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The Wiener-Hopf method applied to dielectric angular regions: the wedge

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Abstract – This paper reports the state of the art on the study of diffraction by a dielectric wedge and it proposes a new method to compute the diffracted field. In particular the paper presents the application of the Wiener-Hopf method to the problem of diffraction of a plane wave by a dielectric wedge immersed in free space. The formulation and the equations are proposed and discussed in the spectral domain.

1 INTRODUCTION

This paper examines the problem of the diffraction by a plane wave on a penetrable wedge immersed in free space.

Several attempts to find the solution has been reported in literature but a general and complete solution of this problem is still not available.

Exact solutions have been obtained only for isorefractive and the ideal double negative (DNG) wedges. The solution of isorefractive wedges has been accomplished in the past by using the Kontorovich-Lebedev transform [1-2] in the frequency domain and the Green function in the time domain [3]. More recently solutions have been obtained [5-10].

The more interesting attempt to solve the dielectric wedge problem was the one proposed by Radlow [11] for the diffraction by the right-angled dielectric wedge. This method was based on multidimensional Wiener-Hopf (W-H) equations, but unfortunately it has been ascertained that this solution is wrong Kraut & Lehaman [12]. The interest to the Radlow method is due to the fact that he introduced multidimensional Wiener-Hopf equations to model the problem. However the factorization of multidimensional W-H equations needs function-theoretic techniques employing two complex variables that are cumbersome to handle.

At present the more interesting results obtained for the penetrable wedge geometries arise from the reduction of the problems to integral equations both one-dimensional and two-dimensional. These equations have been formulated both in the space domain and in the spectral domain [13]-[24]. Several techniques were

used for their solution and many of them are based on regularization approaches.

According to our opinion, the Wiener-Hopf (W-H) technique is the most powerful method for solving field problems in presence of geometrical discontinuities [25-30]. Nevertheless only recently [26, 29] this technique has been successfully applied to wedges with arbitrary aperture angle. In general, the W-H formulation of the wedge problems yields generalized W-H equations (GWHE). GWHE can be reduced to classical W-H equations (CWHE) only for impenetrable wedges; therefore the direct approximation of GWHE is required for the dielectric wedge. In particular similarly to the CWHE also the GWHE can be reduced to Fredholm equations of second kind. The main aim of this work is the application of this technique to solve the dielectric wedge problem.

This technique based on the approximate solution of the GWHE can be extended to solve wedge problems involving anisotropic or bianisotropic media [25]. Apparently this extension is not possible for approximate solution obtained in the framework of the Sommerfeld-Malyuzhinets method [17, 23] since their applicability seems limited only to media where the Helmholtz wave equation holds.

2 THE DIELECTRIC WEDGE

Let us consider a dielectric wedge where we have identified four angular regions, see Fig. 1:

$$0 < \varphi < \Phi, -\Phi < \varphi < 0, \Phi < \varphi < \pi, -\pi < \varphi < -\Phi.$$

The wedge is illuminated by plane wave at skew incidence β and azimuthal incident angle φ_0 .

The Wiener-Hopf technique [26] for angular problems is based on the introduction of the following Laplace transforms (1)-(2), where the subscript + indicates plus functions, *i.e.* functions having regular half-planes of convergence that are upper half-planes in the η -plane.

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$$V_{z+}(\eta, \varphi) = \int_0^\infty E_z(\rho, \varphi) e^{j\eta\rho} d\rho, \quad I_{\rho+}(\eta, \varphi) = \int_0^\infty H_\rho(\rho, \varphi) e^{j\eta\rho} d\rho \quad (1)$$

$$V_{\rho+}(\eta, \varphi) = \int_0^\infty E_\rho(\rho, \varphi) e^{j\eta\rho} d\rho, \quad I_{z+}(\eta, \varphi) = \int_0^\infty H_z(\rho, \varphi) e^{j\eta\rho} d\rho \quad (2)$$

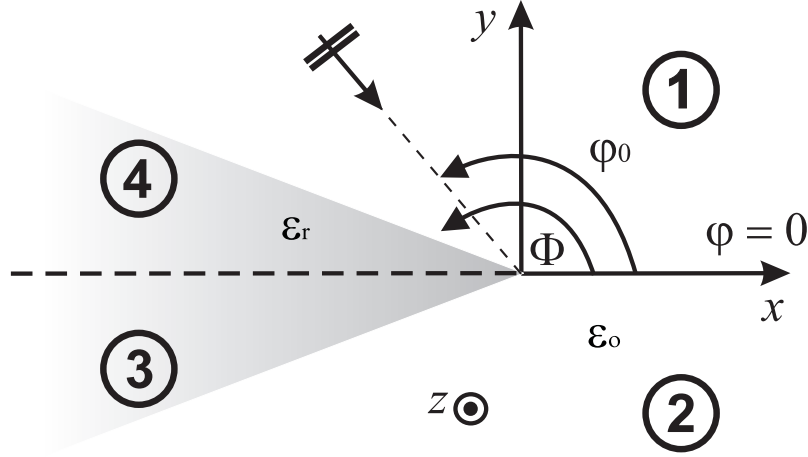


Fig. 1: the dielectric wedge and the four angular regions.

$$\begin{cases} \xi V_{z+}(\eta, 0) - \frac{\tau_o^2}{\omega \varepsilon} I_{\rho+}(\eta, 0) - \frac{\alpha_o \eta}{\omega \varepsilon} I_{z+}(\eta, 0) = -n V_{z+}(-m, \Phi) - \frac{\tau_o^2}{\omega \varepsilon} I_{\rho+}(-m, \Phi) + \frac{\alpha_o m}{\omega \varepsilon} I_{z+}(-m, \Phi) \\ \xi I_{z+}(\eta, 0) + \frac{\tau_o^2}{\omega \mu} V_{\rho+}(\eta, 0) + \frac{\alpha_o \eta}{\omega \mu} V_{z+}(\eta, 0) = -n I_{z+}(-m, \Phi) + \frac{\tau_o^2}{\omega \mu} V_{\rho+}(-m, \Phi) - \frac{\alpha_o m}{\omega \mu} V_{z+}(-m, \Phi) \end{cases} \quad (3)$$

$$\begin{cases} -\xi V_{z+}(-\eta, -\pi) + \frac{\tau_o^2}{\omega \varepsilon} I_{\rho+}(-\eta, -\pi) - \frac{\alpha_o \eta}{\omega \varepsilon} I_{z+}(-\eta, -\pi) = -n V_{z+}(-m, -\Phi) + \frac{\tau_o^2}{\omega \varepsilon} I_{\rho+}(-m, -\Phi) - \frac{\alpha_o m}{\omega \varepsilon} I_{z+}(-m, -\Phi) \\ -\xi I_{z+}(-\eta, -\pi) - \frac{\tau_o^2}{\omega \mu} V_{\rho+}(-\eta, -\pi) + \frac{\alpha_o \eta}{\omega \mu} V_{z+}(-\eta, -\pi) = -n I_{z+}(-m, -\Phi) - \frac{\tau_o^2}{\omega \mu} V_{\rho+}(-m, -\Phi) + \frac{\alpha_o m}{\omega \mu} V_{z+}(-m, -\Phi) \end{cases} \quad (4)$$

For region 1 we obtain the functional equations (3),

where: $\alpha_o = k \cos \beta$, $\tau_o = \sqrt{k^2 - \alpha_o^2}$,

$\text{Im}[\tau_o] \leq 0$, $\xi = \xi(\eta) = \sqrt{\tau_o^2 - \eta^2}$.

Using symmetry and variable substitutions we obtain similar functional equations for the other regions. For example in region 3 the equations (4) hold.

However we need to notice that the quantities involved in equations (3) and (4) depend on the constitutive parameters of the angular region (aperture angle and material), therefore:

for region 1

$\xi = \xi_1 = \sqrt{\tau_1^2 - \eta^2}$, $\tau = \tau_1 = \sqrt{k_1^2 - \alpha_o^2}$,

$\varepsilon = \varepsilon_1$, $\mu = \mu_1$, $k = k_1 = \omega \sqrt{\varepsilon_1 \mu_1}$

$m = m_1 = -\eta \cos \Phi + \xi_1 \sin \Phi$,

$n = n_1 = -\xi_1 \cos \Phi - \eta \sin \Phi$

for region 3

$\xi = \xi_3 = \sqrt{\tau_3^2 - \eta^2}$, $\tau = \tau_3 = \sqrt{k_3^2 - \alpha_o^2}$,

$\varepsilon = \varepsilon_3$, $\mu = \mu_3$, $k = k_3 = \omega \sqrt{\varepsilon_3 \mu_3}$

$m = m_3 = -\eta \cos(-\Phi) + \xi_3 \sin(-\Phi)$,

$n = n_3 = -\xi_3 \cos(-\Phi) - \eta \sin(-\Phi)$

With reference to Fig. 1, let us consider an E-polarized plane wave at normal incidence [31-32]: $\beta = \pi/2$ and $\alpha_o = 0$, $\tau_j = k_j$.

The W-H functional equations assume the following form (5). Because of the symmetry we can rewrite the equations only using two angular regions and therefore by using only two kind of constitutive parameters and functions $m, n \dots$

From (5), after some mathematical manipulation we obtain two uncoupled system of GWHE functional equations of the form presented in (6).

$$\begin{cases} \xi V_{z+}(\eta, 0) - \omega \mu I_{\rho+}(\eta, 0) = -n V_{z+}(-m, \Phi) - \omega \mu I_{\rho+}(-m, \Phi) \\ \xi V_{z+}(\eta, 0) + \omega \mu I_{\rho+}(\eta, 0) = -n V_{z+}(-m, -\Phi) + \omega \mu I_{\rho+}(-m, -\Phi) \\ \xi_1 V_{z+}(-\eta, \pi) + \omega \mu I_{\rho+}(-\eta, \pi) = n_r V_{z+}(-m_r, \Phi) + \omega \mu I_{\rho+}(-m_r, \Phi) \\ \xi_1 V_{z+}(-\eta, -\pi) - \omega \mu I_{\rho+}(-\eta, -\pi) = n_r V_{z+}(-m_r, -\Phi) - \omega \mu I_{\rho+}(-m_r, -\Phi) \end{cases} \quad (5)$$

$$\begin{cases} Y_{i+}(\eta) = X_{i+}(-m_1) - \frac{\xi_{1-}}{n_{1+}} X_{j+}(-m_1) \\ Y_{j+}(\eta) = X_{i+}(-m_2) - \frac{\xi_{2-}}{n_{2+}} X_{j+}(-m_2) \end{cases} \quad (6)$$

where the unknowns are related to the physical quantities (1)-(2). Notice that the unknown are defined into three different complex planes: η, m_1, m_2 . As reported in [26, 29] we can apply a special transformation to map unknowns defined in η, m_i into a new unique complex plane $\bar{\eta}_i$, therefore we obtain CWHE from a GWHE. Each (η, m_i) requires the definition of a new $\bar{\eta}_i$ plane. The dielectric wedge is modeled after the transformations by two uncoupled systems of two functional equations defined into two different complex planes $\bar{\eta}_1, \bar{\eta}_2$:

$$\begin{cases} Y_{i+}(\bar{\eta}_1) = X_{i+}(\bar{\eta}_1) - \sqrt{\frac{k_1 + \bar{\eta}_1}{k_1 - \bar{\eta}_1}} X_{j+}(\bar{\eta}_1) \\ Y_{j+}(\bar{\eta}_2) = X_{i+}(\bar{\eta}_2) - \sqrt{\frac{k_2 + \bar{\eta}_2}{k_2 - \bar{\eta}_2}} X_{j+}(\bar{\eta}_2) \end{cases} \quad (7)$$

In order to solve (6) we can apply the general procedure described in [28-30]: the Fredholm technique.

Since the unknowns are defined into two complex planes, we use the Cauchy formula to relate them:

$$X_{i+}(m_2) = \frac{1}{2\pi j} \oint_{\gamma_1} \frac{X_{i+}(m_1)}{m_1 - m_2} dm_1 \quad (8)$$

The use of the angular plane w and w_1 and of special warping improves the convergence of the numerical discretization of the equations (7)-(8).

Further details on the procedure to get the solution and numerical results in terms of diffraction coefficients of a dielectric wedge will be discussed and presented at the conference.

The solution is given in terms of the diffracted components of the unknown (1)-(2). A complete study

of the field will show the GO, GTD, UTD components.

We note that for different physical parameters the field components will show different spectral properties.

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References

- [1] A.V. Osipov, 1993 "On the method of Kontorovich - Lebedev integrals in problems of wave diffraction in sectorized media." In: Problems of Diffraction and Propagation of Waves, (ed. M.P. Bazarova), vol. 25, pp. 173 - 219, St. Petersburg, Russia: St. Petersburg State university.
- [2] Knockaert, L., F.Olyslager and D. De Zutter. 1997. The diaphaneous wedge. *IEEE Trans. Antennas Propag.*, AP-45:1374-1381
- [3] Scharstein, R.W. and A.M.J. Davis. 1998. Time-domain three-dimensional diffraction by the isorefractive wedge. *IEEE Trans. Antennas Propag.* AP-46: 1148-1158.
- [4] V.G. Daniele, "Rotating Waves in the Laplace Domain for Angular Regions", *Electromagnetics*, vol.23,n.3,2003, pp.223-236.
- [5] V.Daniele, P.L.E. Uslenghi (2007), The Isorefractive Wedge, *Radio Science*, 42, RS6S31,doi:10.1029/2007RS003698, 2007 (9 pages).
- [6] V.Daniele: "Wiener-Hopf solution for isorefractive wedges", *Rapporto Interno ELT-2000-2*. Dipartimento di Elettronica Politecnico di Torino Settembre 2000
<http://www.delen.polito.it/staff/daniele/pubblicazioni.html>.
- [7] P.L.E. Uslenghi, "Exact geometrical optics scattering by a right-angle wedge made of double-negative material", *IEEE Trans. Antennas Propag.*, vol.54, no.8, pp. 2301-2304, August 2006.
- [8] V.G. Daniele, P.L.E. Uslenghi, "The double negative (DNG) wedge", *Electromagnetics in Advanced Applications*, 2007. ICEAA 2007.

- International Conference on 17-21 Sept. 2007
Page(s):68 - 70 Digital Object Identifier
10.1109/ICEAA.2007.4387240
- [9] C. Monzon, D.W. Forester and P. Loschialpo. "Exact solution to the line source scattering by an ideal left-handed wedge" *Physical Review*, 72, 056606 (2005), pp. 056606 -1,5
- [10] V.G. Daniele, P.L.E. Uslenghi: "Scattering by a DNG Wedge" submitted
- [11] J. Radlow, "Diffraction by a right-angled dielectric wedge," *Intern.J.Enging Sci.*, vol.2, 275-290, 1964.
- [12] E.A. Kraut G.W.Lehaman, "Diffraction of electromagnetic waves by a right-angled dielectric wedge," *J. Math. Phys.*, vol.10,1340-1348, 1969.
- [13] A.D. Rawlins, "Diffraction by a dielectric wedge," *J.Inst.Math. Appl.*, vol.19, pp.231-279, 1977
- [14] S. Berntsen, "Diffraction of an electric polarized wave by a dielectric wedge," *SIAM J. Appl. Math.*, vol.43, pp.186-211, 1983
- [15] E. Marx, "Electromagnetic scattering from a dielectric wedge and the single hypersingular integral equation," *IEEE Trans. Antennas Propag.*, vol.AP-41, pp. 1001-1008, 1993.
- [16] J.P.Crosille and G. Lebeau: "Diffraction by an immersed elastic wedge," *Lectures Notes in Mathematics*, n, 1723, 135 pp., Springer-Verlag Berlin Heidelberg 1999
- [17] B.Budaev, "Diffraction by wedges," Longman Scientific & Technical, UK, 1995
- [18] S.Y. Kim and J.W. Ra, "Diffraction by an arbitrary -angled dielectric wedge," *IEEE Trans. Antennas Propag.*, vol.AP-39, pp. 1272-1292, 1991.
- [19] K. Fujii, Rayleigh, "Wave scattering at various corners: Investigation in the wider range of wedge angles," *Bull.Seism. Soc.Am.*, 84(1994), pp.1916-1924.
- [20] M.A. Salem, A. Kamel, A.V. Osipov: "Electromagnetic fields in the presence of an infinite dielectric wedge, Proceedings", Royal Society. Mathematical, Physical and Engineering Sciences, ISSN 1364-5021, 2006, vol. 462, n 2072, pp. 2503-2522
- [21] E. N. Vasil'ev and V. V. Solodukhov "Diffraction of electromagnetic waves by a dielectric wedge" *Radiophysics and Quantum Electronics* , Vol. 17, Number 10, October, 1974 , pp. 1161-1169
- [22] A.K. Gautesen, "Scattering of a Rayleigh wave by an elastic wedge whose angle is greater than 180 degrees," *ASME J.Appl.Mech.*, 68(2001), pp.476-479.
- [23] Buldyrev V.S., M.A. Lyalinov, "Mathematical Methods in Modern Diffraction Theory," Science House Inc, Tokio, 2001
- [24] V.G. Daniele, "The Wiener-Hopf technique for penetrable wedge problems," *URSI General Assembly 2005*, October 23-29,2005,New Delhy, India.
- [25] V.Daniele: "The Wiener-Hopf technique for wedge problems", *Rapporto ELT-2004-2*. Dipartimento di Elettronica Politecnico di Torino. October 2004, <http://www.delen.polito.it/staff/daniele/publicazio ni.html>.
- [26] V.G. Daniele, "The Wiener-Hopf technique for impenetrable wedges having arbitrary aperture angle," *SIAM Journal of Applied Mathematics*, vol. 63, n. 4, pp. 1442-1460, 2003
- [27] V.G. Daniele, G. Lombardi, "Generalized Wiener-Hopf technique for wedges problems involving arbitrary linear media," *International Conference on Electromagnetics in Advanced Applications (ICEAA03)*, Torino (Italy), pp. 761-765, September 8-12, 2003
- [28] V.G. Daniele, G. Lombardi, "The Wiener-Hopf technique for impenetrable wedge problems," invited paper at plenary session, *Proceedings of Days on Diffraction 2005*, Saint Petersburg, Russia, pp. 50-61, June 28-July 1, 2005, Vol. unico, ISBN: 5-9651-0140-6, DOI: 10.1109/DD.2005.204879.
- [29] V.G. Daniele, G. Lombardi, "Wiener-Hopf solution for impenetrable wedges at skew incidence," *IEEE Trans. Antennas Propagat.*, n. 9, Vol. 54, pp. 2472-2485, New York (USA), Sept. 2006, ISSN: 0018-926X, DOI: 10.1109/TAP.2006.880723.
- [30] V.G. Daniele, G. Lombardi, "Fredholm factorization of Wiener-Hopf scalar and matrix kernels," *Radio Science*, Vol. 42, pp. 1-19, 2007, RS6S01, ISSN: 0048-6604, DOI: 10.1029/2007RS003673
- [31] V. Daniele, "The Wiener-Hopf technique for penetrable wedge problems", submitted to *SIAM*, 2008
- [32] V. Daniele, G. Lombardi, "The Wiener-Hopf method applied to multiple angular region problems: the penetrable wedge case ", *Proceedings of IEEE AP-S International Symposium and USNC/CNC/URSI National Radio Science Meeting*, Charleston (SC), USA. June 2009, pp. 1-4.