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Singular higher order vector bases

for wedge-structure MoM-models: the simple recipe

Guido Lombardi[1] Roberto D. Graglia[2]

Abstract – **This paper presents the properties and the rules to define subsectional singular divergence-conforming vector bases that incorporate the edge conditions for wedge structures.**

The use of singular high-order bases in Method of Moment codes provides highly accurate and efficient numerical results for the current and charge density induced on 3D sharp-wedge structures.

1 INTRODUCTION

This paper defines the general properties and the rules to define divergence conforming bases of singular kind to model wedge singularities.

Several attempts to model these behaviors are reported in literature, although only very recently a general and complete definition procedure has been obtained [8].

Method of Moments practitioners were interested in incorporating the edge conditions in finite codes for the last three decades, and interesting developments are reported in [1-8].

In [7, 8] singular higher order complete vector bases of the curl and divergence-conforming kind have been developed. Those two papers report several numerical results for Finite Element Method (FEM) and Method of Moments (MoM) applications that confirm the faster convergence of the singular bases on wedge problems.

A deficiency of the most commonly used regular vector bases, including high-order ones [9], is their inability to accurately model the charge and current densities close to a sharp edge that exhibits singular behavior. Inclusion of the singularity in the basis functions permits one to avoid erroneous results near the edge and improve convergence.

Our singular divergence-conforming bases are defined to properly model impenetrable (conducting) wedges and they are formed by a subset of regular bases [9] plus a Meixner subset of singular bases. The second subset is named after Meixner because it models singular as well as other nonsingular irrational algebraic terms of the Meixner series ([1, 2, 7, 8]).

Notice that the most important features of our singular bases is their additive property, which permits one to model the regular behavior together with multiple singular behaviors of the Meixner kind, associated with different singularity coefficients, as it frequently happens while dealing with penetrable wedge structures [1, 2].

2 SINGULAR LOWEST-ORDER DIVERGENCE CONFORMING BASES

During the last decade our research group focused its attention on the definition of the requirements for singular divergence conforming bases for MoM applications.

With reference to Fig. 1 the singular lowest order divergence-conforming functions must satisfy the following properties:

- they must be complete to the regular zeroth order;
- they must be compatible to adjacent zeroth-order regular elements attached to the nonsingular edges, and to the adjacent singular (edge/vertex) elements of the same order;
- they must be able to model the singular behavior of the current and the charge density along the edge profile;
- they must be able to model the singular irrational algebraic behavior of the current normal to the edge profile.

Figure 1 shows quadrilateral singular elements and two kinds of singular triangular elements: the edge (e) singular triangle and the vertex (v) singular triangle; the latter is considered to be an 'element filler.' These singular elements are made of two subsets of basis functions in order to properly model the regular and the singular properties of the physical quantities.

[1] Dipartimento di Elettronica, Politecnico di Torino, C.so Duca degli Abruzzi 24, 10129 Torino, Italy, e-mail: guido.lombardi@polito.it, tel.: +39 011 564012, fax: +39 011 5644099.

[2] Dipartimento di Elettronica, Politecnico di Torino, C.so Duca degli Abruzzi 24, 10129 Torino, Italy, e-mail: roberto.graglia@polito.it, tel.: +39 011 564056, fax: +39 011 5644099.

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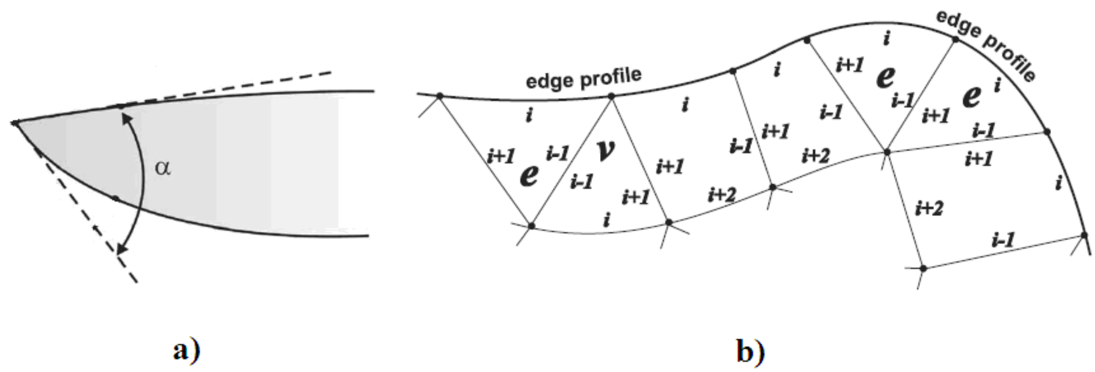

Fig. 1: a) Cross-sectional view of the region around a sharp, but curved edge of aperture angle α, b) Edge profile of the wedge structure as seen from top. Fig. 1b shows the local edge numbering scheme used for the divergence-conforming edge singularity quadrilaterals and edge (e) and vertex (v) singularity triangles.

Singular quantities could or could not be excited by the given electromagnetic sources, and this is why one has to expand the physical (unknowns) quantities by using the additive scheme; i.e. we combine the regular set of bases with the singular one. Multiple singular subsets can be used to model multiple singularities.

The lowest-order ([p, s]=[0,0]) bases are reported in Table I. Singular bases complete to arbitrarily high [p,s] order are described in a unified and consistent manner for curved quadrilateral and triangular elements in [8]. The singular bases guarantee normal continuity along the edges of the elements allowing for the discontinuity of tangential components, adequate modeling of the divergence, and removal of spurious solutions.

These singular high-order bases provide more accurate and efficient numerical solutions for problems modeled by surface integral equations. Several test-case problems have been considered in [8] and new results will be discussed and presented at the conference for 3D cases.

At the conference we will present some important details on how to implement the singular elements in MoM codes. In particular we will show new results relative to a special test case: the spherical shell [10].

2 THE SPHERICAL SHELL

The special test case of a spherical PEC-shell [10] has been studied by using our singular basis functions. This test case has also been considered in [6] and our numerical results will be compared with the ones of [6].

The geometrical parameters of the test case are: radius $a=\lambda/(2\pi)$, aperture angle $\theta_0=120°$. The structure is illuminated by a plane-wave propagating in the positive z direction, while the sphere center is at the origin and the structure is symmetric with respect to the z-axis.

The circular aperture lies in the plane z= -a/2 and has a rim radius $r=a\sqrt{3}/2$. By using a logarithmic 64-color scale, Fig. 2 shows the magnitude of the total current induced on the shell sampled at different times within the (periodic) cycle of the incident plane wave.

Acknowledgments

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References

[1] J. Meixner, "The behavior of electromagnetic fields at edges," IEEE Trans. Antennas Propag., vol. AP-20, no. 4, pp. 442–446, Jul. 1972.

[2] D. R. Wilton and S. Govind, "Incorporation of edge conditions in moment method solutions," IEEE Trans. Antennas Propag., vol. 25, no. 6, pp. 845–850, Nov. 1977.

[3] J. Van Bladel, Singular Electromagnetic Fields and Sources. Oxford, U.K.: Clarendon, 1991.

[4] J. Sercu, N. Fache, F. Libbrecht, and D. De Zutter, "Full-wave spacedomain analysis of open microstrip discontinuities including the singular current-edge behavior," IEEE Trans. Microw. Theory Tech., vol. 41, no. 9, pp. 1581–1588, Sep. 1993.

[5] T. Andersson, "Moment-method calculations on apertures using basis singular functions," IEEE Trans. Antennas Propag., vol. 41, no. 12, pp. 1709–1716, Dec. 1993.

[6] W. J. Brown and D. R. Wilton, "Singular basis functions and curvilinear triangles in the solution of the electric field integral equation," IEEE Trans.

Antennas Propag., vol. 47, no. 2, pp. 347–353, Feb. 1999.

[7] R. D. Graglia and G. Lombardi, "Singular higher order complete vector bases for Finite Methods," IEEE Trans. Antennas Propag., vol. 52, no. 7, pp. 1672–1685, Jul. 2004.

[8] R. D. Graglia and G. Lombardi, "Singular Higher Order Divergence-Conforming Bases of Additive Kind and Moments Method Applications to 3D Sharp-Wedge Structures," IEEE Trans. Antennas Propag., vol. 56, no. 12, pp. 3768–3788, Dec. 2008.

[9] R. D. Graglia, D. R.Wilton, and A. F. Peterson, "Higher order interpolatory vector bases for computational electromagnetics," IEEE Trans. Antennas Propag., Special Issue on Adv.Numer. Tech. Electromagn., vol. 45, no. 3, pp. 329–342, Mar. 1997.

[10] R. W. Ziolkowski and W. A. Johnson, "Electromagnetic scattering of an arbitrary plane wave from a spherical shell with a circular aperture," J. Math. Phys., vol. 28, no. 6, pp. 1293–1314, 1987.

TABLE I

LOWEST-ORDER DIVERGENCE-CONFORMING BASES.

	Basis Functions	Surface Divergences ♡	Dependency Relations
Quadrilateral Base The subscripts are counted modulo 4, for $i = 1, 2, 3$ or 4	Regular Functions [4] $\boldsymbol{\Lambda}_\beta(\boldsymbol{r}) = \frac{\xi_{\beta+2}\,\boldsymbol{\ell}_{\beta-1}}{\mathcal{J}}$ for $\beta = i, i+2, i \pm 1$	$\frac{1}{\mathcal{J}}$	$\xi_{i+1}\,\boldsymbol{\Lambda}_{i+1}(\boldsymbol{r}) + \xi_{i-1}\,\boldsymbol{\Lambda}_{i-1}(\boldsymbol{r}) = 0$ $\xi_i\,\boldsymbol{\Lambda}_i(\boldsymbol{r}) + \xi_{i+2}\,\boldsymbol{\Lambda}_{i+2}(\boldsymbol{r}) = 0$
	Edge Singular Functions ◊ ${}^{e}\boldsymbol{\Lambda}_{i\pm1}(\boldsymbol{r}) = (\nu\xi_i^{\nu-1} - 1)\,\boldsymbol{\Lambda}_{i\pm1}(\boldsymbol{r})$ ${}^{e}\mathbf{V}_{i+2}(\boldsymbol{r}) = (\xi_i^{\nu-1} - 1)\,\boldsymbol{\Lambda}_{i+2}(\boldsymbol{r})$	$\frac{\nu\xi_i^{\nu-1} - 1}{\mathcal{J}}$	$\xi_{i+1}\,{}^{e}\boldsymbol{\Lambda}_{i+1}(\boldsymbol{r}) + \xi_{i-1}\,{}^{e}\boldsymbol{\Lambda}_{i-1}(\boldsymbol{r}) = 0$
Triangular Base The subscripts are counted modulo 3, for $i = 1, 2$ or 3	Regular Functions [4] $\boldsymbol{\Lambda}_\beta(\boldsymbol{r}) = \frac{1}{\mathcal{J}}(\xi_{\beta+1}\boldsymbol{\ell}_{\beta-1} - \xi_{\beta-1}\boldsymbol{\ell}_{\beta+1})$ for $\beta = i, i \pm 1$	$\frac{2}{\mathcal{J}}$	$\xi_{i+1}\,\boldsymbol{\Lambda}_{i+1}(\boldsymbol{r}) + \xi_{i-1}\,\boldsymbol{\Lambda}_{i-1}(\boldsymbol{r})$ $+\,\xi_i\,\boldsymbol{\Lambda}_i(\boldsymbol{r}) = 0$
	Vertex Singular Functions ♣ ${}^{v}\boldsymbol{\Lambda}_{i\pm1}(\boldsymbol{r}) = \chi_a\boldsymbol{\Lambda}_{i\pm1}(\boldsymbol{r}) \mp \chi_b\frac{\xi_{i\mp1}\boldsymbol{\ell}_i}{\mathcal{J}}$ ${}^{v}\mathbf{V}_i(\boldsymbol{r}) = \chi_a\,\boldsymbol{\Lambda}_i(\boldsymbol{r})$ with $\chi_a = (1-\xi_i)^{\nu-1} - 1$ $\chi_b = (1-\nu)\,(1-\xi_i)^{\nu-2}$	$\frac{(1+\nu)\,(1-\xi_i)^{\nu-1} - 2}{\mathcal{J}}$	$\xi_{i+1}\,{}^{v}\boldsymbol{\Lambda}_{i+1}(\boldsymbol{r}) + \xi_{i-1}\,{}^{v}\boldsymbol{\Lambda}_{i-1}(\boldsymbol{r})$ $+\,\xi_i\,{}^{v}\mathbf{V}_i(\boldsymbol{r}) = 0$
	Edge Singular Functions ◊,♠ ${}^{e}\boldsymbol{\Lambda}_{i\pm1}(\boldsymbol{r}) = (\nu\xi_i^{\nu-1} - 1)\,\boldsymbol{\Lambda}_{i\pm1}(\boldsymbol{r})$	$\frac{\nu\,(1+\nu)\,\xi_i^{\nu-1} - 2}{\mathcal{J}}$	$\xi_{i+1}\,{}^{e}\boldsymbol{\Lambda}_{i+1}(\boldsymbol{r}) + \xi_{i-1}\,{}^{e}\boldsymbol{\Lambda}_{i-1}(\boldsymbol{r})$ $+\,\xi_i\,{}^{g}\boldsymbol{\Lambda}_i(\boldsymbol{r}) = 0$

♡ All the basis functions appearing in each row have identical surface divergence. In particular, for the singular functions, (1) yields $\nabla \cdot {}^{e}\boldsymbol{\Lambda}_{i\pm1}(\boldsymbol{r}) = \nabla \cdot {}^{e}\mathbf{V}_{i+2}(\boldsymbol{r}) = \left[\nu\chi^{\nu-1} - 1\right]/\mathcal{J}$ for the quadrilateral base; $\nabla \cdot {}^{v}\boldsymbol{\Lambda}_{i\pm1}(\boldsymbol{r}) = \nabla \cdot {}^{v}\mathbf{V}_i(\boldsymbol{r}) = \left[(1+\nu)\,\chi^{\nu-1} - 2\right]/\mathcal{J}$ for the vertex singularity triangle, and $\nabla \cdot {}^{e}\boldsymbol{\Lambda}_{i\pm1}(\boldsymbol{r}) = \left[\nu\,(1+\nu)\,\chi^{\nu-1} - 2\right]/\mathcal{J}$ for the edge singularity triangle.

◊ The edge singular functions are singular on the i-th edge (where $\xi_i = 0$), and vanish for $\nu = 1$.

♣ The vertex singular functions are singular at the vertex $\xi_i = 1$, and vanish for $\nu = 1$.

♠ The *ghost* function ${}^{g}\boldsymbol{\Lambda}_i(\boldsymbol{r}) = (\nu\xi_i^{\nu-1} - 1)\,\boldsymbol{\Lambda}_i(\boldsymbol{r})$ appearing in the dependency relation at right does not belong to the edge singular triangular basis set because its divergence contains a non-physical $\xi_i^{\nu-2}$ term. Although the divergence of the higher-order edgeless function $\xi_i\,{}^{g}\boldsymbol{\Lambda}_i(\boldsymbol{r})$ is physical (see (17)), the algorithm to construct independent higher-order edge-singular triangular bases has to discard all the functions obtained by multiplying the ghost function times a polynomial of the parent variables because of the reported dependency relation, or because the divergence of these functions contains non-physical hyper-singular terms.

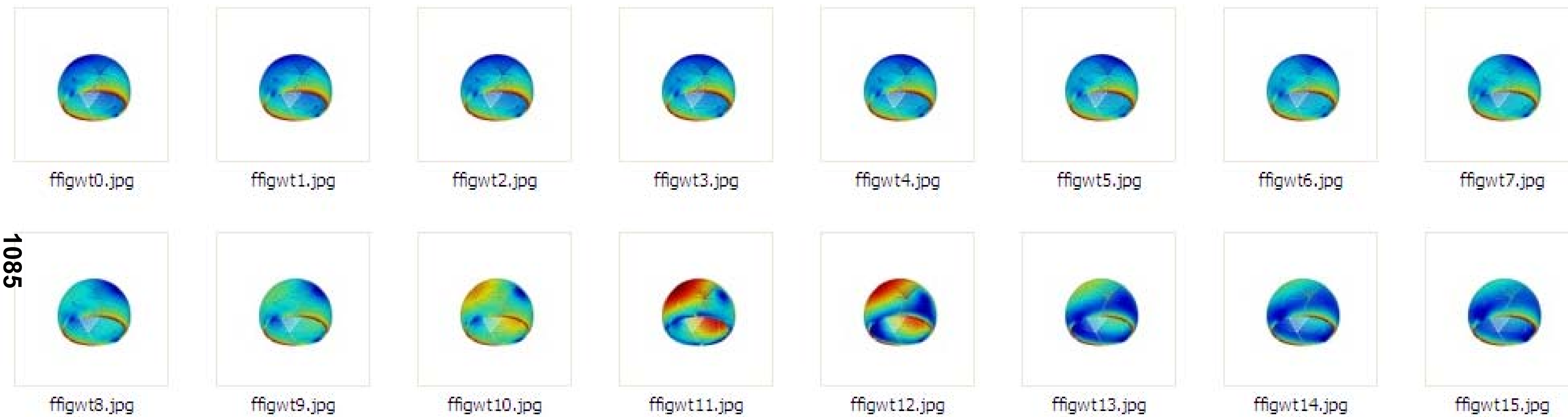

Fig. 2: The spherical shell: color-map of the total current induced on the structure as described in section 2 at different time step.