rho_{\rm m0}$ and $\rho_{\rm r0}$, are connected with observation; the other two are typical of the SST. We may use the energy densities as constraints, then optimize the parameters of the theory in order to fit a number of cosmological quantities. First of all we have considered the dependence of the luminosity of SnIa supernovae on the redshift parameter [1], then we considered also the proportions between light elements in the primordial nucleosynthesis [4], the size of the sound horizon in the CMB [2], and the particle horizon at the recombination influencing the Large Scale Structure formation in the universe [3].

The results of this multiple optimization process are summarized in the numerical values that can be read in table 1 [5]; the maximum likelihood estimates are reported in brackets. It is $B_{a_0} = \frac{8}{9}\kappa\rho_{r0}a_0^4$.

Table 1. Estimated parameter values.

B $(10^{-52}{\rm m}^{-2})$	$\rho_{\rm m0}(10^{-29}{\rm g\ cm}^{-3})$	$B_{a_0}^{-1}(10^{52}{\rm m}^2)$
$2.22\ (2.22) \pm 0.06$	$0.260\ (0.258) \pm 0.009$	$0.011\ (0.009) \pm 0.006$

5. Conclusion

The theory (SST) we have described is based on the idea that spacetime, as a physical continuum, possesses properties similar to the ones of ordinary three-dimensional elastic materials, properly generalized to four dimensions and the Lorentzian signature. We have applied SST to the case of a Robertson-Walker universe with dust and radiation and we have optimized the parameters of the theory by means of four typical cosmological tests: primordial abundances of light elements, acoustic horizon in the CMB, large scale structure formation and the measured luminosity of type Ia supernovae. In order to evaluate whether our estimates are good or not we can compare our results with the ones obtained in the case of the standard ΛCDM theory, where dark energy is represented by a cosmological constant. The comparison has been made with standard statistical parameters and shows that the reliability of SST is more or less the same as the one of ΛCDM [5]. Our task now is to go on working out more implications and consequences of the SST for different symmetries and physical situations.

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